
From DPLL to CDCL SAT solvers

Albert Oliveras and Enric Rodríguez-Carbonell

Logic and Algebra in Computer Science

Session 4

Fall 2009, Barcelona



Overview of the session

• Conflict Analysis

- Motivating example
 - Backjumping
 - Conflict graph
 - Lemma shortening
-
- Lemma removal
 - Decision heuristics
 - Restarts
 - Efficient implementation of UnitProp:
 - Occur lists
 - Two-watched literals

• Final remarks

Motivating Example

$\emptyset \Rightarrow$

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$



Motivating Example

$$\emptyset \implies$$

$$p_{25} \implies$$

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$



Motivating Example

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

$$\emptyset \implies$$

$$p_{25} \implies$$

$$p_{25} \bar{p}_{21}^d \implies$$



Motivating Example

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

$$\emptyset \implies$$

$$p_{25} \implies$$

$$p_{25} \bar{p}_{21}^d \implies$$

$$p_{25} \bar{p}_{21}^d \bar{p}_6 \implies$$



Motivating Example

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

$$\emptyset \implies$$

$$p_{25} \implies$$

$$p_{25} \bar{p}_{21}^d \implies$$

$$p_{25} \bar{p}_{21}^d \bar{p}_6 \implies$$

$$p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} \implies$$



Motivating Example

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

$$\emptyset \implies$$

$$p_{25} \implies$$

$$p_{25} \bar{p}_{21}^d \implies$$

$$p_{25} \bar{p}_{21}^d \bar{p}_6 \implies$$

$$p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} \implies$$

$$p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \implies$$



Motivating Example

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

$$\emptyset \implies$$

$$p_{25} \implies$$

$$p_{25} \bar{p}_{21}^d \implies$$

$$p_{25} \bar{p}_{21}^d \bar{p}_6 \implies$$

$$p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} \implies$$

$$p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \implies$$

$$p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} \implies$$



Motivating Example

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

$$\emptyset \implies$$

$$p_{25} \implies$$

$$p_{25} \bar{p}_{21}^d \implies$$

$$p_{25} \bar{p}_{21}^d \bar{p}_6 \implies$$

$$p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} \implies$$

$$p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \implies$$

$$p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} \implies$$

$$p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 \implies$$



Motivating Example

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

$$\emptyset \implies$$

$$p_{25} \implies$$

$$p_{25} \bar{p}_{21}^d \implies$$

$$p_{25} \bar{p}_{21}^d \bar{p}_6 \implies$$

$$p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} \implies$$

$$p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \implies$$

$$p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} \implies$$

$$p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 \implies$$

$$p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d \implies$$



Motivating Example

$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
 $\bar{p}_{11} \vee p_{13} \vee p_{16}$
 $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
 $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
 $p_{10} \vee \bar{p}_8 \vee p_1$
 $p_{10} \vee p_3$
 $\bar{p}_3 \vee p_{26}$
 $p_{10} \vee \bar{p}_5$
 $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
 $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
 $p_{21} \vee \bar{p}_6$
 $p_{21} \vee \bar{p}_{17}$
 $\bar{p}_{22} \vee \bar{p}_{13}$
 $p_{13} \vee p_8$
 $\bar{p}_4 \vee p_{19}$
 $p_{20} \vee p_{23}$
 $\bar{p}_{20} \vee p_{24}$
 p_{25}

$\emptyset \implies$
 $p_{25} \implies$
 $p_{25} \bar{p}_{21}^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \implies$



Motivating Example

$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
 $\bar{p}_{11} \vee p_{13} \vee p_{16}$
 $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
 $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
 $p_{10} \vee \bar{p}_8 \vee p_1$
 $p_{10} \vee p_3$
 $\bar{p}_3 \vee p_{26}$
 $p_{10} \vee \bar{p}_5$
 $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
 $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
 $p_{21} \vee \bar{p}_6$
 $p_{21} \vee \bar{p}_{17}$
 $\bar{p}_{22} \vee \bar{p}_{13}$
 $p_{13} \vee p_8$
 $\bar{p}_4 \vee p_{19}$
 $p_{20} \vee p_{23}$
 $\bar{p}_{20} \vee p_{24}$
 p_{25}

$\emptyset \implies$
 $p_{25} \implies$
 $p_{25} \bar{p}_{21}^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d \implies$



Motivating Example

$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
 $\bar{p}_{11} \vee p_{13} \vee p_{16}$
 $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
 $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
 $p_{10} \vee \bar{p}_8 \vee p_1$
 $p_{10} \vee p_3$
 $\bar{p}_3 \vee p_{26}$
 $p_{10} \vee \bar{p}_5$
 $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
 $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
 $p_{21} \vee \bar{p}_6$
 $p_{21} \vee \bar{p}_{17}$
 $\bar{p}_{22} \vee \bar{p}_{13}$
 $p_{13} \vee p_8$
 $\bar{p}_4 \vee p_{19}$
 $p_{20} \vee p_{23}$
 $\bar{p}_{20} \vee p_{24}$

 p_{25}

$\emptyset \implies$
 $p_{25} \implies$
 $p_{25} \bar{p}_{21}^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23} \implies$



Motivating Example

$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
 $\bar{p}_{11} \vee p_{13} \vee p_{16}$
 $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
 $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
 $p_{10} \vee \bar{p}_8 \vee p_1$
 $p_{10} \vee p_3$
 $\bar{p}_3 \vee p_{26}$
 $p_{10} \vee \bar{p}_5$
 $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
 $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
 $p_{21} \vee \bar{p}_6$
 $p_{21} \vee \bar{p}_{17}$
 $\bar{p}_{22} \vee \bar{p}_{13}$
 $p_{13} \vee p_8$
 $\bar{p}_4 \vee p_{19}$
 $p_{20} \vee p_{23}$
 $\bar{p}_{20} \vee p_{24}$
 p_{25}

$\emptyset \implies$
 $p_{25} \implies$
 $p_{25} \bar{p}_{21}^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23} \implies$
 $\underbrace{p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}}_M p_{11}^d \implies$



Motivating Example

$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
 $\bar{p}_{11} \vee p_{13} \vee p_{16}$
 $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
 $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
 $p_{10} \vee \bar{p}_8 \vee p_1$
 $p_{10} \vee p_3$
 $\bar{p}_3 \vee p_{26}$
 $p_{10} \vee \bar{p}_5$
 $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
 $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
 $p_{21} \vee \bar{p}_6$
 $p_{21} \vee \bar{p}_{17}$
 $\bar{p}_{22} \vee \bar{p}_{13}$
 $p_{13} \vee p_8$
 $\bar{p}_4 \vee p_{19}$
 $p_{20} \vee p_{23}$
 $\bar{p}_{20} \vee p_{24}$
 p_{25}

$\emptyset \implies$
 $p_{25} \implies$
 $p_{25} \bar{p}_{21}^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23} \implies$
 $\underbrace{p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}}_M p_{11}^d \implies$
 $M p_{11}^d \implies$



Motivating Example

$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
 $\bar{p}_{11} \vee p_{13} \vee p_{16}$
 $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
 $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
 $p_{10} \vee \bar{p}_8 \vee p_1$
 $p_{10} \vee p_3$
 $\bar{p}_3 \vee p_{26}$
 $p_{10} \vee \bar{p}_5$
 $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
 $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
 $p_{21} \vee \bar{p}_6$
 $p_{21} \vee \bar{p}_{17}$
 $\bar{p}_{22} \vee \bar{p}_{13}$
 $p_{13} \vee p_8$
 $\bar{p}_4 \vee p_{19}$
 $p_{20} \vee p_{23}$
 $\bar{p}_{20} \vee p_{24}$
 p_{25}

$\emptyset \implies$
 $p_{25} \implies$
 $p_{25} \bar{p}_{21}^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d \implies$
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23} \implies$
 $\underbrace{p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}}_M p_{11}^d \implies$
 $M p_{11}^d \implies$

Before we continue...Some notation:

Literal p_{25} belongs to **decision level 0**

Literals $\bar{p}_{21}^d \bar{p}_6 \bar{p}_{17}$ belong to **decision level 1**

...



Motivating Example (2)

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$



Motivating Example (2)

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$$M p_{11}^d \implies$$



Motivating Example (2)

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$$M p_{11}^d \implies$$

$$M p_{11}^d \bar{p}_{12} \implies$$



Motivating Example (2)

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$$M p_{11}^d \implies$$

$$M p_{11}^d \bar{p}_{12} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \implies$$



Motivating Example (2)

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$$M p_{11}^d \implies$$

$$M p_{11}^d \bar{p}_{12} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \implies$$



Motivating Example (2)

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$$M p_{11}^d \implies$$

$$M p_{11}^d \bar{p}_{12} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} \implies$$



Motivating Example (2)

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$$M p_{11}^d \implies$$

$$M p_{11}^d \bar{p}_{12} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 \implies$$



Motivating Example (2)

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$$M p_{11}^d \implies$$

$$M p_{11}^d \bar{p}_{12} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 \implies$$



Motivating Example (2)

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$$M p_{11}^d \implies$$

$$M p_{11}^d \bar{p}_{12} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \implies$$



Motivating Example (2)

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$$M p_{11}^d \implies$$

$$M p_{11}^d \bar{p}_{12} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \bar{p}_5 \implies$$



Motivating Example (2)

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$$M p_{11}^d \implies$$

$$M p_{11}^d \bar{p}_{12} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \bar{p}_5 \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \bar{p}_5 p_{18} \implies$$



Motivating Example (2)

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$$M p_{11}^d \implies$$

$$M p_{11}^d \bar{p}_{12} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \bar{p}_5 \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \bar{p}_5 p_{18} \implies$$

fail



Motivating Example (2)

$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
 $\bar{p}_{11} \vee p_{13} \vee p_{16}$
 $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
 $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
 $p_{10} \vee \bar{p}_8 \vee p_1$
 $p_{10} \vee p_3$
 $\bar{p}_3 \vee p_{26}$
 $p_{10} \vee \bar{p}_5$
 $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
 $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
 $p_{21} \vee \bar{p}_6$
 $p_{21} \vee \bar{p}_{17}$
 $\bar{p}_{22} \vee \bar{p}_{13}$
 $p_{13} \vee p_8$
 $\bar{p}_4 \vee p_{19}$
 $p_{20} \vee p_{23}$
 $\bar{p}_{20} \vee p_{24}$
 p_{25}

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$M p_{11}^d \implies$
 $M p_{11}^d \bar{p}_{12} \implies$
 $M p_{11}^d \bar{p}_{12} p_{16} \implies$
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \implies$
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} \implies$
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 \implies$
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 \implies$
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \implies$
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \bar{p}_5 \implies$
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \bar{p}_5 p_{18} \implies$
fail

- Let's try to find out the causes of the conflict
- First of all we will compute, for each literal, the reason why it is true

Motivating Example (3)

1. $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2. $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4. $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5. $p_{10} \vee \bar{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\bar{p}_3 \vee p_{26}$
8. $p_{10} \vee \bar{p}_5$
9. $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11. $p_{21} \vee \bar{p}_6$
12. $p_{21} \vee \bar{p}_{17}$
13. $\bar{p}_{22} \vee \bar{p}_{13}$
14. $p_{13} \vee p_8$
15. $\bar{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\bar{p}_{20} \vee p_{24}$
18. p_{25}

Motivating Example (3)

1. $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2. $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4. $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5. $p_{10} \vee \bar{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\bar{p}_3 \vee p_{26}$
8. $p_{10} \vee \bar{p}_5$
9. $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11. $p_{21} \vee \bar{p}_6$
12. $p_{21} \vee \bar{p}_{17}$
13. $\bar{p}_{22} \vee \bar{p}_{13}$
14. $p_{13} \vee p_8$
15. $\bar{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\bar{p}_{20} \vee p_{24}$
18. p_{25}

Literal:	p_{11}^d	\bar{p}_{12}	p_{16}	\bar{p}_2	\bar{p}_{10}	p_1	p_3	p_{26}	\bar{p}_5	p_{18}
Reason:	\emptyset	1	2	3	4	5	6	7	8	9

Motivating Example (3)

1. $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2. $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4. $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5. $p_{10} \vee \bar{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\bar{p}_3 \vee p_{26}$
8. $p_{10} \vee \bar{p}_5$
9. $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11. $p_{21} \vee \bar{p}_6$
12. $p_{21} \vee \bar{p}_{17}$
13. $\bar{p}_{22} \vee \bar{p}_{13}$
14. $p_{13} \vee p_8$
15. $\bar{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\bar{p}_{20} \vee p_{24}$
18. p_{25}

Literal:	p_{11}^d	\bar{p}_{12}	p_{16}	\bar{p}_2	\bar{p}_{10}	p_1	p_3	p_{26}	\bar{p}_5	p_{18}
Reason:	\emptyset	1	2	3	4	5	6	7	8	9

Let us take the **conflicting** clause $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$.

The reason why p_{18} is true is clause 9. Resolution gives:

$$\frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17}}$$

Motivating Example (3)

1. $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2. $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4. $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5. $p_{10} \vee \bar{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\bar{p}_3 \vee p_{26}$
8. $p_{10} \vee \bar{p}_5$
9. $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11. $p_{21} \vee \bar{p}_6$
12. $p_{21} \vee \bar{p}_{17}$
13. $\bar{p}_{22} \vee \bar{p}_{13}$
14. $p_{13} \vee p_8$
15. $\bar{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\bar{p}_{20} \vee p_{24}$
18. p_{25}

Literal:	p_{11}^d	\bar{p}_{12}	p_{16}	\bar{p}_2	\bar{p}_{10}	p_1	p_3	p_{26}	\bar{p}_5	p_{18}
Reason:	\emptyset	1	2	3	4	5	6	7	8	9

Let us take the **conflicting** clause $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$.
 The reason why p_{18} is true is clause 9. Resolution gives:

$$\frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17}}$$

Now, last assigned lit false in the resulting clause is p_5 .
 The reason why p_5 is false is clause 8. Again, resolution:

$$\frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5}{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10}}$$

Motivating Example (3)

1. $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2. $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4. $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5. $p_{10} \vee \bar{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\bar{p}_3 \vee p_{26}$
8. $p_{10} \vee \bar{p}_5$
9. $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11. $p_{21} \vee \bar{p}_6$
12. $p_{21} \vee \bar{p}_{17}$
13. $\bar{p}_{22} \vee \bar{p}_{13}$
14. $p_{13} \vee p_8$
15. $\bar{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\bar{p}_{20} \vee p_{24}$
18. p_{25}

Literal:	p_{11}^d	\bar{p}_{12}	p_{16}	\bar{p}_2	\bar{p}_{10}	p_1	p_3	p_{26}	\bar{p}_5	p_{18}
Reason:	\emptyset	1	2	3	4	5	6	7	8	9

Let us take the **conflicting** clause $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$.
The reason why p_{18} is true is clause 9. Resolution gives:

$$\frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17}}$$

Now, last assigned lit false in the resulting clause is p_5 .
The reason why p_5 is false is clause 8. Again, resolution:

$$\frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5}{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10}}$$

The process is now iterated...

Motivating Example (4)

1. $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2. $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4. $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5. $p_{10} \vee \bar{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\bar{p}_3 \vee p_{26}$
8. $p_{10} \vee \bar{p}_5$
9. $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11. $p_{21} \vee \bar{p}_6$
12. $p_{21} \vee \bar{p}_{17}$
13. $\bar{p}_{22} \vee \bar{p}_{13}$
14. $p_{13} \vee p_8$
15. $\bar{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\bar{p}_{20} \vee p_{24}$
18. p_{25}

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

Literal:	p_{11}^d	\bar{p}_{12}	p_{16}	\bar{p}_2	\bar{p}_{10}	p_1	p_3	p_{26}	\bar{p}_5	p_{18}
Reason:	0	1	2	3	4	5	6	7	8	9

Motivating Example (4)

1. $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2. $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4. $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5. $p_{10} \vee \bar{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\bar{p}_3 \vee p_{26}$
8. $p_{10} \vee \bar{p}_5$
9. $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11. $p_{21} \vee \bar{p}_6$
12. $p_{21} \vee \bar{p}_{17}$
13. $\bar{p}_{22} \vee \bar{p}_{13}$
14. $p_{13} \vee p_8$
15. $\bar{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\bar{p}_{20} \vee p_{24}$
18. p_{25}

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

Literal:	p_{11}^d	\bar{p}_{12}	p_{16}	\bar{p}_2	\bar{p}_{10}	p_1	p_3	p_{26}	\bar{p}_5	p_{18}
Reason:	0	1	2	3	4	5	6	7	8	9

$$\begin{array}{rcl}
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} & \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18} & \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} & & p_{10} \vee \bar{p}_5 \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} & &
 \end{array}$$

Motivating Example (4)

1. $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2. $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4. $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5. $p_{10} \vee \bar{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\bar{p}_3 \vee p_{26}$
8. $p_{10} \vee \bar{p}_5$
9. $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11. $p_{21} \vee \bar{p}_6$
12. $p_{21} \vee \bar{p}_{17}$
13. $\bar{p}_{22} \vee \bar{p}_{13}$
14. $p_{13} \vee p_8$
15. $\bar{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\bar{p}_{20} \vee p_{24}$
18. p_{25}

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

Literal:	p_{11}^d	\bar{p}_{12}	p_{16}	\bar{p}_2	\bar{p}_{10}	p_1	p_3	p_{26}	\bar{p}_5	p_{18}
Reason:	\emptyset	1	2	3	4	5	6	7	8	9

$$\begin{array}{c}
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18} \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5 \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee p_3 \\
 \hline
 \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10}
 \end{array}$$

Motivating Example (4)

1. $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2. $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4. $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5. $p_{10} \vee \bar{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\bar{p}_3 \vee p_{26}$
8. $p_{10} \vee \bar{p}_5$
9. $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11. $p_{21} \vee \bar{p}_6$
12. $p_{21} \vee \bar{p}_{17}$
13. $\bar{p}_{22} \vee \bar{p}_{13}$
14. $p_{13} \vee p_8$
15. $\bar{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\bar{p}_{20} \vee p_{24}$
18. p_{25}

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

Literal:	p_{11}^d	\bar{p}_{12}	p_{16}	\bar{p}_2	\bar{p}_{10}	p_1	p_3	p_{26}	\bar{p}_5	p_{18}
Reason:	0	1	2	3	4	5	6	7	8	9

$$\begin{array}{c}
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18} \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5 \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee p_3 \\
 \hline
 \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10}
 \end{array}$$

$$\begin{array}{c}
 \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee \bar{p}_8 \vee p_1 \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee p_{10} \vee \bar{p}_8
 \end{array}$$



Motivating Example (4)

1. $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2. $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4. $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5. $p_{10} \vee \bar{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\bar{p}_3 \vee p_{26}$
8. $p_{10} \vee \bar{p}_5$
9. $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11. $p_{21} \vee \bar{p}_6$
12. $p_{21} \vee \bar{p}_{17}$
13. $\bar{p}_{22} \vee \bar{p}_{13}$
14. $p_{13} \vee p_8$
15. $\bar{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\bar{p}_{20} \vee p_{24}$
18. p_{25}

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

Literal:	p_{11}^d	\bar{p}_{12}	p_{16}	\bar{p}_2	\bar{p}_{10}	p_1	p_3	p_{26}	\bar{p}_5	p_{18}
Reason:	0	1	2	3	4	5	6	7	8	9

$$\begin{array}{r}
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18} \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5 \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee p_3 \\
 \hline
 \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10}
 \end{array}$$

$$\begin{array}{r}
 \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee \bar{p}_8 \vee p_1 \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee p_{10} \vee \bar{p}_8 \quad p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10} \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_2 \vee \bar{p}_4 \vee p_{20}
 \end{array}$$

Motivating Example (4)

1. $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2. $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4. $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5. $p_{10} \vee \bar{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\bar{p}_3 \vee p_{26}$
8. $p_{10} \vee \bar{p}_5$
9. $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11. $p_{21} \vee \bar{p}_6$
12. $p_{21} \vee \bar{p}_{17}$
13. $\bar{p}_{22} \vee \bar{p}_{13}$
14. $p_{13} \vee p_8$
15. $\bar{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\bar{p}_{20} \vee p_{24}$
18. p_{25}

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

Literal:	p_{11}^d	\bar{p}_{12}	p_{16}	\bar{p}_2	\bar{p}_{10}	p_1	p_3	p_{26}	\bar{p}_5	p_{18}
Reason:	0	1	2	3	4	5	6	7	8	9

$$\begin{array}{c}
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18} \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5 \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee p_3 \\
 \hline
 \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10}
 \end{array}$$

$$\begin{array}{c}
 \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee \bar{p}_8 \vee p_1 \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee p_{10} \vee \bar{p}_8 \quad p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10} \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_2 \vee \bar{p}_4 \vee p_{20} \quad p_{12} \vee \bar{p}_{16} \vee \bar{p}_2 \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16}
 \end{array}$$

Motivating Example (4)

1. $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2. $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4. $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5. $p_{10} \vee \bar{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\bar{p}_3 \vee p_{26}$
8. $p_{10} \vee \bar{p}_5$
9. $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11. $p_{21} \vee \bar{p}_6$
12. $p_{21} \vee \bar{p}_{17}$
13. $\bar{p}_{22} \vee \bar{p}_{13}$
14. $p_{13} \vee p_8$
15. $\bar{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\bar{p}_{20} \vee p_{24}$
18. p_{25}

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

Literal:	p_{11}^d	\bar{p}_{12}	p_{16}	\bar{p}_2	\bar{p}_{10}	p_1	p_3	p_{26}	\bar{p}_5	p_{18}
Reason:	0	1	2	3	4	5	6	7	8	9

$$\begin{array}{c}
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18} \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5 \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee p_3 \\
 \hline
 \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \\
 \\
 \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee \bar{p}_8 \vee p_1 \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee p_{10} \vee \bar{p}_8 \quad p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10} \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_2 \vee \bar{p}_4 \vee p_{20} \quad p_{12} \vee \bar{p}_{16} \vee \bar{p}_2 \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16} \\
 \\
 \bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16} \quad \bar{p}_{11} \vee p_{13} \vee p_{16} \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{11} \vee p_{13}
 \end{array}$$

Motivating Example (4)

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

1. $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2. $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4. $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5. $p_{10} \vee \bar{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\bar{p}_3 \vee p_{26}$
8. $p_{10} \vee \bar{p}_5$
9. $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11. $p_{21} \vee \bar{p}_6$
12. $p_{21} \vee \bar{p}_{17}$
13. $\bar{p}_{22} \vee \bar{p}_{13}$
14. $p_{13} \vee p_8$
15. $\bar{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\bar{p}_{20} \vee p_{24}$
18. p_{25}

Literal:	p_{11}^d	\bar{p}_{12}	p_{16}	\bar{p}_2	\bar{p}_{10}	p_1	p_3	p_{26}	\bar{p}_5	p_{18}
Reason:	0	1	2	3	4	5	6	7	8	9

$$\begin{array}{l}
 \frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5} \\
 \frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5}{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee p_3} \\
 \frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee p_3}{\bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10}} \\
 \\
 \frac{\bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee \bar{p}_8 \vee p_1}{\bar{p}_{19} \vee p_{17} \vee p_{10} \vee \bar{p}_8 \quad p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}} \\
 \frac{\bar{p}_{19} \vee p_{17} \vee p_{10} \vee \bar{p}_8 \quad p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_2 \vee \bar{p}_4 \vee p_{20} \quad p_{12} \vee \bar{p}_{16} \vee \bar{p}_2} \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_2 \vee \bar{p}_4 \vee p_{20} \quad p_{12} \vee \bar{p}_{16} \vee \bar{p}_2}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16}} \\
 \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16} \quad \bar{p}_{11} \vee p_{13} \vee p_{16}}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{11} \vee p_{13} \quad \bar{p}_{11} \vee p_6 \vee \bar{p}_{12}} \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{11} \vee p_{13} \quad \bar{p}_{11} \vee p_6 \vee \bar{p}_{12}}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{11} \vee p_{13} \vee p_6}
 \end{array}$$



Motivating Example (4)

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

1. $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2. $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4. $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5. $p_{10} \vee \bar{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\bar{p}_3 \vee p_{26}$
8. $p_{10} \vee \bar{p}_5$
9. $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11. $p_{21} \vee \bar{p}_6$
12. $p_{21} \vee \bar{p}_{17}$
13. $\bar{p}_{22} \vee \bar{p}_{13}$
14. $p_{13} \vee p_8$
15. $\bar{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\bar{p}_{20} \vee p_{24}$
18. p_{25}

Literal:	p_{11}^d	\bar{p}_{12}	p_{16}	\bar{p}_2	\bar{p}_{10}	p_1	p_3	p_{26}	\bar{p}_5	p_{18}
Reason:	0	1	2	3	4	5	6	7	8	9

$$\begin{array}{c}
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18} \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5 \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee p_3 \\
 \hline
 \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \\
 \\
 \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee \bar{p}_8 \vee p_1 \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee p_{10} \vee \bar{p}_8 \quad p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10} \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_2 \vee \bar{p}_4 \vee p_{20} \quad p_{12} \vee \bar{p}_{16} \vee \bar{p}_2 \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16} \\
 \\
 \bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16} \quad \bar{p}_{11} \vee p_{13} \vee p_{16} \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{11} \vee p_{13} \quad \bar{p}_{11} \vee p_6 \vee \bar{p}_{12} \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{11} \vee p_{13} \vee p_6
 \end{array}$$

Note that process now can't continue

Motivating Example (4)

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

1. $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2. $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4. $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5. $p_{10} \vee \bar{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\bar{p}_3 \vee p_{26}$
8. $p_{10} \vee \bar{p}_5$
9. $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11. $p_{21} \vee \bar{p}_6$
12. $p_{21} \vee \bar{p}_{17}$
13. $\bar{p}_{22} \vee \bar{p}_{13}$
14. $p_{13} \vee p_8$
15. $\bar{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\bar{p}_{20} \vee p_{24}$
18. p_{25}

Literal:	p_{11}^d	\bar{p}_{12}	p_{16}	\bar{p}_2	\bar{p}_{10}	p_1	p_3	p_{26}	\bar{p}_5	p_{18}
Reason:	0	1	2	3	4	5	6	7	8	9

$$\begin{array}{c}
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18} \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5 \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee p_3 \\
 \hline
 \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \\
 \\
 \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee \bar{p}_8 \vee p_1 \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee p_{10} \vee \bar{p}_8 \quad p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10} \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_2 \vee \bar{p}_4 \vee p_{20} \quad p_{12} \vee \bar{p}_{16} \vee \bar{p}_2 \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16} \\
 \\
 \bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16} \quad \bar{p}_{11} \vee p_{13} \vee p_{16} \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{11} \vee p_{13} \quad \bar{p}_{11} \vee p_6 \vee \bar{p}_{12} \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{11} \vee p_{13} \vee p_6
 \end{array}$$

Note also that all obtained clauses are false in assignment.

Motivating Example (4)

Remember M is $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

1. $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2. $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4. $\bar{p}_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5. $p_{10} \vee \bar{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\bar{p}_3 \vee p_{26}$
8. $p_{10} \vee \bar{p}_5$
9. $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11. $p_{21} \vee \bar{p}_6$
12. $p_{21} \vee \bar{p}_{17}$
13. $\bar{p}_{22} \vee \bar{p}_{13}$
14. $p_{13} \vee p_8$
15. $\bar{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\bar{p}_{20} \vee p_{24}$
18. p_{25}

Literal:	p_{11}^d	\bar{p}_{12}	p_{16}	\bar{p}_2	\bar{p}_{10}	p_1	p_3	p_{26}	\bar{p}_5	p_{18}
Reason:	0	1	2	3	4	5	6	7	8	9

$$\begin{array}{l}
 \frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5} \\
 \frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee p_3}{\bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10}} \\
 \\
 \frac{\bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee \bar{p}_8 \vee p_1}{\bar{p}_{19} \vee p_{17} \vee p_{10} \vee \bar{p}_8 \quad p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}} \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_2 \vee \bar{p}_4 \vee p_{20} \quad p_{12} \vee \bar{p}_{16} \vee \bar{p}_2}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16}} \\
 \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16} \quad \bar{p}_{11} \vee p_{13} \vee p_{16}}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{11} \vee p_{13} \quad \bar{p}_{11} \vee p_6 \vee \bar{p}_{12}} \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{11} \vee p_{13} \vee p_6}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{11} \vee p_{13} \vee p_6}
 \end{array}$$

Now, in blue lits false at the current decision level (5)

Motivating Example (5)

- Three clauses with only one literal assigned at the last DL (5):
 - $\bar{p}_{19} \vee p_{17} \vee p_{10} \vee \bar{p}_8$ (max DL of others:3)
 - $\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_2 \vee \bar{p}_4 \vee p_{20}$ (max DL of others:4)
 - $\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{11} \vee p_{13} \vee p_6$ (max DL of others:4)
- If we had had those clauses:
 - At DL. 3 we could've propagated p_{10}
 - At DL. 4 we could've propagated p_2
 - At DL. 4 we could've propagated \bar{p}_{11}
- In practice, procedure stops when first such clause is found, because:
 - It is the cheapest one to find
 - It can propagate lits at a lower DL

Backjump rule

This examples motivates us to introduce the rule:

Backjump

$$Ml^d N \parallel F \implies Ml' \parallel F \text{ if } \left\{ \begin{array}{l} \text{for some clause } C \vee l' : \\ F \models C \vee l' \text{ and } M \models \neg C \\ l' \text{ is undefined in } M \\ l' \text{ or } \neg l' \text{ occurs in } F \end{array} \right.$$

The only thing we need is a **backjump clause** $C \vee l'$ such that:

1. It is a logical consequence of the rest of the clauses
2. All its literals are false at some previous decision level d , except one which was undefined at d

Conflict Analysis

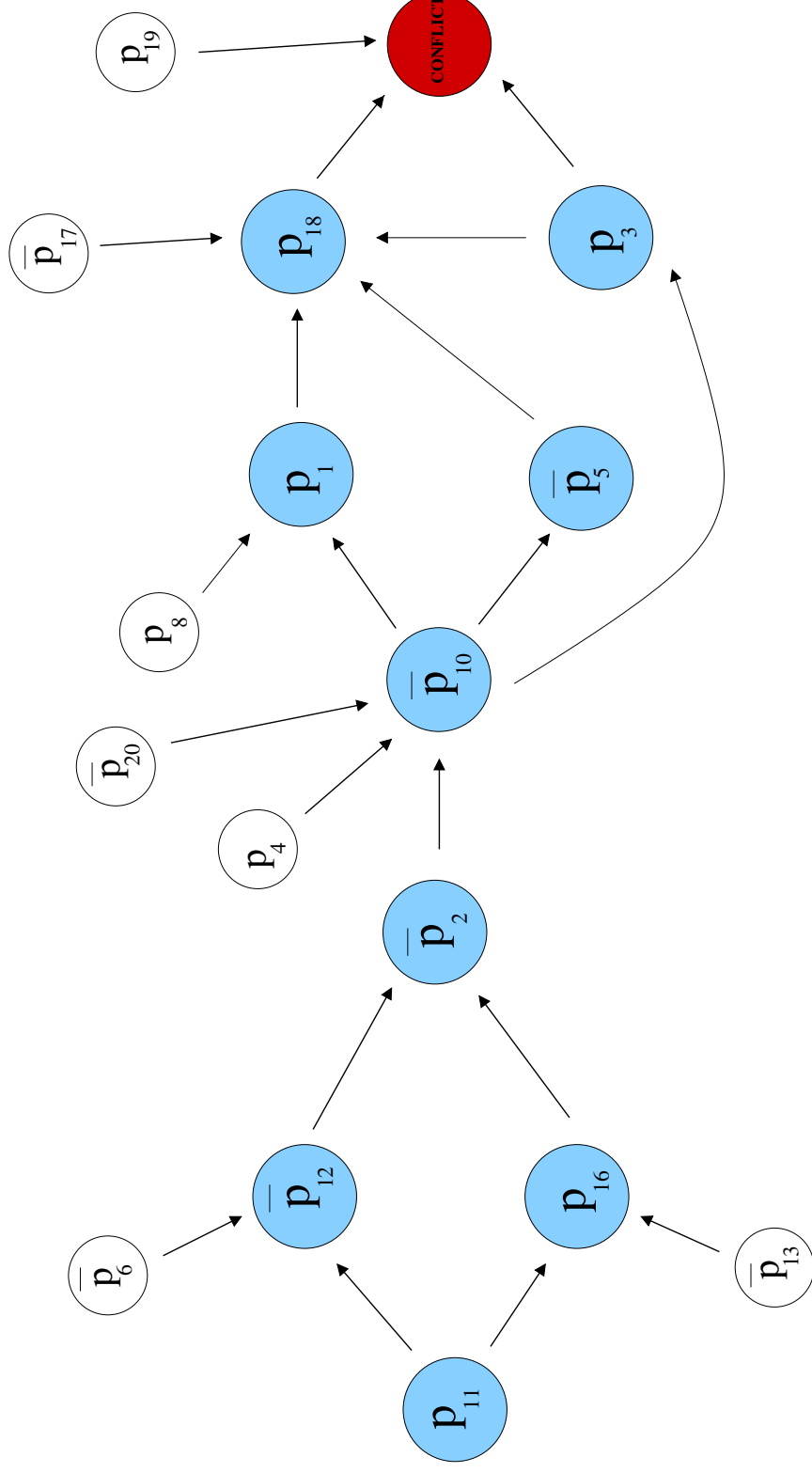
- The procedure shown in the example is called **conflict analysis**
- Why the obtained clause is a logical consequence of the input?
 - Because resolution is correct

Conflict Analysis (2)

- The procedure shown in the example is called **conflict analysis**
- Why always a **false** clause with only one lit set at the last decision level (dl) is obtained?
 - Conflicting clause has at least two lits false at dl (provided unit propagation applied before any decision)
 - Each non-decision lit l false at dl can be resolved away
 - l is replaced by lits l_1, \dots, l_n such that
 1. All of them are false
 2. All of them have been added to the assignment before l (hence their decision level is $\leq dl$)
 3. At least one was set at dl (again, provided)
 - By 3, obtained clauses contain at least one lit false at dl
 - Procedure terminates because of 2. In the worst case, with last decision lit being the only set to false at dl

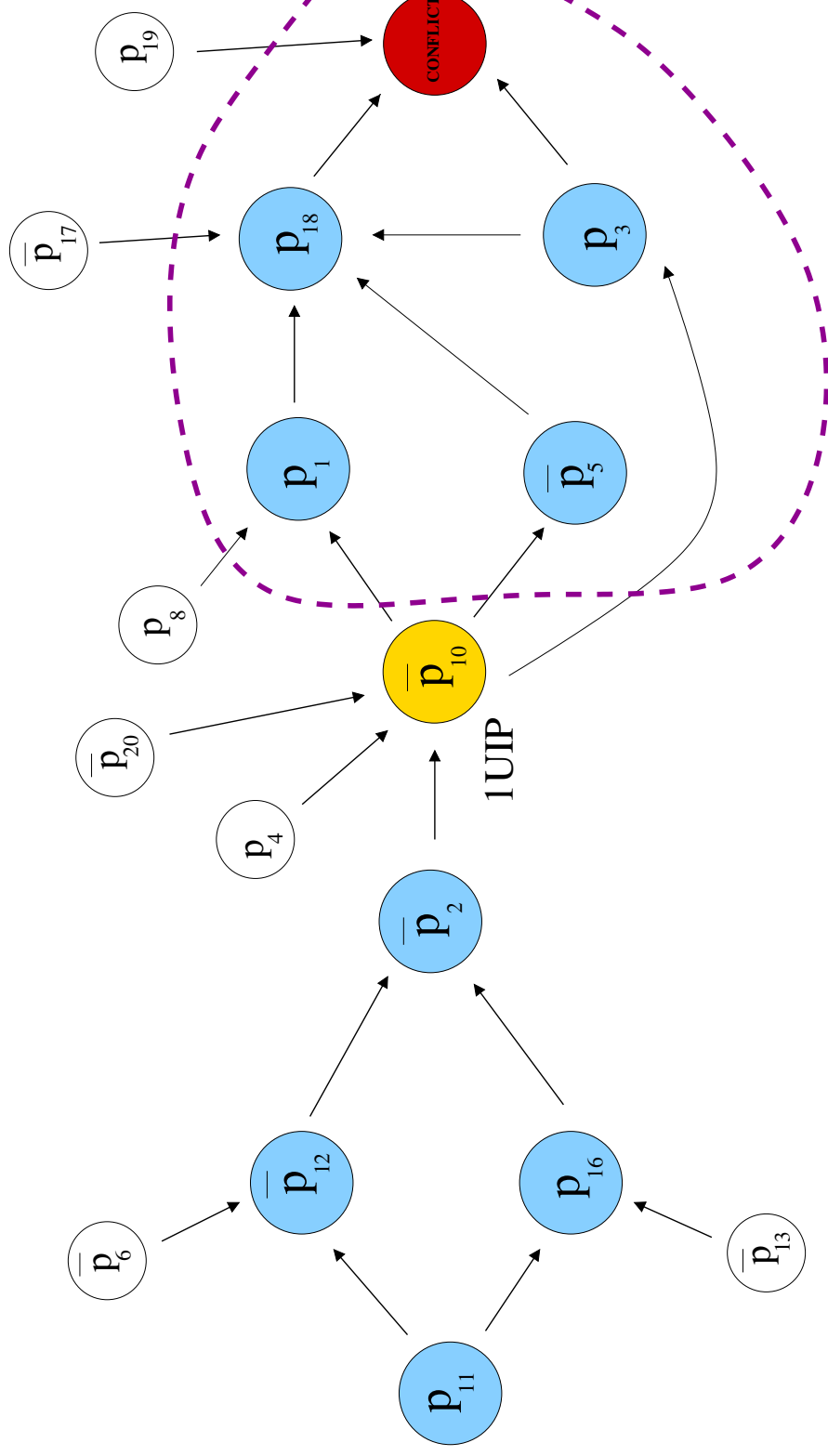
Conflict Analysis - Conflict Graph

- 🔴 Situation is best observed in the **conflict graph**:



Conflict Analysis - Conflict Graph (2)

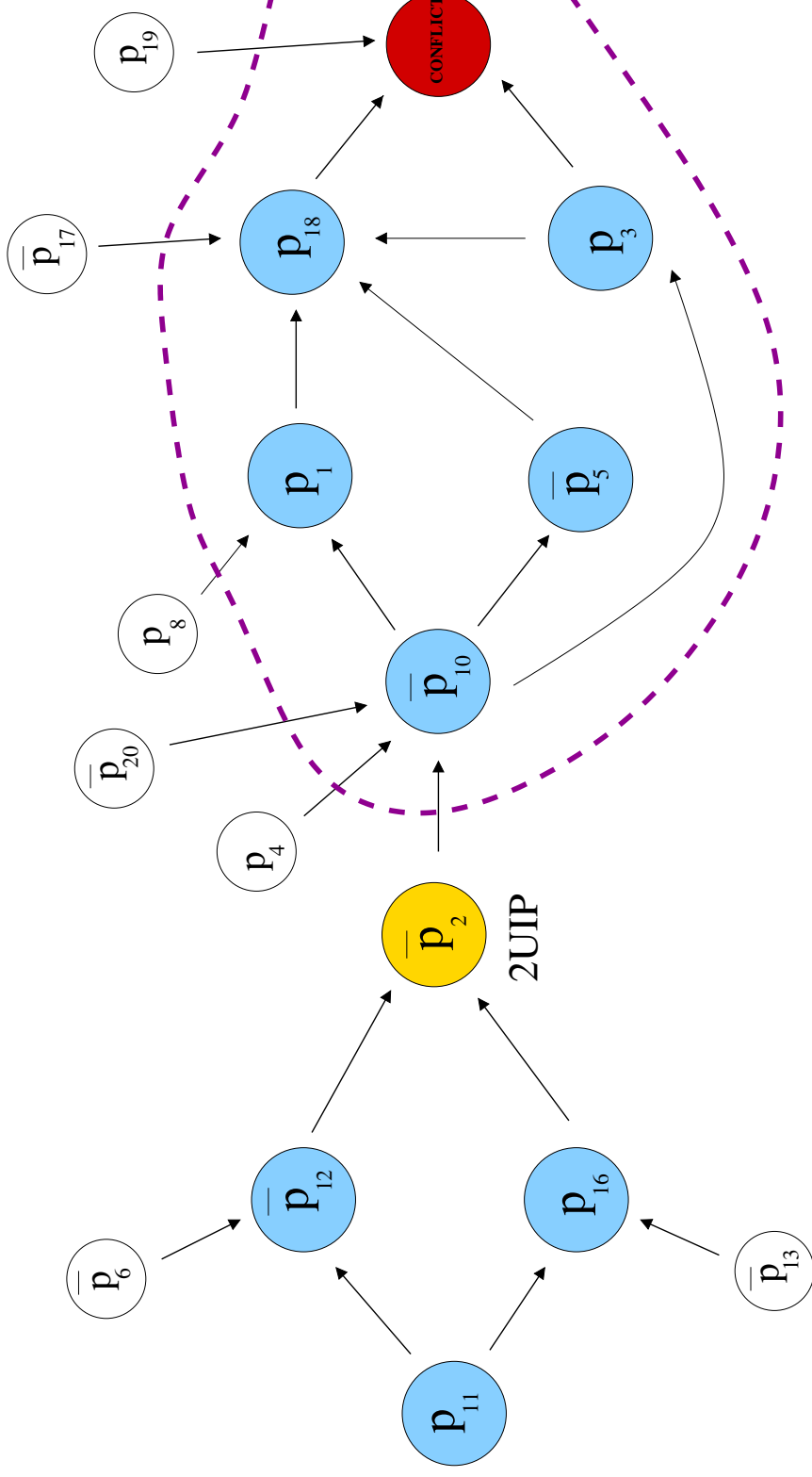
- This cut represents the 1-UIP learning scheme (Unique Implication Point)



Clause learned is $\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_{10}$

Conflict Analysis - Conflict Graph (3)

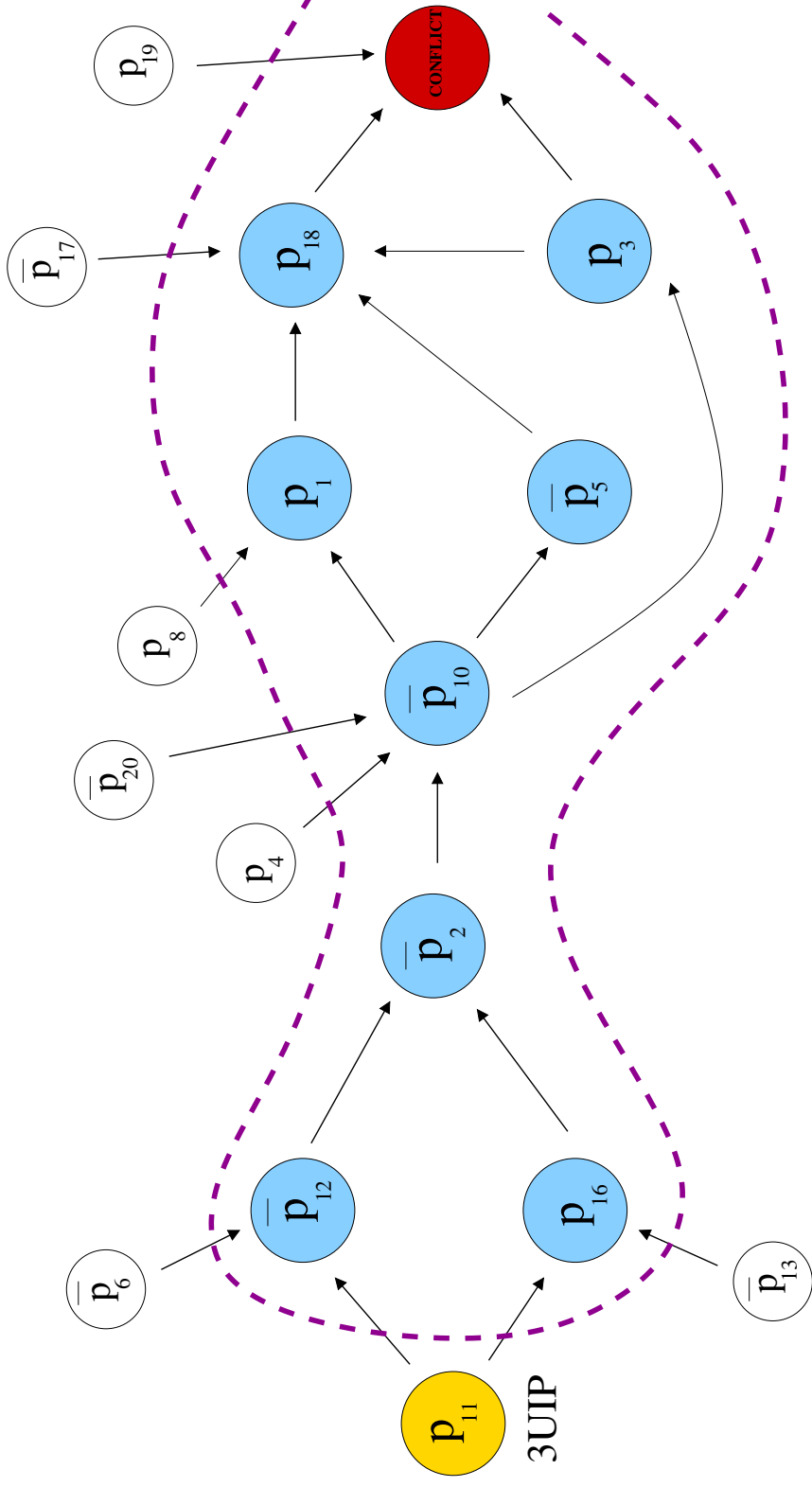
- This cut represents the 2-UIP learning scheme



- Clause learned is $\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_{20} \vee \bar{p}_4 \vee p_2$

Conflict Analysis - Conflict Graph (4)

- This cut represents the 3-UIP learning scheme



- Clause learned is $\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_{20} \vee \bar{p}_4 \vee p_6 \vee \bar{p}_{11} \vee p_{13}$