

Fibonacci algorithm - $O(\log n)$

It is not difficult to prove the following matrix equality

$$\begin{pmatrix} F_{n-2} \\ F_{n-1} \end{pmatrix} * \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} F_{n-1} \\ F_n \end{pmatrix}$$

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$$F_{n-1} = F_{n-2} * 0 + F_{n-1} * 1 = F_{n-1}$$

$$F_n = F_{n-2} * 1 + F_{n-1} * 1 = F_{n-2} + F_{n-1} = F_n$$

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The matrix representation gives the following closed expression for the Fibonacci numbers:

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n = \begin{pmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{pmatrix}.$$