DATA130004: Homework 6

Due via eLearning at 23:59 on November 20, 2023

1. Rizzo book (1st edition) Exercises 7.1, 7.4, 7.6, 7.7 and 7.8.

2. Given a set of numbers $X_1, \ldots, X_N$ of size N. Denote the sample mean and sample standard deviation by $\bar{X}$ and S, where

$$\bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_i, \qquad S = \left\{ \frac{1}{N} \sum_{i=1}^{N} (X_i - \bar{X})^2 \right\}^{1/2}.$$

   (a) A sample $x_1, \ldots, x_n$ of size n is selected from $X_1, \ldots, X_N$ by random sampling with replacement. The standard deviation of the sample average $\bar{x} = \sum_{i=1}^{n} x_i/n$ is called *the standard error* of $\bar{x}$, denoted by $\mathrm{se}(\bar{x})$. Show that $\mathrm{se}(\bar{x}) = S/\sqrt{n}$.

   (b) Suppose $n < N$ and $x_1, \ldots, x_n$ is selected by random sampling *without* replacement, show that
   $$\mathrm{se}(\bar{x}) = \frac{S}{\sqrt{n}} \left(\frac{N-n}{N-1} \right)^{1/2}.$$

   (c) A standard Bootstrap sample is obtained in (a) by further assuming $n = N$. Given $X_1, \ldots, X_N$ are distinct with each other, what is the number of distinct bootstrap samples?