Mar. 27, 2023 (Due: 08:00 Apr. 3, 2023)

1. Interpolate the following data set and visualize your solution on $[-1,1]\times[-1,1]$.

x_i	y_i	z_i
-1.0000	-1.0000	1.6389
-1.0000	1.0000	0.5403
1.0000	-1.0000	-0.9900
1.0000	1.0000	0.1086
-0.7313	0.6949	0.9573
0.5275	-0.4899	0.8270
-0.0091	-0.1010	1.6936
0.3031	0.5774	1.3670

Note that different interpolation strategies will lead to different results. You are encouraged to try different 2D interpolation strategies learned from the lecture. (If Delaunay triangularization is used, you can make use of the MATLAB/Octave function `delaunay`.)

2. Sometimes we are interested in finding a curve that (approximately) passes through the given data points (x_1, y_1), (x_2, y_2), $\ldots$, (x_n, y_n). The curve is not necessarily of the form $y = f(x)$ because a straight line parallel to the y-axis may intersect with the curve at multiple points. One strategy is to perform two cubic spline interpolations (or cubic spline fittings) $x = x(t)$ and $y = y(t)$ by choosing an appropriate sequences of t_i's. According to differential geometry, the best parameterization is to choose t as the arc length. In our case, we can replace arc length by straight line distance since we only have a discrete data set. Use this strategy to interpolate the following data sets and visualize your results.

(1) A smooth curve that connects (in turn)

$$(1,1),\ (0,2),\ (-1,1),\ (0,0),\ (1,-1),\ (0,-2),\ (-1,-1).$$

(2) A smooth closed curve that connects (in turn)

$$(3,0),\ (2,2),\ (0,3),\ (-2,2),\ (-3,0),\ (-2,-2),\ (0,-3),\ (2,-2).$$

3. In the homework on March 6, you have been asked to find the root of $x^{64} - 0.1 = 0$ in $[0,1]$ using bisection and *regula falsi*. Try to fit the history of residuals using a simple model.

4. (optional) Launch an image processing program (e.g., `mspaint` on Windows). Open your left hand naturally, and put it on the computer screen. Then use the mouse to sketch the outline of your hand. (A few discrete points already suffice.) Use the technique from Exercise 2 to reconstruct the outline of your hand with (piecewise) algebraic curves. Visualize the result.