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Log likelihood

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XXX

JIS

Input estimation:

$$\bar{R}_k = G P^x_{k|k-1} G^{\mathrm{T}} + R$$

$$M_k = (J^{\mathrm{T}} \bar{R}_k^{-1} J)^{-1} J^{\mathrm{T}} \bar{R}_k^{-1}$$

$$\hat{p}_{k|k} = M_k (y_k - G\hat{x}_{k|k-1})$$

$$P^p_{k|k} = (J^{\mathrm{T}} \bar{R}_k^{-1} J)^{-1}$$

Measurement update:

$$K_k = P^x_{k|k-1} G^{\mathrm{T}} \bar{R}_k^{-1}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (y_k - G\hat{x}_{k|k-1} - J\hat{p}_k)$$

$$P^x_{k|k} = P^x_{k|k-1} - K_k (\bar{R}_k - J P^p_{k|k} J^{\mathrm{T}}) K_k^{\mathrm{T}}$$

$$P^{xp}_{k|k} = -K_k J P^p_{k|k}$$

Time update:

$$\bar{S}_k = A P_{x,k|k-1} G^{\mathrm{T}} + S$$

$$\bar{K}_k = \bar{S}_k \bar{R}_k^{-1} + (B - \bar{S}_k \bar{R}_k^{-1} J) M_k$$

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k-1} + \bar{K}_k (y_k - G\hat{x}_{k|k-1})$$

$$P^x_{k+1|k} = A P^x_{k|k-1} A^{\mathrm{T}} + Q - \bar{S}_k \bar{K}_k^{\mathrm{T}} - \bar{K}_k \bar{S}_k^{T} + \bar{K}_k \bar{R}_k \bar{K}_k^{\mathrm{T}}$$

log(L)

Joint input and state estimation:

$$\hat{p}_{k|k} = M_k(y_k - G\hat{x}_{k|k-1})$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(y_k - G\hat{x}_{k|k-1} - J\hat{p}_{k|k})$$

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k-1} + \bar{K}_k(y_k - G\hat{x}_{k|k-1})$$

$$\Omega_k = \begin{bmatrix} G & J \end{bmatrix} \mathrm{cov}\left(\begin{bmatrix} x_k - \hat{x}_{k|k-1} \\ p_k - \hat{p}_{k|k} \end{bmatrix}\right) \begin{bmatrix} G & J \end{bmatrix}^{\mathrm{T}} + R$$

Residual sequence:

$$i_k = y_k - G\hat{x}_{k|k-1} - J\hat{p}_{k|k} = y_k - \begin{bmatrix} G & J \end{bmatrix} \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{p}_{k|k} \end{bmatrix} = \begin{bmatrix} G & J & I \end{bmatrix} \begin{bmatrix} x_k - \hat{x}_{k|k-1} \\ p_k - \hat{p}_{k|k} \\ v_k \end{bmatrix}$$

Log likelihood:

$$\log(L) = \sum_k \log(|\Omega_k|) + i_k^{\mathrm{T}} \Omega_k^{-1} i_k$$

$$\Omega_k = \begin{bmatrix} G & J & I \end{bmatrix} \mathrm{cov}\left(\begin{bmatrix} x_k - \hat{x}_{k|k-1} \\ p_k - \hat{p}_{k|k} \\ v_k \end{bmatrix}\right) \begin{bmatrix} G & J & I \end{bmatrix}^{\mathrm{T}}$$

$$\mathrm{cov}\left(\begin{bmatrix} x_k - \hat{x}_{k|k-1} \\ p_k - \hat{p}_{k|k} \\ v_k \end{bmatrix}\right) = \mathrm{E}\left[\begin{bmatrix} \tilde{x}_{k|k-1}\tilde{x}_{k|k-1}^{\mathrm{T}} & \tilde{x}_{k|k-1}\tilde{p}_{k|k}^{\mathrm{T}} & \tilde{x}_{k|k-1}v_k^{\mathrm{T}} \\ \tilde{p}_{k|k}\tilde{x}_{k|k-1}^{\mathrm{T}} & \tilde{p}_{k|k}\tilde{p}_{k|k}^{\mathrm{T}} & \tilde{p}_{k|k}v_k^{\mathrm{T}} \\ v_k\tilde{x}_{k|k-1}^{\mathrm{T}} & v_k\tilde{p}_{k|k}^{\mathrm{T}} & v_kv_k^{\mathrm{T}} \end{bmatrix}\right] = \begin{bmatrix} P^x_{k|k-1} & E[\tilde{x}_{k|k-1}\tilde{p}_{k|k}^{\mathrm{T}}] & 0 \\ E[\tilde{p}_{k|k}\tilde{x}_{k|k-1}^{\mathrm{T}}] & P^p_{k|k-1} & E[\tilde{p}_{k|k}v_k^{\mathrm{T}}] \\ 0 & E[v_k\tilde{p}_{k|k}^{\mathrm{T}}] & R \end{bmatrix}$$

Two cross-covariance terms need be derived

Derivation covariance

Rewrite measurement update for state:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(y_k - G\hat{x}_{k|k-1}J\hat{p}_k) = \hat{x}_{k|k-1} + K_k(y_k - G\hat{x}_{k|k-1}) - K_k J\hat{p}_k = \hat{x}_{k|k-1} + K_k(y_k - G\hat{x}_{k|k-1}) - K_k JM_k(y_k - G\hat{x}_{k|k-1})$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(I - JM_k)(y_k - G\hat{x}_{k|k-1})$$

Derive a relation between the state prediction error and state filter error:

$$\tilde{x}_{k|k} = x_k - \hat{x}_{k|k} = x_k - \hat{x}_{k|k-1} - K_k(I - JM_k)(Gx_k + Jp_k + v_k - G\hat{x}_{k|k-1})$$

$$\tilde{x}_{k|k} = \tilde{x}_{k|k-1} - K_k(I - JM_k)G\tilde{x}_{k|k-1} - K_k(I - JM_k)Gv_k - K_k(I - JM_k)Jp_k$$

The matrix term in front of the force above is zero (Gillijns)

We then rearrange:

$$\tilde{x}_{k|k} = (I - K_k(I - JM_k)G)\tilde{x}_{k|k-1} - K_k(I - JM_k)v_k$$

$$\tilde{x}_{k|k} + K_k(I - JM_k)v_k = (I - K_k(I - JM_k)G)\tilde{x}_{k|k-1}$$

Derivation covariance

Multiply the equation by the input error transposed:

$$\tilde{x}_{k|k}\tilde{p}_{k|k}{}^{\mathrm{T}} + K_k(I - JM_k)v_k\tilde{p}_{k|k}{}^{\mathrm{T}} = (I - K_k(I - JM_k)G)\tilde{x}_{k|k-1}\tilde{p}_{k|k}{}^{\mathrm{T}}$$

We use that (Gillijns):

$$\tilde{p}_{k|k} = -M_k(G\tilde{x}_{k|k-1} + v_k)$$

We find the first term of interest:

$$E\left[v_k\tilde{p}_{k|k}{}^{\mathrm{T}}\right] = -RM_k{}^{\mathrm{T}}$$

And the second term:

$$E\left[\tilde{x}_{k|k-1}\tilde{p}_{k|k}{}^{\mathrm{T}}\right] = (I - K_k(I - JM_k)G)^{-1}\left(P_{k|k}^{xp} - K_k(I - JM_k)RM_k{}^{\mathrm{T}}\right)$$

Not used

Next we define:

$$\tilde{y}_k = y_k - G\hat{x}_{k|k-1} = Gx_k + Jp_k + v_k - G\hat{x}_{k|k-1} = G\tilde{x}_{k|k-1} + v_k + Jp_k = e_k + Jp_k, \quad e_k = G\tilde{x}_{k|k-1} + v_k$$