Problem 7.1:

Assuming that the charged particle is a proton, we have that $q = 1.602 \times 10^{-19}C$ and $m = 1.672 \times 10^{-27}kg$. Then, assuming $B = 1$ tesla:

$$\nu = \frac{qB_z}{2\pi m} = \frac{1.602 \times 10^{-19} \times 1}{2\pi \times 1.672 \times 10^{-27}} \approx 1.525 \times 10^7 hz$$

Now we compute the speed of the proton, assuming it has an initial kinetic energy of 100 keV. We have:

$$\begin{aligned} \frac{mv^2}{2} &= q \times 10^3 \text{eV} \\ v^2 &= \frac{2q \times 10^3 \text{eV}}{m} \\ v &= \sqrt{\frac{2q \times 10^3 \text{eV}}{m}} = \sqrt{\frac{2 \times 1.602 \times 10^{-19}C \times 10^3 \text{eV}}{1.672 \times 10^{-27}kg}} \\ &\approx 4.378 \times 10^5 ms^{-1} \end{aligned}$$

The radius of the proton's trajectory can then be calculated, using the fact that the norm of the speed vector is given by:

$$\begin{aligned} V &= 2\pi R\nu \\ R &= \frac{V}{2\pi\nu} = \frac{4.378 \times 10^5 ms^{-1}}{2\pi \times 1.525 \times 10^7 hz} \\ &\approx 4.569 \times 10^{-3} m = 4.569\, mm \end{aligned}$$

Problem 7.2:

We have $m\ddot{x} = q(E_x + \dot{y}B_z)$ and $m\ddot{y} = q(E_y - \dot{x}B_z)$. We must rearrange these equations so that each contains the derivatives of only one variable. To do this, we first solve for $(\dot{x}, \dot{y})$:

$$\dot{y} = \frac{m\ddot{x} - qE_x}{qB_z}, \;\; \dot{x} = \frac{m\ddot{y} - qE_y}{qB_z}$$

Now substituting these into each other gives:

$$
\begin{aligned}
m\frac{d}{dt}\left(\frac{m\ddot{y}-qE_y}{qB_z}\right) &= q(E_x+\dot{y}B_z) \\
\frac{m^2}{qB_z}\dddot{y} &= q(E_x+\dot{y}B_z) \\
\dddot{y} &= \frac{q^2B_z(E_x+\dot{y}B_z)}{m^2} \\
&= \left(\frac{q}{m}\right)^2 B_zE_x + \left(\frac{qB_z}{m}\right)^2 \dot{y}
\end{aligned}
$$

Similarly, one can show that:

$$
\dddot{x} = \left(\frac{q}{m}\right)^2 B_zE_y - \left(\frac{qB_z}{m}\right)^2 \dot{x}
$$

So our equations now only involve constants and derivatives of the same variable. We proceed by denoting $\underline{g}(t) = \begin{pmatrix} -v_x \\ v_y \end{pmatrix} = \begin{pmatrix} -\dot{x} \\ \dot{y} \end{pmatrix}$. Our system of equations then becomes:

$$
\ddot{v}_x = \left(\frac{q}{m}\right)^2 B_zE_y + \left(\frac{qB_z}{m}\right)^2(-v_x),\ \ \ddot{v}_y = \left(\frac{q}{m}\right)^2 B_zE_x + \left(\frac{qB_z}{m}\right)^2 v_y
$$

Which can be written as:

$$
\frac{d\underline{g}(t)}{dt} = \left(\frac{q}{m}\right)^2 B_z\begin{pmatrix} E_y \\ E_x \end{pmatrix} + \left(\frac{qB_z}{m}\right)^2 \underline{g}(t)
$$

So, finally, we get the following first order ODEs:

$$\begin{aligned}
\frac{dv_x}{dt} &= \left(\frac{q}{m}\right)^2 B_z E_y - \left(\frac{qB_z}{m}\right)^2 v_x \\
\frac{dv_y}{dt} &= \left(\frac{q}{m}\right)^2 B_z E_x + \left(\frac{qB_z}{m}\right)^2 v_y \\
\frac{dx}{dt} &= v_x \\
\frac{dy}{dt} &= v_y
\end{aligned}$$

As required.