Report Lab 1

Experimental Physics for AI 2

G, C, D, O

First semester 2024 - 2025

Chapter 1

Measurement of the current-voltage characteristic of a resistor

Chapter 2

Measurement of the current-voltage characteristic of a diode

2.1 Goal

Now we want to measure the current-voltage characteristic of a diode, which should not be linear. Indeed, according to Shockley's law, it is exponential:

$$I = I_0 \left(e^{\frac{qV}{gkT}} - 1 \right)$$

where I_0 is the reverse saturation current, q is the electron charge, k is the Boltzmann constant, T is the temperature, and g is the diode type-dependent constant. In this chapter we will try to verify this law.

Moreover for practical applications it's common practice to define the diode's *threshold voltage* as the voltage at which the diode starts conducting a "significant" current. We will try to measure this value as well.

2.2 Method

Using a similar setup as the one in part one, we recorded the measured values of current at different voltages. The setup is shown in figure 2.1, where the voltmeter is a handheld Fluke multimeter and the ammeter is a Agilent bench multimeter.

Later, in section 2.4, we will perform various fits to the data to verify the exponential relation and estimate the values of the parameters.

2.3 Data

The data we collected is shown in table and represented graphically in figure 2.2. The bench multimeter for the current measurements had an accuracy of $\pm 0.05\% + 0.05\mu A$ in the $500\mu A$ range; and the handheld multimeter had an accuracy of $\pm 0.5\% + 0.002V$ in the $2V$ range.

from the graph in figure 2.2 we can already see that after a certain voltage value (about 2.4) the graph follows accurately an exponential relationship, as expected. The previous values are very low in current and have a greater error. We will perform the analysis only on the part of the data which shows a clear exponential behavior, that is for voltages greater than 2.399 and currents greater than $1.74\mu A$ (medium values).

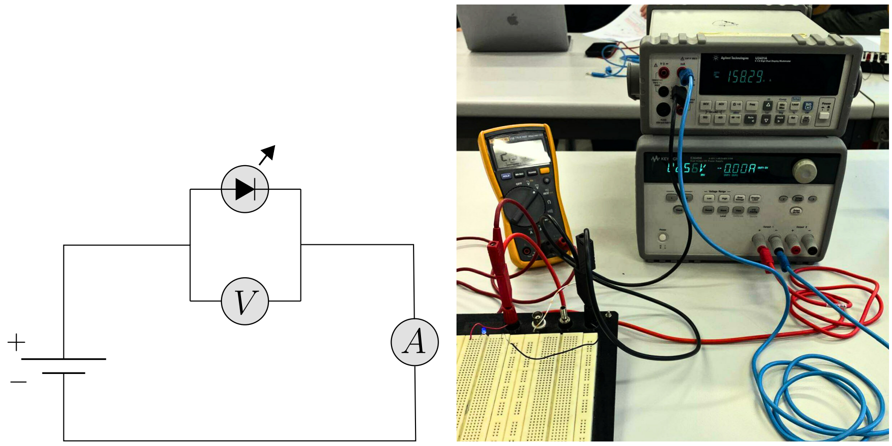

Figure 2.1: Setup of the diode experiment: on the left the diagram showing the circuit made, on the right a photo of the setup

2.4 Analysis

2.4.1 Shockley's law

We used the following approximation:

$$\ln\left(e^{x}-1\right)\approx x \quad \text{for } x\gg 1 \tag{2.4.1}$$

so that we could linearize the relationship as

$$I = I_0\left(e^{\frac{qV}{gkT}}-1\right)\approx I_0 e^{\frac{qV}{gkT}} \implies \log I \approx \frac{q}{gkT}V+\log I_0$$

Then we performed a linear regression on the data, with the errors on V. We decided to keep the error only in the independent variable since it was much greater than the one in the dependent variable. in `R` we ran the following commands:

```r
diode.data <- read.csv("data.csv")
diode.selected <- diode.data[diode.data$Vdetapprox >= 2.399,]
logIerr <- diode.selected$errIdet/diode.selected$Idetapprox
diode.lm <- lm(log(Idetapprox) ~ Vdetapprox, data=diode.selected,
                                            weights=1/logIerr^2)
summary(diode.lm)
```

obtaining the following result:

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -66.698      1.294  -51.55   <2e-16 ***
Vdetapprox    28.215      0.509   55.44   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 44.5 on 19 degrees of freedom
Multiple R-squared:  0.9939, Adjusted R-squared:  0.9935
F-statistic:  3073 on 1 and 19 DF,  p-value: < 2.2e-16
```

Voltage (V)		Current (μA)	
0.499	$\pm$ 0.004	0.03	$\pm$ 0.05
0.999	$\pm$ 0.007	0.08	$\pm$ 0.05
1.499	$\pm$ 0.009	0.12	$\pm$ 0.05
1.999	$\pm$ 0.012	0.16	$\pm$ 0.05
2.099	$\pm$ 0.012	0.17	$\pm$ 0.05
2.200	$\pm$ 0.013	0.19	$\pm$ 0.05
2.299	$\pm$ 0.013	0.26	$\pm$ 0.05
2.399	$\pm$ 0.014	1.74	$\pm$ 0.05
2.409	$\pm$ 0.014	2.33	$\pm$ 0.05
2.419	$\pm$ 0.014	3.16	$\pm$ 0.05
2.428	$\pm$ 0.014	4.33	$\pm$ 0.05
2.438	$\pm$ 0.014	5.95	$\pm$ 0.05
2.448	$\pm$ 0.014	8.19	$\pm$ 0.05
2.458	$\pm$ 0.014	11.28	$\pm$ 0.06
2.467	$\pm$ 0.014	15.47	$\pm$ 0.06
2.477	$\pm$ 0.014	21.16	$\pm$ 0.06
2.486	$\pm$ 0.014	28.55	$\pm$ 0.06
2.495	$\pm$ 0.014	38.42	$\pm$ 0.07
2.504	$\pm$ 0.015	51.03	$\pm$ 0.08
2.512	$\pm$ 0.015	66.50	$\pm$ 0.08
2.520	$\pm$ 0.015	84.54	$\pm$ 0.09
2.528	$\pm$ 0.015	105.8	$\pm$ 0.1
2.535	$\pm$ 0.015	130.0	$\pm$ 0.1
2.542	$\pm$ 0.015	158.8	$\pm$ 0.1
2.549	$\pm$ 0.015	189.2	$\pm$ 0.1
2.556	$\pm$ 0.015	222.6	$\pm$ 0.2
2.562	$\pm$ 0.015	258.5	$\pm$ 0.2
2.568	$\pm$ 0.015	296.5	$\pm$ 0.2

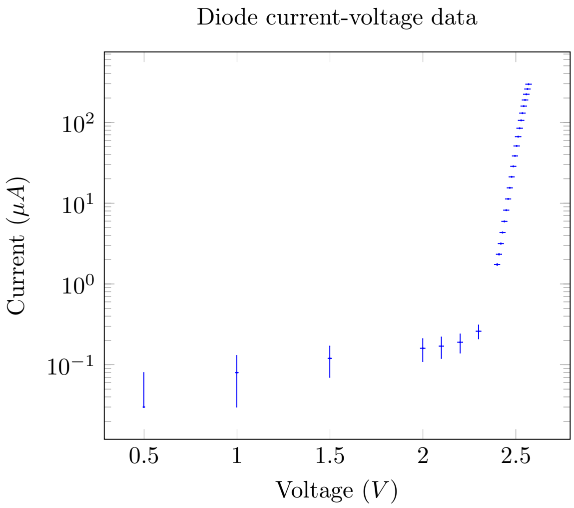

Figure 2.2: Data collected for the diode

So we can very confidently say that the relation between the current and the voltage is exponential. The formula is then:

$$\log I_0 = -67 \pm 1 \implies I_0 = 10^{-29} \pm 10^{-29} \mu A$$

$$\frac{q}{gkT} = 28.2 \pm 0.5 \implies g = \frac{38.6}{28.2} \mp \frac{0.5}{28.2^2} = 1.368 \pm 0.006$$

$$I \approx 1e - 29 e^{\frac{q}{kT} \cdot \frac{1}{1.368} \cdot V} = 10^{-29} e^{28.2V} \mu A$$

(We did not take the measurement for the temperature, so actually 38.6 here should not obviously be a constant, but as we don't have the measurement, we considered it as such, consequently the error on g is much smaller than the one it should be). In figure 2.3 we can see in a graph the accuracy of the fit. Now we can justify the approximation (2.4.1) as indeed for $V \geq 2.4$ we clearly have that the exponent $\frac{qV}{gkT} \geq 67.68 \gg 1$, more precisely, if $f, g : [67.68, \infty)$ are respectively the functions $x \mapsto \ln(e^x - 1)$ and $x \mapsto x$ then

$$\|f - g\|_\infty = |(f - g)(67.68)| \approx 4.05 \cdot 10^{-30}$$

Chi-squared test Finally, we performed a χ^2 test on the residuals of the fit to see if the errors were independent and normally distributed (assuming homoscedasticity), which is what we expect:

```
res <- exp(diode.lm$fitted.values) - diode.selected$Idetapprox
chisq <- sum(res^2 / var(res)) # assume homoscedasticity
```

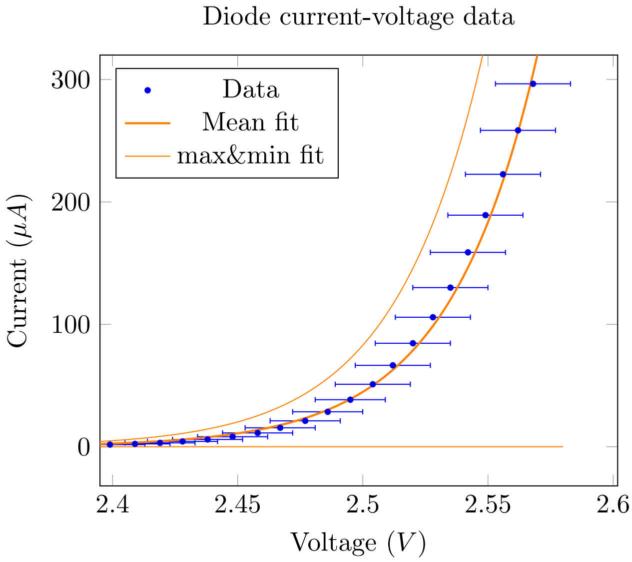

Figure 2.3: Exponential fit of the data

```r
df <- length(diode.lm$residuals) - length(diode.lm$coefficients)
pval <- pchisq(chisq, df, lower.tail=FALSE)
pval
```

Which returns a p-value of $0.29 > 0.05$, so we can't reject the null hypothesis that the errors are independent and the fit is good.

2.4.2 Threshold voltage

As explained before, the *threshold voltage* is said to be the voltage at which a diode starts conducting a "significant" current. It's obtained by fitting the data as a linear relationship and then taking the x-intercept of the line. To decide how many points to take out of the selected ones, we will perform the χ^2 test to see if the residuals are independent normal random variables. The null hypothesis for each value of l (the number of points) is that the errors are independent and normally distributed, while the alternative hypothesis is that they are not. We will then take the biggest l such that we can't reject the null hypothesis. In R code this is:

```r
l <- length(diode.selected$Vdetapprox)+1
pval <- 0
while (pval < 0.05) {
    l = l - 1
    diode.sel_thresh <- tail(diode.selected, l)
    diode.lm_threshold <- lm(Idetapprox ~ Vdetapprox,
                             data=diode.sel_thresh)
    r = diode.lm_threshold$residuals
    chisq <- sum(r^2 / abs(diode.lm_threshold$fitted.values))
    pval <- pchisq(chisq, l - 2, lower.tail=FALSE)
}
summary(diode.lm_threshold)
```

obtaining that we need to keep the last 9 points. Finally we calculated the V-coordinate of the fitting line of those 9 points. Since we know from the linear model that the line has equation $i = mv + q$, with m and q parameters saved in the variable

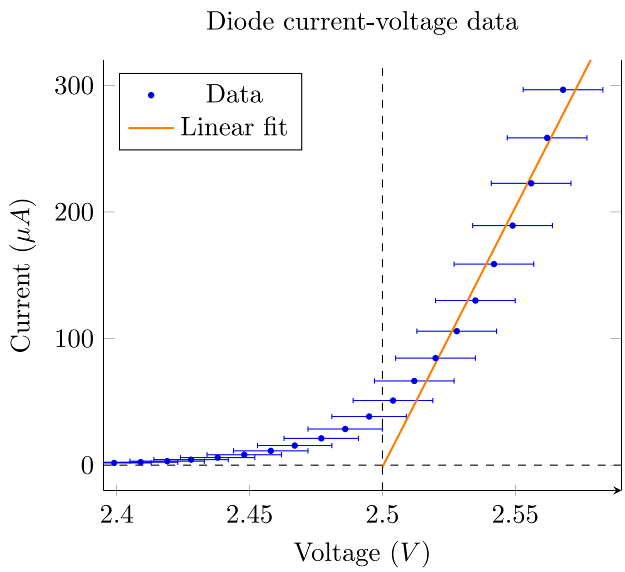

Figure 2.4: Threshold voltage

`diode.lm_threshold$coefficients`, and we obtain

$$q = -10.3 \pm 0.6\, mA \quad m = 4.1 \pm 0.2\, mA/V$$

$$\implies V_{\text{threshold}} = \frac{10.3}{4.1} \pm \sqrt{\left(\frac{0.6}{-10.3}\right)^2 + \left(\frac{0.2}{4.1}\right)^2} = 2.50 \pm 0.8\, V$$

2.5 Conclusion

A clear improvement would be to measure the temperature of the room in order to have a more precise value of the constant g. Moreover, If we had more datapoints, we could have performed a more accurate fit and have smaller errors.

That said, we have successfully verified Shockley's law for a diode and estimated the value of the parameters. We also measured the threshold voltage of the diode. The results are consistent with the theory: for silicon diodes the ideality factor g should be between 1 and 2, and the threshold voltage makes sense that is around $2.5V$, since our diode was a blue LED, so if we see the threshold voltage as the voltage at which the LED starts emitting light, the first photon has energy $E = h\nu$ with $\nu = c/\lambda$, c the speed of light and h Planck's constant. If we add $E = qV_{\text{threshold}}$ and solve for the wavelength λ we get

$$\lambda = \frac{hc}{qV_{\text{threshold}}} \approx 496\, nm$$

which is in the range of blue light.