

LIS - Longest increasing subsequence

Array's member	29	6	14	31	39	78	63	50	13	65	61	62	19	64	20	70	43	84	35	98
Index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19

LIS = {6, 14, 31, 39, 50, 61, 62, 64, 70, 84, 98}

$$A_{i+1} > A_i$$

Problem: Given sequence $X = \langle x_1, x_2, \dots, x_n \rangle$,
find the **longest subsequence**
 $Z = \langle z_1, z_2, \dots, z_k \rangle$ that is $z_{i+1} > z_i$.

A subsequence is a subset of elements from the sequence with **strictly increasing** order (**not necessarily contiguous**).

1. Dynamic Programming

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LIS dynamic programming (length of subsequence).

29	6	14	31	39	78	63	50	13	65	61	62	19	64	20	70	43	84	35	98
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29 index 0

29	6	14	31	39	78	63	50	13	65	61	62	19	64	20	70	43	84	35	98
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6 index 0

29	6	14	31	39	78	63	50	13	65	61	62	19	64	20	70	43	84	35	98
----	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

6 14 index 1

29	6	14	31	39	78	63	50	13	65	61	62	19	64	20	70	43	84	35	98
----	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

6 14 31

index 2

29	6	14	31	39	78	63	50	13	65	61	62	19	64	20	70	43	84	35	98
----	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

6 14 31 39

index 3

29	6	14	31	39	78	63	50	13	65	61	62	19	64	20	70	43	84	35	98
----	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

6 14 31 39 78

index 4

29	6	14	31	39	78	63	50	13	65	61	62	19	64	20	70	43	84	35	98
----	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

6 14 31 39 63

index 4 $39 < 63 < 78$

29	6	14	31	39	78	63	50	13	65	61	62	19	64	20	70	43	84	35	98
----	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

6 14 31 39 50

index 4 $39 < 50 < 63$

29	6	14	31	39	78	63	50	13	65	61	62	19	64	20	70	43	84	35	98
----	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

6 13 31 39 50

index 1

29	6	14	31	39	78	63	50	13	65	61	62	19	64	20	70	43	84	35	98
----	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

6 13 31 39 50 65

index 5

29	6	14	31	39	78	63	50	13	65	61	62	19	64	20	70	43	84	35	98
----	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

6 13 31 39 50 61

index 5 $50 < 61 < 65$

29	6	14	31	39	78	63	50	13	65	61	62	19	64	20	70	43	84	35	98
----	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

6 13 31 39 50 61 62

index 6

29	6	14	31	39	78	63	50	13	65	61	62	19	64	20	70	43	84	35	98
----	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

6 13 19 39 50 61 62

index 2

29	6	14	31	39	78	63	50	13	65	61	62	19	64	20	70	43	84	35	98
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6 13 19 39 50 61 62 64

index 7

29	6	14	31	39	78	63	50	13	65	61	62	19	64	20	70	43	84	35	98
----	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

6 13 19 20 50 61 62 64

index 3

29	6	14	31	39	78	63	50	13	65	61	62	19	64	20	70	43	84	35	98
----	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

6 13 19 20 50 61 62 64 70

index 8

29	6	14	31	39	78	63	50	13	65	61	62	19	64	20	70	43	84	35	98
----	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

6 13 19 20 **43** 61 62 64 70

index 4

29	6	14	31	39	78	63	50	13	65	61	62	19	64	20	70	43	84	35	98
----	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	-----------	----	----

6 13 19 20 43 61 62 64 70 **84**

index 9

29	6	14	31	39	78	63	50	13	65	61	62	19	64	20	70	43	84	35	98
----	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	-----------	----

6 13 19 20 **35** 61 62 64 70 84

index 4

29	6	14	31	39	78	63	50	13	65	61	62	19	64	20	70	43	84	35	98
----	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	-----------

6 13 19 20 35 61 62 64 70 84 **98**

index 10

6 **13 19 20 35** 61 62 64 70 84 **98**

6, 14, 31, 39, 50, 61, 62, 64, 70, 84, 98

LIS dynamic programming

6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 13, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 13, 19, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 13, 19, 20, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 13, 19, 20, 35, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 50, 61, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 50, 61, 62, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 50, 61, 62, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 50, 61, 62, 64, 70, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 50, 61, 62, 64, 70, 84, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 50, 61, 62, 64, 70, 84, 98, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,

6, 14, 31, 39, 50, 61, 62, 64, 70, 84, 98

int binarySearchBetweenArr(**int** []arr, **int** end, **int** value)

Pseudocode:

```
low = 0
high = end
if (value < arr[0])           return 0
if (value > arr[end])        return end+1

Loop ( low <= high )
  loop begin
  middle = (low + high)/2
  if (low == high)
    return low
  else
    if ( arr[middle] == value)
      return middle
    if ( arr[middle] < value)
      high = middle
    else
      low = middle+1
  loop end
return -1
```


int binarySearchBetweenMat(**int** [][]arr, **int** end, **int** value)

Pseudocode:

```
low = 0
high = end
if (value < arr[0][0])           return 0
if (value > arr[end][end])       return end+1

Loop ( low <= high )
    loop begin
        middle = (low + high)/2
        if (low == high)
            return low
        else
            if ( arr[middle][middle] == value)
                return middle
            if ( arr[middle][middle] > value)
                high = middle
            else
                low = middle+1
```

```
    loop end  
    return -1
```

```
int LISLength(int [] arr)
```

Pseudocode:

```
    size = length of arr
```

```
    array d[size]
```

```
    d[0] = arr[0]
```

```
    end = 0
```

```
    Loop ( i from 1 to size step 1 )
```

```
        loop begin
```

```
            index = binarySearchBetweenArr(d, end, arr[i])
```

```
            d[index] = arr[i]
```

```
            if (index > end)
```

```
                end = end + 1
```

```
            loop end
```

```
    return end+1
```

```
int[] LIS2(int [] arr)
```

Pseudocode:

size = length of arr

array mat[size][size]

mat[0][0] = arr[0]

end = 0

Loop (i from 1 to size step 1)

loop begin

index = *binarySearchBetweenMat*(d, end, arr[i])

mat[index][index] = arr[i]

Loop (j from 0 to index step 1)

loop begin

mat[index][j] = mat[index-1][j]

loop end

mat[index][index] = arr[i]

if (index > end)

```
        end = end + 1
    loop end
    array ans[end+1]
    Loop ( j from 0 to index step 1 )
        loop begin
            ans[j] = mat[end][j]
        return ans
```

[6, 14, 31, 39, 50, 61, 62, 64, 70, 84, 98]

[29, 6, 14, 31, 39, 78, 63, 50, 13, 65, 61, 62, 19, 64, 20, 70, 43, 84, 35, 98]

public static int[] LIS2(**int** [] arr)

pseudocode :

size = arr.length

mat [size][size]

end = 0

Loop (i from 1 to size -1 step 1)

index = *binarySearchBetweenMat*(mat, end, arr[i])

mat[index][index] = arr[i];

Loop (j from 0 to index step 1)

mat[index][j]=mat[index-1][j]

loop end

```
if (index > end )
```

```
    end = end + 1
```

```
loop end
```

```
ans[end+1]
```

```
Build ans
```


index = 0	mat[0][0]	= 6
index = 1	mat[1][1]	= 14
index = 2	mat[2][2]	= 31
index = 3	mat[3][3]	= 39
index = 4	mat[4][4]	= 78
index = 4	mat[4][4]	= 63
index = 4	mat[4][4]	= 50
index = 1	mat[1][1]	= 13
index = 5	mat[5][5]	= 65
index = 5	mat[5][5]	= 61
index = 6	mat[6][6]	= 62
index = 2	mat[2][2]	= 19
index = 7	mat[7][7]	= 64
index = 3	mat[3][3]	= 20
index = 8	mat[8][8]	= 70
index = 4	mat[4][4]	= 43
index = 9	mat[9][9]	= 84
index = 4	mat[4][4]	= 35
index = 10	mat[10][10]	= 98

index = 0 mat[0][0] = 29

mat.length = 20

end = 0

6, 0,
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index = 1 mat[1][1] = 14

end = 1

mat.length = 20

6, 0,
6, 14, 0,
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0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,

index = 2 mat[2][2] = 31

end = 2

mat.length = 20

6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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index = 3 mat[3][3] = 39

end = 3

mat.length = 20

6, 0,
6, 14, 0,
6, 14, 31, 0,
6, 14, 31, 39, 0,
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index = 4 mat[4][4] = 78

end = 4

mat.length = 20

6, 0,
6, 14, 0,
6, 14, 31, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 78, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,

index = 4 mat[4][4] = 63

mat.length = 20

6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 63, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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index = 4 mat[4][4] = 50

mat.length = 20

6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 50, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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index = 1 mat[1][1] = 13

mat.length = 20

6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **13**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,

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index = 5 mat[5][5] = 65

end = 5

mat.length = 20

6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 13, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 50, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 50, 65, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,

index = 5 mat[5][5] = 61

mat.length = 20

6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **13**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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index = 6 mat[6][6] = 62

end = 6

mat.length = 20

6, 0,
6, **13**, 0,
6, **14**, **31**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, **62**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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index = 2 mat[2][2] = 19

mat.length = 20

6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **13**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **13**, **19**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, **62**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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index = 7 mat[7][7] = 64

end = 7

mat.length = 20

6, 0,
6, **13**, 0,
6, **13**, **19**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, **62**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, **62**, **64**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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index = 3 mat[3][3] = 20

mat.length = 20

6, 0,
6, **13**, 0,
6, **13**, **19**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **13**, **19**, **20**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, **62**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, **62**, **64**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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index = 8 mat[8][8] = 70

end = 8

mat.length = 20

6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **13**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **13**, **19**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **13**, **19**, **20**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, **62**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, **62**, **64**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, **62**, **64**, **70**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,

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0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,

index = 4 mat[4][4] = 43

mat.length = 20

6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **13**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **13**, **19**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **13**, **19**, **20**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **13**, **19**, **20**, **43**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, **62**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, **62**, **64**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, **62**, **64**, **70**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,

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index = 9 mat[9][9] = 84

end = 9

mat.length = 20

6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **13**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **13**, **19**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **13**, **19**, **20**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **13**, **19**, **20**, **43**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, **62**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, **14**, **31**, **39**, **50**, **61**, **62**, **64**, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,

[illegible]

```
index = 4  mat[4][4] = 35
mat.length = 20
```

6, 0,
6, 13, 0,
6, 13, 19, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 13, 19, 20, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 13, 19, 20, 35, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 50, 61, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 50, 61, 62, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 50, 61, 62, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,

6, 14, 31, 39, 50, 61, 62, 64, 70, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 50, 61, 62, 64, 70, 84, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,

index = 10 mat[10][10] = 98

end = 10

mat.length = 20

```
6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 13, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 13, 19, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 13, 19, 20, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 13, 19, 20, 35, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 50, 61, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 50, 61, 62, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 50, 61, 62, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 50, 61, 62, 64, 70, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 50, 61, 62, 64, 70, 84, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 14, 31, 39, 50, 61, 62, 64, 70, 84, 98, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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RESULT :

6, 14, 31, 39, 50, 61, 62, 64, 70, 84, 98,
[6, 14, 31, 39, 50, 61, 62, 64, 70, 84, 98]