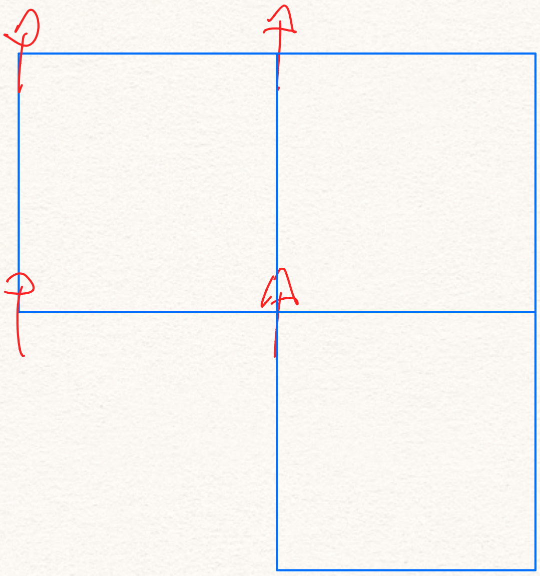


$$\boxed{E = -J\left(s_1 s_2 + s_2 s_1\right)}$$

s_i = spin

$\uparrow = 1$

$\downarrow = -1$

$$E_{\uparrow\uparrow} = -J\left(1\cdot 1 + 1\cdot 1\right)$$

$$= -2J$$

$$E_{\uparrow\downarrow} = -J\left(1\cdot(-1) + (-1)\cdot 1\right)$$

$$= 2J$$

$$E_{\downarrow\uparrow} = -J\left(-1\cdot 1 + 1\cdot(-1)\right)$$

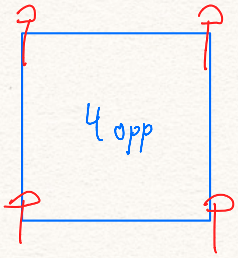
$$= 2J$$

$$E_{\downarrow\downarrow} = -J\left(-1\cdot(-1) + -1\cdot(-1)\right)$$

$$= -2J$$

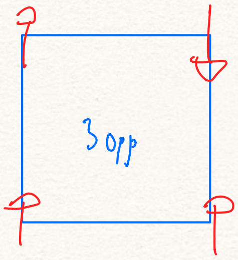
de

degeneracy



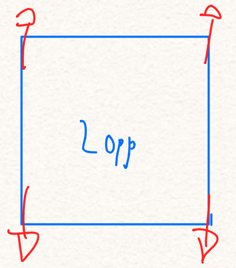
1

$$E = (-2J) + (-2J) + (-2J) + (-2J)$$
$$= -8J \qquad \text{mag} = 4$$



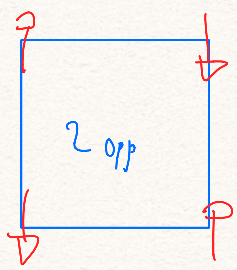
4

$$E = 2J + (-2J) + 2J + (-2J)$$
$$= 0 \qquad \text{mag} = 2$$



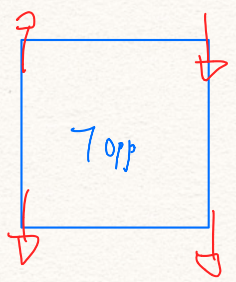
4

$$E = (-2J) + 2J + (-2J) + 2J$$
$$= 0 \qquad \text{mag} = 0$$



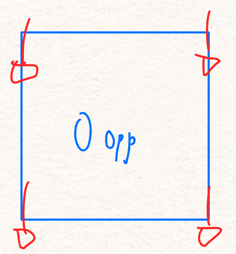
2

$$E = 2J + 2J + 2J + 2J$$
$$= 8J \qquad \text{mag} = 0$$



4

$$E = 2J + 2J + (-2J) + (-2J)$$
$$= 0 \qquad \text{mag} = -2$$



1

$$E = (-2J) + (-2J) + (-2J) + (-2J)$$
$$= -8J \qquad \text{mag} = -4$$

Number spins up	Degeneracy	Energy	Magnetization
4	1	$-8J$	4
3	4	0	2
2	4	0	0
2	2	$8J$	0
1	4	0	-2
0	1	$-8J$	-4

$n=4$

degeneracy

$$\boxed{Z} = \sum_{i=1}^{2^n} e^{-\beta E_i}$$

$$= e^{-\frac{(-8J)}{kT}} + 4 \cdot e^{-\frac{0J}{kT}} + 4 \cdot e^{-\frac{0J}{kT}} + 2e^{-\frac{8J}{kT}} + 4 \cdot e^{-\frac{0J}{kT}} + e^{-\left(-\frac{8J}{kT}\right)}$$

$$= 2e^{\beta 8J} + 2e^{-\beta 8J} + 12 = 4\,\frac{e^{\beta 8J} + e^{-\beta 8J}}{2} + 12$$

$$= 4\cosh(8\beta J) + 12$$

$$\boxed{C_V} = \frac{1}{k_B T^2}\left(\langle E^2\rangle - \langle E\rangle^2\right)$$

Trenger $\langle E^2 \rangle$

side 10

$$\boxed{\langle E \rangle} = \sum_{i=1}^{2^n} E_i P_i(T) = \frac{1}{Z}\sum_{i=1}^{16} E_i e^{-\frac{E_i}{kT}} = -\frac{\partial \ln(Z(T))}{\partial \beta}$$

$$= -\frac{\partial}{\partial \beta}\left(\ln\left(4\cosh(8\beta J) + 12\right)\right)$$

$$= -\frac{32\sinh(8\beta J)}{4\cosh(8\beta J) + 12}$$

$$\boxed{\langle E^2 \rangle} = \sum_{i=1}^{2^n} E_i^2 P_i(T) = \frac{1}{Z} \sum_{i=1}^{16} E_i^2 e^{-\frac{E_i}{kT}}$$

$$= \frac{1}{Z}\left((-8)^2 e^{-\frac{(-8J)}{kT}} + \cancel{0^2 \cdot e^{-\frac{0J}{kT}}} + \cancel{0^2 \cdot e^{-\frac{0J}{kT}}} + 2\cdot 8^2 \cdot e^{-\frac{8J}{kT}} + \cancel{0^2 \cdot e^{\frac{0J}{kT}}} + (-8)^2 e^{-\frac{(-8J)}{kT}} \right)$$

$$= \frac{1}{Z}\left(64 J^2 e^{\beta 8 J} + 2 \cdot 64 J^2 e^{-\beta 8 J} + 64 J^2 e^{\beta 8 J} \right)$$

$$= \frac{128 J^2 e^{\beta 8 J} + 128 J^2 e^{-\beta 8 J}}{4 \cosh(8\beta J) + 12} = \frac{256 J^2 \cdot \frac{e^{\beta 8 J} + e^{\beta 8 J}}{2}}{4 \cosh(8 \beta J) + 12}$$

$$= \frac{256 J^2 \cosh(8 \beta J)}{4 \cosh(8\beta J) + 12} = \frac{64 J^2 \cosh(\beta 8 J)}{\cosh(8 \beta J) + 3}$$

$$\boxed{C_V} = \frac{1}{k_B T^2}\left(\langle E^2 \rangle - \langle E \rangle^2 \right)$$

$$= \frac{1}{k_B T^2} \underbrace{\left(\frac{64 J^2 \cosh(\beta 8 J)}{\cosh(8 \beta J) + 3} - \left(- \frac{32 \sinh(8 \beta J)}{4 \cosh(8 \beta J) + 12} \right)^2 \right)}_{\sigma_E^2}$$

$$\boxed{\chi} = \frac{\left(\langle M^2 \rangle - \langle M(T) \rangle^2 \right)}{k_B T}$$

trenger $\langle M^2(T) \rangle$ og $M(T)$

$$\boxed{\langle M^2 \rangle} = \frac{1}{Z} \sum_i M_i^2 e^{-\beta E_i}$$

$$= \frac{1}{Z} \left(\cancel{4^2 e^{-\frac{(-8J)}{kT}}} + \cancel{4 \cdot 2^2 e^{-\frac{0J}{kT}}} + \cancel{4 \cdot 0 \cdot e^{-\frac{0J}{kT}}} + \cancel{2 \cdot 0 \cdot e^{\frac{8J}{kT}}} + \cancel{4 \cdot (-2)^2 e^{\frac{0J}{kT}}} + \cancel{(-4)^2 \cdot e^{-\frac{(-8J)}{kT}}} \right)$$

$$= \frac{32 e^{\beta 8 J} + 32}{4 \cosh(8 \beta J) + 12} = \frac{8 e^{\beta 8 J} + 8}{\cosh(8 \beta J) + 3}$$

$$\boxed{\langle M \rangle} = \sum_{i=1}^{M} M_i P_i(\beta) = \frac{1}{Z} \sum M_i e^{-\beta E_i}$$

$$= \frac{1}{Z} \left(\cancel{4 e^{-\frac{(-8J)}{kT}}} + \cancel{4 \cdot 2 \cdot e^{-\frac{0J}{kT}}} + \cancel{4 \cdot 0 \cdot e^{-\frac{0J}{kT}}} + \cancel{2 \cdot 0 \cdot e^{\frac{8J}{kT}}} + \cancel{4 \cdot (-2) e^{\frac{0J}{kT}}} + \cancel{(-4) e^{-\frac{(-8J)}{kT}}} \right)$$

$$= 0$$

$$\boxed{\chi} = \frac{\langle M^2 \rangle - \langle |M| \rangle^2}{kT}$$

$$= \frac{\langle M^2 \rangle - 0^2}{kT} = \frac{\left(\frac{8e^{\beta 8J} + 8}{\cosh(8\beta J) + 3} \right)}{kT}$$

$$= \frac{8e^{\beta 8J} + 8}{\left(\cosh(8\beta J) + 3\right) kT}$$

Men vi ble bedt spesifikt om å regne ut $|M|$

$$\boxed{|M|} = \sum_{i=1}^{M} \left| M_i P_i(\beta) \right| = \frac{1}{Z} \sum \left| M_i e^{-\beta E_i} \right|$$

$$= \frac{1}{Z} \left(\cancel{4e^{-\frac{(-8J)}{kT}}} + \cancel{4 \cdot 2 \cdot e^{-\frac{0J}{kT}}} + \cancel{4 \cdot 0 \cdot e^{-\frac{0J}{kT}}} + \cancel{2 \cdot 0 \cdot e^{-\frac{8J}{kT}}} + \left| \cancel{4 \cdot (-2) e^{\frac{0J}{kT}}} \right| + \left| \cancel{(-4) \cdot e^{-\frac{(-8J)}{kT}}} \right| \right)$$

$$= \frac{1}{4\cosh(8\beta J) + 12} \left(8e^{\beta 8J} + 16 \right) = \frac{\cancel{4}\left(2e^{\beta 8J} + 4 \right)}{\cancel{4}\left(\cosh(8\beta J) + 3 \right)}$$

$$= \frac{2e^{\beta 8J} + 4}{\cosh(8\beta J) + 3}$$

Og da regner jeg med vi skal sette den inn i suseptibilitien, men det e ikke sikkert.

$$\boxed{\chi} = \frac{\left(\langle M^2 \rangle - \langle |M| \rangle^2 \right)}{k_B T}$$

$$= \frac{\dfrac{8e^{\beta 8J} + 8}{\cosh(8\beta J) + 3} - \left(\dfrac{2e^{\beta 8J} + 4}{\cosh(8\beta J) + 3} \right)^2}{k_B T}$$