# OSKAR Theory of Operation

## 1 Introduction

This document describes the methods that OSKAR uses to perform simulations, and all users of the software should read it carefully. If there are any problems, please notify us by sending an email to oskar@oerc.ox.ac.uk so that the simulation software can be updated.

### 1.1 Interferometer Simulation

OSKAR 2 uses the Radio Interferometer Measurement Equation of Hamaker, Bregman & Sault (1996) to produce simulated visibility data. This equation describes the effect due to all visible sources (indexed by $s$) on the complex, polarised visibility $V_{p,q}$ for a baseline between stations $p$ and $q$.

The main terms in the Measurement Equation are Jones matrices: these are two-by-two complex quantities that effectively modify the original signal from each radio source, to give the signal that would be produced from each station due to that source. Each source has an additive effect on the measured visibilities, so the observed visibility on baseline $p,q$ is given by the sum over all visible sources of the product of each set of Jones matrices with the source coherency matrix $\mathbf{B}$. The Measurement Equation currently implemented in OSKAR takes the form

$$\mathbf{V}_{p,q} = \sum_s \mathbf{K}_{p,s}\mathbf{E}_{p,s}\mathbf{G}_{p,s}\mathbf{R}_{p,s}\left\langle \mathbf{B}_s \right\rangle \mathbf{R}^H_{q,s}\mathbf{G}^H_{q,s}\mathbf{E}^H_{q,s}\mathbf{K}^H_{q,s}$$

where the superscript-H denotes Hermitian transpose.

The Jones matrices currently included in OSKAR are:

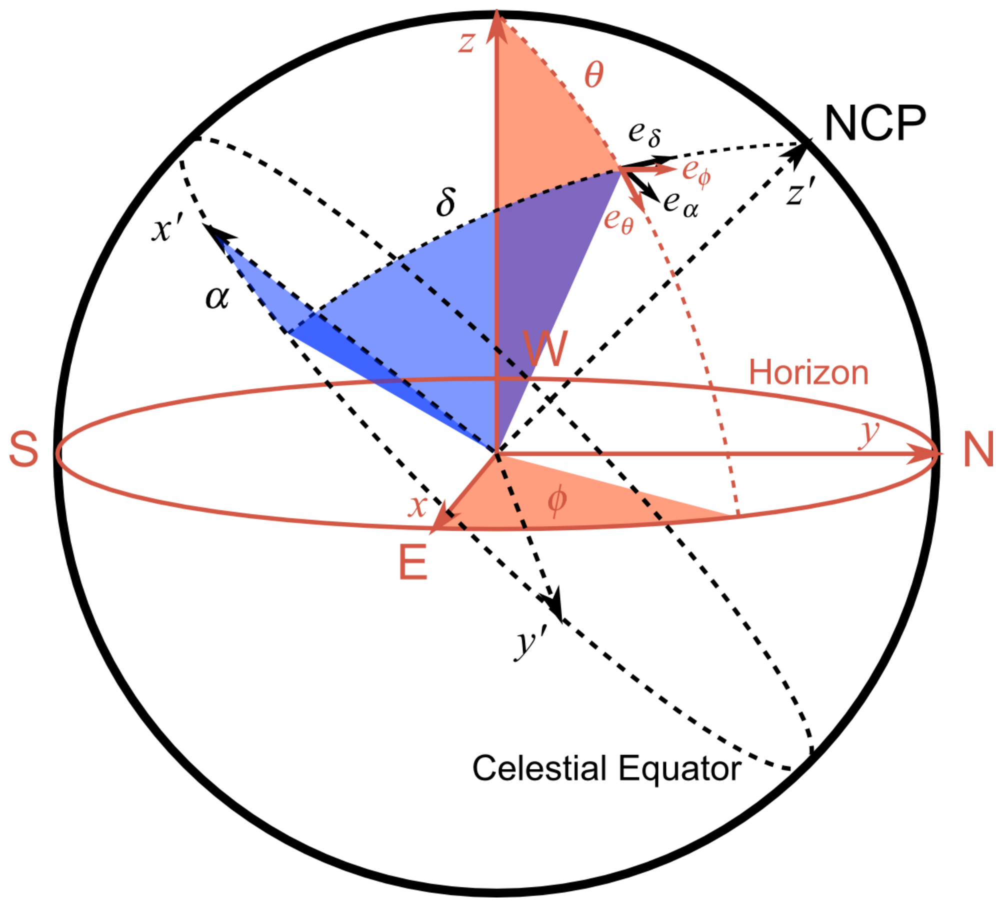
- Parallactic angle rotation matrix $\mathbf{R}$.
- Element factor, or antenna field pattern matrix $\mathbf{G}$. This is factorised out where possible, but is strictly part of the station beam response.
- Array factor, or station beamforming matrix $\mathbf{E}$.
- Interferometer phase matrix $\mathbf{K}$.

For the following sections, in which the Jones terms are described, it will be helpful to introduce the coordinate systems used.

## 2 Coordinate Systems

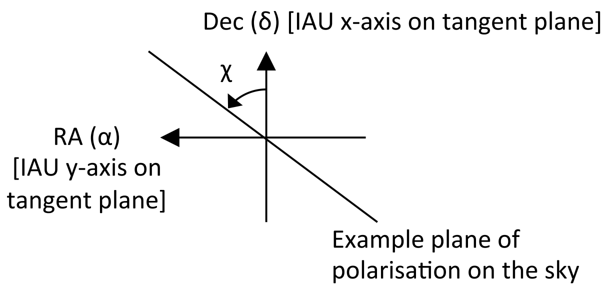
Sources are specified in the equatorial system and have positions Right Ascension $\alpha$ and Declination $\delta$. This spherical system has Cartesian $(x',y',z')$ axes, where the $x'$ axis points towards $\alpha = 0$, the $z'$ axis points towards the North Celestial Pole (NCP) and the $y'$ axis is perpendicular to both to make a right-handed system. The angle $\alpha$ increases from $x'$ towards $y'$, and the angle $\delta$ increases from the $x'y'$-plane towards $z'$. The equatorial system is shown in dashed black in the figure below.

Antennas are specified in the local horizontal coordinate system. This spherical system has Cartesian $(x,y,z)$ axes, where the x-axis points to the observer's geographic East, the y-axis points to geographic North, and the z-axis points to the observer's zenith. The local horizon is therefore in the xy-plane. The angle $\phi$ is the co-azimuth, and increases from $x$ towards $y$, and the angle $\theta$ is the polar angle, zenith distance, or co-elevation, which increases from z towards the xy-plane. The horizontal system is shown in red in the figure below.




## 2.1 Source Brightness Matrix (B)

The source brightness (or coherency) matrix represents the intrinsic, unmodified radiation from an astronomical source. It is constructed using the source Stokes parameters $(I,Q,U,V)$, which completely describe the polarisation state of the radiation. Using the standard polarisation convention adopted by the International Astronomical Union for radio astronomy (IAU, 1974; see figure below), the polarisation axes are defined using the tangent plane to the sphere at a point in the equatorial system. The polarisation angle is measured due east (counter-clockwise) from the direction to the North Celestial Pole, so that 100% Stokes +Q corresponds to North-to-South polarisation, 100% Stokes +U corresponds to North-East-to-South-West polarisation, and 100% Stokes +V corresponds to right-handed circular polarisation.




Using this convention, Hamaker & Bregman (1996) show that

$$\langle \mathbf{B} \rangle = \begin{bmatrix} I+Q & U+iV \\ U-iV & I-Q \end{bmatrix}$$

## 2.2 Parallactic Angle Rotation (R)

The emission from each source must ultimately be expressed in the antenna frame, so the equatorial Stokes parameters must be transformed from components along the $(\hat{\mathbf{e}}_\delta, \hat{\mathbf{e}}_\alpha)$ directions to components along the $(\hat{\mathbf{e}}_\theta, \hat{\mathbf{e}}_\phi)$ directions. This involves a rotation (intermixing) of only the Stokes Q and U parameters, using the parallactic angle at the position of the source: Stokes I and V remain unchanged.

The parallactic angle at a source position is defined as the angle between the direction of the North Celestial Pole and the local vertical on the sky (measured from north toward east), and depends on the observer's latitude $\varphi$ and the source hour angle $H$ and declination $\delta$. The parallactic angle is

$$\psi_p = \arctan\left(\frac{\cos\varphi\sin H}{\sin\varphi\cos\delta - \cos\varphi\sin\delta\cos H}\right)$$

and the $\mathbf{R}$ matrix corresponds to a normal 2D rotation by the parallactic angle.

$$\mathbf{R} = \begin{bmatrix} \cos\psi_p & -\sin\psi_p \\ \sin\psi_p & \cos\psi_p \end{bmatrix}$$

## 2.3 Element Factor, or Antenna Field Pattern (G)

The polarisation response of each antenna is the main component of the system that will corrupt the true source polarisation. Assuming two dipoles labelled X and Y, which have their respective axes nominally along the x and y axes shown in the figure above, then the matrix consists of four complex values that correspond to the average response of the X and Y dipoles in the $\theta$ and $\phi$ directions for all the antennas in the station. These four values are obtained by evaluating the fitted antenna patterns at the $\theta$ and $\phi$ positions of the source.

$$\mathbf{G} = \begin{bmatrix} g_\theta^X & g_\phi^X \\ g_\theta^Y & g_\phi^Y \end{bmatrix}$$

The $\theta$-coordinate appears first here, since in the limit of $\theta = \phi = \psi_p = 0$ (near the zenith of the Earth's North Pole, where the equatorial and horizontal systems are effectively aligned), the $\hat{\mathbf{e}}_\theta$-direction is parallel to a dipole on the ground oriented along the x-axis, and at this point the $\hat{\mathbf{e}}_\theta$-direction is also anti-parallel to the $\hat{\mathbf{e}}_\delta$-direction, which is the axis of zero polarisation according to the IAU (1974) definition. See the examples later in this document for further discussion on this topic.

## 2.4 Array Factor (E)

The shape of the station beam is primarily governed by the projected spacing between individual antennas (the array factor), which is polarisation-independent. The E-Jones matrix in this case is a scalar matrix, which corresponds to the required value of the complex beam pattern: this is given by the weighted (beam-formed) sum of all antennas in the station, evaluated at the source position.

$$\mathbf{E} = e\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Using all antennas $a$, the complex scalar value $e$ at the source position is given by the sum of the complex beamforming weights $w$ multiplied by the complex antenna signals $S$ due to the test source:

$$e = \sum_a w_a S_a$$

If the antennas in the station cannot be described by an average element pattern, then the $\mathbf{G}$ matrix cannot be treated separately, and it is absorbed in the computation of $\mathbf{E}$.

## 2.5 Interferometer Phase (K)

The interferometer phase matrix depends only on the projected spacing between stations. This is polarisation-independent, so $\mathbf{K}$ is a scalar matrix.

$$\mathbf{K} = k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The phase $k$ is (e.g. Thompson, Moran & Swenson, 2001)

$$k = \exp\{-2\pi i [ul + vm + w(n-1)]\}$$

where $(u, v, w)$ are the *station* coordinates in the plane perpendicular to the phase centre, and $(l, m, n)$ are the direction cosines of the source relative to the phase centre. Using the normal conventions in radio astronomy, the $u$ and $l$ directions increase towards the East, the $v$ and $m$ directions increase towards the North, and the $w$ and $n$ directions increase towards the phase centre.

## 2.6 Visibilities to Stokes Parameters

Having obtained the simulated visibility correlation matrix

$$\mathbf{V_{p,q}} = \begin{bmatrix} XX & XY \\ YX & YY \end{bmatrix} = \begin{bmatrix} I+Q & U+iV \\ U-iV & I-Q \end{bmatrix}$$

the Stokes parameters can then be recovered for the purposes of making images by rearranging the diagonal and off-diagonal elements:

$$\begin{aligned} I &= \frac{1}{2}(XX+YY) \\ Q &= \frac{1}{2}(XX-YY) \\ U &= \frac{1}{2}(XY+YX) \\ V &= -\frac{1}{2}i(XY-YX) \end{aligned}$$

Note, however, that this conversion does not involve polarisation calibration in any way: additional corrections for the parallactic angle and antenna response would need to be made in order to recover the true source polarisation in the equatorial frame.

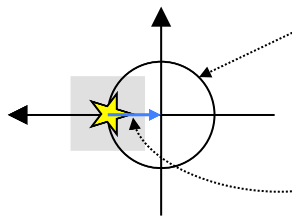
## 2.7 Interferometer Polarisation Example

To test that the polarisation simulation behaves as expected, we describe some simple examples where the geometry of the system is known in advance. In all cases, we choose the telescope longitude to be 0 degrees, and the simulation date to be close to the autumnal equinox (21 September 2000), so that the local sidereal time is approximately equal to the solar time. For these examples, we observe at midnight UTC, so that objects with zero right ascension will be on the local meridian.

At a selection of points in the sky and on the Earth, we simulate the observation of four different sources, emitting in pure (100% polarised) +Q, -Q, +U and -U directions in the equatorial (IAU) frame. We then make images of these sources in Stokes Q and U to show how the simulator will corrupt the source polarisation, even when using ideal dipoles.

### 2.7.1 Telescope near North Pole, observing towards (RA, Dec) = (90, 87)

Let the telescope be sited at latitude 89.9 degrees, observing in the direction of RA = 90 degrees, Dec = 87 degrees. (These numbers were chosen to be close to the pole and the zenith, but not precisely at these points, so that all directions remain well-defined.) Almost everywhere in the sky at this latitude, the parallactic angle is very close to zero: for a source at this declination, $\mathbf{R}$ is almost an identity matrix, so it can be ignored safely. The source at RA = 90 degrees lies in the Easterly direction at this point in time, or along the x-axis in the horizontal frame. This is shown in the figure below which shows the geometry looking towards the zenith and pole from the observer's location. The shaded grey square shows an area of the sky that will be imaged using the normal conventions, and the blue arrow shows the direction that would actually appear as 'up' in the image (and hence defines the IAU axis of polarisation for the source, corresponding to Stokes +Q).




When making an image of this source, the usual conventions apply: The direction to the North Celestial Pole will be 'up' and the direction of increasing right ascension will be 'left,' so using the figure as a guideline, the marked x-axis would be pointing 'down,' and the blue arrow would be pointing 'up.' (The marked x-axis is in the horizon plane, and is unrelated to the image l-coordinate or baseline u-coordinate.)

Ignoring all except the G-Jones term, the measurement equation for this source is given by:

$$\begin{bmatrix} XX & XY \\ YX & YY \end{bmatrix} = \begin{bmatrix} g_\theta^X & g_\phi^X \\ g_\theta^Y & g_\phi^Y \end{bmatrix} \begin{bmatrix} I+Q & U+iV \\ U-iV & I-Q \end{bmatrix} \begin{bmatrix} g_\theta^X & g_\theta^Y \\ g_\phi^X & g_\phi^Y \end{bmatrix}$$

At this source position, the $\mathbf{G}$ matrix is approximately:

$$\mathbf{G} \approx \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The $\mathbf{G}$ matrix is almost an identity matrix here, because the dipole oriented along the x-axis will respond maximally to radiation polarised in the $\theta$-direction (which is essentially parallel to x for this source), and the dipole oriented along the y-axis will respond maximally to radiation polarised in the $\phi$-direction (parallel to y for this source).

If the source emits in pure Stokes +Q ( $I = 1$, $Q = 1$, IAU frame of the sky), then the measured values will be

$$\begin{aligned} Q_m &= \frac{1}{2}(XX - YY) = \frac{1}{2}(I + Q - (I - Q)) \approx \frac{1}{2}(1 + 1 - (1 - 1)) \approx 1 \\ U_m &= \frac{1}{2}(XY + YX) \approx \frac{1}{2}(0 + 0) \approx 0 \end{aligned}$$

If the source emits in pure Stokes -Q ( $I = 1$, $Q = -1$, IAU frame of the sky), then the measured values will be

$$Q_m = \frac{1}{2}(XX - YY) = \frac{1}{2}(I + Q - (I - Q)) \approx \frac{1}{2}(1 - 1 - (1 - (-1))) \approx -1$$

$$U_m = \frac{1}{2}(XY + YX) \approx \frac{1}{2}(0+0) \approx 0$$

If the source emits in pure Stokes +U ( $I = 1$, $U = 1$, IAU frame of the sky), then the measured values will be

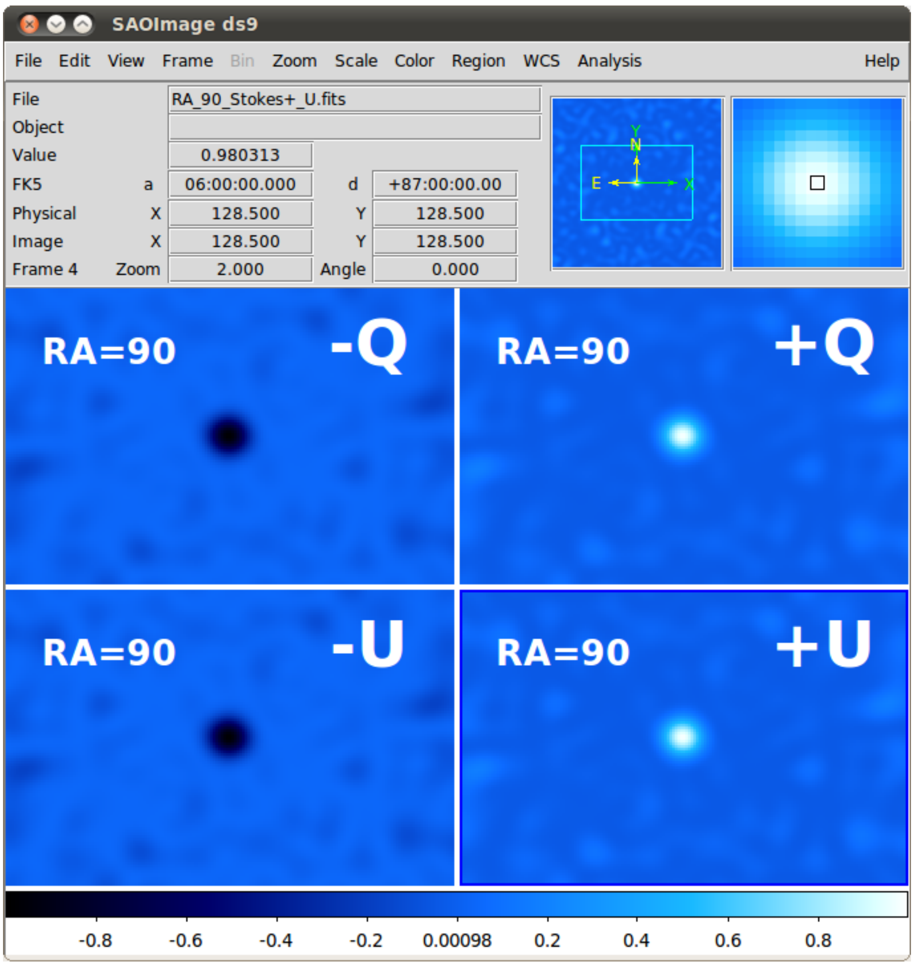
$$\begin{aligned} Q_m &= \frac{1}{2}(XX - YY) = \frac{1}{2}(I + Q - (I - Q)) \approx \frac{1}{2}(1-1) \approx 0 \\ U_m &= \frac{1}{2}(XY + YX) = \frac{1}{2}(U + U) \approx \frac{1}{2}(1+1) \approx 1 \end{aligned}$$

If the source emits in pure Stokes -U ( $I = 1$, $U = -1$, IAU frame of the sky), then the measured values will be

$$\begin{aligned} Q_m &= \frac{1}{2}(XX - YY) = \frac{1}{2}(I + Q - (I - Q)) \approx \frac{1}{2}(1-1) \approx 0 \\ U_m &= \frac{1}{2}(XY + YX) = \frac{1}{2}(U + U) \approx \frac{1}{2}(-1-1) \approx -1 \end{aligned}$$

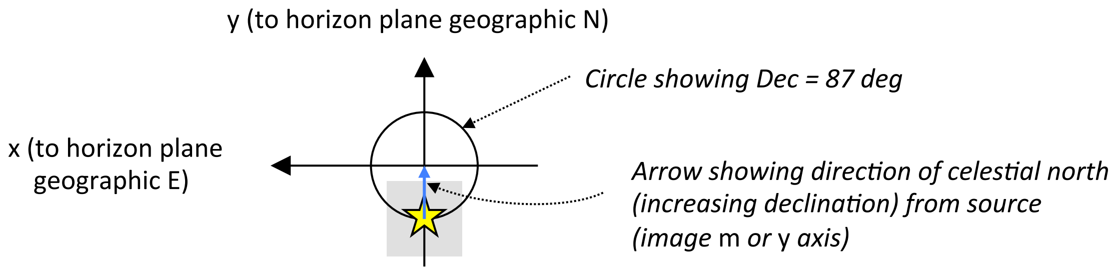
This is one of the few examples where the $\mathbf{G}$-matrix is almost an identity matrix, and the instrumental response closely corresponds to the intrinsic polarisation properties of the source.

The figure below shows images of a source at (RA, Dec) = (90, 87) for an observer near the North Pole. The four non-zero polarisations in the images correspond closely to the intrinsic source polarisation, labelled in the top-right of each panel.



### 2.7.2 Telescope near North Pole, observing towards (RA, Dec) = (0, 87)

Now let the telescope be placed at the same latitude of 89.9 degrees, but observing in the direction of RA = 0 degrees, Dec = 87 degrees. The source at RA = 0 degrees now lies in the Southerly direction, or along the -y-axis in the horizontal frame. The orientation of each dipole has not changed, but the axis of polarisation angle (as defined by the IAU in the tangent plane to the sky) has now been effectively rotated by 90 degrees with respect to the antennas, so the response of the instrument will no longer correspond to the intrinsic source polarisation. This is shown in the figure below which shows the geometry looking towards the zenith and pole from the observer's location. The shaded grey square shows an area of the sky that will be imaged using the normal conventions, and the blue arrow shows the direction that would actually appear as 'up' in the image (and hence defines the IAU axis of polarisation for the source, corresponding to Stokes +Q).




Ignoring all except the G-Jones term, the measurement equation for this source is given by:

$$\begin{bmatrix} XX & XY \\ YX & YY \end{bmatrix} = \begin{bmatrix} g_\theta^X & g_\phi^X \\ g_\theta^Y & g_\phi^Y \end{bmatrix} \begin{bmatrix} I+Q & U+iV \\ U-iV & I-Q \end{bmatrix} \begin{bmatrix} g_\theta^X & g_\theta^Y \\ g_\phi^X & g_\phi^Y \end{bmatrix}$$

At this source position, the $\mathbf{G}$ matrix is approximately:

$$\mathbf{G} \approx \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

The $\mathbf{G}$ matrix takes this form at this point, because the dipole oriented along the x-axis will respond maximally to radiation polarised in the $\phi$-direction (which is essentially parallel to x for this source), and the dipole oriented along the y-axis will respond maximally to radiation polarised in the $\theta$-direction (anti-parallel to y for this source, hence the negative sign).

If the source emits in pure Stokes +Q ( $I = 1$, $Q = 1$, IAU frame of the sky), then the measured values will be

$$\begin{bmatrix} XX & XY \\ YX & YY \end{bmatrix} \approx \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1+1 & 0 \\ 0 & 1-1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \approx \begin{bmatrix} 0 & 0 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \approx \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{aligned} Q_m &= \frac{1}{2}(XX - YY) \approx \frac{1}{2}(0-2) \approx -1 \\ U_m &= \frac{1}{2}(XY + YX) \approx 0 \end{aligned}$$

If the source emits in pure Stokes -Q ( $I = 1$, $Q = -1$, IAU frame of the sky), then the measured values will be

$$\begin{bmatrix} XX & XY \\ YX & YY \end{bmatrix} \approx \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1+-1 & 0 \\ 0 & 1--1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \approx \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \approx \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$
\begin{aligned}
Q_m &= \frac{1}{2}(XX - YY) \approx \frac{1}{2}(2-0) \approx 1 \\
U_m &= \frac{1}{2}(XY + YX) \approx 0
\end{aligned}
$$

If the source emits in pure Stokes +U ( $I = 1$, $U = 1$, IAU frame of the sky), then the measured values will be

$$
\begin{bmatrix} XX & XY \\ YX & YY \end{bmatrix} \approx \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \approx \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \approx \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}
$$

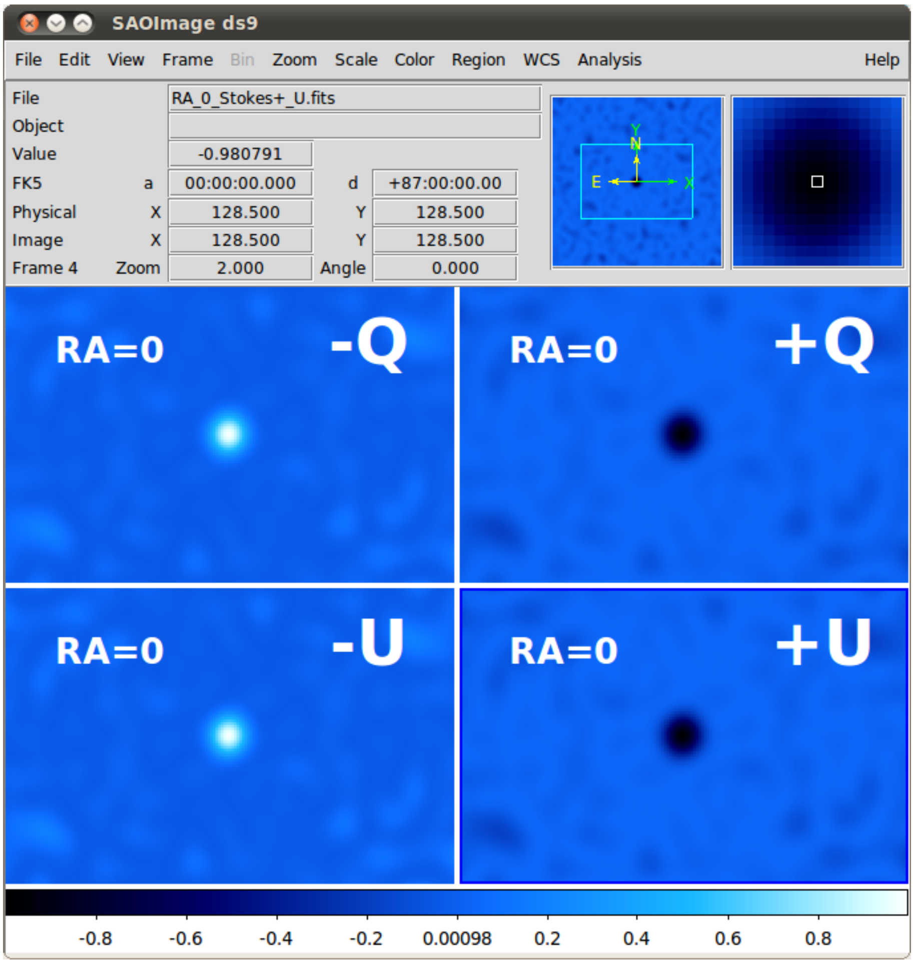
$$
\begin{aligned}
Q_m &= \frac{1}{2}(XX - YY) \approx \frac{1}{2}(1-1) \approx 0 \\
U_m &= \frac{1}{2}(XY + YX) \approx \frac{1}{2}(-1-1) \approx -1
\end{aligned}
$$

If the source emits in pure Stokes -U ( $I = 1$, $U = -1$, IAU frame of the sky), then the measured values will be

$$
\begin{bmatrix} XX & XY \\ YX & YY \end{bmatrix} \approx \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \approx \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \approx \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}
$$

$$
\begin{aligned}
Q_m &= \frac{1}{2}(XX - YY) \approx \frac{1}{2}(1-1) \approx 0 \\
U_m &= \frac{1}{2}(XY + YX) \approx \frac{1}{2}(1+1) \approx 1
\end{aligned}
$$

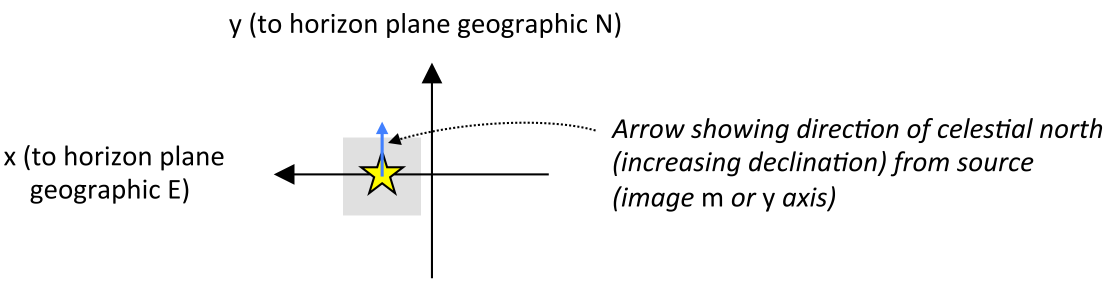
The figure below shows images of a source at (RA, Dec) = (0, 87) for an observer near the North Pole. The four non-zero polarisations in the images show a sign change with respect to the intrinsic source polarisation, labelled in the top-right of each panel.




### 2.7.3 Telescope on Equator, observing towards (RA, Dec) = (1, 0)

In this final example, the telescope is at latitude 0 degrees (on the Earth's equator), observing in the direction of RA = 1 degree, Dec = 0 degrees (again, almost directly overhead). The source at RA = 1 degree again lies in the Easterly direction, or along the x-axis in the horizontal frame. The main difference between this example and the first example is that the parallactic angle is no longer nearly zero, but is now -90 degrees.

This is shown in the figure below which shows the geometry looking towards the zenith from the observer's location. The shaded grey square shows an area of the sky that will be imaged using the normal conventions, and the blue arrow shows the direction that would actually appear as 'up' in the image (and hence defines the IAU axis of polarisation for the source, corresponding to Stokes +Q).



Ignoring all except the R- and G-Jones terms, the measurement equation for this source is given by:

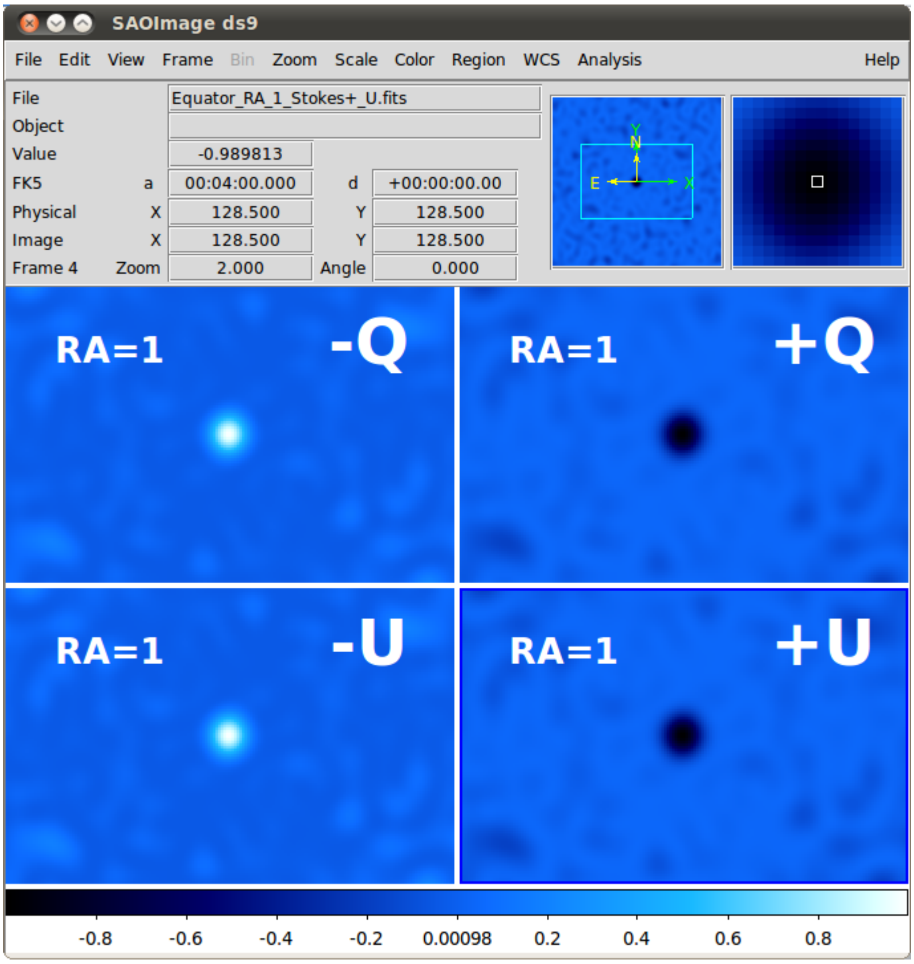
$$\begin{bmatrix} XX & XY \\ YX & YY \end{bmatrix} = \begin{bmatrix} g_\theta^X & g_\phi^X \\ g_\theta^Y & g_\phi^Y \end{bmatrix} \begin{bmatrix} \cos\psi_p & -\sin\psi_p \\ \sin\psi_p & \cos\psi_p \end{bmatrix} \begin{bmatrix} I+Q & U+iV \\ U-iV & I-Q \end{bmatrix} \begin{bmatrix} \cos\psi_p & \sin\psi_p \\ -\sin\psi_p & \cos\psi_p \end{bmatrix} \begin{bmatrix} g_\theta^X & g_\theta^Y \\ g_\phi^X & g_\phi^Y \end{bmatrix}$$

While the $\mathbf{G}$ matrix is almost equal to the identity matrix for the same reasons as described in the first example, the $\mathbf{R}$ matrix is now given by:

$$\mathbf{R} \approx \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Since $\mathbf{R}$ in this case is equal to $\mathbf{G}$ in the second example, it follows that the source polarisation will be modified in the same way.

The figure below shows images of a source at (RA, Dec) = (1, 0) for an observer on the Equator. The four non-zero polarisations in the images show a sign change with respect to the intrinsic source polarisation, labelled in the top-right of each panel.



# 3 Beam Pattern Polarisation Order

When computing a station beam pattern, the result returned by OSKAR is the combination of the array factor and the element factor. The four polarisation planes in the image cube are in the same order as that used to construct the $\mathbf{G}$ matrix, so the order is $X_\theta, X_\phi, Y_\theta, Y_\phi$.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} e \cdot g_\theta^X & e \cdot g_\phi^X \\ e \cdot g_\theta^Y & e \cdot g_\phi^Y \end{bmatrix}$$

# 4 Addition of Uncorrelated System Noise

When performing interferometer simulations, OSKAR provides the option of adding uncorrelated Gaussian noise, $\varepsilon$, to the simulated visibilities, $\mathbf{V}_0$.

$$\mathbf{V} = \mathbf{V}_0 + \varepsilon$$

This is achieved by adding randomly generated values, drawn from a zero-mean Gaussian distribution, to the complex visibility amplitudes for each baseline, time integration, frequency channel and polarisation. Gaussian distributions are defined as a function of frequency, and can be given a different value for each station in the interferometer. Noise values are expressed as the RMS flux level of an unresolved, unpolarised source measured in a single polarisation of the receiver.

While OSKAR requires that the the noise is expressed as a RMS in Jy, one can easily convert to this value from a measure of the noise in terms of system sensitivity or system temperature and effective area using the standard formulae described by Thompson, Moran & Swenson and Wrobel & Walker.

The noise power per unit bandwidth, received in one polarisation of an antenna from an unpolarised source of system equivalent flux density $S_{\rm sys}$, is given by

$$k_{\rm B} T_{\rm sys} = \frac{S_{\rm sys} A_{\rm eff} \eta}{2}$$

Here, $T_{\rm sys}$ is the system temperature, $A_{\rm eff}$ is the effective area of the antenna, $\eta$ is the system efficiency, and $k_{\rm B}$ is the Boltzmann constant.

The RMS noise on a given baseline can then be expressed in terms of the system equivalent flux densities $S_p$ and $S_q$ of antennas (or stations) $p$ and $q$ that make up the baseline by

$$\sigma_{p,q} = \sqrt{\frac{S_p S_q}{2 \Delta\nu \, \tau_{\rm acc}}}$$

Here, $\Delta\nu$ is the bandwidth and $\tau_{\rm acc}$ is the correlator accumulation time. Note the term $2\Delta\nu\,\tau_{\rm acc}$ represents the number of independent samples of the signal for a band-limited signal sampled at the Nyquist rate.

This equation can be re-expressed in terms of the individual system temperatures $T_p$ and $T_q$, effective areas $A_p$ and $A_q$ and system efficiencies $\eta_p$ and $\eta_q$ of antennas (or stations) which make up the baseline as

$$\sigma_{p,q} = k_{\rm B} \sqrt{\frac{2\, T_p T_q}{A_p A_q \, \eta_p \eta_q \, \Delta\nu \, \tau_{\rm acc}}}$$

Equally, given values of the RMS on individual baselines $\sigma_p$ and $\sigma_q$, the baseline RMS is given by

$$\sigma_{p,q} = \sqrt{\sigma_p \sigma_q}$$

Noise fluctuations in the real and imaginary parts of the complex correlator outputs are uncorrelated. The RMS uncertainty in the visibility, $\varepsilon_{p,q}$, obtained from combining the real and imaginary outputs of the correlator will therefore be

$$\varepsilon_{p,q} = \sqrt{\langle \varepsilon \cdot \varepsilon \rangle} = \sqrt{2}\sigma_{p,q}.$$

## 4.1 Noise in the Synthesised Map

For an array with $n_b$ antenna pairs which observes for a length of total observation time $\tau_0$, the total number of independent data points in the $(u,v)$ plane for a single polarisation is

$$n_d = n_b \frac{\tau_0}{\tau_{\rm acc}}$$

and therefore the noise in the image or map will decrease by a factor $\sqrt{n_d}$.

If we consider the special case where the system temperature, effective area, and system efficiency are the same for an array of $n_a$ antennas observing for total time $\tau_0$, the following equation describes the total noise in the image plane of a single polarisation image.

$$\sigma_{\rm im} = \frac{2\,k_{\rm B}\,T_{\rm sys}}{A_{\rm eff}\eta\,\sqrt{n_a(n_a-1)\Delta\nu\,\tau_0}}$$

This can be expressed in terms of the RMS noise on a given baseline as

$$\sigma_{\rm im} = \frac{\sigma_{p,q}}{\sqrt{\frac{n_a(n_a-1)}{2}\frac{\tau_0}{\tau_{\rm acc}}}} = \frac{\sigma_{p,q}}{\sqrt{n_d}}$$

Note that for measurements comprised of combinations of single polarisation data (such as Stokes-I,Q,U,V) the RMS will be reduced by a further factor of $\sqrt{2}$.

# 5 References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**Revision History**

| Revision | Date | Modification |
|---|---|---|
| 1 | 2012-04-27 | Creation. |
| 2 | 2012-05-03 | Clarified polarisation convention, updated definition of R and G matrices. |
| 3 | 2012-05-14 | Added polarisation examples. |
| 4 | 2012-10-22 | Added description of the addition of uncorrelated system noise. |
| 5 | 2013-11-26 | Fixed description of interferometer phase term. |
| 6 | 2014-08-12 | Updated description of addition of uncorrelated noise. |
| 7 | 2015-05-11 | Revised description of addition of uncorrelated noise. |