# 4
# Spatial Statistics:
## Geographically weighted regression

GEOG-325: Applied Spatial Statistics and Urban Modelling

Athanasios Votsis

# Spatial effects

are divided in:

1. **Spatial dependence**
   - With issues surrounding Tobler's law, spatial correlation and clustering, spatial interaction and spatial spillover effects.
   - Is explored with global and local spatial correlation tests.
   - Is addressed and explained with spatial regression models.

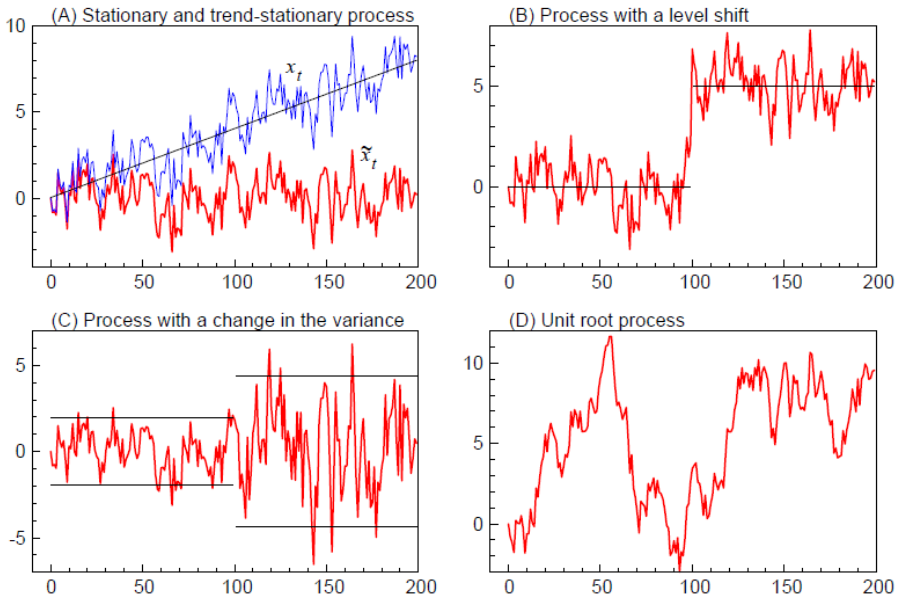
2. **Spatial heterogeneity…**

# … Spatial heterogeneity

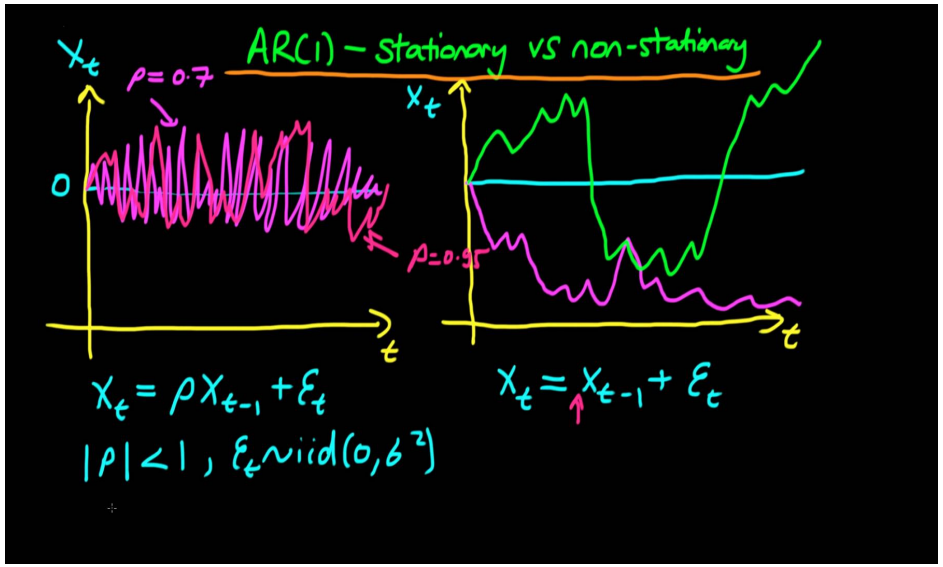
- Deals with issues surrounding the geographical variation of functional relationships between variables.
    - That is, the behavior of your phenomenon does not only vary geographically, but:
    - The **mechanisms** by which certain factors affect the phenomenon **vary geographically**.

- It is formally addressed with:
    - selecting study areas that are homogenous,
    - sub-setting the data to create homogenous study areas,
    - or representing geographical niches in model specification.

# Stationary vs. non-stationary time variables




Nielsen (2007) (in terms of the variable's mean)



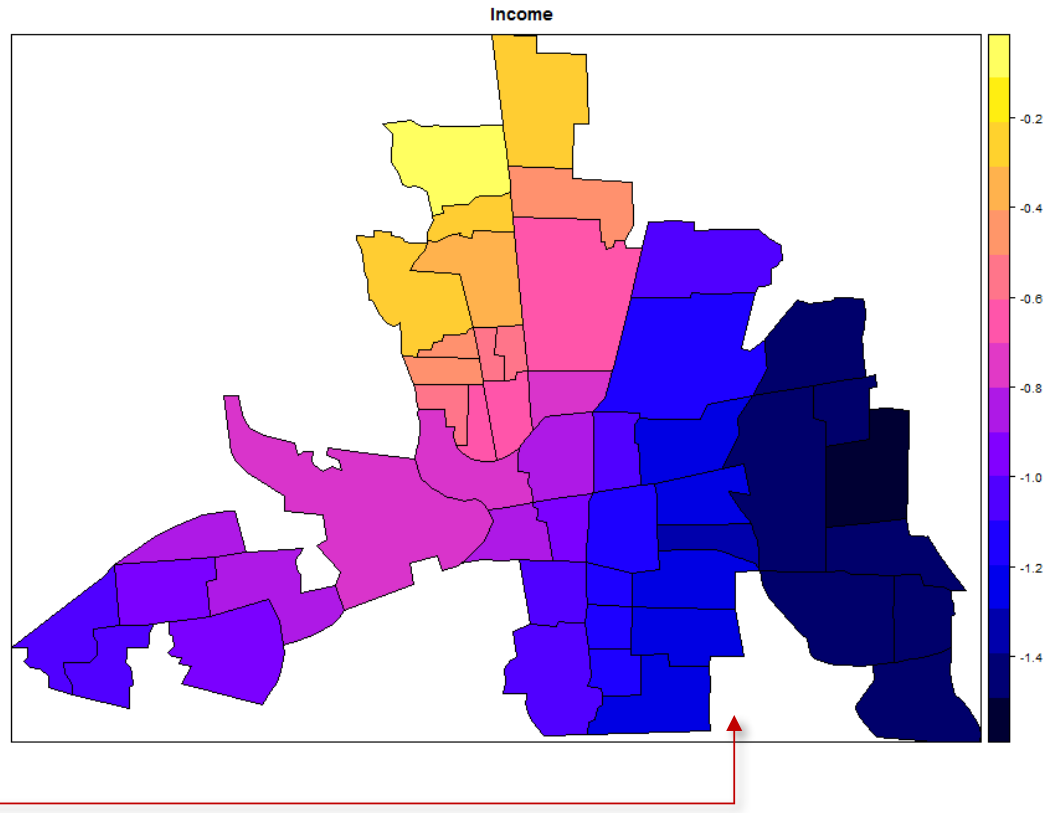

Ben Lambert - https://www.youtube.com/watch?v=v70-kLB3BLM (in terms of "amplification")

# Spatial non-stationarity in marginal effects

Is an aspect of spatial heterogeneity.

Refers specifically to the situation where the regression coefficients deviate locally from those estimated globally for the whole region.

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  70.5766     4.1227  17.119  < 2e-16 ***
INC          -0.9677     0.3278  -2.952  0.00500 **
HOVAL        -0.1733     0.0926  -1.872  0.06773 .
DISCBD       -5.2157     1.2829  -4.066  0.00019 ***
---
```



# Geographically weighted regression

Geographically weighted regression (GWR) is a method proposed by Fotheringham et al. (2002)[1] to explore spatial non-stationarity in estimated marginal effects.

- It uses a filter that moves over each observation,
- Puts larger weights to nearby observations,
- And estimates a regression for each locus.

spatial weights mentality

- This is repeated for all the observations, and provides a geographically variable collection of regression coefficients.

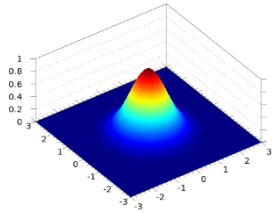
[1] Fotheringham AS, Brundson C, Charlton ME (2002), *Geographically Weighted Regression* (Wiley).

# Geographically weighted regression

The standard OLS model $\mathbf{y} = \mathbf{X}\beta + \boldsymbol{\varepsilon}$ is decomposed in $i$ local instances, where $i$ is the number of observations (polygons or points): $\mathbf{y}_i = \mathbf{X}\beta_i + \boldsymbol{\varepsilon}$

$\beta_i$ is estimated by using a Gaussian weighting filter, in which the weight between observation at location $i$ and another observation at location $j$ is set not by the user, but as:

$$w_{ij} = e^{\left(-d_{ij}/B\right)^2}$$



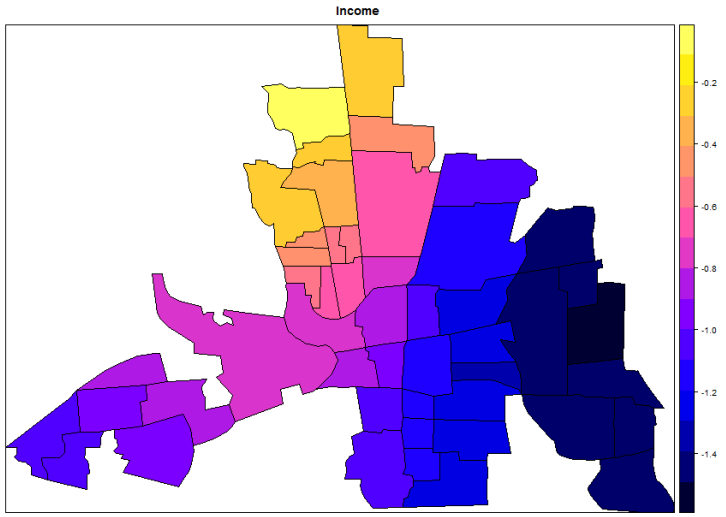
$d_{ij}$ is the Euclidean distance between locations $i$ and $j$

$B$ is the bandwidth and can be set by the user or searched by error minimization or information criteria.

# Geographically weighted regression

The results of GWR are locally parameterized coefficient estimates, able to explore spatial non-stationarity of the effects of X on y and spatial heterogeneity in general.

For instance, in this session's exercise we are assuming that the effect of income on crime in the city of Columbus OH varies across neighborhoods. The GWR results indicate that the beta coefficient varies from -1.5 to -0.1. The influence is stronger in south-southeastern neighborhoods.



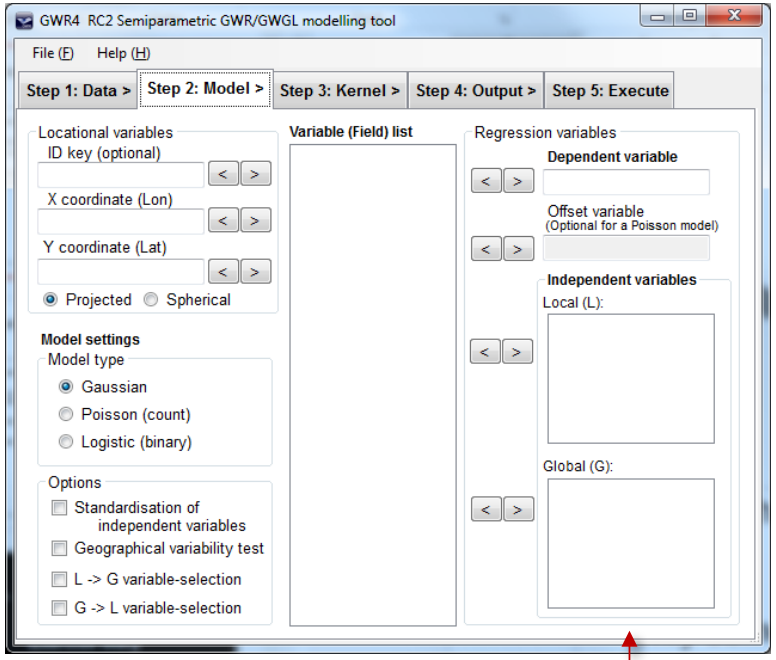

# Global and local variables

You can further assume that:

- Only some of the independent variables will have locally varying coefficients.
- While others will have stationary coefficients.

A combination of test regressions and theory can guide you in selecting global and local variables.

This option is currently implemented in software GWR4



# Software implementations of GWR

- In R with package `spgwr`.

- In ArcGIS with the tool `Geographically Weighted Regression (GWR)` in the Spatial Statistics toolbox.
  - Attention: “Parameter estimates and predicted values for GWR are computed using the following spatial weighting function: exp(-d^2/b^2). There may be differences in this weighting function among various GWR software implementations. Consequently, results from the ESRI GWR tool may not match results of other GWR software packages exactly.” (ArcGIS help text)

- In the stand-alone software `GWR4`.

- Increasingly available in other statistical software, too: e.g. in STATA and SAS.

# Questions for this session

1. What is spatial non-stationarity in regression coefficients and how does geographically weighted regression address it?

2. Can you give some examples of spatial non-stationarity in real-world phenomena?