

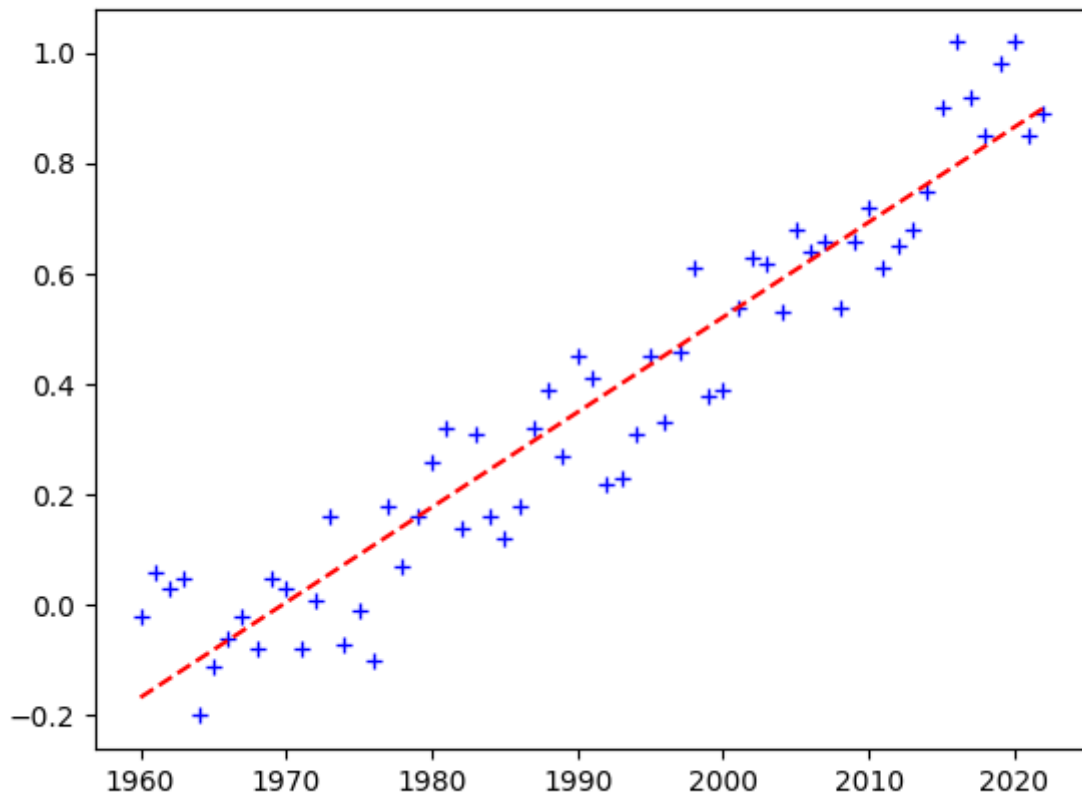
## Problem 3.6 and 4.1

### Problem 3.6:

$\Delta T$  is likely to reach 1.5 in the year 2056

The parameter  $a$  represents the amount of temperature the Earth will change by without the effects of time

The parameter  $b$  represents the increased amount of temperature the Earth will change by due to the human influences that year



### Problem 4.1:

$$\frac{dN}{dt} = R_0 N \exp(-R_1 N) - \frac{Y}{2} \left( 1 + \tanh \left( \frac{N - N_h}{K} \right) \right)$$

Require  $R_0 N = \hat{N}$  and  $-R_1 N = -\hat{R}_1 \hat{N}$ . This implies  $N = \frac{1}{R_0} \hat{N}$  and  $R_1 = \frac{\hat{R}_1 \hat{N}}{N} = \frac{\hat{R}_1 \hat{N}}{\frac{1}{R_0} \hat{N}} = R_0 \hat{R}_1$ . So we have the first two changes of variables,  $N = \frac{1}{R_0} \hat{N}$  and  $R_1 = R_0 \hat{R}_1$ .

Now  $\frac{d}{dt}(N) = \frac{d}{dt}(\frac{1}{R_0} \hat{N}) = \frac{1}{R_0} \frac{d\hat{N}}{dt}$ . We require that this is equal to  $\frac{d\hat{N}}{dt}$  so let  $t = \frac{1}{R_0} \hat{t}$  to make these equal.

Require  $N - N_h = \hat{N} - 1$ . Using the substitution from before,  $\frac{1}{R_0} \hat{N} - N_h = \hat{N} - 1$ . Rearranging for  $N_h$  gives  $N_h = (\frac{1}{R_0} - 1)\hat{N} + 1$ . Finally let  $Y = \hat{Y}$  and  $K = \hat{K}$ .