Report Lab 1

Experimental Physics for AI 2

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Abstract

In the first laboratory the general objectives were to appropriately configure instruments to make resistance measurements, verify Ohm's law and to measure the current voltage characterization of a non-linear device. In order to make the measurement we studied DC-fed circuits constructing the voltage current characteristic of the two different devices, that is to say an Ohmic conductor and a non-Ohmic conductor, the first one a resistor, the latter a diode.

Chapter 1

Measurement of the current-voltage characteristic of a resistor

1.1 Goal

In the first part of the experiment, we want to evaluate the best instrument configuration for the circuits given, verify Ohm's law using the results obtained, and measure the composite resistances in parallel and in series of the circuit.

1.2 Setup

The setup for the experiment is shown in figure 1.1.

1.3 Method

1.3.1 Evaluation of the best instrument configuration

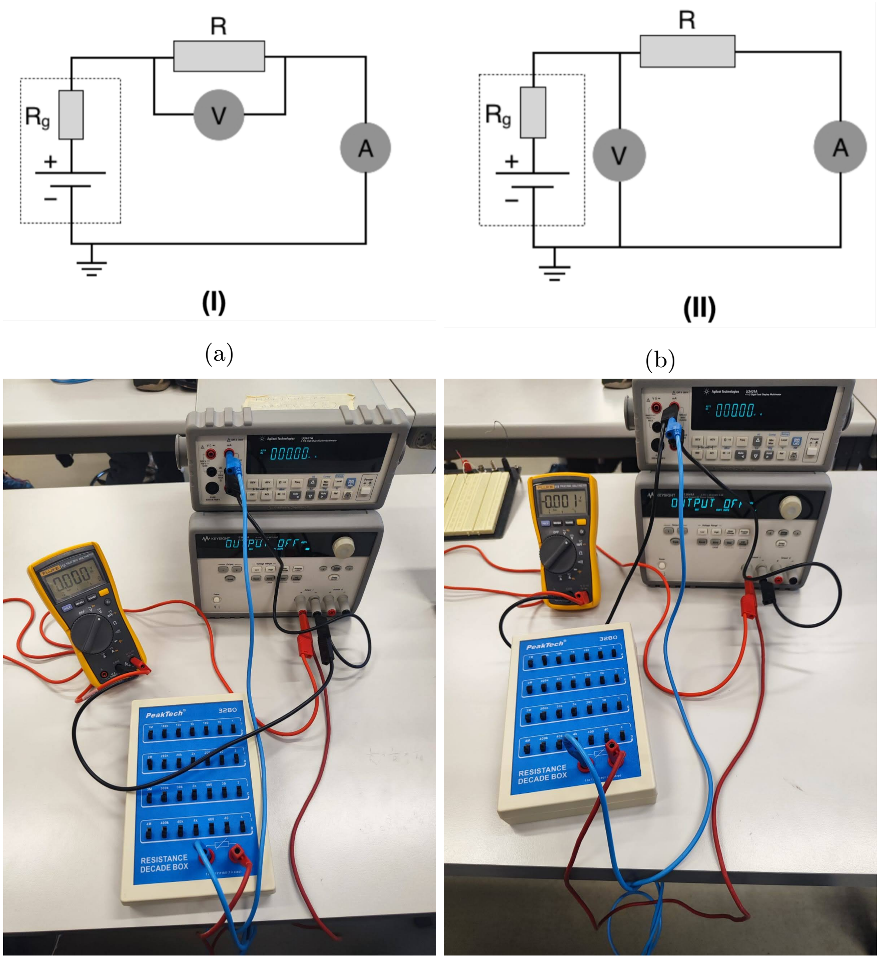
In order to do the evaluation, we set up all the devices necessary to recreate the first circuit (image 1.1c). With the aim of analyzing the best instrument configuration, we need to find the value of the voltmeter's internal resistance R_V for setup 1 (image 1.1a) and the ammeter's internal resistance R_A for setup 2 (image 1.1b). For R_V, we need to apply Ohm's law to the parallel of resistors R and R_V, and for R_A, to the series of resistors R and R_A.

After connecting every element, we started to try different values of the resistance (R_{box}) and the $V_{battery}$ produced by the power supply, in both setups. We obtained some values of V (detected voltage) on the fluke and of the current (detected current). With these detections, we calculated firstly the resistance of the circuit using Ohm's law:

$$R_{\text{detected}} = \frac{V_{\text{detected}}}{I_{\text{detected}}}$$

Using the found values of R, it was possible to calculate the voltmeter's internal resistance R_V and the ammeter's internal resistance R_A. We arrived at these results using the equivalent resistance derived from Ohm's law. For R_V, we used the formula for resistors in parallel, and for R_A, the formula for resistors in series:

$$R_V = \left(\frac{1}{R_{\text{detected}}} - \frac{1}{R_{\text{box}}} \right)^{-1} \tag{1.3.1}$$



(a) (b)

(c) (d)

Figure 1.1: Setup of the first part of the experiment: on the top are shown two diagrams, one for each of the two circuits. On the bottom are shown two photos, one for each of the two setups

$$R_A = R_{\text{detected}} - R_{\text{box}} \tag{1.3.2}$$

1.3.2 Data

The data collected is shown in Table 1.2.

1.3.3 Which of the two circuits is more suitable for measuring "small" resistances and which for "large" resistances

Circuit (I) is more suitable for small resistances, while circuit (II) is better for large resistances because in this way their internal resistances are irrelevant. To prove this point, we analyzed the opposite case:

- For circuit (I), we used large resistances and found that we have a constant internal resistance of the voltmeter (R_V). Since we found an internal resistance, we proved that, to have a negligible one, we need to use small resistances.
- For circuit (II), we used small resistances and found that we have a constant

Voltage (V)	**Current** (mA)	$R_{\textbf{expected}}$ (Ω)	$R_{\textbf{detected}}$ (Ω)
0.10	0.47	110 (2 in parallel of 220)	212
0.20	0.93	110 (2 in parallel of 220)	213
0.50	2.35	110 (2 in parallel of 220)	212
1.00	8.81	110 (2 in parallel of 220)	113
1.99	17.62	110 (2 in parallel of 220)	113
2.99	26.44	110 (2 in parallel of 220)	113
3.99	35.25	110 (2 in parallel of 220)	113
4.98	44.06	110 (2 in parallel of 220)	113
9.97	88.10	110 (2 in parallel of 220)	113
0.10	0.18	440 (2 in series)	541
0.20	0.37	440 (2 in series)	542
0.50	0.92	440 (2 in series)	542
1.00	1.84	440 (2 in series)	541
2.00	3.69	440 (2 in series)	542
3.00	6.77	440 (2 in series)	443
4.00	9.03	440 (2 in series)	443
5.00	11.28	440 (2 in series)	443
9.99	22.56	440 (2 in series)	443

Table 1.1: Data collected for the measurements of the composite resistances

internal resistance of the ammeter (R_A). Since we found an internal resistance, we proved that, to have a negligible one, we need to use large resistances.

1.3.4 Verification of Ohm's law

To check the validity of Ohm's law, we used the values of $V_{battery}$ and R_{box}. Then, we calculated the current using the inverse of Ohm's law:

$$I = \frac{V}{R}$$

We observed that increasing the resistance results in a larger difference from the calculated and expected values.

1.3.5 Measurement of composite resistances

We picked resistors with values of 220. When in parallel, they become 110; when in series, they become 440. With the values obtained using all the instruments, we compared the two values of the resistance. The data table is shown in Table 1.1

Voltage (V)	**Current** (μA)	$R_{\textbf{box}}$ ($M\Omega$)	$R_{\textbf{detected}}$ ($M\Omega$)	R_V ($M\Omega$)
0.246	24.70	0.01	0.01	2.46
0.247	12.43	0.02	0.02	3.29
0.248	4.98	0.05	0.05	12.40
0.248	2.51	0.10	0.10	9.92
0.248	1.26	0.20	0.20	16.53
0.248	0.52	0.50	0.48	13.05
0.248	0.27	1.00	0.92	11.27
0.248	0.14	2.00	1.67	10.20
0.248	0.07	5.00	3.34	10.10
0.248	0.05	10.00	5.03	10.10
0.494	49.65	0.01	0.01	1.98
0.496	24.97	0.02	0.02	2.92
0.498	10.00	0.05	0.05	12.45
0.498	5.03	0.10	0.10	9.96
0.499	2.52	0.20	0.20	19.96
0.499	1.03	0.50	0.48	15.59
0.499	0.53	1.00	0.94	16.10
0.499	0.29	2.00	1.67	10.20
0.499	0.14	5.00	3.34	10.10
0.499	0.10	10.00	5.03	10.10
0.741	74.46	0.01	0.01	2.06
0.744	37.45	0.02	0.02	2.98
0.747	15.01	0.05	0.05	10.67
0.747	7.55	0.10	0.10	9.34
0.748	3.78	0.20	0.20	18.70
0.748	1.55	0.50	0.48	13.85
0.748	0.80	1.00	0.94	14.38
0.748	0.44	2.00	1.67	10.20
0.748	0.21	5.00	3.34	10.10
0.748	0.14	10.00	5.03	10.10
0.988	99.42	0.01	0.01	1.61
0.993	49.99	0.02	0.02	2.92
0.997	20.03	0.05	0.05	11.08
0.998	10.06	0.10	0.10	12.48
0.998	5.04	0.20	0.20	19.96
0.998	2.07	0.50	0.48	13.49
0.998	1.08	1.00	0.92	12.17
0.999	0.59	2.00	1.67	10.20
0.999	0.29	5.00	3.34	10.10
0.999	0.19	10.00	5.03	10.10
1.978	198.98	0.01	0.01	1.68
1.989	100.05	0.02	0.02	3.31
1.995	40.10	0.05	0.05	9.98
1.997	20.19	0.10	0.10	9.29
1.998	10.18	0.20	0.20	10.80
1.998	4.15	0.50	0.48	12.97
1.988	2.16	1.00	0.92	11.56
1.999	1.18	2.00	1.67	10.20
1.999	0.58	5.00	3.34	10.10
1.999	0.38	10.00	5.03	10.10
2.968	298.52	0.01	0.01	1.73
2.983	151.10	0.02	0.02	1.53
2.992	60.17	0.05	0.05	9.21
2.995	30.23	0.10	0.10	10.89
2.997	15.15	0.20	0.20	18.16
2.998	6.22	0.50	0.48	13.38
2.998	3.24	1.00	0.93	12.39
2.998	1.77	2.00	1.67	10.20
2.998	0.87	5.00	3.34	10.10
2.998	0.57	10.00	5.03	10.10
3.959	398.21	0.01	0.01	1.71
3.979	200.23	0.02	0.02	3.11
3.991	80.26	0.05	0.05	9.07
3.995	40.32	0.10	0.10	10.80
3.997	20.21	0.20	0.20	17.76
3.998	8.30	0.50	0.48	13.15
3.999	4.32	1.00	0.93	12.46
3.999	2.36	2.00	1.67	10.20
3.999	1.16	5.00	3.34	10.10
3.999	0.76	10.00	5.03	10.10
4.949	497.71	0.01	0.01	1.76
4.973	250.27	0.02	0.02	3.07
4.989	100.32	0.05	0.05	9.24
4.994	50.39	0.10	0.10	11.10
4.996	25.27	0.20	0.20	17.23
4.998	10.38	0.50	0.48	13.19
4.998	5.40	1.00	0.93	12.43
4.998	2.95	2.00	1.69	11.08
4.999	1.45	5.00	3.45	11.10
4.999	0.95	10.00	5.26	11.11
9.9	995.60	0.01	0.01	1.77
9.95	500.70	0.02	0.02	3.11
9.98	200.74	0.05	0.05	8.79
9.99	100.87	0.10	0.10	10.30
9.99	50.63	0.20	0.20	14.80
10	20.84	0.50	0.48	11.98
10	10.89	1.00	0.92	11.24
10	5.98	2.00	1.67	10.20
10	2.99	5.00	3.34	10.10
10	1.99	10.00	5.03	10.10

Voltage (V)	**Current** (μA)	$R_{\textbf{box}}$ ($k\Omega$)	$R_{\textbf{detected}}$ ($k\Omega$)	R_A ($k\Omega$)
0.248	2.32	0.001	0.107	0.106
0.248	2.31	0.002	0.107	0.105
0.248	2.23	0.005	0.111	0.106
0.248	2.11	0.010	0.118	0.108
0.248	1.95	0.020	0.127	0.107
0.248	1.58	0.050	0.157	0.107
0.248	1.22	0.100	0.204	0.104
0.248	0.82	0.200	0.304	0.104
0.248	0.41	0.500	0.601	0.101
0.248	0.23	1.000	1.098	0.098
0.248	0.12	2.000	2.093	0.093
0.248	0.05	5.000	5.084	0.084
0.248	0.02	10.000	10.077	0.077
0.499	4.70	0.001	0.106	0.105
0.499	4.70	0.002	0.106	0.104
0.499	4.53	0.005	0.110	0.105
0.499	4.34	0.010	0.115	0.105
0.499	4.02	0.020	0.124	0.104
0.499	3.21	0.050	0.155	0.105
0.499	2.43	0.100	0.205	0.105
0.499	1.63	0.200	0.305	0.105
0.499	0.83	0.500	0.601	0.101
0.499	0.45	1.000	1.101	0.101
0.499	0.24	2.000	2.103	0.103
0.499	0.10	5.000	5.098	0.098
0.499	0.05	10.000	10.093	0.093
0.996	115.90	0.001	0.009	0.008
0.996	111.60	0.002	0.009	0.007
0.997	75.80	0.005	0.013	0.008
0.997	58.90	0.010	0.017	0.007
0.998	37.74	0.020	0.026	0.006
0.998	17.49	0.050	0.057	0.007
0.999	4.87	0.100	0.205	0.105
0.999	3.29	0.200	0.303	0.103
0.999	1.66	0.500	0.601	0.101
0.999	0.91	1.000	1.100	0.100
0.999	0.47	2.000	2.104	0.104
0.999	0.20	5.000	5.102	0.102
0.999	0.10	10.000	10.097	0.097
1.999	186.30	0.005	0.011	0.006
1.997	128.20	0.010	0.016	0.006
1.997	79.10	0.020	0.025	0.005
1.998	35.97	0.050	0.056	0.006
1.999	18.95	0.100	0.105	0.005
1.999	9.80	0.200	0.204	0.004
1.999	3.32	0.500	0.601	0.101
1.999	1.82	1.000	1.099	0.099
1.999	0.95	2.000	2.104	0.104
1.999	0.39	5.000	5.100	0.100
1.999	0.20	10.000	10.095	0.095
4.991	276.11	0.010	0.018	0.008
4.994	177.80	0.020	0.028	0.008
4.996	85.17	0.050	0.059	0.009
4.998	46.10	0.100	0.108	0.008
4.998	24.09	0.200	0.208	0.008
4.998	9.88	0.500	0.506	0.006
4.999	4.53	1.000	1.103	0.103
4.999	2.37	2.000	2.106	0.106
4.999	0.98	5.000	5.102	0.102
4.999	0.50	10.000	10.054	0.054
9.99	414.40	0.020	0.024	0.004
9.99	183.20	0.050	0.055	0.005
10	95.30	0.100	0.105	0.005
10	49.30	0.200	0.203	0.003
10	19.94	0.500	0.502	0.002
10	9.99	1.000	1.001	0.001
10	4.76	2.000	2.102	0.102
10	1.96	5.000	5.098	0.098
10	0.99	10.000	10.053	0.053

Table 1.2: Data collected for the measurements of R_V (left) and R_A (right)

Chapter 2

Measurement of the current-voltage characteristic of a diode

2.1 Goal

Now we want to measure the current-voltage characteristic of a diode, which should not be linear. Indeed, according to Shockley's law, it is exponential:

$$I = I_0 \left(e^{\frac{qV}{gkT}} - 1 \right)$$

where I_0 is the reverse saturation current, q is the electron charge, k is the Boltzmann constant, T is the temperature, and g is the diode type-dependent constant. In this chapter we will try to verify this law.

Moreover for practical applications it's common practice to define the diode's *threshold voltage* as the voltage at which the diode starts conducting a "significant" current. We will try to measure this value as well.

2.2 Method

Using a similar setup as the one in part one, we recorded the measured values of current at different voltages. The setup is shown in figure 2.1, where the voltmeter is a handheld Fluke multimeter and the ammeter is a Agilent bench multimeter.

Later, in section 2.4, we will perform various fits to the data to verify the exponential relation and estimate the values of the parameters.

2.3 Data

The data we collected is shown in table and represented graphically in figure 2.2. The bench multimeter for the current measurements had an accuracy of $\pm 0.05\% + 0.05\mu A$ in the $500\mu A$ range; and the handheld multimeter had an accuracy of $\pm 0.5\% + 0.002V$ in the $2V$ range.

from the graph in figure 2.2 we can already see that after a certain voltage value (about 2.4) the graph follows accurately an exponential relationship, as expected. The previous values are very low in current and have a greater error. We will perform the analysis only on the part of the data which shows a clear exponential behavior, that is for voltages greater than 2.399 and currents greater than $1.74\mu A$ (medium values).

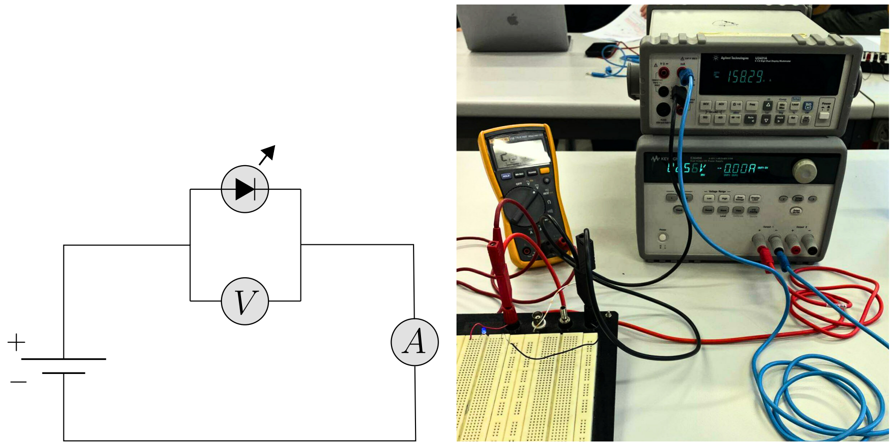


Figure 2.1: Setup of the diode experiment: on the left the diagram showing the circuit made, on the right a photo of the setup

2.4 Analysis

2.4.1 Shockley's law

We used the following approximation:

$$\ln\left(e^{x}-1\right) \approx x \quad \text{for } x \gg 1 \tag{2.4.1}$$

so that we could linearize the relationship as

$$I = I_0\left(e^{\frac{qV}{gkT}}-1\right) \approx I_0 e^{\frac{qV}{gkT}} \implies \log I \approx \frac{q}{gkT}V + \log I_0$$

Then we performed a linear regression on the data, with the errors on V. We decided to keep the error only in the independent variable since it was much greater than the one in the dependent variable. in `R` we ran the following commands:

```r
diode.data <- read.csv("data.csv")
diode.selected <- diode.data[diode.data$Vdetapprox >= 2.399,]
logIerr <- diode.selected$errIdet/diode.selected$Idetapprox
diode.lm <- lm(log(Idetapprox) ~ Vdetapprox, data=diode.selected,
                                          weights=1/logIerr^2)
summary(diode.lm)
```

obtaining the following result:

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -66.698      1.294  -51.55   <2e-16 ***
Vdetapprox    28.215      0.509   55.44   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 44.5 on 19 degrees of freedom
Multiple R-squared:  0.9939, Adjusted R-squared:  0.9935
F-statistic:  3073 on 1 and 19 DF,  p-value: < 2.2e-16
```

Voltage (V)		Current (μA)	
0.499	$\pm$ 0.004	0.03	$\pm$ 0.05
0.999	$\pm$ 0.007	0.08	$\pm$ 0.05
1.499	$\pm$ 0.009	0.12	$\pm$ 0.05
1.999	$\pm$ 0.012	0.16	$\pm$ 0.05
2.099	$\pm$ 0.012	0.17	$\pm$ 0.05
2.200	$\pm$ 0.013	0.19	$\pm$ 0.05
2.299	$\pm$ 0.013	0.26	$\pm$ 0.05
2.399	$\pm$ 0.014	1.74	$\pm$ 0.05
2.409	$\pm$ 0.014	2.33	$\pm$ 0.05
2.419	$\pm$ 0.014	3.16	$\pm$ 0.05
2.428	$\pm$ 0.014	4.33	$\pm$ 0.05
2.438	$\pm$ 0.014	5.95	$\pm$ 0.05
2.448	$\pm$ 0.014	8.19	$\pm$ 0.05
2.458	$\pm$ 0.014	11.28	$\pm$ 0.06
2.467	$\pm$ 0.014	15.47	$\pm$ 0.06
2.477	$\pm$ 0.014	21.16	$\pm$ 0.06
2.486	$\pm$ 0.014	28.55	$\pm$ 0.06
2.495	$\pm$ 0.014	38.42	$\pm$ 0.07
2.504	$\pm$ 0.015	51.03	$\pm$ 0.08
2.512	$\pm$ 0.015	66.50	$\pm$ 0.08
2.520	$\pm$ 0.015	84.54	$\pm$ 0.09
2.528	$\pm$ 0.015	105.8	$\pm$ 0.1
2.535	$\pm$ 0.015	130.0	$\pm$ 0.1
2.542	$\pm$ 0.015	158.8	$\pm$ 0.1
2.549	$\pm$ 0.015	189.2	$\pm$ 0.1
2.556	$\pm$ 0.015	222.6	$\pm$ 0.2
2.562	$\pm$ 0.015	258.5	$\pm$ 0.2
2.568	$\pm$ 0.015	296.5	$\pm$ 0.2

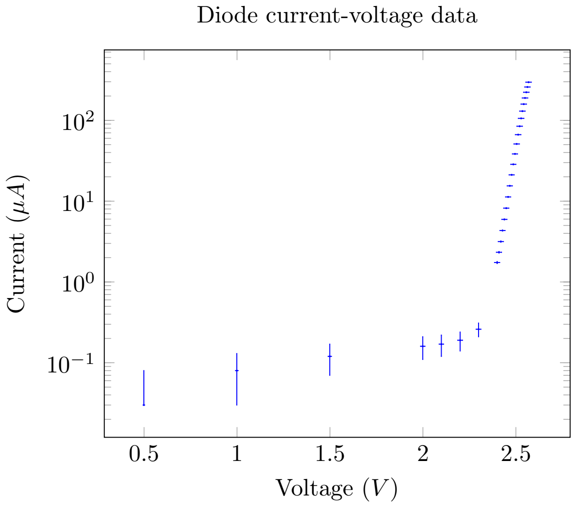


Figure 2.2: Data collected for the diode

So we can very confidently say that the relation between the current and the voltage is exponential. The formula is then:

$$\log I_0 = -67 \pm 1 \implies I_0 = 10^{-29} \pm 10^{-29} \mu A$$
$$\frac{q}{gkT} = 28.2 \pm 0.5 \implies g = \frac{38.6}{28.2} \mp \frac{0.5}{28.2^2} = 1.368 \pm 0.006$$
$$I \approx 1e - 29 e^{\frac{q}{kT} \cdot \frac{1}{1.368} \cdot V} = 10^{-29} e^{28.2V} \mu A$$

(We did not take the measurement for the temperature, so actually 38.6 here should not obviously be a constant, but as we don't have the measurement, we considered it as such, consequently the error on g is much smaller than the one it should be). In figure 2.3 we can see in a graph the accuracy of the fit. Now we can justify the approximation (2.4.1) as indeed for $V \geq 2.4$ we clearly have that the exponent $\frac{qV}{gkT} \geq 67.68 \gg 1$, more precisely, if $f, g : [67.68, \infty)$ are respectively the functions $x \mapsto \ln(e^x - 1)$ and $x \mapsto x$ then

$$\|f - g\|_\infty = |(f - g)(67.68)| \approx 4.05 \cdot 10^{-30}$$

Chi-squared test Finally, we performed a χ^2 test on the residuals of the fit to see if the errors were independent and normally distributed (assuming homoscedasticity), which is what we expect:

```
res <- exp(diode.lm$fitted.values) - diode.selected$Idetapprox
chisq <- sum(res^2 / var(res)) # assume homoscedasticity
```

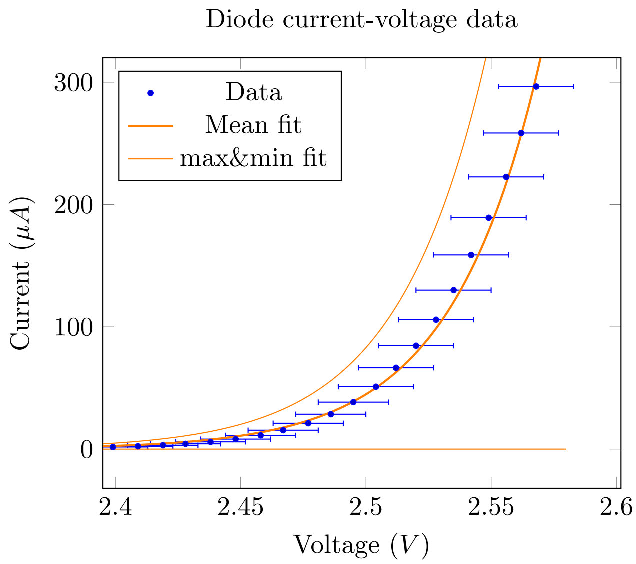


Figure 2.3: Exponential fit of the data

```
df <- length(diode.lm$residuals) - length(diode.lm$coefficients)
pval <- pchisq(chisq, df, lower.tail=FALSE)
pval
```

Which returns a p-value of $0.29 > 0.05$, so we can't reject the null hypothesis that the errors are independent and the fit is good.

2.5 Threshold voltage

As explained before, the *threshold voltage* is said to be the voltage at which a diode starts conducting a "significant" current. It's obtained by fitting the data as a linear relationship and then taking the x-intercept of the line. To decide how many points to take out of the selected ones, we will perform the χ^2 test to see if the residuals are independent normal random variables. The null hypothesis for each value of l (the number of points) is that the errors are independent and normally distributed, while the alternative hypothesis is that they are not. We will then take the biggest l such that we can't reject the null hypothesis. In R code this is:

```
l <- length(diode.selected$Vdetapprox)+1
pval <- 0
while (pval < 0.05) {
    l = l - 1
    diode.sel_thresh <- tail(diode.selected, l)
    diode.lm_threshold <- lm(Idetapprox ~ Vdetapprox,
                             data=diode.sel_thresh)
    r = diode.lm_threshold$residuals
    chisq <- sum(r^2 / abs(diode.lm_threshold$fitted.values))
    pval <- pchisq(chisq, l - 2, lower.tail=FALSE)
}
summary(diode.lm_threshold)
```

obtaining that we need to keep the last 9 points. Finally we calculated the V-coordinate of the fitting line of those 9 points. Since we know from the linear model that the line has equation $i = mv + q$, with m and q parameters saved in the variable

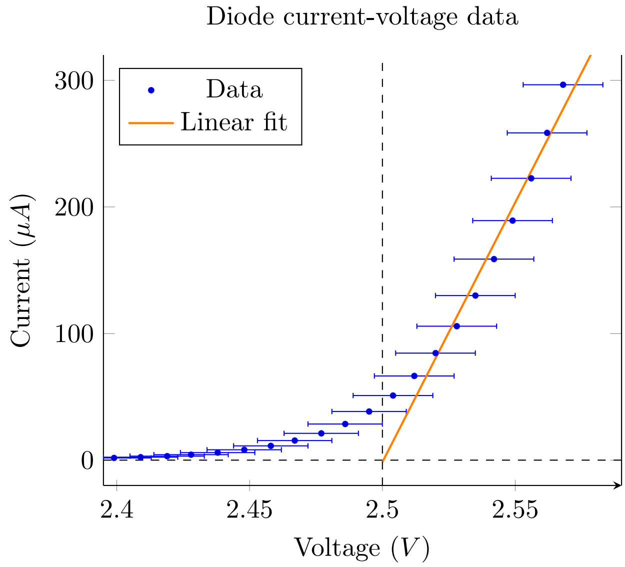


Figure 2.4: Threshold voltage

`diode.lm_threshold$coefficients`, and we obtain

$$q = -10.3 \pm 0.6\, mA \quad m = 4.1 \pm 0.2\, mA/V$$

$$\implies V_{\text{threshold}} = \frac{10.3}{4.1} \pm \sqrt{\left(\frac{0.6}{-10.3}\right)^2 + \left(\frac{0.2}{4.1}\right)^2} = 2.50 \pm 0.8\, V$$

2.6 Conclusion

A clear improvement would be to measure the temperature of the room in order to have a more precise value of the constant g. Moreover, If we had more datapoints, we could have performed a more accurate fit and have smaller errors.

That said, we have successfully verified Shockley's law for a diode and estimated the value of the parameters. We also measured the threshold voltage of the diode. The results are consistent with the theory: for silicon diodes the ideality factor g should be between 1 and 2, and the threshold voltage makes sense that is around $2.5V$, since our diode was a blue LED, so if we see the threshold voltage as the voltage at which the LED starts emitting light, the first photon has energy $E = h\nu$ with $\nu = c/\lambda$, c the speed of light and h Planck's constant. If we add $E = qV_{\text{threshold}}$ and solve for the wavelength λ we get

$$\lambda = \frac{hc}{qV_{\text{threshold}}} \approx 496\, nm$$

which is in the range of blue light.