

In order for a car traveling at $v \text{ ms}^{-1}$ to come to complete rest at a maximal rate of deceleration a_{max} , we require:

$$\frac{0 - v}{\Delta t} = -a_{\text{max}} \implies \Delta t = \frac{v}{a_{\text{max}}}$$

The car will take Δt time to stop, given as above. To find the distance the car travels in this time, observe:

$$\ddot{x} = -a_{\text{max}} \implies \dot{x} = -a_{\text{max}}t + v \implies x = \int_{t_0}^{t_1} -a_{\text{max}}t + v dt$$

Now we are interested in the distance travelled between $t_0 = 0$ and $t_1 = \Delta t$:

$$\begin{aligned} \Delta x &= \int_0^{\Delta t} -a_{\text{max}}t + v dt \\ &= \left[-\frac{1}{2}a_{\text{max}}t^2 + vt \right]_0^{\Delta t} \\ &= -\frac{1}{2}a_{\text{max}}(\Delta t)^2 + v\Delta t \\ &= -\frac{1}{2}a_{\text{max}}\frac{v^2}{a_{\text{max}}^2} + \frac{v^2}{a_{\text{max}}} \\ &= \frac{v^2}{2a_{\text{max}}} \end{aligned}$$

Assuming the cars are spaced Δx apart, we can compute the flow past a fixed point on the road:

$$\begin{aligned} q &= \frac{v_0}{\Delta L} = \frac{v_0}{\Delta x + l_c} \\ &= \frac{v_0}{\frac{v_0^2}{2a_{\text{max}}} + l_c} \\ &= \frac{2v_0a_{\text{max}}}{v_0^2 + 2l_ca_{\text{max}}} \end{aligned}$$

Setting $a_{\max} = 5\text{ms}^{-2}$ and $l_c = 5\text{m}$ we can plot q as a function of v_0 :

