# Missing Scores

## Part 1

Allan had an aggregate mark of 24 and Ellen topped Maths, therefore he cannot get a 5 in Maths. His scores are as follows (as well as Ellen's known scores):

|         | English | History | French | Maths | Science |
|---------|---------|---------|--------|-------|---------|
| Alan    | 5       | 5       | 5      | 4     | 5       |
| Barbara |         |         |        |       |         |
| Charles |         |         |        |       |         |
| David   |         |         |        |       |         |
| Ellen   |         |         |        | 5     | 3       |

We know that Charles scored the same in 4 out of 5 subjects. If Charles scored 4 in 4 out of 5 subjects, the minimum score he could achieve would be 4, 4, 4, 1, 4 = 17. This is not possible, as the maximum score Barbara could then achieve would be 3, 3, 3, 3, 2 = 14. If Charles scored 2 in 4 out of 5 subjects. The maximum score he could achieve would be 4, 2, 2, 2, 2 = 12. This is not possible, as the next lowest possible score is 11 for David, leaving no lower score for Ellen to achieve, as her minimum possible score is also 11 (if she scored all 1s in her remaining subjects).

We can therefore list the valid permutations for Charles:

3, 3, 3, 3, 1 = 13
3, 3, 3, 3, 2 = 14
3, 3, 3, 3, 4 = 16

We know that Barbara has to score a minimum of 14 i.e. she must score higher than Charles (min 13) who must score higher than David (min 12) who must score higher than Ellen (min 11 as shown above), so she must have scored 4 in at least 2 subjects. For her other three subjects French, Maths and Science, the permutations she can score are:

4, 2, 4 = 18 (total)
4, 1, 4 = 17
4, 2, 2 = 16
4, 1, 2 = 15

2, 2, 2 = 14

If she scores 4, 4, 4, 2, 4 or 4, 4, 4, 1, 4; David's maximum score is then 2, 2, 2, 2, 2 = 10 or 2, 2, 2, 1, 2 = 9. This does not beat Ellen's minimum score of 11 so these can be eliminated. If she scores 4, 4, 2, 2, 2; then the only way David can beat Ellen is by scoring 2, 2, 4, 1, 4 = 13. This leaves only one valid permutation for Charles: 3, 3, 3, 3, 1 = 13. These aggregates are the same so this permutation can also be eliminated. The only valid permutations remaining for Barbara are 4, 4, 4, 1, 2 or 4, 4, 4, 2, 2. Our table now looks like this:

|  | English | History | French | Maths | Science |
|---|---|---|---|---|---|
| Alan | 5 | 5 | 5 | 4 | 5 |
| Barbara | 4 | 4 | 4 |  | 2 |
| Charles | 3 | 3 | 3 | 3 |  |
| David |  |  |  |  |  |
| Ellen |  |  |  | 5 | 3 |

Given the scores remaining, David has to score the maximum of what is left 2, 2, 2, 2, 4 = 12, in order to beat Ellen's minimum score of 11. With this information we can deduce the rest of the table:

|  | English | History | French | Maths | Science |
|---|---|---|---|---|---|
| Alan | 5 | 5 | 5 | 4 | 5 |
| Barbara | 4 | 4 | 4 | 1 | 2 |
| Charles | 3 | 3 | 3 | 3 | 1 |
| David | 2 | 2 | 2 | 2 | 4 |
| Ellen | 1 | 1 | 1 | 5 | 3 |

**Barbara's score for Mathematics was 1; and Alan, Charles, and David scored the same in 4 out of 5 subjects.**

# Part 2

We can start by filling in what we know about E. This will help us in the future. E cannot have won any games because they have 2 points from 2 draws. They have played a total of 4 games, 2 of which they have drawn, so they must have lost 2 as well. If they have lost 2 games from 2 goals against, each of those games must have been 0 - 1, meaning that the games they drew must have been 0 - 0, meaning they had 0 goals for. Here is E now:

| Played | Won | Lost | Drew | For | Against | Pts |
|--------|-----|------|------|-----|---------|-----|
| 4 | 0 | 2 | 2 | 0 | 2 | 2 |

Given that E has 4 games and B has 1 game, B's only game must have been against E. E scored no goals so B had 0 goals against. This gives us a total of 7 goals scored. Given that B has only played E, C can have played a maximum of 3 games achieving 6 points. This means C must have won all of their games. Here is C now:

| Played | Won | Lost | Drew | For | Against | Pts |
|--------|-----|------|------|-----|---------|-----|
| 3 | 3 | 0 | 0 | 5 | 0 | 6 |

Given that A has 4 points over a maximum of 3 games, they must have either won 2 (and possibly lost 1) or won 1 and drew 2. Each of these scenarios require team A to score at least 2 goals, so the 2 remaining 'for' goals go to A. Now we know that A and C are the only teams that scored goals, they must have both won against E, meaning that B and D both drew with E. Here is our table now:

| Team | Played | Won | Lost | Drew | For | Against | Pts |
|------|--------|-----|------|------|-----|---------|-----|
| A | | | | | 2 | 1 | 4 |
| B | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| C | 3 | 3 | 0 | 0 | 5 | 0 | 6 |
| D | | | | | 0 | 4 | |
| E | 4 | 0 | 2 | 2 | 0 | 2 | 2 |

The only other scoring team was C, so A must have lost once to C giving us the 2 wins 1 loss scenario we discovered earlier. Given that C scored 1 of their goals against E and one of their goals against A, their other three goals must have been against D. **This means that C has played D, and the score for the game was 3 - 0.**

For interest, this means that the final goal against D must have been scored by A, so the table can be finished like so:

| Team | Played | Won | Lost | Drew | For | Against | Pts |
|------|--------|-----|------|------|-----|---------|-----|
| A    | 3      | 2   | 1    | 0    | 2   | 1       | 4   |
| B    | 1      | 0   | 0    | 1    | 0   | 0       | 1   |
| C    | 3      | 3   | 0    | 0    | 5   | 0       | 6   |
| D    | 3      | 0   | 2    | 1    | 0   | 4       | 1   |
| E    | 4      | 0   | 2    | 2    | 0   | 2       | 2   |