CD Convert Continuous Stochastic to Discrete VI

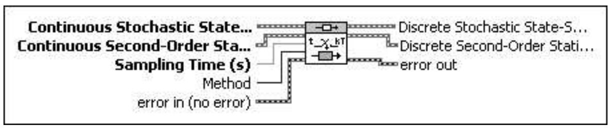
Updated 2023-03-14 | 5 minute(s) read # LabVIEW Control Design and Simulation Module # API Reference # LabVIEW G

Owning Palette: Stochastic Systems VIs

Requires: Control Design and Simulation Module

Converts a continuous stochastic state-space model and the associated continuous noise model to a discrete stochastic state-space model and discrete noise model.

Details



- **Continuous Stochastic State-Space Model** specifies a mathematical representation of a continuous stochastic system.
- **Continuous Second-Order Statistics Noise Model** specifies a continuous mathematical representation of the noise model of the **Continuous Stochastic State-Space Model**. A noise model defines the expected behavior of the noise vectors ***w*** and ***v***. You can use the CD Construct Noise Model VI to construct a noise model for a given stochastic state-space system.
- **Sampling Time (s)** specifies the discrete sampling time of the **Discrete Stochastic State-Space Model**. The default value is 1 second.
- **Method** specifies the method this VI uses to calculate the discrete equivalent of the process noise covariance matrix ***Q***.

0	**Truncation of Taylor Series—** Computes the discrete equivalent of the process noise covariance matrix by using the Truncation of Taylor Series Expansion method.
1	**Numerical Integration** (default)—Computes the discrete equivalent of the process noise covariance matrix by using the numerical integration method as proposed by Van Loan.

- **error in** describes error conditions that occur before this node runs. This input provides standard error in functionality.
- **Discrete Stochastic State-Space Model** returns a mathematical representation of the discrete equivalent of the **Continuous Stochastic State-Space Model**.
- **Discrete Second-Order Statistics Noise Model** returns a mathematical representation of the discrete equivalent of the **Continuous Second-Order Statistics Noise Model**.
- **error out** contains error information. This output provides standard error out functionality.

CD Convert Continuous Stochastic to Discrete Details

This VI assumes the noise vectors ***w*** and ***v*** are temporally uncorrelated. This VI also assumes the **Sampling Time** T you specify is much smaller than the Shannon period of the input signal $\boldsymbol{u}(t)$. Therefore, the following relationship is true:

$$\boldsymbol{u}(t) \approx \boldsymbol{u}(kT),\ kT \le t < (k+1)T$$

where t is continuous time and k is discrete time.

This VI assumes the **Continuous Stochastic State-Space Model** is of the following form:

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{G}\boldsymbol{w}(t)\\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t) + \boldsymbol{H}\boldsymbol{w}(t) + \boldsymbol{v}(t)\end{aligned}$$

This VI also assumes the **Continuous Second-Order Statistics Noise Model** is of the following form:

$$E\{\boldsymbol{w}(t)\} = \boldsymbol{m}_w(t)$$

$$E\{\boldsymbol{w}(t) \cdot \boldsymbol{w}^T(\tau)\} = \boldsymbol{Q}(t) \cdot \delta(t - \tau)$$

$$E\{\boldsymbol{v}(t)\} = \boldsymbol{m}_v(t)$$

$E\{\boldsymbol{v}(t) \cdot \boldsymbol{v}^T(\tau)\} = \boldsymbol{R}(t) \cdot \delta(t - \tau)$

$E\{\boldsymbol{w}(t) \cdot \boldsymbol{v}^T(\tau)\} = \boldsymbol{N}(t) \cdot \delta(t - \tau)$

where d(t) is the Dirac delta function. This function is defined as d(t) = ∞ when $x = 0$; d(t) = 0 when $x \neq 0$.

This VI returns the **Discrete Stochastic State-Space Model** in the following form:

$\boldsymbol{x}[(k+1)T] = \boldsymbol{A}_d\boldsymbol{x}(kT) + \boldsymbol{B}_d\boldsymbol{u}(kT) + \boldsymbol{n}(\boldsymbol{kT})$

$\boldsymbol{y}(kT) = \boldsymbol{C}_d\boldsymbol{x}(kT) + \boldsymbol{D}_d\boldsymbol{u}(kT) + \boldsymbol{r}(kT), k = 0, 1, 2 \ldots$

where

$$\begin{aligned} \boldsymbol{A}_d &= e^{\boldsymbol{A}T} \\ \boldsymbol{B}_d &= \int_0^T e^{\boldsymbol{A}\eta}\boldsymbol{B}d\eta \\ \boldsymbol{C}_d &= \boldsymbol{C} \\ \boldsymbol{D}_d &= \boldsymbol{D} \end{aligned}$$

$\boldsymbol{n}(kT)$ and $\boldsymbol{r}(kT)$ are the discrete equivalents of the noise vectors.

This VI returns the **Discrete Second-Order Statistics Noise Model** in the following form:

$E\{\boldsymbol{n}(kT)\} = [\boldsymbol{A}_d - \boldsymbol{I}]\boldsymbol{A}^{-1}\boldsymbol{G}\boldsymbol{m}_w(kT)$

$E\{\boldsymbol{n}(kT) \cdot \boldsymbol{n}^T(lT)\} = \boldsymbol{P}(kT)\delta(kT - lT), k, l = 0, 1, 2 \ldots$

$E\{\boldsymbol{r}(kT)\} = \boldsymbol{H}\boldsymbol{m}_w + \boldsymbol{m}_v(kT)$

$E\{\boldsymbol{r}(kT) \cdot \boldsymbol{r}^T(lT)\} = \boldsymbol{S}(kT)\delta(kT - lT)$

$E\{\boldsymbol{n}(kT) \cdot \boldsymbol{r}^T(lT)\} = \boldsymbol{O}(kT)\delta(kT - lT)$

where

$$\begin{aligned} \boldsymbol{S}(kT) &= \tfrac{1}{T}[\boldsymbol{H}\boldsymbol{Q}(kT)\boldsymbol{H}^T + \boldsymbol{H}\boldsymbol{N}(kT) + \boldsymbol{N}^T(kT)\boldsymbol{H}^T + \boldsymbol{R}(kT)] \\ \boldsymbol{O}(kT) &= \tfrac{1}{T}[\boldsymbol{A}_d - \boldsymbol{I}]\boldsymbol{A}^{-1}\boldsymbol{G}[\boldsymbol{Q}(kT)\boldsymbol{H}^T + \boldsymbol{N}(kT)] \end{aligned}$$

If you specify **Numerical Integration** for the **Method** parameter, then

$$\boldsymbol{P}(kT) = \int_0^T e^{\boldsymbol{A}\eta}\boldsymbol{G}\boldsymbol{Q}\boldsymbol{G}^T e^{\boldsymbol{A}^T\eta}d\eta$$

If you specify **Truncation of TSE** for the **Method** parameter, then

$$\boldsymbol{P}(kT) \approx \int_0^T \boldsymbol{G}\boldsymbol{Q}\boldsymbol{G}^T d\eta = \boldsymbol{G}\boldsymbol{Q}\boldsymbol{G}^T \cdot T$$

where	n is the number of states
	m is the number of inputs
	r is the number of outputs
	$\boldsymbol{x}$ is the state vector.
	$\boldsymbol{u}$ is the input vector.
	$\boldsymbol{y}$ is the output vector.
	$\boldsymbol{w}$ is the process noise vector.
	$\boldsymbol{v}$ is the observation noise vector.
	A is an $n \times n$ state matrix of the given system.
	B is an $n \times m$ input matrix of the given system.
	C is an $r \times n$ output matrix of the given system.
	D is an $r \times m$ direct transmission matrix of the given system.
	G is a matrix relating **w** to the states.

H is a matrix relating **w** to the outputs.

Q is the auto-covariance matrix of **w**.

R is the auto-covariance matrix of **v**.

N is the cross-covariance matrix between **w** and **v**.

E{} denotes the expected mean or value of the enclosed term(s).

This approximation becomes less accurate as the value of T increases.