Report Lab 3

Experimental Physics for AI 2

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Abstract

This laboratory experience consisted of three different experiments concerning electromagnetism. The first one is a visualization of the electric field lines through a capacitor. The second one is a measurement of earth's magnetic field using the phone's magnetometer. The third one is a measurement of the magnetic field induced by a coil.

Chapter 1

Visualization of Electric Potential and Electric Field

1.1 Goal

The main goal of this experiment is to visualize the field lines of the electric field inside an insulator. While we can't directly observe the electric field, we can measure the electric potential at different points inside the insulator and thus infer the electric field lines, as we know that

$$\vec{E} = -\nabla V$$

Thus what we want to do at first is to measure accurately the electric potential in a grid of points inside the insulator.

We expect to see that the vector field is mostly aligned vertically inside the insulator while on the edges it should be slightly more curved due to the lack of charges on the outside of the insulator, the result is a rectangle with rounded shapes on the east and west sides.

1.2 Method

Firstly, the chosen insulator is a graphite sheet placed on a flat wooden board, with a grid of points marked on it. This choice is because actually graphite is conductive, but as we are working with extremely small currents (approximately $0muA$), it will act as an insulator for our purposes, but we know that the very low current that will actually move, will move through the graphite, since it's much more conductive than air. The sheet has the north and south sides connected with a metal bar. A low current, 5V DC power supply is connected to the sheet. The electric potential inside it is then measured using a multimeter. The whole setup is shown in Figure 1.1.

It's useful to place a reference system on the board to be able to address each point with a pair of coordinates (x, y). In particular we will be be considering y to be the vertical axis and x to be the horizontal axis, with the origin in the bottom left corner of the board, as shown in the figure.

In theory it would be enough to measure the electric potential at each point relative to a fixed reference point. However in practice, since we will be just needing the partial derivatives, we directly measured the potential difference between each point (x, y) and the point $(0, y)$ (reference point, once fixed a y value) and between each point (x, y) and the point $(x, 0)$ (reference point, once fixed an x value). This way leads us to have a description of each point in the grid with two values $V_x(x, y)$

and $V_y(x, y)$ which represent the difference in potential from the projection of such point on the x and y axis, respectively.

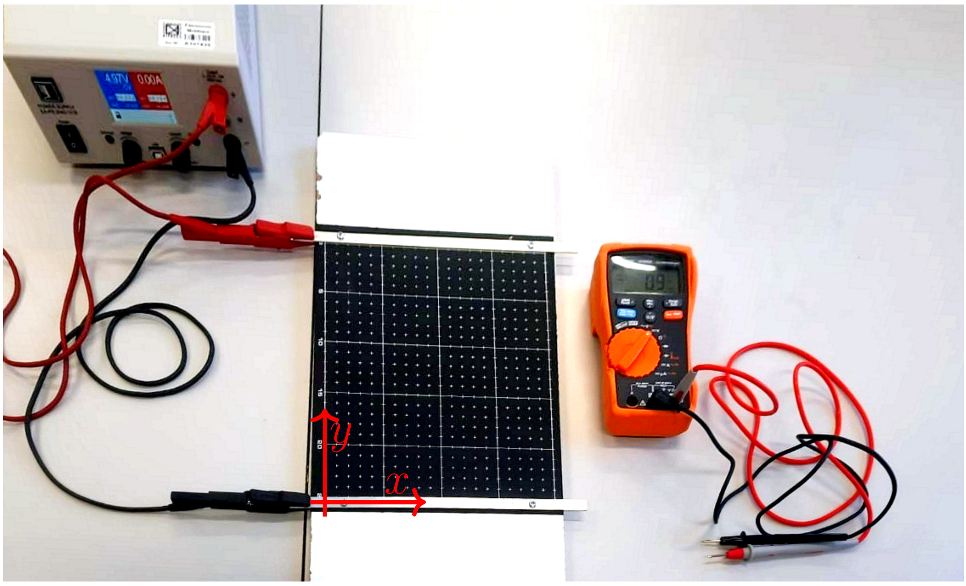

Figure 1.1: Experimental setup for recording electric potential inside an insulator. In red are shown the axes used for addressing points. The unit is the same as the smallest grid width on the board, which marks centimeters. The origin $0, 0$ is placed at the intersection of the axes

1.3 Data

The data is imported from a CSV file, and its content is shown in Table 1.1 for a sample of the points (Only points of coordinates non-zero, multiples of 6 are shown for brevity). Note that as a consequence of the configuration of the board, points on the x axis are all of same potential, so each of their V_y is zero, and similarly for points of coordinates $(x, 25)$.

We used a hand-held Fluke multimeter to measure the potential differences. Its accuracy specification on the DC Volts is $0.5\% + 0.002V$ in the $6V$ range.

It has to be noted that, since the setup was not perfectly controlled and calibrated, and we changed person taking the measurements during the experiment, and it was a bit uncomfortable to measure the potential at each point, the measures might have a (bigger) degree of statistical uncertainty due to human error. However we didn't take measures multiple times so we can't estimate the size of this error. We will proceed with the visualization of the field lines without taking errors into account.

1.4 Analysis

We analyzed the collected data to calculate the electric field at each point restricted to the xy plane. The python script used to calculate and then plot the field is shown in Listing 1.1. The script achieves this by calculating at each point the (backwards)

x	y	$V_x(x,y)$ (V)		$V_y(x,y)$ (V)	
6	6	-0.098	± 0.002	1.60	± 0.01
12	6	-0.139	± 0.003	1.62	± 0.01
18	6	-0.067	± 0.002	1.54	± 0.01
6	12	-0.004	± 0.002	2.51	± 0.01
12	12	-0.011	± 0.002	2.47	± 0.01
18	12	-0.013	± 0.002	2.50	± 0.01
6	18	0.052	± 0.002	3.40	± 0.02
12	18	0.070	± 0.002	3.35	± 0.02
18	18	0.019	± 0.002	3.42	± 0.02
6	24	0.286	± 0.001	4.27	± 0.02
12	24	0.405	± 0.000	4.09	± 0.02
18	24	0.058	± 0.002	4.54	± 0.02

Table 1.1: Data collected for the electric potential inside the graphite sheet

incremental ratios of the potential with respect to x and y explicitly, this means:

$$E_x(x,y) = -\frac{\partial V}{\partial x} \approx -\frac{\Delta V}{\Delta x} = \frac{V_x(x-3,y) - V_x(x,y)}{3\,cm} \tag{1.4.1}$$

$$E_y(x,y) = -\frac{\partial V}{\partial y} \approx -\frac{\Delta V}{\Delta y} = \frac{V_y(x,y-3) - V_y(x,y)}{3\,cm} \tag{1.4.2}$$

where the 3 comes from how we sampled the points, which were 3 cm apart. The field is so calculated for each point in the grid except for the axes.

```python
#!/usr/bin/env python
import numpy as np

# importing the csv with data
import pandas as pd
df = pd.read_csv("data.csv")
Vx = dict(zip(zip(df["x"], df["y"]), df["Vx"]))
Vy = dict(zip(zip(df["x"], df["y"]), df["Vy"]))

# calculating Ex and Ey
def getEx(x, y):
    assert x > 0, "x must be positive"
    dx = 3 - (3 - x)%3 # previous multiple of 3
    return (Vx[(x-dx, y)] - Vx[(x,y)]) / (.01 * dx)

def getEy(x, y):
    assert y > 0, "y must be positive"
    dy = 3 - (3 - y)%3 # previous multiple of 3
    return (Vy[(x, y-dy)] - Vy[(x,y)]) / (.01 * dy)
```
Listing 1.1: Python script for calculating the electric field from the potential data

Finally we plotted the electric vector field using the *quiver* plot function from the matplotlib library. The resulting plot is shown in Figure 1.2.

1.5 Results

We expected the electric field to be mostly vertical, and directed from north to south inside the sheet. The down direction is because the anode is placed on the $y = 25$ line and the cathode on the $y = 0$ line, as can be seen in figure 1.1. The field lines are slightly curved on the edges toward the outside of the sheet, as we can see in the plot.

The result is consistent with our knowledge of real capacitors.

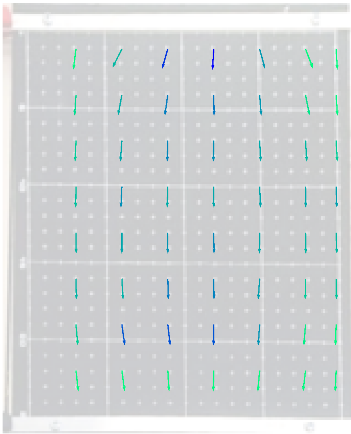

Figure 1.2: Electric field lines inside the insulator

1.6 Conclusion

As we expected, the electric field lines are mostly vertical from north to south inside the board and slightly curved on the edges. When modeling in a naive way the electric field, we usually think of it as completely vertical, as it would be in an infinite parallel plate capacitor. However, in a real plane (or in this case segment) capacitor, the field lines are more curved on the edges, as we observed qualitatively in this experiment.

Chapter 2

Measuring the Earth's Magnetic Field using a phone

2.1 Goal

The main objective of our experiment is to determine the components of the Earth's magnetic field. The principal components that we are talking about are:

- B_h, the horizontal component of the magnetic field;
- B_v, the vertical component of the magnetic field;
- B, the magnetic field;
- θ, the angle between the horizontal and the vertical component.

2.2 Method

In order to achieve our goal, we performed the measurements using different tools:

- A smartphone's magnetometer and the Phyphox app;
- A rotating surface (chair), to better perform the rotation of the device;
- GPS Coordinates to determine geographic location;
- NOAA Magnetic Calculator to compute theoretical magnetic field values.

The last two tools were used to compare data with the ones that we collected in order to verify the reliability of the smartphone's magnetometer.

2.3 Data and error analysis

2.3.1 Determine B_h

In order to calculate the horizontal component of the magnetic field, we apply the following equation to our data:

$$B_h = \sqrt{B_x^2 + B_y^2}$$

where B_x and B_y are the magnetic field's components along the x-axis and y-axis of the phone.

For the purpose of analyzing B_h, we have to determine B_x and B_y from the detected graphs on the Phyphox app. We downloaded the CSV file from the app, and thanks to Excel tools, we were able to estimate the specific values. Then inserting the values, we obtain our result:

$$B_h = \sqrt{20.62^2 + 11.25^2} = 21.62$$

2.3.2 Determine B

In order to calculate B (the magnetic field), combining B_h (the horizontal component of the magnetic field) and B_v (the vertical component), we have to apply the following formula:

$$B = \sqrt{B_h^2 + B_v^2}$$

However, for the type of data that we used, B_v is simply the mean of all the collected values of B_z (the vertical component of the magnetic field along the z-axis of the phone). Obtaining that value, we calculate:

$$B = \sqrt{21.62^2 + 60.18^2} = 64.42$$

The formula that we used till now is the Pythagorean theorem viewed in a three-dimensional situation where, as we said before, B_h is the horizontal component and B_v is the average of the vertical component values. B is just the vector formed by combining the horizontal and vertical components of the magnetic field. Analyzing our results, we see a difference in the values of the total magnetic field and the horizontal component. This discrepancy represents the mutability of the Earth's magnetic field being neither absolutely horizontal nor absolutely vertical but varying based on the geographical position.

2.3.3 Determine θ and B_v

In order to calculate the dip angle θ, namely the angle between the horizontal component of B (B_h) and B itself, we have to derive it from a more general formula $B_h = B\cos(\theta)$. From this and the data that we calculated in the previous points, we can extrapolate the angle:

$$\theta = \arccos\left(\frac{B_h}{B}\right) = \arccos\left(\frac{21.62}{64.42}\right) = 70.39°$$

Then, thanks to the angle that we found and the general formula stated before, we can calculate B_v:

$$B_v = B\sin(\theta) = 64.42 \times \sin(70.39°) = 60.68$$

2.3.4 Determine the amplitude of the signal

The amplitude of the components of the Earth's magnetic field is fundamental. With the app that we used, Phyphox, the recorded data could be analyzed directly on the device or exported as a CSV file for additional processing. In order to calculate the specific amplitude, we have to sum, as we have done in the previous formula, all the amplitudes of the components along the x-, y-, z-axis:

$$A = \sqrt{B_x^2 + B_y^2 + B_z^2} = 60.89$$

2.4 Conclusion

In this experiment, we successfully measured the components of the Earth's magnetic field, taking advantage of the help of a smartphone and the Phyphox app. All the results that we obtained for each component (B_h, B_v, B) matched our expectations.

All the data that we collected, however, were subject to errors caused by different elements:

- Magnetic interference from nearby electronic devices or metallic objects;
- A non-constant rotation of the device due to human imprecision.

In the end, all the data were collected and analyzed correctly, giving us a precise and well-rounded analysis of Earth's electromagnetic field.

Chapter 3

Measuring the Magnetic Field of a Coil

3.1 Goal

The goal of this experiment is to determine the permeability of free space (μ_0) by measuring the magnetic field produced by a current-carrying coil. This is achieved by varying the current intensity (I) and the number of turns (N) of the coil, while keeping the distance (z) from the center of the coil constant. The magnetic field strength (B) is measured using a smartphone magnetometer.

3.2 Method

The experiment uses a circular coil of radius $R = 7\,\text{cm}$ with a known number of loops (N). The magnetic field along the axis of the coil at a distance z from its center is modeled by the equation:

$$B(z) = \frac{\mu_0 N I R^2}{2(R^2 + z^2)^{3/2}},$$

where $B(z)$ is the magnetic field at distance z (measured in μT), μ_0 is permeability of free space (to be determined), N number of loops in the coil, I the current through the coil (A), R the radius of the coil (m) and z the distance from the center of the coil along its axis (m).

The experimental setups with different number of loops are shown in figures 3.1 and 3.2.

We used a coil wound on a plastic support, a current generator, a multimeter to measure current, and the Phyphox app on a smartphone to measure B.

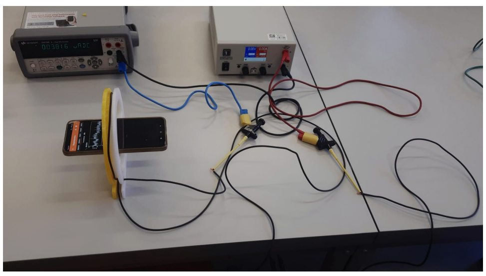

Figure 3.1: Magnetometer Setup with 1 Loop

Figure 3.2: Magnetometer Setup with 10 Loops

The magnetic field B was recorded for different values of I and N, keeping z constant. The collected data was analyzed to calculate μ_0 by rearranging the formula for $B(z)$.

3.3 Data

The data in table 3.1 was collected during the experiment:

Current (I) [A]	Turns (N)	Distance (z) [m]	Avg. Abs. Field (B) [μT]
0.000006	1	0.06	0.00003
2.0	5	0.06	34.5
3.0	5	0.06	62.2
0.000006	10	0.06	0.0004
2.0	10	0.06	78.7
3.0	10	0.06	118.2

Table 3.1: Collected data showing magnetic field measurements at different currents and coil configurations.

The Absolute Magnetic Field (B) [μT] is the magnitude of the magnetic field vector, calculated using the formula:

$$B = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

It is calculated using all three (x, y and z) coordinates recorded in the Phyphox app. In the table 3.1 we calculated the averages of Absolute Magnetic Fields for each row using the Excel data from the Phyphox magnetometer.

We found the distance (z) value is 4 cm by using a ruler to measure the axial distance from the center of the coil to the point where the magnetic field is being measured.

3.4 Analysis

We rearrange the formula to compute μ_0:

$$\mu_0 = \frac{2B(z)\left(R^2 + z^2\right)^{3/2}}{NIR^2}.$$

Calculate the average and standard deviation of μ_0 across all measurements to account for experimental variability. Compare the experimental value of μ_0 with the theoretical value of $4\pi \times 10^{-7}$ T·m/A.

3.4.1 Results

```python
import numpy as np

# Constants
R = 0.07  # Radius of the coil (in meters)
mu_0_theoretical = 4 * np.pi * 1e-7  # Theoretical value of mu_0 in Tm/A

# Data (Current, Number of Turns, Distance, Magnetic Field)
data = [
    (0.000006, 1, 0.06, 0.00003),
    (2.0, 5, 0.06, 34.5),
    (3.0, 5, 0.06, 62.2),
    (0.000006, 10, 0.06, 0.0004),
    (2.0, 10, 0.06, 78.7),
    (3.0, 10, 0.06, 118.2)
]

# Function to calculate mu_0 from the formula
def calculate_mu_0(B, N, I, z, R):
    return (2 * B * (R**2 + z**2)**(3/2)) / (N * I * R**2)

# Calculate mu_0 for each measurement
mu_0_values = []
for I, N, z, B in data:
    mu_0 = calculate_mu_0(B * 1e-6, N, I, z, R)  # Convert B to Tesla (since B is in microteslas)
    mu_0_values.append(mu_0)

# Calculate the average and standard deviation of mu_0
mu_0_average = np.mean(mu_0_values)
mu_0_std_dev = np.std(mu_0_values)

# Output the results
print(f"Average mu_0: {mu_0_average:.5e} Tm/A")
print(f"Standard deviation of mu_0: {mu_0_std_dev:.5e} Tm/A")
print(f"Theoretical mu_0: {mu_0_theoretical:.5e} Tm/A")
```

Listing 3.1: Python code for calculating μ_0

The computed values of magnetic permeability μ_0 are as follows:
Average value of μ_0:

$$\mu_0 \approx 1.4569 \times 10^{-6}\,\mathrm{T{\cdot}m/A}$$

Standard deviation of μ_0:

$$\sigma_{\mu_0} \approx 3.281 \times 10^{-7}\,\mathrm{T{\cdot}m/A}$$

Theoretical value of μ_0:

$$\mu_0^{\text{theoretical}} = 1.257 \times 10^{-6}\,\mathrm{T{\cdot}m/A}$$

The experimental results are in reasonable agreement with the theoretical value of μ_0. The difference is within the experimental uncertainty, as indicated by the standard deviation. While the experimental value is slightly higher than the theoretical value, this discrepancy is small enough to be considered within the expected experimental error, especially in light of the standard deviation.