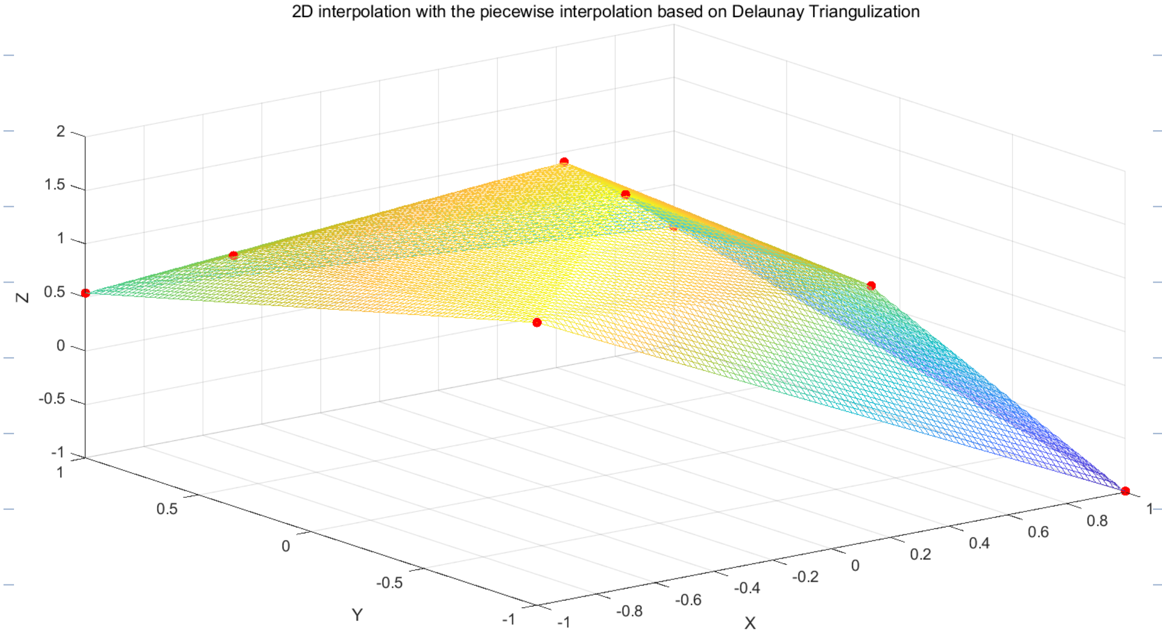
3月27日

1

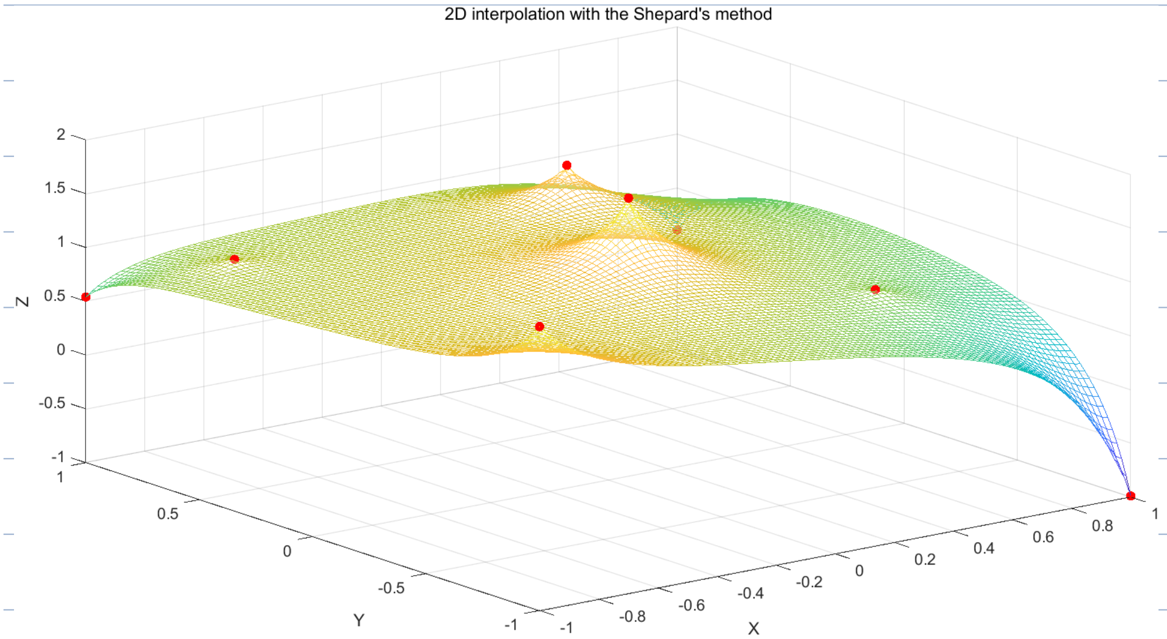
1. Interpolate the following data set and visualize your solution on $[-1,1]\times[-1,1]$.

x_i	y_i	z_i
-1.0000	-1.0000	1.6389
-1.0000	1.0000	0.5403
1.0000	-1.0000	-0.9900
1.0000	1.0000	0.1086
-0.7313	0.6949	0.9573
0.5275	-0.4899	0.8270
-0.0091	-0.1010	1.6936
0.3031	0.5774	1.3670

使用分片线性插值效果如下：



使用 Shepard's method 插值效果如下：



2

2. Sometimes we are interested in finding a curve that (approximately) passes through the given data points (x_1, y_1), (x_2, y_2), $\ldots$, (x_n, y_n). The curve is not necessarily of the form $y = f(x)$ because a straight line parallel to the y-axis may intersect with the curve at multiple points. One strategy is to perform two cubic spline interpolations (or cubic spline fittings) $x = x(t)$ and $y = y(t)$ by choosing an appropriate sequences of t_i's. According to differential geometry, the best parameterization is to choose t as the arc length. In our case, we can replace arc length by straight line distance since we only have a discrete data set. Use this strategy to interpolate the following data sets and visualize your results.

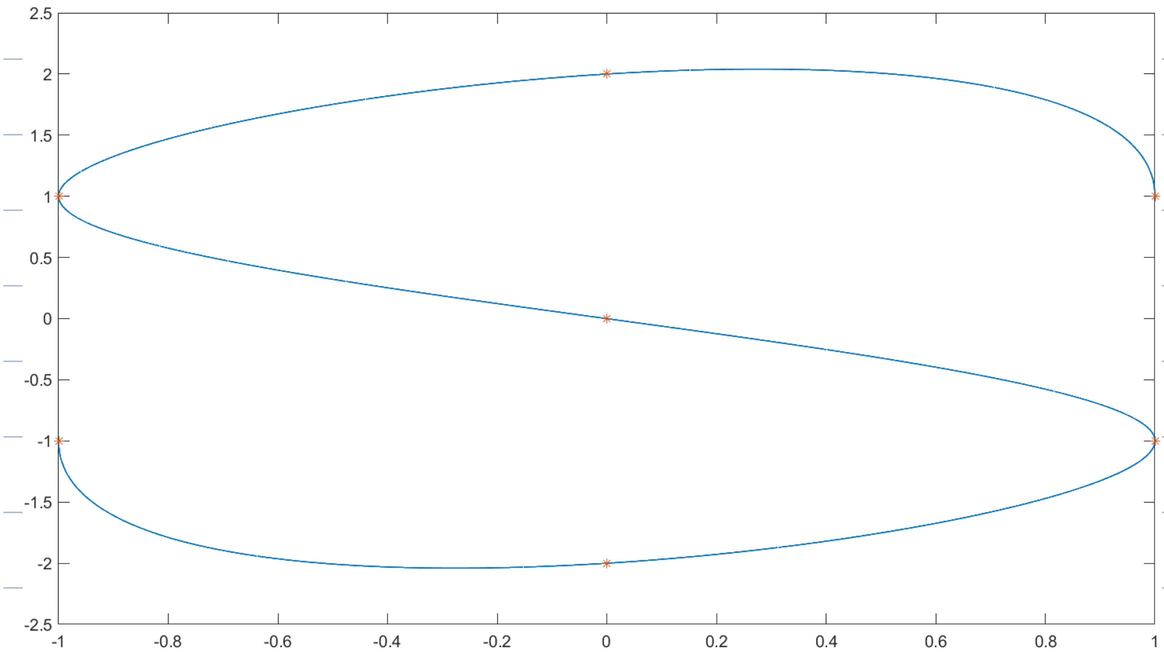
(1) A smooth curve that connects (in turn)

$$(1,1),\ (0,2),\ (-1,1),\ (0,0),\ (1,-1),\ (0,-2),\ (-1,-1).$$

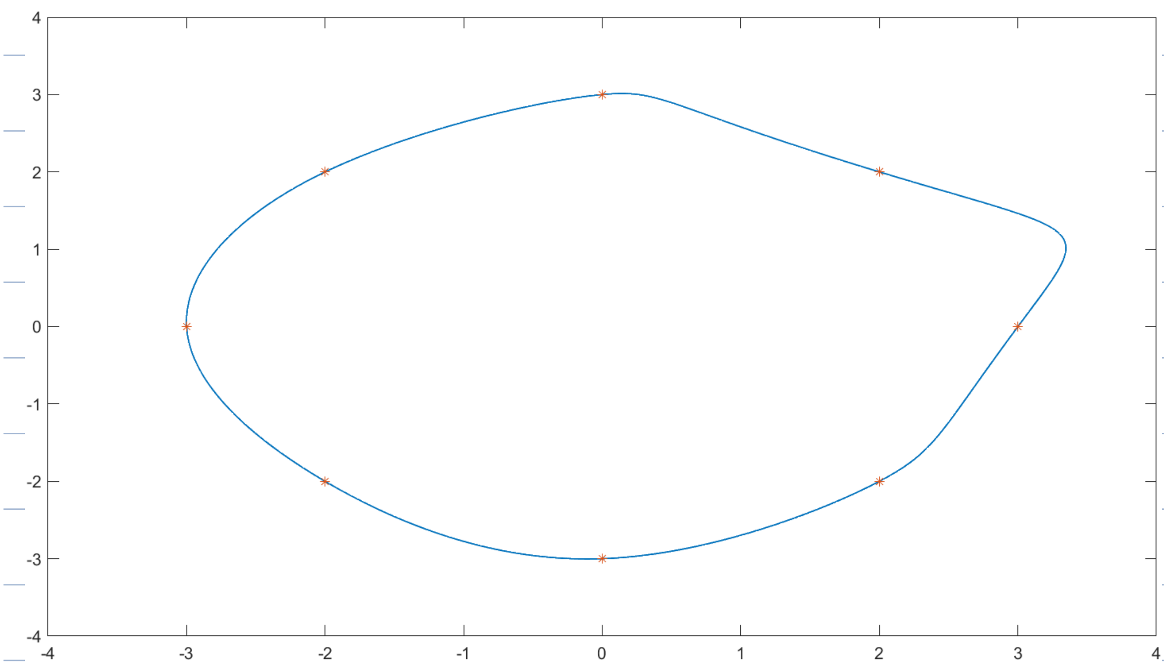
(2) A smooth closed curve that connects (in turn)

$$(3,0),\ (2,2),\ (0,3),\ (-2,2),\ (-3,0),\ (-2,-2),\ (0,-3),\ (2,-2).$$

(1) 连接各点的光滑曲线如下:



(2) 连接各点的光滑闭曲线如下:



3. In the homework on March 6, you have been asked to find the root of $x^{64} - 0.1 = 0$ in $[0, 1]$ using bisection and *regula falsi*. Try to fit the history of residuals using a simple model.

解：在我目前学过的"simple model"之中，我认为最小2乘法是一个比较明智的选取。由于在半对数图中可以看出残差在取对数后有线性下降的特点，那么，可以设：

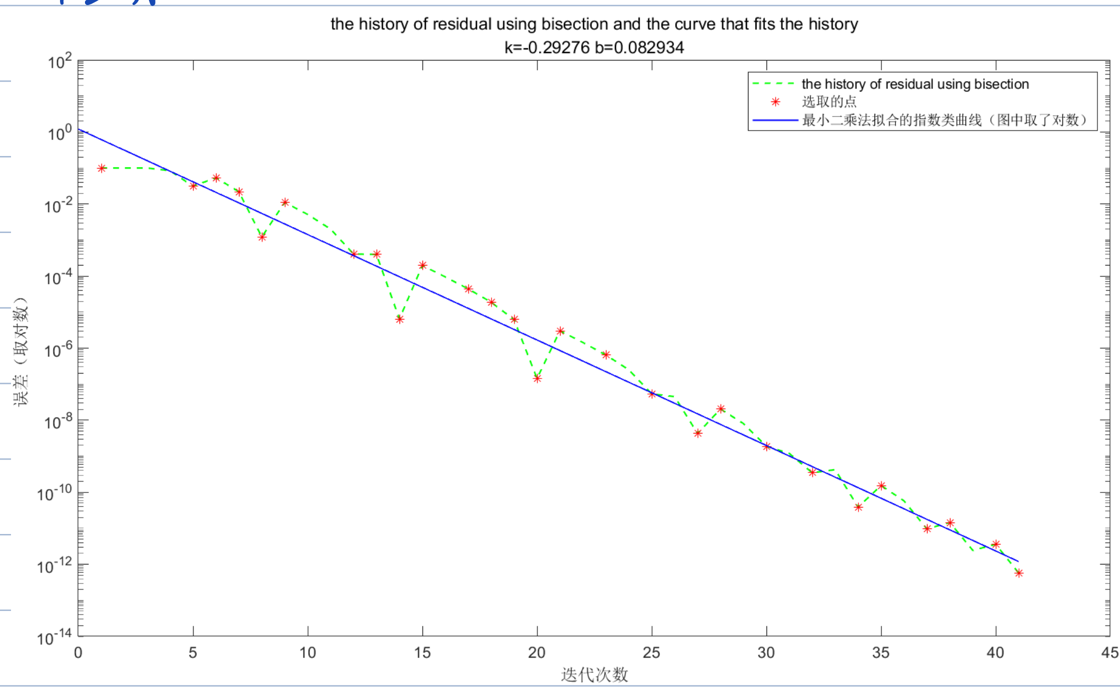
$$y = 10^{kx+b}$$

令 $z = \log_{10} y \Rightarrow z = kx+b$

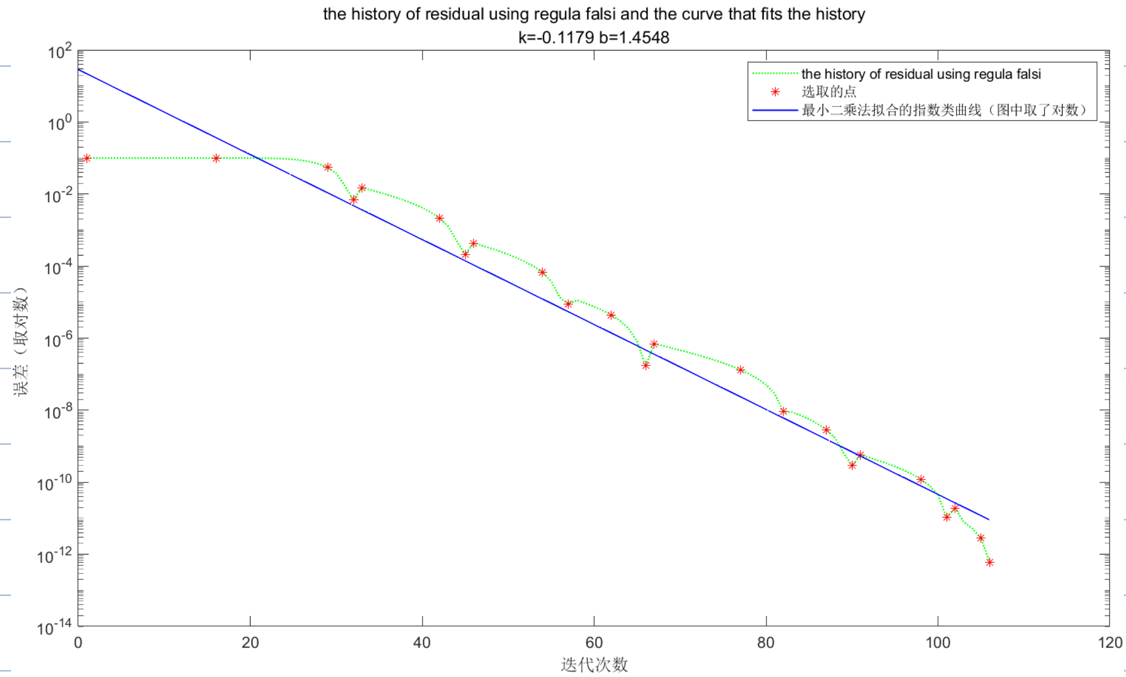
对选取的点作变换：

$$(x_i, y_i) \rightarrow (x_i, \log_{10} y_i) = (x_i, z_i)$$

对 (x_i, z_i) 进行最小2乘直线拟合之后得到如下结果：



$$\Rightarrow y = 10^{-0.29276x + 0.082934}$$



$$\Rightarrow \quad y = 10^{-0.1179x + 1.4548}$$

4. (optional) Launch an image processing program (e.g., `mspaint` on Windows). Open your left hand naturally, and put it on the computer screen. Then use the mouse to sketch the outline of your hand. (A few discrete points already suffice.) Use the technique from Exercise 2 to reconstruct the outline of your hand with (piecewise) algebraic curves. Visualize the result.

解：对手的重构如下：