Stock Market Model

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Abstract

In this article, we look at different models for a closed economic system, with N agents performing a number of pair-wise money transactions. We use Monte Carlo methods for performing random transactions between random agents, and measure the wealth distribution at points of equilibrium. We present our results as probability distributions of wealth.

Our findings include (maybe not surprisingly) that the majority of the population ends up with little money, whilst a few become "filthy rich". By adjusting various parameters and adjusting who trades with who, the distribution of wealth can be skewed either towards more equal or more unequal distributions. We also show that the Pareto distribution is made plausible by many of our results, though we are not able to confirm it.

Finally, it is concluded that the most dramatic changes in wealth distribution depends on the amount of money people save, and hence, how much they spend in each transaction. Also, the same applies in the cases where the population is subject to different forms of taxation. These parameters seem to lead to more "fair" distributions of wealth.

Contents

1 Introduction

There are multiple articles written on the use of statistical computational methods for modelling economic systems. Such models are necessarily quite simple and not a full picture of the complex reality of micro- and macro economics. Still, it is quite conceivable that they could tell us something about how different trends affect a closed economic system. Pareto (1897) claims that most economic systems follow the so called Pareto law, that is

$$\omega(m) = m^{-(1+\nu)}$$
$$\log(\omega(m)) = -(1+\nu)\log(m)$$

Goswami & Sen (2014) adds that ν tends to be between 1 and 3.

In this article, our goal is to reproduce results from some of the articles already published. This involves running calculations for a number of different models: The DY model (Drăgulescu & Yakovenko, 2000), the saving propensity model, the nearest neighbours model, and the past transactions model. In addition, we present two rough taxation models that could be the basis for more precise work. We present our results as plots showing the equilibrium wealth distributions for all these models, and discuss our interpretation of them, as well as their implications for the economic system. We will also investigate if Pareto's claim seems plausible or not.

Also, amongst other things, we will to a greater extent than previous writers leave our source code and computational methods open to the reader. We will discuss some potential strengths and weaknesses of our implementations of the models, and point to further possible research.

2 Theory

Our different models of a closed economy are all based on the framework introduced by (Drăgulescu & Yakovenko (2000). We assume N number of agents, and that every agent starts out with the same amount of money, $m_0 \geq 0$. At a given time step a pair of agents, (m_i, m_j), are chosen at random to make a change of wealth so that agent i's money changes from m_i to m_i', and similarly, m_j changes to m_j'. The transaction is made by summing the two amounts the agents have and then redistribute the money by multiplying with a random number, $\epsilon \in [0, 1]$, extracted from a uniform distribution.

This means that the total amount of money is conserved in the transaction:

$$m_i + m_j = m'_i + m'_j$$

and that the redistribution is given by:

$$m'_i = \epsilon(m_i + m_j)$$
$$m'_j = (1 - \epsilon)(m_i + m_j)$$

After enough transactions and through the conservation of money, the system comes to an equilibrium, w_m. In the following, we present the theory for our different models, all of which can be seen as adjustments to the one described above.

2.1 DY Model

For the simplest model, all pairs of agents are chosen at random, and all of their total wealth is redistributed between the two. From Patriarca (2004), we know that the state of equilibrium is given by the exponential Gibbs distribution:

$$w_m = \beta e^{(-\beta m)}$$

where $\beta = \frac{1}{\langle m \rangle}$, and $\langle m \rangle = \Sigma_i \frac{m_i}{N} = m_0$ is the average money. Note that taking the logarithm of the Gibbs distribution gives a linear equation and hence a linear curve in a log plot.

$$ln(w_m) = ln(\beta) - \beta m$$

2.2 Savings Model

In this model, we introduce a saving criterion meaning that agents save a fraction, $\lambda \in [0, 1]$, of the amount of money they have before the transaction. With this new saving rate introduced, the new distribution of wealth for each transaction is given by:

$$m'_i = \lambda m_i + \epsilon(1 - \lambda)(m_i + m_j)$$
$$m'_j = \lambda m_j + (1 - \epsilon)(1 - \lambda)(m_i + m_j)$$

and as seen in the previous model, the total amount of money is still conserved for each transaction as:

$$
\begin{aligned}
m_i' + m_j' &= \lambda m_i + \epsilon(1-\lambda)(m_i + m_j) + \lambda m_j + (1-\epsilon)(1-\lambda)(m_i) + m_j) \\
&= \lambda(m_i + m_j) + (1-\lambda)(m_i + m_j) \\
&= m_i + m_j
\end{aligned}
$$

The equilibrium distribution thus gives a quite different curve for different values of λ, as also noted by Patriarca (2004).

2.3 Nearest Neighbor And Former Transaction Model

As described by Goswami & Sen (2014), one could add further aspects to the model. In stead of looking at how a transaction takes place, one could look at which agents interact. Goswami & Sen propose that the probability for a transaction between agents i and j taking place, is given by

$$P_{ij} \propto |m_i - m_j|^{-\alpha} \cdot (c_{ij} + 1)^{\gamma}$$

Here, m_i is the wealth of one agent, c_{ij} is the number of times agent i has interacted with agent j, and α and γ are parameters to be chosen.

The first term makes it more probable that two agents interact with each other if the difference of their wealth is small, or in other terms, if they are "close financial neighbours". The second term indicates that the probability for two agents interacting increases with the amount of times they have already interacted. Note that for $\alpha = \gamma = 0$, the probability is just 1, and so the model reverts to the DY model described above. For our implementation of this model, see the methods section.

3 Methods

For our calculations, we have used the function `Financial_Analysis`. This function performs a Monte Carlo calculation for a number of Monte Carlo Cycles inputted by the user.

Mainly, the function calls on two other functions to perform the calculation of each cycle, `Sampling_Rule` and `Transaction`. The first one provides two random indexes for choosing agents.

The other one performs the transaction of money between two given agents. All of these functions can be found in the file `Pro5_Functions.cpp`.

The general idea is to use these functions to take the closed economic system from an initial condition ($m_i = 1$ for all i) to equilibrium. Once this state has been reached, we measure the wealth distribution. To get sufficient data, we have in most cases run 1000 independent experiments, consisting of a number of Monte Carlo Cycles, or transactions. We then make a histogram (using `matplotlib.pyplot.hist`) of all agents and their wealth, from all the experiments. This gives $1000N$ points of data.

In the following, we describe some of the finer points of our methods.

3.1 Reaching equilibrium

In order to determine the number of transactions needed to reach the equilibrium state, tests were applied, plotting the median for only one experiment as a function of the number of transactions. These tests quickly let us approximate how many transactions the system would need to reach the equilibrium state. As an example, we show the time development of the median for the simplest case in figure 1. All produced median plots can be found in the GitHub repository.

For the first four models, the number of transactions needed for reaching equilibrium is 10^5, but the more complex the models get, the more transactions are needed, as shown in the appendix. The number of transactions needed for the model considering Former Transactions, is 10^7.

It turns out our method applied on this case would take close to 80 hours to run, which was not doable in the time available to us. We therefore made a change to our calculations, making them more similar to the approach taken by Goswami and Sen (2014). Instead of performing 1000 independent experiments, we performed only one. After reaching equilibrium, we measured the wealth distribution after every 1000th transaction. This significantly shortens the computation time, and with four cores working in parallel, we were able to conduct four of these experiments, yielding $4000N$ data points in about half an hour. In hindsight, this seems a better method altogether, and could probably have been used for all models.

3.2 Fitting of the Saving Model

Introducing the reducible variable, $x = \frac{m}{\langle m \rangle}$, like Patriarca (2004), we are able to fit the wealth distributions for different values of λ with the analytic expression:

$$P_n(x) = a_n x^{-1-n} exp(-nx)$$

where

$$n(\lambda) = 1 + \frac{3\lambda}{1-\lambda}$$

and

$$a_n = \frac{n^n}{\Gamma(n)}$$

using Γ, as the Gamma function. This is implemented in the code as the function `P`. Note that in our case, $\langle m \rangle = 1$, and so $x = m$.

3.3 Fitting of other Models

Like Goswami & Sen (2014), we are interested to see if our result in any way resemble Pareto distributions. To do this, we have plotted our wealth distributions as log-log plots. In such a plot, a linear curve corresponds to a power law. The power is the same as the slope of the curve, as is easily shown:

$$y = x^a$$

$$\log_{10}(y) = \log_{10}(x^a)$$

$$\log_{10}(y) = a \log_{10}(x)$$

Mainly, we are interested to see if the tail of our distribution follows a power law, and so we have made a cutoff. This is done manually, by trial and error, and by simply observing the plots. Therefore, our results for the power law expressions are merely indicative.

3.4 Nearest neighbours and former transactions

For Goswami & Sen's transaction probability distribution we have assumed that there is a probability for any two agents interacting with each other, given as

$$P_{ij} = |m_i - m_j|^{-\alpha} \cdot C(c_{ij} + 1)^{\gamma}$$

Note that we have added a scaling factor C to the expression given by Goswami & Sen. This is to make sure that the random number r is not always smaller than P, even though $(c_{ij} + 1) > 1$ - see below.

The implementation of this probability is done in the function `sampling_rule`, which provides two agents from the probability distribution. The function is written such that it always provides two indexes - meaning that an experiment with e.g. ten cycles, always consists of ten transactions. In pseudo-code, it looks like this:

- generate two random integers i and $j \in [0, N-1]$
- calculate P with given i and j
- generate a uniformly distributed random double $r \in [0, 1]$
- if $r < P$, accept integers, else, repeat from top

The function also makes sure that $i \neq j$, and updates c_{ij}.

3.4.1 A note on probability scaling

In the literature (Goswami & Sen, 2014), the probability is given as being proportional to the expression above, like this:

$$P_{ij} \propto |m_i - m_j|^{-\alpha} \cdot (c_{ij} + 1)^{\gamma}$$

For the results presented, we have chosen to do a simple scaling of the second term, $C = N/Cycles$, where *Cycles* is the amount of Monte Carlo cycles for each experiment. In other words, C is the inverse of the number of transactions performed by each agent, given that they all perform the same number of transactions. Clearly they do not, but it seems a reasonable scaling parameter. Note that this way of doing things in many cases gives $P > 1$, and since $r < 1$, many proposed transactions are guaranteed to take place.

Other, more elaborate attempts have been made for the scaling of probabilities, with the goal to make them normalized. That is, to make sure $P <= 1$, by introducing two scaling parameters, A and C:

$$P_{ij} = A|m_i - m_j|^{-\alpha} \cdot C(c_{ij} + 1)^{\gamma}$$

Briefly explained, for the first term we assumed that the probability for a random chosen agent m_i to interact with *someone*, sums to one, or (with A as the normalization factor):

$$\sum_k \frac{1}{A_i} |m_i - m_k|^{-\alpha} = 1$$
$$\implies A_i = \sum_k |m_i - m_k|^{-\alpha}$$

For an agent m_j, the same applies. The probability that they interact with each other, could then scale with the factor

$$A = \frac{2}{A_i + A_j}$$

The same approach was taken for the second term:

$$\sum_k \frac{1}{C_i} (c_{ik} + 1)^{\gamma} = 1$$
$$\implies C_i = \sum_k (c_{ik} + 1)^{\gamma}$$

yielding $C = \frac{2}{C_i + C_j}$.

However - sadly - these parameters did not seem to yield any improvement in the results we got. Or, rather, they did not yield the same results as Goswami & Sen (2014). Also, limited time has prohibited us from more testing and adjusting of the scaled algorithm. Therefore we have chosen to stick with the simplified version presented above. Still, it could well be that the scaled version (or a modified one) has some merit to it, and therefore we leave the code in the GitHub-repository, for anyone interested. The scaled version is written as the functions `Financial_Analysis_scaled_prob` and `Sampling_Rule_scaled`.

3.5 Taxes Model

As a final point of interest, we have taken a look at how taxation affects the system. There are two obvious, different ways of performing a taxation. Either the agents pay a tax depending on the magnitude of their transaction, also known as Value Added Tax, or VAT, or the agents pay based on their total wealth, also called a Wealth Tax. Paid tax is uniformly distributed back to all agents - this seemed the easiest way of doing things with limited time.

We have implemented computational algorithms for both of these scenarios. For the first one, we have written a function called `transaction_VAT`, which calculates the due tax for every transaction. We assume that after a transaction has taken place (defined by the random number ϵ as before), a percentage of the performed transaction is paid as VAT. The VAT is paid by the receiving agent. We have run calculations for four values of VAT percentages, results of which can be seen in figure 9.

For the second tax model, we have used the function `Wealth_Tax`. For every transaction, we add a step where all agents pay a certain percentage of their wealth to the fellowship. This is then paid back in equal measure to all agents.

Due to limited time, the results are presented in a simple way, and the discussion of them is quite brief. Also, there are many things we wanted to do, but could not, as can be seen in the discussion. Note also that the tax model has been applied on the DY model only, so no other adjusting factors apply.

4 Results

In this section, we're presenting how the wealth distribution is changing for different models, ranging from the simple to the more complex.

4.1 Results for the DY Model

Figure 1 shows a time development of the median as a function of Monte Carlos cycles for one experiment to indicate when the system has reached the equilibrium state.

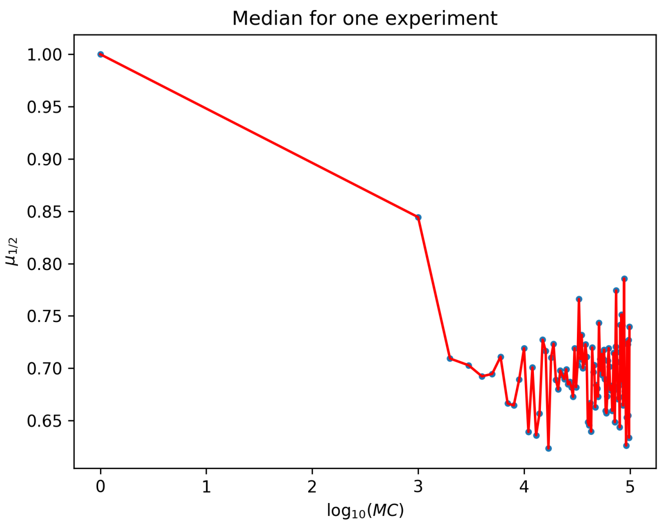
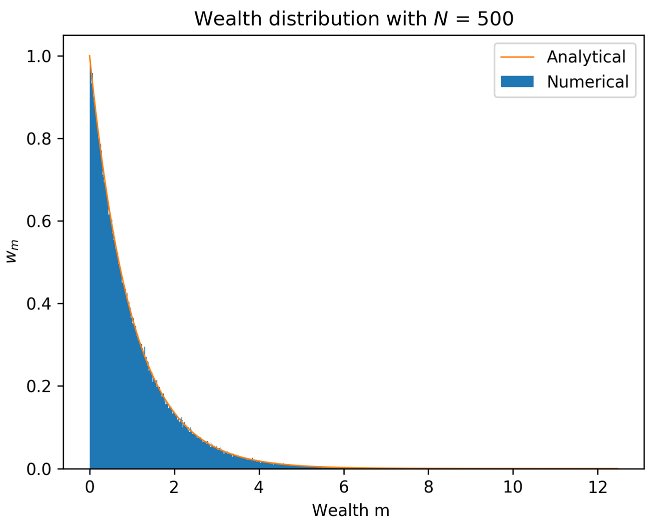
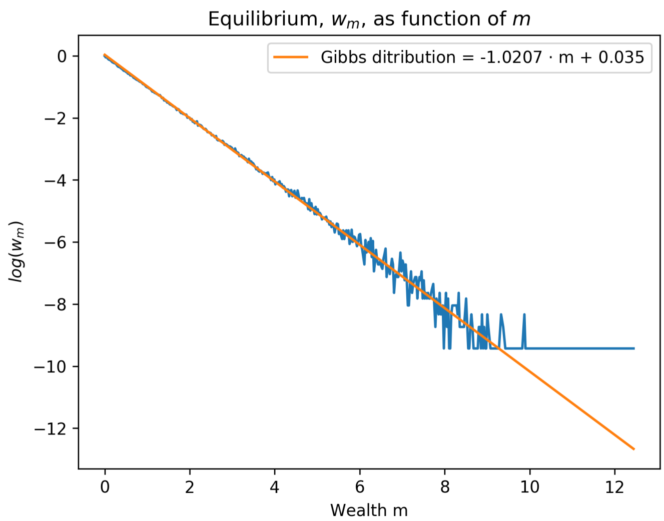


Figure 1: *Time development for the median in the DY Model.*

Figure 2a gives a graphical presentation of the wealth distribution of the first and most simple model, following the analytical inverse exponential Gibbs distribution, showing that the number of agents is decreasing as the wealth increases. Figure 2b shows a log plot of the same distribution, where we see the linear graph as described in the theory section, 2.1.



(a) $N = 500$, $m_0 = 1$, *MC-cycles* $= 10^5$



(b) $N = 500$, $m_0 = 1$, *MC-cycles* $= 10^5$

Figure 2: *a) Histogram of the wealth distribution and the analytical Gibbs distribution with 500 agents and no savings or other criterion. b) Log plot of the same numerical equilibrium distribution as in figure 1a, with a linear fitted line giving the slope of the distribution.*

4.2 Results for the Savings Model

Figure 3 displays wealth distributions for different degrees of saving propensity. For no saving rate, $\lambda = 0$, the Gibbs distribution as depicted in figure 2a is reproduced. For increased savings, we see a peak building around $m = 1 = m_0$, and how the model deviates more and more from the Gibbs distribution.

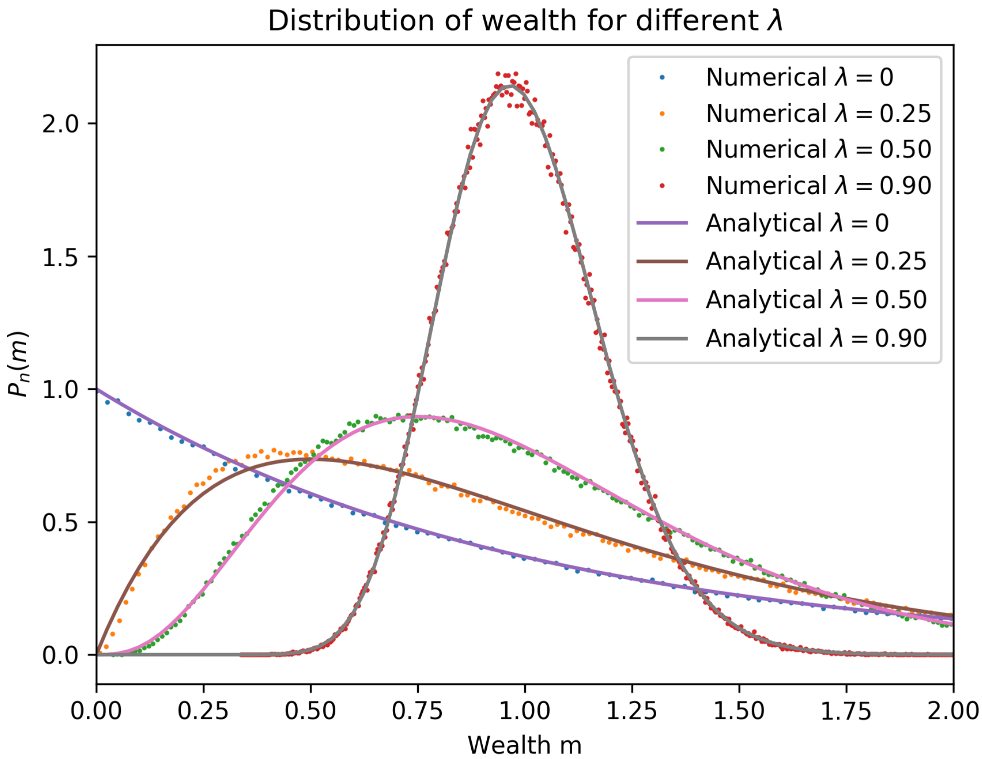
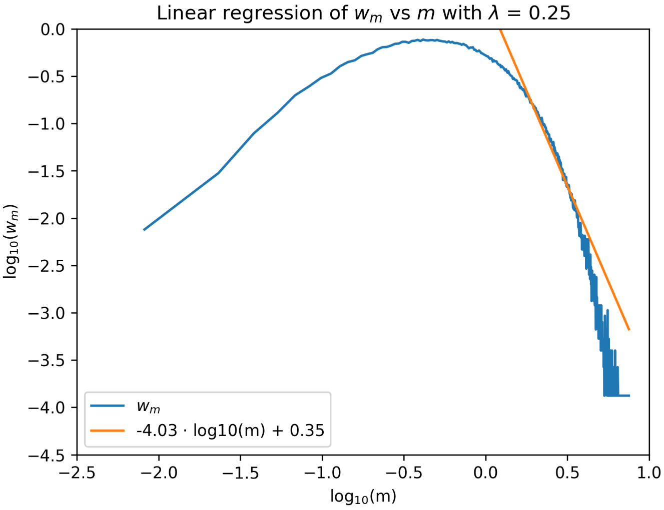
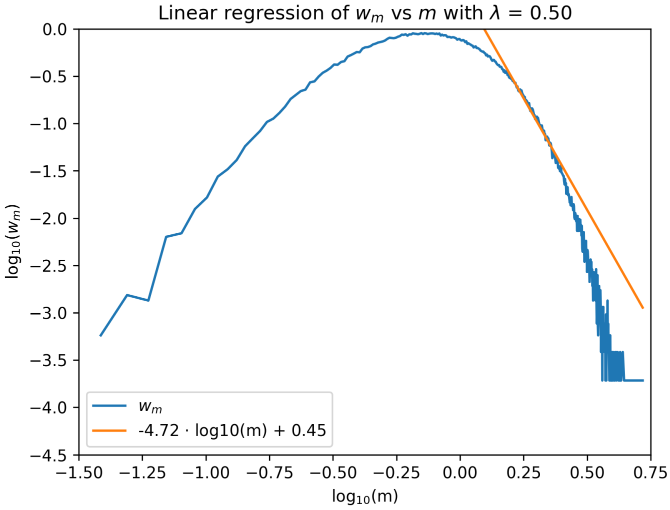


Figure 3: *a) Wealth distribution with 500 agents and a varied savings rate for both numerical and analytical* $\lambda \in [0, 0.25, 0.5, 0.9]$.

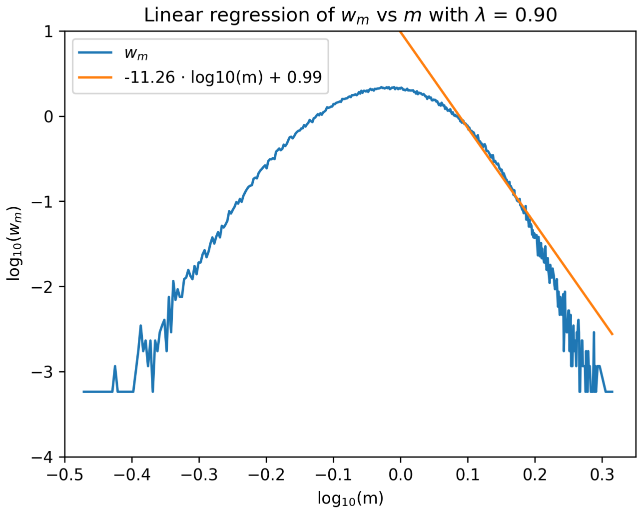
The high-end tails for the three saving rates are shown below in figure 4, together with a power law fit for each plot.



(a) $\lambda = 0.25$



(b) $\lambda = 0.50$



(c) $\lambda = 0.90$

Figure 4: *Log-log plot of the wealth distribution for selected* λ*, together with a power law fit for the high-end tails.*

4.3 Results for the Nearest Neighbor Model

Figure 5 shows the wealth distribution for various combinations of N, λ and α. In figure 6, the distributions are plotted separately, together with linear regressions of the distribution tails.

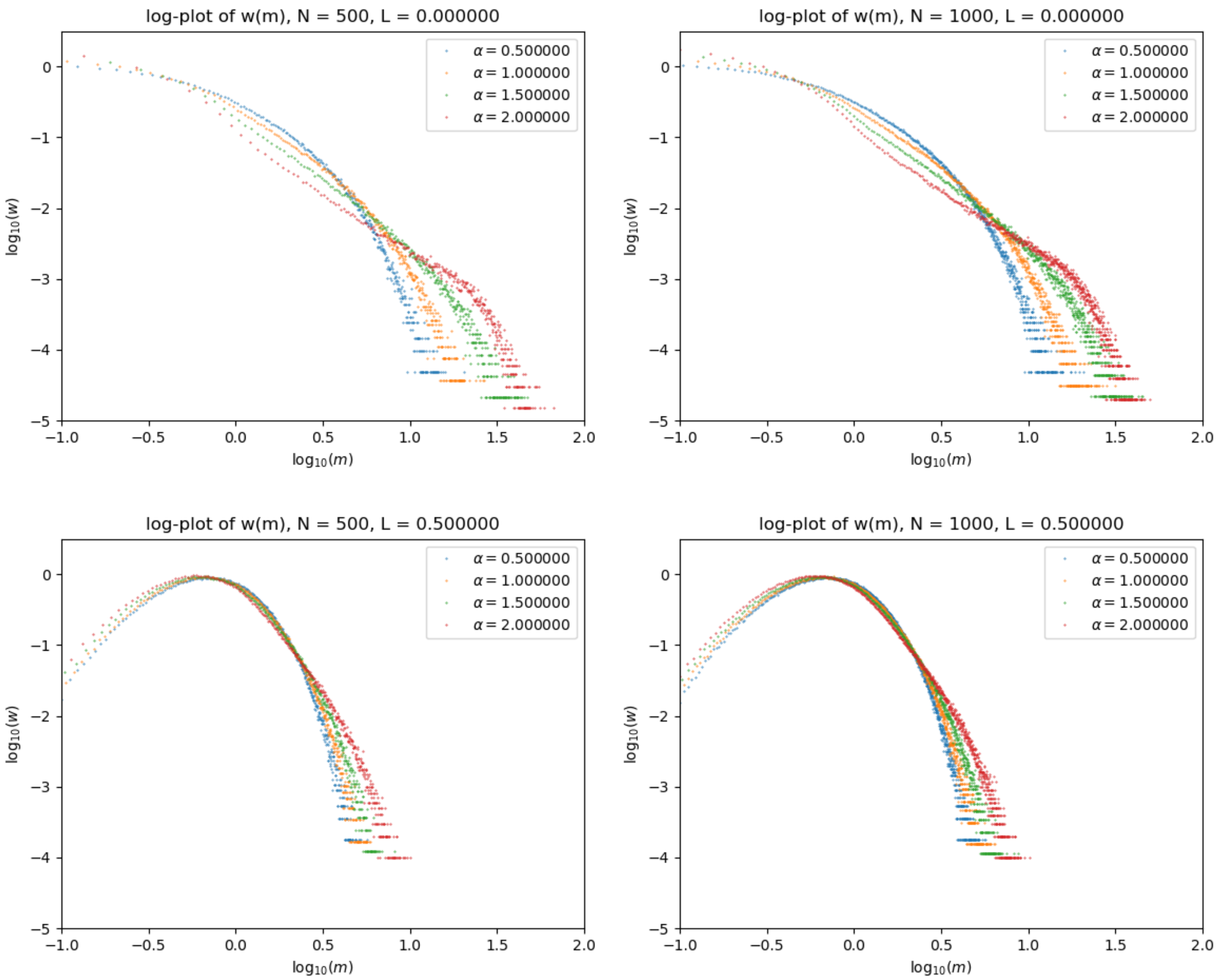


Figure 5: *Log-log plots of the wealth distribution $\omega(m)$, for different N, λ (L in the titles) and α. The legends read "blue, yellow, green, red".*

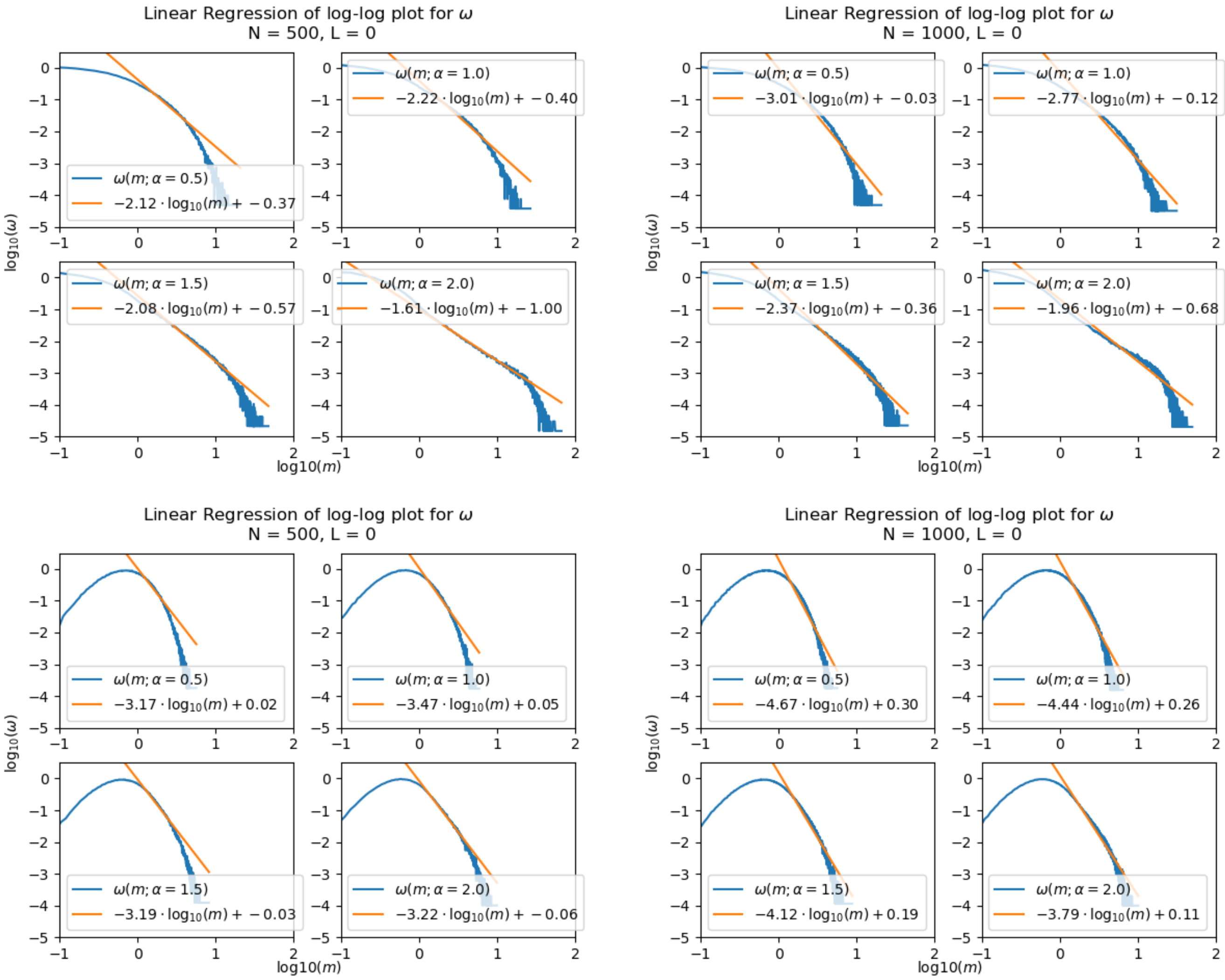


Figure 6: *Wealth distributions for the Nearest Neighbour Model, plotted together with the linear regression of the tail. The slope of the regression is written in the label.*

4.4 Results for the Nearest Neighbor And Former Transaction Model

As in the previous section, figures 7 and 8 show the wealth distribution for this model, together with the linear tail regressions.

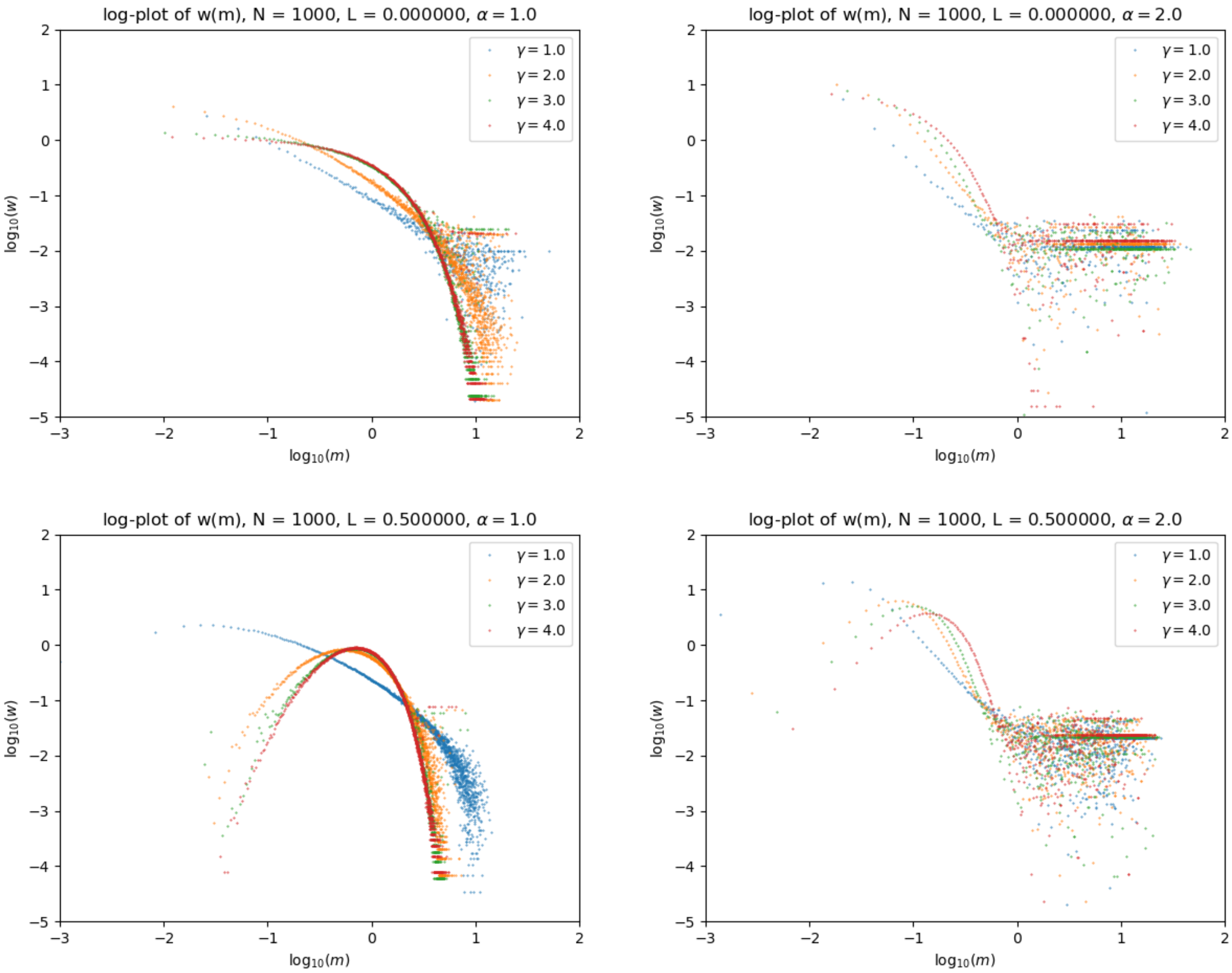


Figure 7: *Log-log plots of the wealth distribution $\omega(m)$, for different N, λ (L in the titles), α and γ. The legends read "blue, yellow, green, red".*

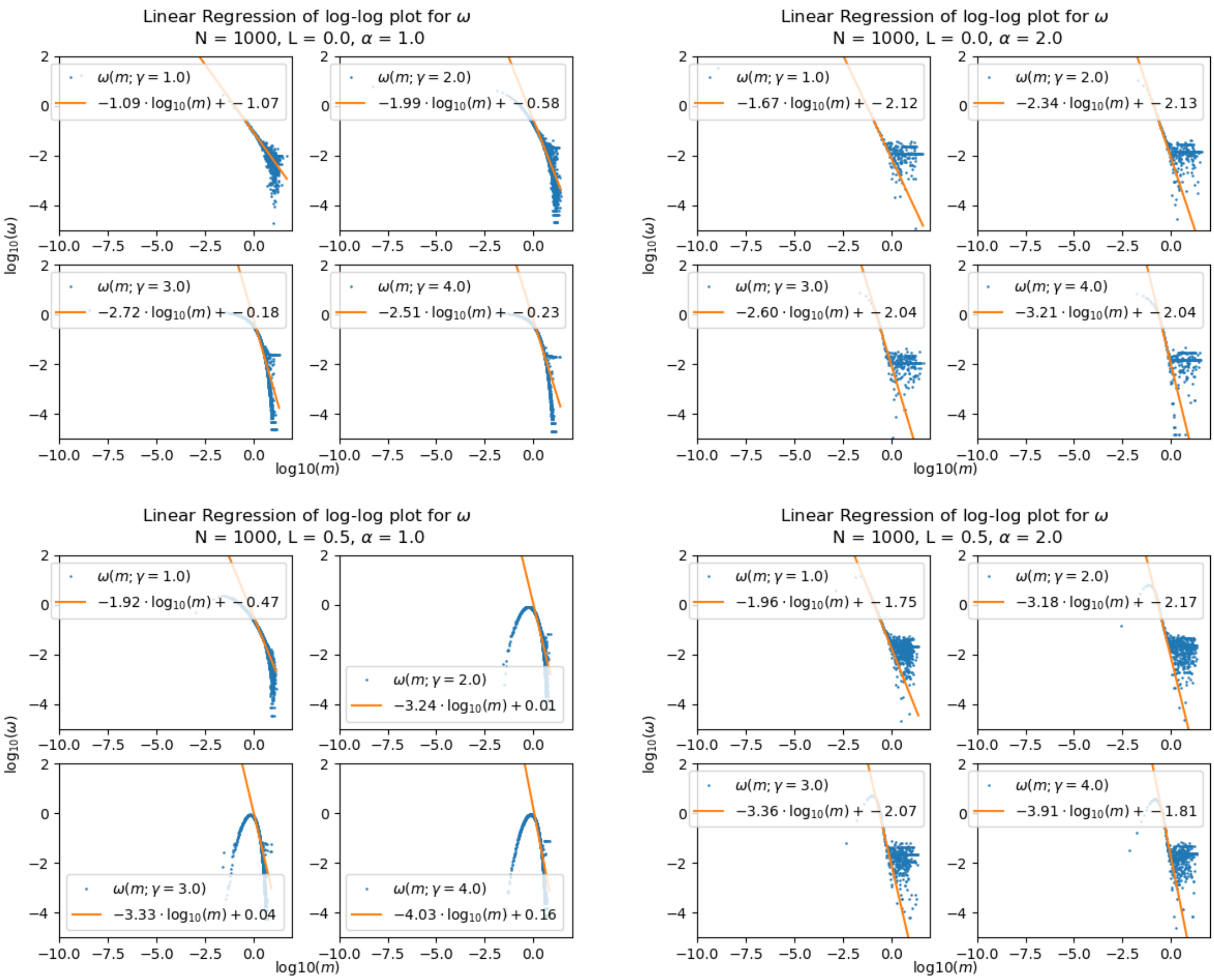


Figure 8: *Wealth distributions for the Nearest Neighbour and Former Transactions Model, plotted together with the linear regression of the tail. The slope of the regression is written in the label.*

5 Results for Taxation models

Figures 9 and 10 show the wealth distribution at equilibrium for different taxation models.

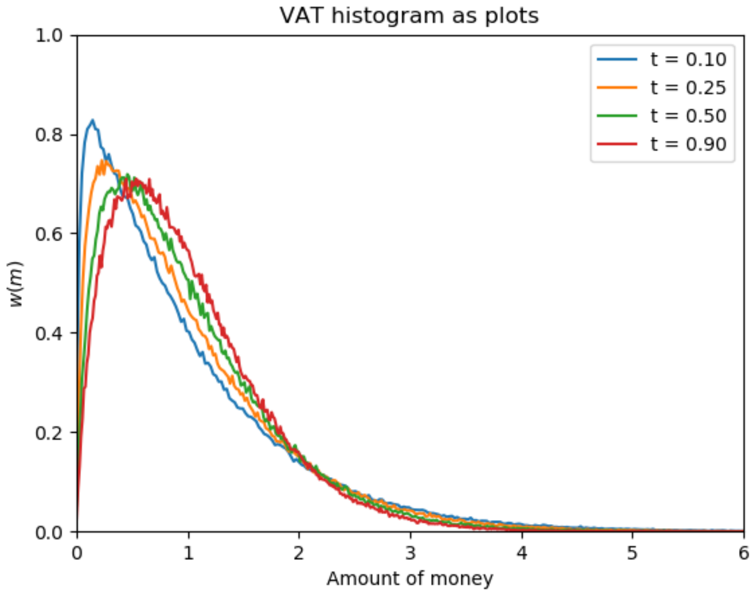


Figure 9: *The wealth distribution at equilibrium for different values of t, the VAT percentage. The calculations were run with 500 agents.*

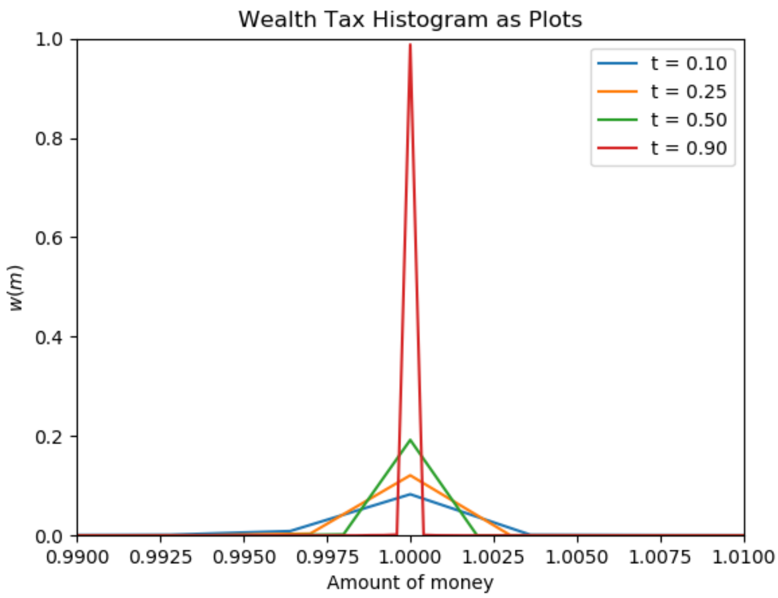


Figure 10: *The wealth distribution at equilibrium for different values of t, the wealth taxation percentage. The calculations were run with 500 agents.*

6 Discussion

6.1 Discussion of the DY Transactions Model

For the simple model of the wealth distribution in figure 2a, it is easy to see that it follows the Gibbs distribution, an inverse exponential curve. Many agents have a small amount of money, and a few agents have a lot. It is also demonstrated in figure 2b that the Gibbs distribution follows a linear curve in a log plot, where we see that the gradient of the curve indeed is $\sim$ -1 as expected, given that the agents start with $m_0 = 1$.

6.2 Discussion of the Savings Model

With the introduction of a saving rate for each transaction, the distribution starts to look a bit more fair for the agents. The higher the saving rate, the more agents have a substantial amount of wealth, as displayed in figure 3, where we also see that the peak approaches m_0 the higher the saving rate. We also see that the numerical model fit the analytical solution provided by Patriarca (2004) pretty well, as shown in the separate histograms in figure 3.

The drastic change of the distribution shape is explained by the fact that agents now hold on to more of their money in each transaction. That means that it takes more than one transaction to get either very poor or very rich, which in turn means that fewer people will approach extreme wealth values.

In order to make a parametrization for the power law, we used linear regression, as shown in figure 4. What we see is that the slope of the linear regression fits pretty well with the Pareto distribution with a $\nu = -3.03$ and $\nu = -3.72$ for $\lambda = 0.25, 0.50$, respectively. For the saving rate, $\lambda = 0.9$, the Pareto exponent is pretty high with a $\nu = -10.26$, but we are not quite sure why. It could be that the model just does not fit very well at higher saving rates, as it is just that, a model.

6.3 Discussion of the Nearest Neighbour Model

In figure 5, the wealth distributions for the Nearest Neighbour model are shown. Firstly, we see that the number of agents in the experiment does not seem to matter much for the distribution. However, if we vary the parameter α, we see interesting things happen. In short, we see that the larger the value of α, the more "unfair" the distribution gets, by which we mean that the distribution is skewed towards the extremes. This means less people of average wealth, and more people that are either rich or poor. This is the case both with and without the saving propensity, but the effect seems clearer for the cases without. The results make sense, because after the system has reached equilibrium, we would expect the poor to only interact with the poor, and the rich to interact with the rich. That makes it harder for someone of low wealth to accumulate much money and become rich, since the potential gain from any transaction is small.

On the other hand, if two rich agents interact, one of them could quite possibly become very poor after the transaction. This could explain why the α-factor does not massively change the distribution shape, like the saving propensity does - there are still many more poor agents than there are rich.

Comparing our results with figure 1 in Goswami & Sen (2014), we see that the shape of our tails look similar to theirs. However, the units of the axes do not quite transfer from their plot to ours. It seems that they have more data points for higher levels of wealth, although with very low probability. The reason for this is not immediately clear. Goswami & Sen (2014) are not clear on how their calculations are performed, and so it is hard to discern any differences between their research and ours that would cause this change. We see that the crossing of our distribution plots (by which we mean what happens at $\log_{10}(m) \approx 1$ for $N = 1000$, $\lambda = 0$) corresponds well with Goswami & Sen (2014) on the x-axis, but not on the y-axis.

In figure 6, the distributions are shown separately, together with linear regressions for their tails. It seems that higher values of α takes the distribution closer to a power law tail. The values of the slopes are between -2 and -4, which matches well with the Pareto distribution described by Goswami & Sen (2014). Note that the linear regressions performed are not done by any strict definition of the cutoff point for the tails. Rather, we have approximated the cutoff points at which the tail starts, and where the data becomes "chaotic" at the tail's end, by trial and error, measuring by eye. This means that the slope values should be taken with a pinch of salt. Still, they seem to give a good indication that the Pareto distribution is a sensible description of the economic system.

6.4 Discussion of the Nearest Neighbour and Former Transactions Model

Figure 7 shows the wealth distributions once we have added a probability factor dependent on the number of past transactions between agents i and j. As you can see, the effect of the new term seems quite prominent. Where as increasing α made the distribution more linear (in the loglog-plot), increasing γ increases the probability of being of "medium" wealth, as opposed to very poor or very rich. The explanation is not immediately clear, but most likely has to do with the evolution of the system from the initial state to equilibrium. Once the system has reached equilibrium, the nearest neighbour term makes the poor trade with the poor, as described in the previous section. However, in some time after the initial state, some agents might interact with other agents that end up with very different wealth than themselves once equilibrium has been reached. As the most basic example, after the very first interaction, one agent could became very poor, and the other agent correspondingly rich. This transaction is stored, so that later, the probability of these agents interacting again is greater than it would have been without the past interaction term.

This increased probability of agents of different wealth interacting leads to the wealth distribution increasing towards the mean value. A real life illustration may serve to make this plausible. Consider two people, Tom and Hank, becoming friends in university. Tom drops out, travels the world and spends his money, living on minimum wage for the rest of his life. Hank finishes education, invests his money in a successful company and lives like a king. The two remain friends however, and so Tom may occasionally interact with Hank and benefit from this.

We see that adding the saving propensity changes the distribution significantly, just like before. However, the trends remain for different values of γ. For $\alpha = 2$, the data seems flawed for higher values of m. It is not easy to find a reason for this, other than to conclude that for all γ values, we measure roughly the same amount of agents for many of the higher wealth levels. This could of course be a result of not running the simulation for long enough. It could also be a weakness of the one-experiment model, since the measurements are all dependent on one another. It could also be some other computational mistake, leading to many different, more or less unique wealth levels with low probability.

What we *do* see, is that for $\alpha = 2$, we once again seem to get Pareto tails, this time with exponents roughly between -1.7 and -4. The linear regressions are shown in figure 8. Note that once again, the cutoff for the tail is manually approximated, and so the values for the Pareto exponent are not to be taken as exact. Also, the chaotic end of the tail is omitted from the regression.

6.5 Discussion of Taxation Models

Although our implementation of taxation in the model should probably be refined, there are some trends to be seen in figures 9 and 10. For the VAT model, increased tax makes the distribution peak move towards higher values, as one would expect. The goal of taxation is to reduce differences, and we see that this is the case - in addition to the peak moving, we also see that the tail goes more quickly to zero.

Relating to real world, this is exactly what the Democratic presidential candidate, Andrew Yang, is running for with the 2020 campaign. Yang is suggesting a 10% VAT and a uniformly distributed payment and is as described in the Methods section 3.5 the easiest way to implement. This is true for the implementation in real life as well as one would not need the bureaucracy of deciding who is eligible and who is not. This is what is often called a Universal Basic Income, UBI, and Yang is suggesting $1000 monthly payment for all agents over year 18. Everyone that wants gets it, no question asked. A VAT would tax the heavy spenders most and benefit the poorest in the society and seems like a good suggestion for the economy when considering the coming of artificial intelligent and robots.

The Wealth Tax model yields dramatic results, where every agent retains the same amount of money at equilibrium as they did initially. The effect is more pronounced for higher values of tax percentage t, but basically, the effect is the same even for a seemingly small tax percentage (note the values on the x-axis). This could be interpreted as extreme socialism, or even communism. A wealth tax is what the candidates Bernie Sanders and Elisabeth Warren are proposing, but as shown in other countries the wealth tax is easily avoidable, as rich agents easily can evade the wealth tax only by investing instead of holding money in an account.

One challenge of the Wealth Tax model is to choose when to levy taxes. We have chosen to levy a tax every time step, and this causes every agent to pay significantly more often than they make a transaction with another agent. This gives much time to move an agent towards the centre of the distribution between the times he makes a transaction, and could explain why the results are so similar for different t-values. Maybe the model would be less dramatic if the time span between tax payments was greater, for example 500 or 1000 cycles. One could also envision a model were all of the tax was paid at the very end of an experiment. Or, one could let the agents pay a wealth tax every time they perform a transaction.

There are many ways implement tax payment in the model. Another interesting aspect is the case of redistribution of tax payers' money. In both our models, tax is redistributed uniformly. But realistically, one should think that poor people tend to benefit more from welfare than rich people. It would therefore be interesting to look at a model where agents of less wealth receive a larger percentage of the tax money than rich people. Most likely, this would yield further "equalization", were the peak of figure 9 moves even further to the right. Also, one should implement the taxation models with the other models presented here, to try and make a more complete picture.

7 Conclusion

In this article, we have discussed different models for a closed economic system, namely the DY model, the Savings model, the Nearest Neighbour model and the Former Transactions model, in addition to two rough taxation models. We have seen that the simplest model yields a wealth distribution similar to the Gibbs distribution. Adding the parameters for nearest neighbours and former transactions adjusts the distribution slightly. In short, nearest neighbours seems to take the distribution towards a Pareto distribution, with more poor agents, more rich agents, and fewer agents in between. In contrast, adding significance to former transactions leads to a distribution with slightly less agents with extreme wealth values, and more agents in between.

The most drastic change to the distribution however, occurs when we add either a saving propensity or a taxation model. This significantly reduces the amount of both poor and rich agents, whilst increasing the amount of agents closer to the mean. The greater the propensity, the more concentrated around the mean the distribution gets. Inspired by Patriarca (2004), we have shown that the resulting distribution takes the form of a gamma-distribution. For the tax model, we have indicated that the distributions tend towards a higher mean value and smaller extreme values.

Further research could involve trying to find a sensible scaling method for the probability models introduced by Goswami & Sen (2014). One could also look closer at the taxation models and try to implement these in more sophisticated ways. In addition, it would make sense to investigate possible reasons for the "complete mess" in the plots for Former Transactions and $\alpha = 2$.

8 Rererences

All code files, plots and result files can be found in the GitHub-repository at: https://github.com/HolyWaters95/FYS4150_Project5 (Note that the Python files for plotting are a slight mess. It is not elegant, but it gets the job done. Mainly, `Pro5_Plotting.py` and `Py_Functions.py` have been used, but the file `Py_Functions_Improved.py` will give you the best idea of what is going on.

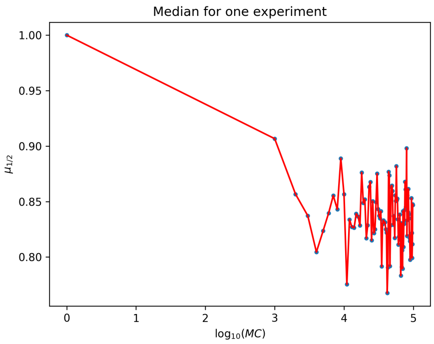
Drăgulescu, A., Yakovenko, V.M. (2000), "Statistical mechanics of money". In European Physical Journal B, Volume 17, Issue 4, 2 October 2000, Pages 723-729. Department of Physics, University of Maryland, College Park, MD 20742-4111, United States.

Goswami, Sanchari, and Parongama Sen. "Agent Based Models for Wealth Distribution with Preference in Interaction." Physica A: Statistical Mechanics and its Applications 415 (2014): 514–524.

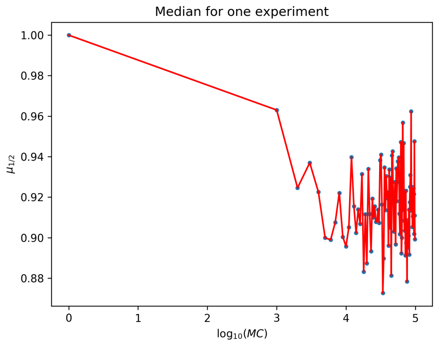
Patriarca, Marco, Anirban Chakraborti, and Kimmo Kaski. "Gibbs Versus Non-Gibbs Distributions in Money Dynamics." Physica A: Statistical Mechanics and its Applications 340.1-3 (2004): 334–339.

9 Appendix

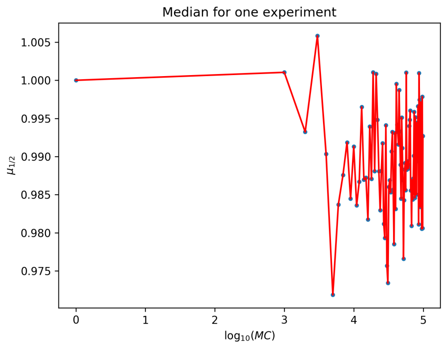
9.1 Median plots for the Savings Model



(a) $\lambda = 0.25$



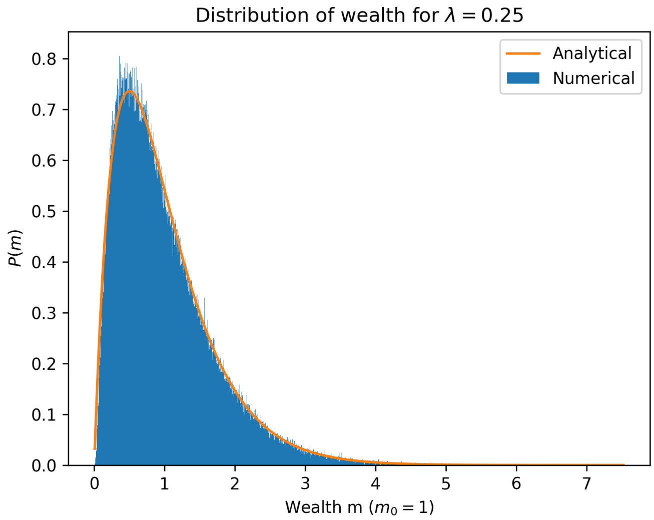
(b) $\lambda = 0.50$



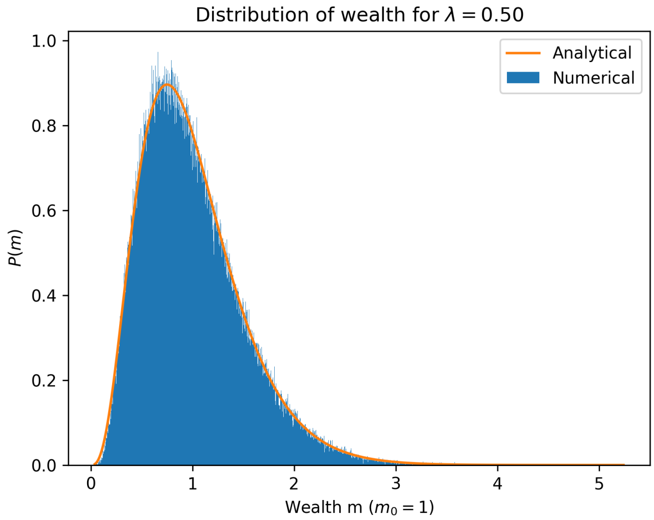
(c) $\lambda = 0.90$

Figure 11: *Time development plots of the median for different saving fractions, λ.*

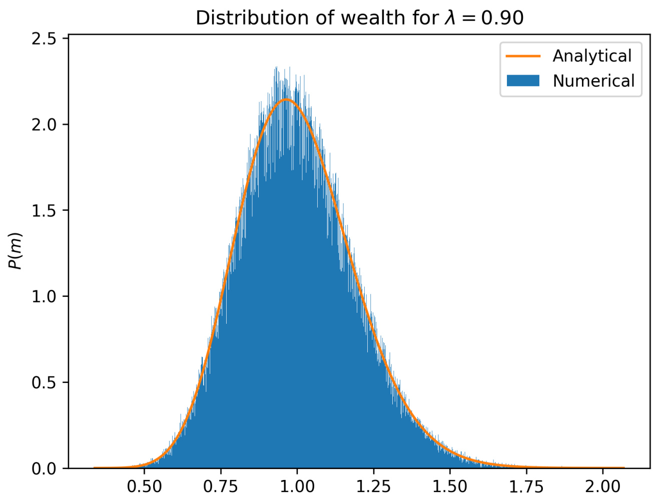
9.2 Histograms of the Savings Model



(a) $\lambda = 0.25$



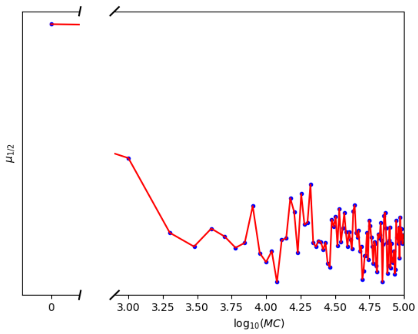
(b) $\lambda = 0.50$



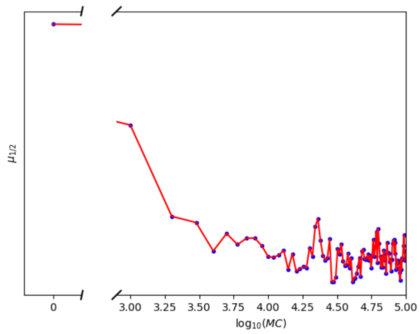
(c) $\lambda = 0.90$

Figure 12: *Time development plots of the median for different saving fractions, λ.*

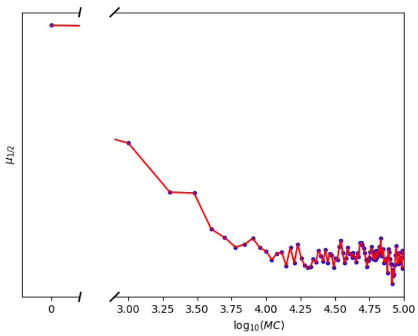
9.3 Median plots for Nearest Neighbour Model



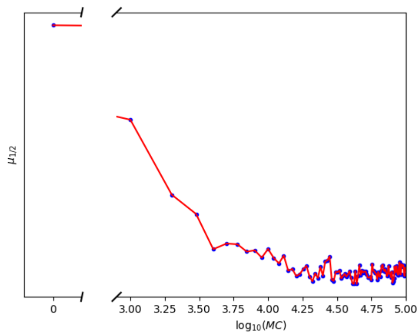
(a) $N = 500 \quad \lambda = 0.0 \quad \alpha = 0.5$



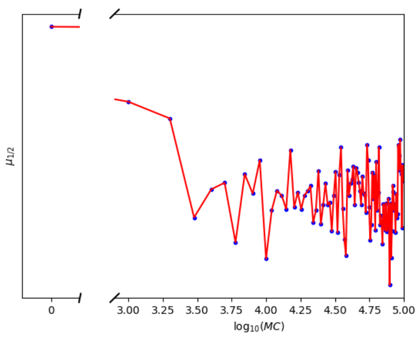
(b) $N = 500 \quad \lambda = 0.0 \quad \alpha = 1.0$



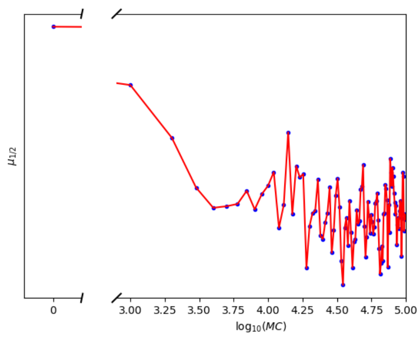
(c) $N = 500 \quad \lambda = 0.0 \quad \alpha = 1.5$



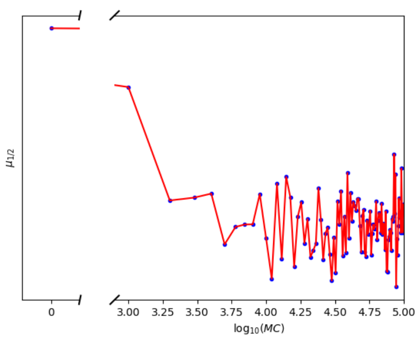
(d) $N = 500 \quad \lambda = 0.0 \quad \alpha = 2.0$



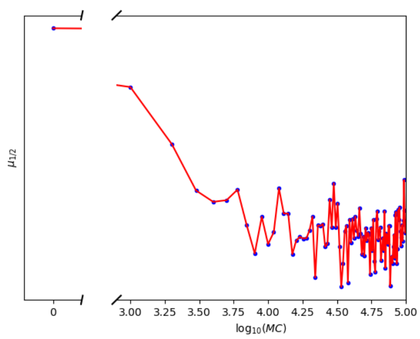
(e) $N = 500 \quad \lambda = 0.5 \quad \alpha = 0.5$



(f) $N = 500 \quad \lambda = 0.5 \quad \alpha = 1.0$

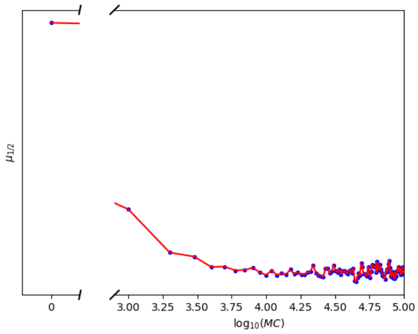


(g) $N = 500 \quad \lambda = 0.5 \quad \alpha = 1.5$

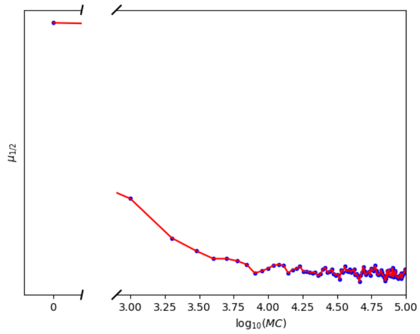


(h) $N = 500 \quad \lambda = 0.5 \quad \alpha = 2.0$

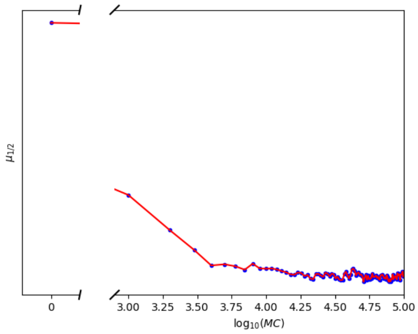
Figure 13: *Here we present the Median plots used to determine equilibrium for the Nearest Neighbour Model. It is clear that the system is in equilibrium after 10^5 cycles.*



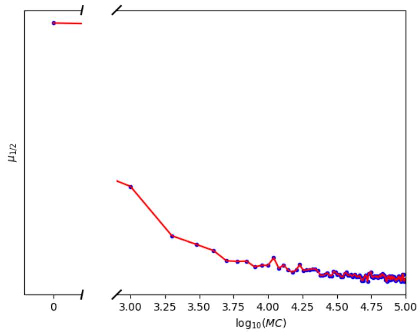
(a) $N = 1000 \quad \lambda = 0.0 \quad \alpha = 0.5$



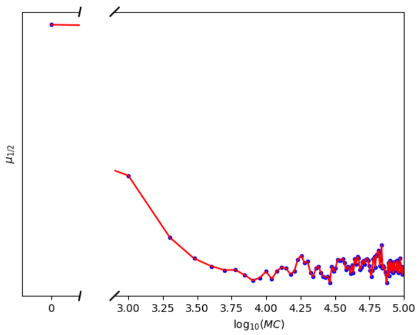
(b) $N = 1000 \quad \lambda = 0.0 \quad \alpha = 1.0$



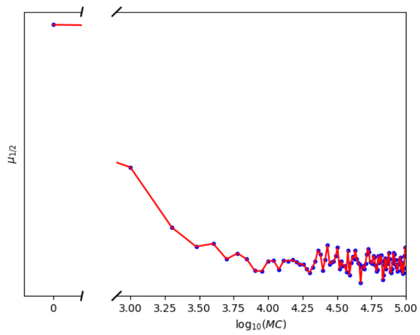
(c) $N = 1000 \quad \lambda = 0.0 \quad \alpha = 1.5$



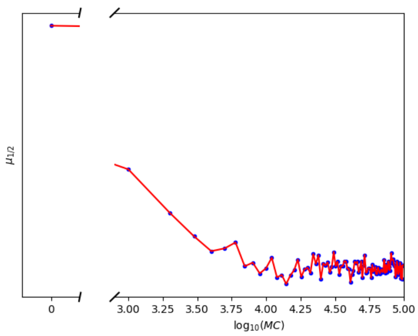
(d) $N = 1000 \quad \lambda = 0.0 \quad \alpha = 2.0$



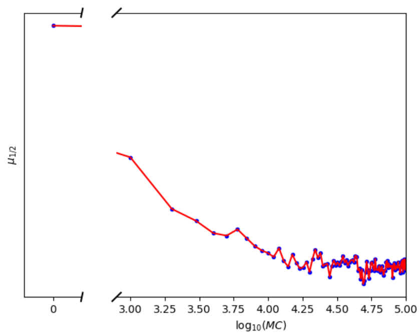
(e) $N = 1000 \quad \lambda = 0.5 \quad \alpha = 0.5$



(f) $N = 1000 \quad \lambda = 0.5 \quad \alpha = 1.0$



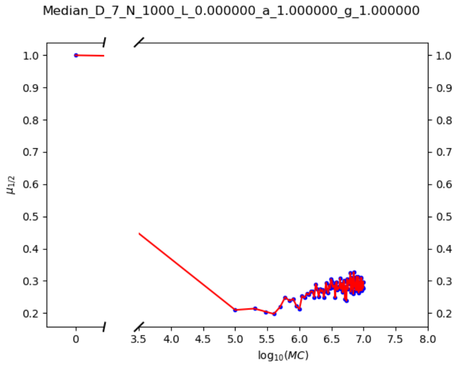
(g) $N = 1000 \quad \lambda = 0.5 \quad \alpha = 1.5$



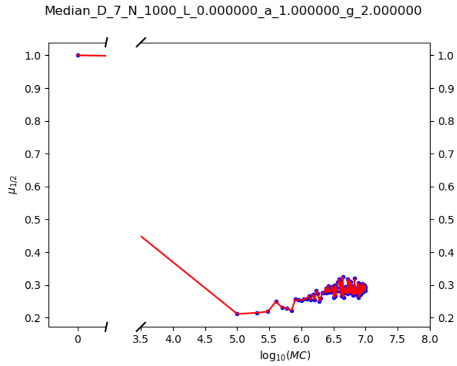
(h) $N = 1000 \quad \lambda = 0.5 \quad \alpha = 2.0$

Figure 14: *Here we present the Median plots used to determine equilibrium for the Nearest Neighbour Model. It is clear that the system is in equilibrium after* 10^5 *cycles.*

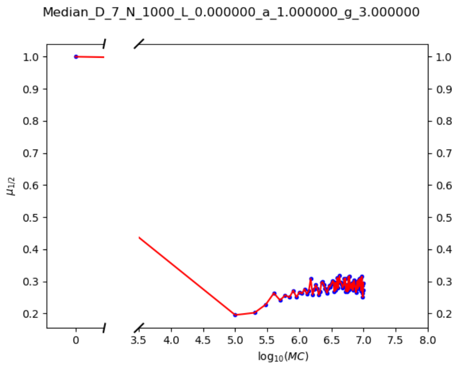
9.4 Median plots for Nearest Neighbour and Former Transactions Model



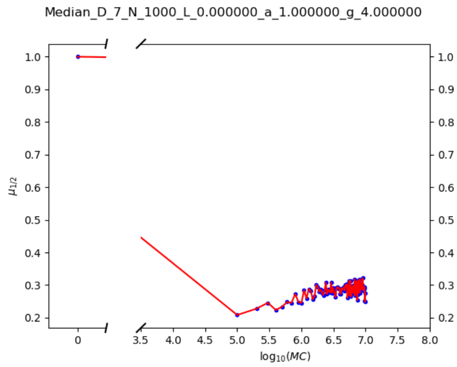
(a) $\lambda = 0.0 \quad \alpha = 1.0 \quad \gamma = 1.0$



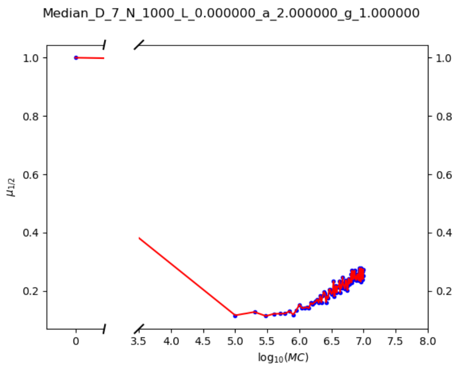
(b) $\lambda = 0.0 \quad \alpha = 1.0 \quad \gamma = 2.0$



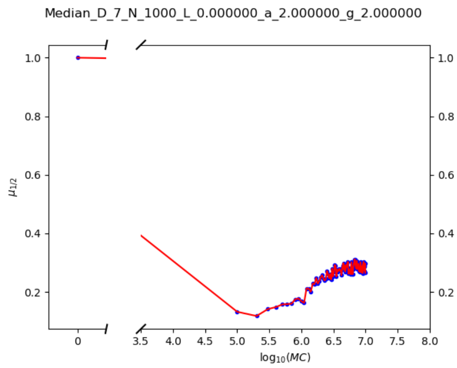
(c) $\lambda = 0.0 \quad \alpha = 1.0 \quad \gamma = 3.0$



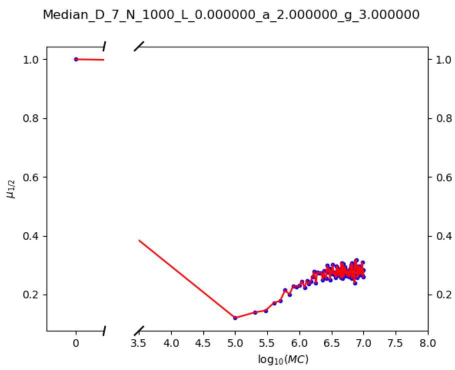
(d) $\lambda = 0.0 \quad \alpha = 1.0 \quad \gamma = 4.0$



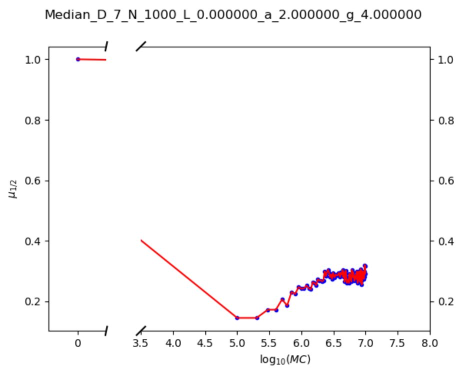
(e) $\lambda = 0.0 \quad \alpha = 2.0 \quad \gamma = 1.0$



(f) $\lambda = 0.0 \quad \alpha = 2.0 \quad \gamma = 2.0$

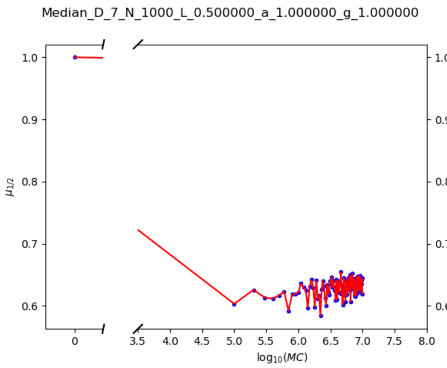


(g) $\lambda = 0.0 \quad \alpha = 2.0 \quad \gamma = 3.0$

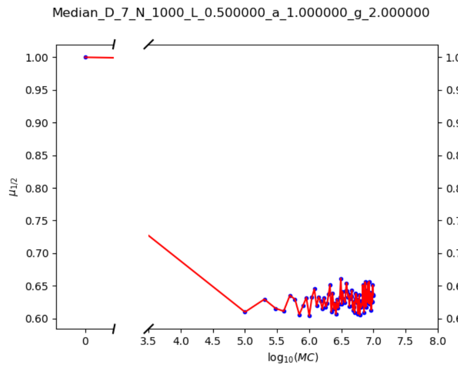


(h) $\lambda = 0.0 \quad \alpha = 2.0 \quad \gamma = 4.0$

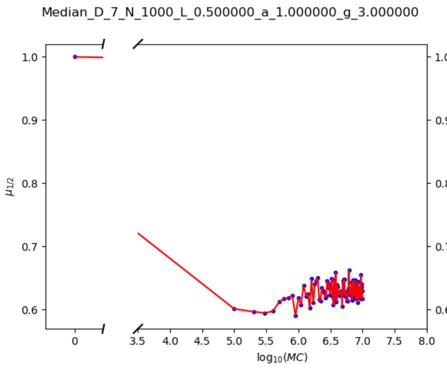
Figure 15: *Here we present the Median plots used to determine equilibrium for the Nearest Neighbour and Former Transactions Model. The system reaches equilibrium after* 10^7 *cycles.*



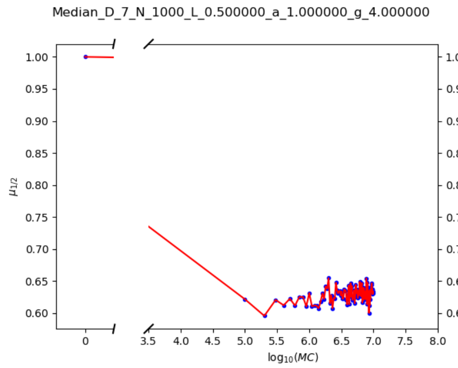
(a) $\lambda = 0.5 \quad \alpha = 1.0 \quad \gamma = 1.0$



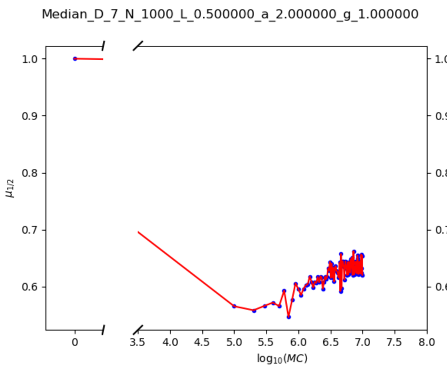
(b) $\lambda = 0.5 \quad \alpha = 1.0 \quad \gamma = 2.0$



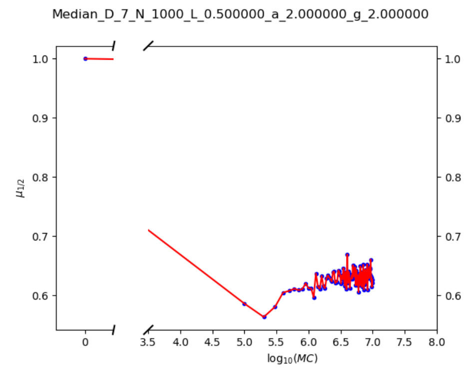
(c) $\lambda = 0.5 \quad \alpha = 1.0 \quad \gamma = 3.0$



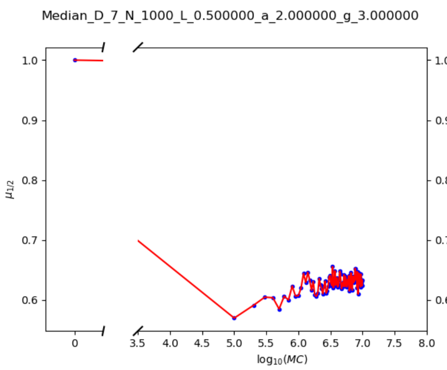
(d) $\lambda = 0.5 \quad \alpha = 1.0 \quad \gamma = 4.0$



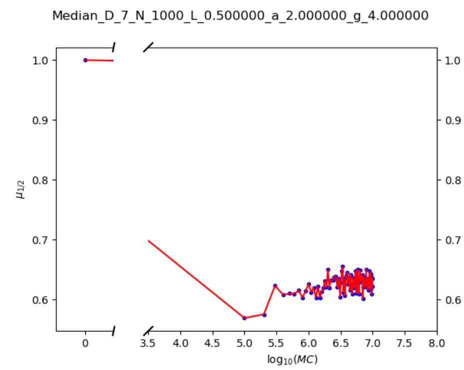
(e) $\lambda = 0.5 \quad \alpha = 2.0 \quad \gamma = 1.0$



(f) $\lambda = 0.5 \quad \alpha = 2.0 \quad \gamma = 2.0$



(g) $\lambda = 0.5 \quad \alpha = 2.0 \quad \gamma = 3.0$



(h) $\lambda = 0.5 \quad \alpha = 2.0 \quad \gamma = 4.0$

Figure 16: *Here we present the Median plots used to determine equilibrium for the Nearest Neighbour and Former Transactions Model. The system reaches equilibrium after* 10^7 *cycles.*