Report Lab 1

Experimental Physics for AI 2

G, C, D, O

First semester 2024 - 2025

Chapter 1

Measurement of the current-voltage characteristic of a resistor

Chapter 2

Measurement of the current-voltage characteristic of a diode

2.1 Goal

Now we want to measure the current-voltage characteristic of a diode, which should not be linear. Indeed, according to Shockley's law, it is exponential:

$$I = I_0 \left(e^{\frac{qV}{gkT}} - 1 \right)$$

where I_0 is the reverse saturation current, q is the electron charge, k is the Boltzmann constant, T is the temperature, and g is the diode type-dependent constant. In this chapter we will try to verify this law.

Moreover for practical applications it's common practice to define the diode's *threshold voltage* as the voltage at which the diode starts conducting a "significant" current. We will try to measure this value as well.

2.2 Method

Using a similar setup as the one in part one, we recorded the measured values of current at different voltages. The setup is shown in figure 2.1, where the voltmeter is a handheld Fluke multimeter and the ammeter is a Agilent bench multimeter.

Later, in section 2.4, we will perform various fits to the data to verify the exponential relation and estimate the values of the parameters.

2.3 Data

The data we collected is shown in table and represented graphically in figure 2.2. The bench multimeter for the current measurements had an accuracy of $\pm 0.05\% + 0.05\mu A$ in the $500\mu A$ range; and the handheld multimeter had an accuracy of $\pm 0.5\% + 0.002V$ in the $2V$ range.

from the graph in figure 2.2 we can already see that after a certain voltage value (about 2.4) the graph follows accurately an exponential relationship, as expected. The previous values are very low in current and have a greater error. We will perform the analysis only on the part of the data which shows a clear exponential behavior, that is for voltages greater than 2.399 and currents greater than $1.74\mu A$ (medium values).

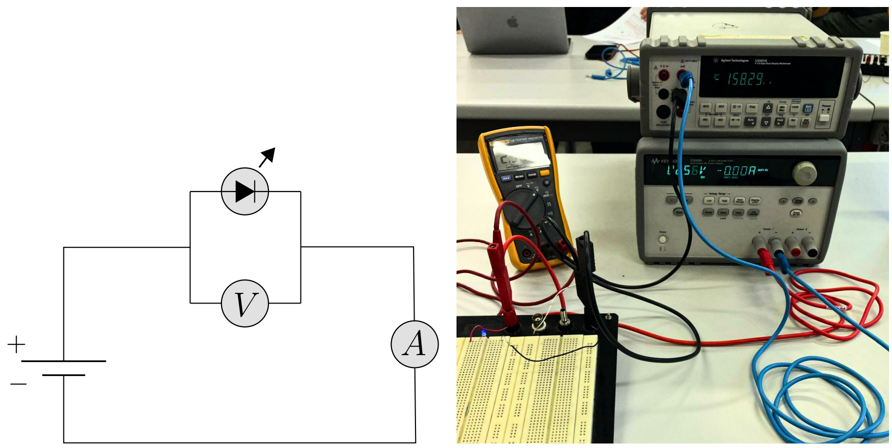

Figure 2.1: Setup of the diode experiment: on the left the diagram showing the circuit made, on the right a photo of the setup

2.4 Analysis

2.4.1 Shockley's law

We used the following approximation:

$$\ln\left(e^{x} - 1\right) \approx x \quad \text{for } x \gg 1 \tag{2.4.1}$$

so that we could linearize the relationship as

$$I = I_0\left(e^{\frac{qV}{gkT}} - 1\right) \approx I_0 e^{\frac{qV}{gkT}} \implies \log I \approx \frac{q}{gkT}V + \log I_0$$

Then we performed a linear regression on the data, with the errors on V. We decided to keep the error only in the independent variable since it was much greater than the one in the dependent variable. in `R` we ran the following commands:

```
diode.data <- read.csv("data.csv")
diode.selected <- diode.data[diode.data$Vdetapprox >= 2.399,]
logIerr <- diode.selected$errIdet/diode.selected$Idetapprox
diode.lm <- lm(log(Idetapprox) ~ Vdetapprox, data=diode.selected,
                                             weights=1/logIerr^2)
summary(diode.lm)
```

obtaining the following result:

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -66.698      1.294  -51.55   <2e-16 ***
Vdetapprox    28.215      0.509   55.44   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 44.5 on 19 degrees of freedom
Multiple R-squared:  0.9939, Adjusted R-squared:  0.9935
F-statistic:  3073 on 1 and 19 DF,  p-value: < 2.2e-16
```

Voltage (V)		Current (μA)	
0.499	± 0.004	0.03	± 0.05
0.999	± 0.007	0.08	± 0.05
1.499	± 0.009	0.12	± 0.05
1.999	± 0.012	0.16	± 0.05
2.099	± 0.012	0.17	± 0.05
2.200	± 0.013	0.19	± 0.05
2.299	± 0.013	0.26	± 0.05
2.399	± 0.014	1.74	± 0.05
2.409	± 0.014	2.33	± 0.05
2.419	± 0.014	3.16	± 0.05
2.428	± 0.014	4.33	± 0.05
2.438	± 0.014	5.95	± 0.05
2.448	± 0.014	8.19	± 0.05
2.458	± 0.014	11.28	± 0.06
2.467	± 0.014	15.47	± 0.06
2.477	± 0.014	21.16	± 0.06
2.486	± 0.014	28.55	± 0.06
2.495	± 0.014	38.42	± 0.07
2.504	± 0.015	51.03	± 0.08
2.512	± 0.015	66.50	± 0.08
2.520	± 0.015	84.54	± 0.09
2.528	± 0.015	105.8	± 0.1
2.535	± 0.015	130.0	± 0.1
2.542	± 0.015	158.8	± 0.1
2.549	± 0.015	189.2	± 0.1
2.556	± 0.015	222.6	± 0.2
2.562	± 0.015	258.5	± 0.2
2.568	± 0.015	296.5	± 0.2

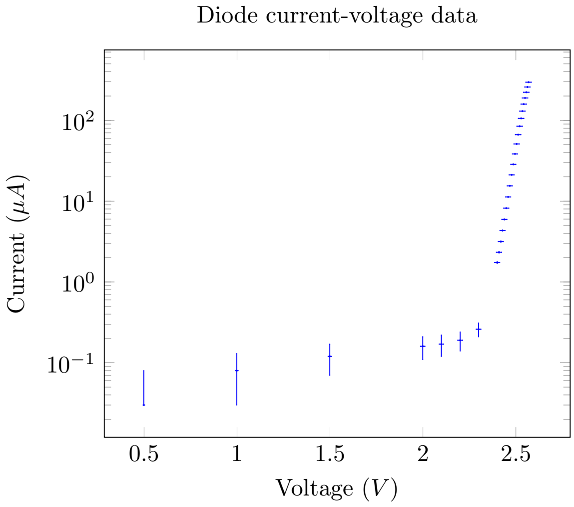

Figure 2.2: Data collected for the diode

So we can very confidently say that the relation between the current and the voltage is exponential. The formula is then:

$$\log I_0 = -67 \pm 1 \implies I_0 = 10^{-29} \pm 10^{-29}$$
$$\frac{q}{gkT} = 28.2 \pm 0.5 \implies g = \frac{38.6}{28.2} \mp \frac{0.5}{28.2^2} = 1.368 \pm 0.006$$
$$I \approx 1e - 29e^{\frac{q}{kT} \cdot \frac{1}{1.368} \cdot V} = 10^{-29} e^{28.2V}$$

In figure 2.3 we can see in a graph the accuracy of the fit. Now we can justify the approximation (2.4.1) as indeed for $V \geq 2.4$ we clearly have that the exponent $\frac{qV}{gkT} \geq 67.68 \gg 1$, more precisely, if $f, g : [67.68, \infty)$ are respectively the functions $x \mapsto \ln(e^x - 1)$ and $x \mapsto x$ then

$$\|f - g\|_\infty = |(f - g)(67.68)| \approx 4.05 \cdot 10^{-30}$$

2.4.2 Threshold voltage

As explained before, the *threshold voltage* is said to be the voltage at which a diode starts conducting a "significant" current. It's obtained by fitting the data as a linear relationship and then taking the x-intercept of the line. To decide how many points to take out of the selected ones, we will perform the Durbin-Watson test to see if the residuals are correlated. If it turns out that they are correlated, then one of

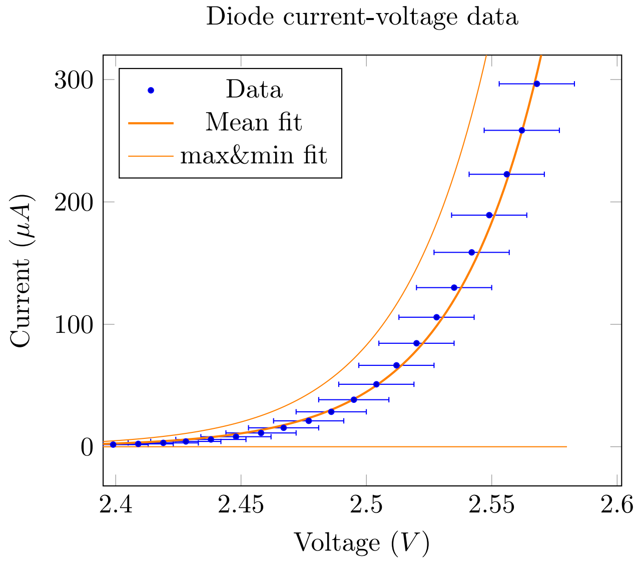

Figure 2.3: Exponential fit of the data

the hypothesis of the linear regression is not satisfied. We will take the largest set of points, out of the right part of the graph, such that $|d - 2|$, where d is the test statistic of the Durbin-Watson test, is less than 0.5. In `R` code this translates to

```
l <- length(diode.selected$Vdetapprox)
d <- 1
while (abs(d - 2) > .5) {
    l = l - 1
    diode.sel_thresh = tail(diode.selected, l)
    diode.lm_threshold <- lm(Idetapprox ~ Vdetapprox,
                             data=diode.sel_thresh)
    r = diode.lm_threshold$residuals
    SSR = sum(diode.lm_threshold$residuals^2)
    DW_sum = sum((r[2:length(r)] - r[1:length(r)-1])^2)
    d = DW_sum / SSR
}
```

obtaining that we need to keep the last 4 points. Finally we calculated the $V-$coordinate of the fitting line of those 4 points. Since we know from the linear model that the line has equation $i = mv + q$, with m and q parameters saved in the variable `diode.lm_threshold$coefficients`, and because we know the error must be

$$v = -\frac{q}{m} \implies \delta v = -\frac{\delta q}{m} + \frac{q\,\delta m}{m^2}$$

obtaining a value of

$$q = -14.2 \pm 0.7mA \quad m = 5.7 \pm 0.3mA/V$$
$$\implies V_{threshold} = \frac{14.2}{5.7} \mp \frac{0.7}{5.7} \mp \frac{-14.2 \cdot 0.3}{5.7^2} = 2.51 \pm 0.24$$