Dec 13, 2022 (Due: 08:00 Dec 22, 2022)

1. Implement the Lanczos algorithm for symmetric eigenvalue problems. Test it with some examples of size several thousands. What do you observe for the convergence of Ritz pairs and the orthogonality of the Lanczos vectors?

2. Implement the Lanczos algorithm for computing the matrix functional $v^* f(A)v$, where A is Hermitian. Test it with some matrices of size several thousands and some sufficiently smooth functions. Plot the convergence history.

3. Suppose that you have a black-box function that computes $v^\top f(A)v$, where $A \in \mathbb{R}^n$ is symmetric and $v \in \mathbb{R}^n$ are the inputs. Make use of this black-box function to compute $u^\top f(A)v$.

4. (optional) Implement the preconditioned inverse iteration. Test it with the exact shift-and-invert preconditioner and compare the result with that produced by the usual inverse iteration.

Can you improve the preconditioned inverse iteration for symmetric eigenvalue problems using the Rayleigh–Ritz procedure?

5. (optional) Implement the LSQR algorithm. Test it with some examples and visualize the convergence history.

6. (optional) Implement the Arnoldi algorithm for the unsymmetric eigenvalue problem. Test it with some examples of size several thousands. Which eigenvalues can you obtain?

(H, optional) Implement implicitly restarted Arnoldi and Krylov–Schur algorithms and test them for computing eigenvalues in a certain region.