To Cross or Not to Cross the Spread: *That* is the Question

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Aggressive vs Passive Orders

Aggressive orders cross the bid-ask spread. Market orders can only be aggressive (outside an auction). Limit orders can passive or aggressive. Typically aggressive orders are immediate or cancel (IOC). An example of an aggressive limit order is one where a limit price is greater than (less than) the best price on the corresponding side of the limit order book (LOB). A passive order is an order which adds to the depth of the LOB and can only be a limit order. A "buy initiated trade" has an aggressive order submitted on the bid side, and vice versa.

STOXX 600 Data Processing

Index members are equally divided into three market cap groups: small, mid and large. All trades that happen within a 100 μs interval on the same side are aggregated. This addresses the issue of multiple trades reacting to the same state of the LOB. The aggregated trades are then matched with the state of the LOB before the first trade to form pairs $\Theta(t)$.

LOB Imbalance

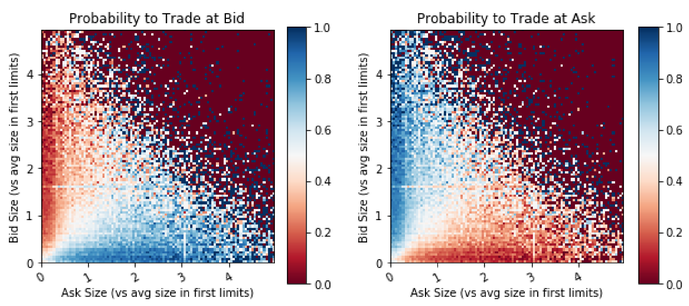
We build on [1] which found LOB imbalances allow for reliable forecasts regarding the side and size of the next trade. On average, aggressive traders will mainly target the side with the smallest size and also consume a larger percentage share when the smaller limit is hit rather than the larger limit. This insight in respect to the next trade enables estimation of the passive posting risk.
LOB imbalance $OBI(t)$ is calculated by,

$$OBI(t) = \frac{V^b(t) - V^a(t)}{V^b(t) + V^a(t)}$$

LOB Shape Drives the Side & Size of Next Trade

We calculate empirical probabilities of the side of the next trade by binning observations $\Theta(t)$ across the relative first bid and ask sizes, which are defined as the proportions of the bid/ask limit to the average inside size over the period (daily average)

$$V^{b,a}_{adj}(t) = \frac{V^{b,a}(t)}{\frac{1}{2}avg(V^b(\tau) + V^a(\tau))}$$



The effect of $OBI(t)$ on the probability of the trade occurring on a given side is roughly linear.

$p(\text{Side of Next Trade} \mid \mathbf{\Theta})$

$p(\text{Side of Next Trade} \mid OBI(t))$

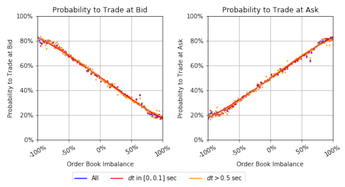
$p(\text{Side of Next Trade} \mid OBI(t), \text{mkt. cap.})$

$p(\text{Side of Next Trade} \mid OBI(t), \text{mkt. cap.})$

	Ticker	Name	Bid Size	Ask Size	Order Book Imbalance	Probability to Trade at Bid	Probability to Trade at Ask
0	ULVR LN	UNILVER	10	20000	-100.0	83.0	17.0
1	ULVR LN	UNILVER	1200	4800	-60.0	72.0	28.0
2	ULVR LN	UNILVER	1200	1200	0.0	50.0	50.0
3	ULVR LN	UNILVER	4800	1200	60.0	28.0	72.0
4	ULVR LN	UNILVER	20000	10	100.0	15.0	85.0

A practical example for a large cap stock highlights the results

$p(\text{Side of Next Trade} \mid \text{time to next agg. trade})$



The side of the next trade is independent from the duration to the next trade.

$p(\text{Size of Next Trade} \mid \mathbf{\Theta})$

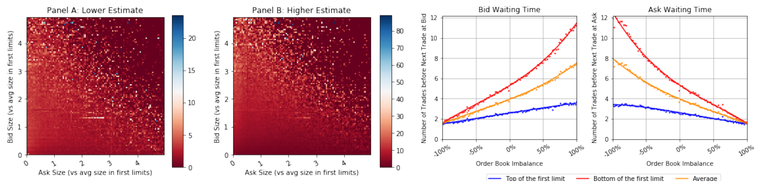
$p(\text{Size of Next Trade} \mid OBI(t)$

$p(\%$ of price level matched $\mid OBI(t))$

	Ticker	Name	Bid Size	Ask Size	Order Book Imbalance	Pct of Bid Limit Consumed if Bid is Hit	Pct of Ask Limit Consumed if Ask is Hit
0	ULVR LN	UNILVER	10	20000	-100.0	96.0	53.0
1	ULVR LN	UNILVER	1200	4800	-60.0	83.0	60.0
2	ULVR LN	UNILVER	1200	1200	0.0	69.0	70.0
3	ULVR LN	UNILVER	4800	1200	60.0	59.0	83.0
4	ULVR LN	UNILVER	20000	10	100.0	55.0	97.0

Passive Waiting Time in Number of Trades

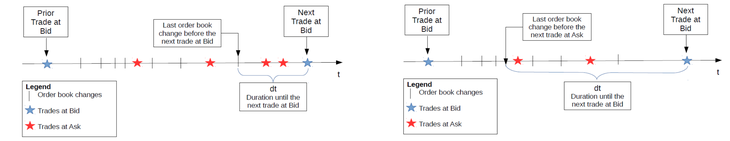
The risk incurred by posting a passive order at the inside of the LOB is estimated using the expected number of trades before the passive order is executed. Two scenarios provide lower and upper bounds for risk estimate. The lower (higher) estimate occurs when the passive order is at the top (bottom) of the first limit price queue.



In the heatmap; bucket trade count observations by normalized bid/ask sizes and show mean wait time in number of trades. In the line plots; expected number of trades until the next execution can be described using only $OBI(t)$ without considering actual bid/ask sizes, allowing estimation of risk incurred by a passive posting as a function of the current $OBI(t)$.

Execution Scenario Schematic

In this example, there are three trades until a passive order posted at first bid is executed in the optimistic scenario,



Outperforming Percentage of Volume (POV)

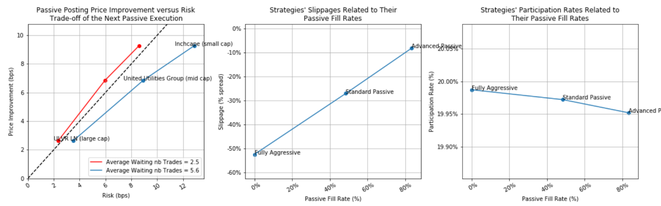
Average waiting time until the next execution is the ratio of expected number of trades before execution $C^{b,a}(t)$ to expected daily number of trades D, $AVGWT = {}^{C(t)}/_{D}$. A measure of risk R incurred by waiting for the execution of a passive order is the product of the ADV and the square root of the average waiting time until execution $R = \sigma\sqrt{AVGWT}$.
Define ratio $S(t)$ to decide when to submit passive versus aggressive orders,

- $S(t) = {}^{a(t)-b(t)}/_{R} = {}^{a(t)-b(t)}/_{\sigma\sqrt{AVGWT}}$
- If $S(t) \geq 1$, the $OBI(t)$ is favorable to a passive posting, given the level of the bid-ask spread. Else unfavorable so cross the spread.

Define three variants of the POV execution strategy,

- Fully aggressive - aggressive order sent when lag between real and target POV participation.
- Standard passive - passive orders sent with sizes equal to the lag of the POV, unless the lag exceeds N median trade sizes then send aggressive orders.
- Advanced passive - as above except that orders are not sent aggressively if their passive execution probability $> 30\%$ as per volume imbalance model.

Three measures of performance are used to test the above (1) Slippage as % of bid-ask spread (2) Passive fill rate (3) Tracking of the participation rate. Results are as follows,



This shows more passive algorithms have smaller slippage. Although the advanced passive posting strategy captures the spread more often, it may not track the participation rate closely as a passive posting so inducing execution risk.

Conclusions, Future Work & References

- Results are similar to [1] except that we find waiting time is higher in the pessimistic scenario. In future work we plan to;
- Use a wider set of LOB states, not just prior to a trade.
- Look at the return of a passive order, not just the risk of a passive order.
- Measure passive waiting time directly (μs), not just indirectly (number of trades).

[1] Paul Besson, Stéphanie Pelin, and Matthieu Lasnier. To cross or not to cross the spread: That is the question. *The Journal of Trading*, 11(4):77–91, 2016.