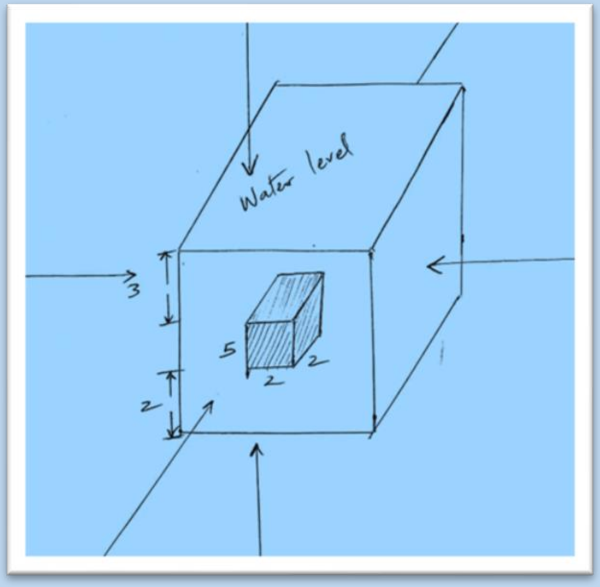
Application of integration in calculating fluid pressure

The pressure p at a depth h in a fluid of mass density ρ is given by $p = h\rho g$. The pressure acts on each point on an object in the direction perpendicular to the objects surface at the point. Assume g=10N/Kg^2



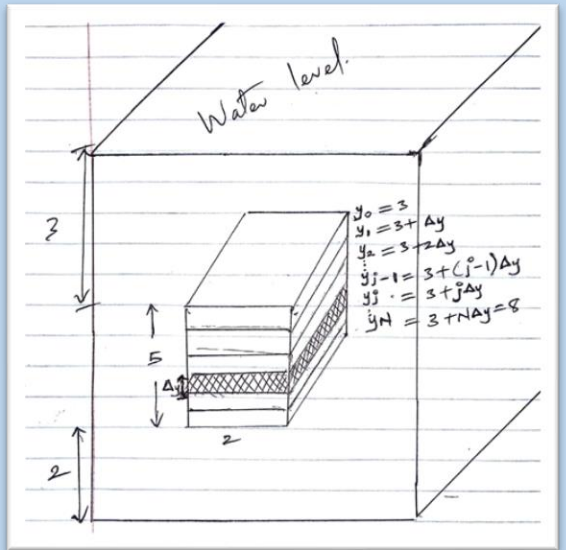
Force on the top = **Pressure×Area = PA = hρg×Area** = (3×1000×10) × (2×2) = 120000N

Force on the top = **Pressure×Area = PA = hρg×Area** = (8×1000×10) × (2×2) = 320000N

But this method is not accurate since the pressure varies with depth it is essential to calculate force as an integral.

Calculating the force using integration.

Since the pressure varies with depth, we divide the side of the box into N thin horizontal strips. Let F_j be the force on the j^{th} strip. The total force, F is equal t the sum of the forces on the strips.



$\mathbf{F_1+F_2+F_3+F_4 \ldots F_{j-1}+F_j+\ldots+F_N}$

Approximating the force on a strip, we will use the variable y to denote the depth where **y = 0** at the water level on top of the immersed box. It is also important to note that y is positive in the downward direction

Thus, a larger value of y denotes a greater depth. Each strip is a rectangle of height $\Delta_y = \frac{5}{N}$ and length **2**, so the area of a strip is $\mathbf{2\Delta_y}$. The bottom edge of the $\mathbf{j^{th}}$ strip has depth $\mathbf{y_j = 3 + j(\Delta_y)}$.

If Δ_y is small, then the pressure on the j^{th} strip is nearly constant with the value $\boldsymbol{\rho g y_j}$, (from the formula **p=hρg**), because all points on the strip lie at nearly the same depth y_j

So, we can approximate the force for the **j^{th}** strip as

$$\mathbf{F_j = \rho g y_j \times 2\Delta_y = \rho g 2 y_j \Delta_y}$$

Approximating the total force as a Reimann sum we have,

$$\mathbf{F_{total} = F_1 + F_2 + \ldots + F_N = \rho g \sum_{j=0}^{N} 2 y_j \Delta_y}$$

The sum on the right is a Reimann sum that converges to the integral

$$\mathbf{F_{total} = \rho g \int_3^8 2y\,dy}$$

The interval of integration is [3,8] because the box extends from y=0 to y=8 (the Reimann sum has been set with $y_0 = 3$ and $y_N = 8$

$$\mathbf{F_{total} = 1000 \times 10 \int_3^8 2y\,dy = 10000 y^2 \big|_3^8 = 10000(64-9) = 550000\ N}$$