LCS dynamic programming

X = "abcbdab" X {1, 2, 3, 2, 4, 1, 2}

Y = "adcaba" Y {1, 4, 3, 1, 2, 1}

a	1
b	2
C	3
d	4

Length of X - 7 (Lx)
Length of Y - 6 (Ly)

Build matrix M[Lx+1][Ly+1]

Formula הנוסחה

$$M[i][j]=\begin{cases} 0 & \text{if } i = 0 \text{ or } j = 0 \\ M[i-1][j-1] + 1 & \text{if } X[i-1]=Y[j-1] \\ \max(M[i-1][j],M[i][j-1]) & \text{if } X[i-1]\neq Y[j-1] \end{cases}$$

		Y	1	4	3	1	2	1
	index	0	1	2	3	4	5	6
X	0	0	0	0	0	0	0	0
1	1	0						
2	2	0						
3	3	0						
2	4	0						
4	5	0		M[i-1,j-1]	M[i-1,j]			
1	6	0		M[i][j-1]	M[i,j]			
2	7	0						

$$M[i][j]=\begin{cases} M[i-1][j-1]+1 & \text{if } X[i-1]=Y[j-1] \end{cases}$$

		Y	1	4	3	1	2	1
	index	0	1	2	3	4	5	6
X	0	0	0	0	0	0	0	0
1	1	0						
2	2	0						
3	3	0						
2	4	0						
4	5	0						
1	6	0		M[i-1,j-1]	M[i-1,j]			
2	7	0		M[i][j-1]	M[i,j]			

$$M[i][j]=\begin{cases} \\ \\ \\ \\ \\ \\ \\ \\ \max(M[i-1][j],M[i][j-1]) \quad \text{if } X[i-1]\neq Y[j-1] \end{cases}$$

	Y	1	4	3	1	2	1
X	0,	0,	0,	0,	0,	0,	0
1	0,	1,	1,	1,	1,	1,	1
2	0,	1,					
3	0,	1,					
2	0,	1,					
4	0,	1,					
1	0,	1,					
2	0,	1,					

	Y	1	4	3	1	2	1
X	0,	0,	0,	0,	0,	0,	0
1	0,	1,	1,	1,	1,	1,	1
2	0,	1,	1,	1,	1,	2,	2
3	0,	1,	1,				
2	0,	1,	1,				
4	0,	1,	2,				
1	0,	1,	2,				
2	0,	1,	2,				

	Y	1	4	3	1	2	1
X	0,	0,	0,	0,	0,	0,	0
1	0,	1,	1,	1,	1,	1,	1
2	0,	1,	1,	1,	1,	2,	2
3	0,	1,	1,	2,	2,	2,	2
2	0,	1,	1,	2,			
4	0,	1,	2,	2,			
1	0,	1,	2,	2,			
2	0,	1,	2,	2,			

	Y	1	4	3	1	2	1
X	0,	0,	0,	0,	0,	0,	0
1	0,	1,	1,	1,	1,	1,	1
2	0,	1,	1,	1,	1,	2,	2
3	0,	1,	1,	2,	2,	2,	2
2	0,	1,	1,	2,	2,	3,	3
4	0,	1,	2,	2,	2,		
1	0,	1,	2,	2,	3,		
2	0,	1,	2,	2,	3,		

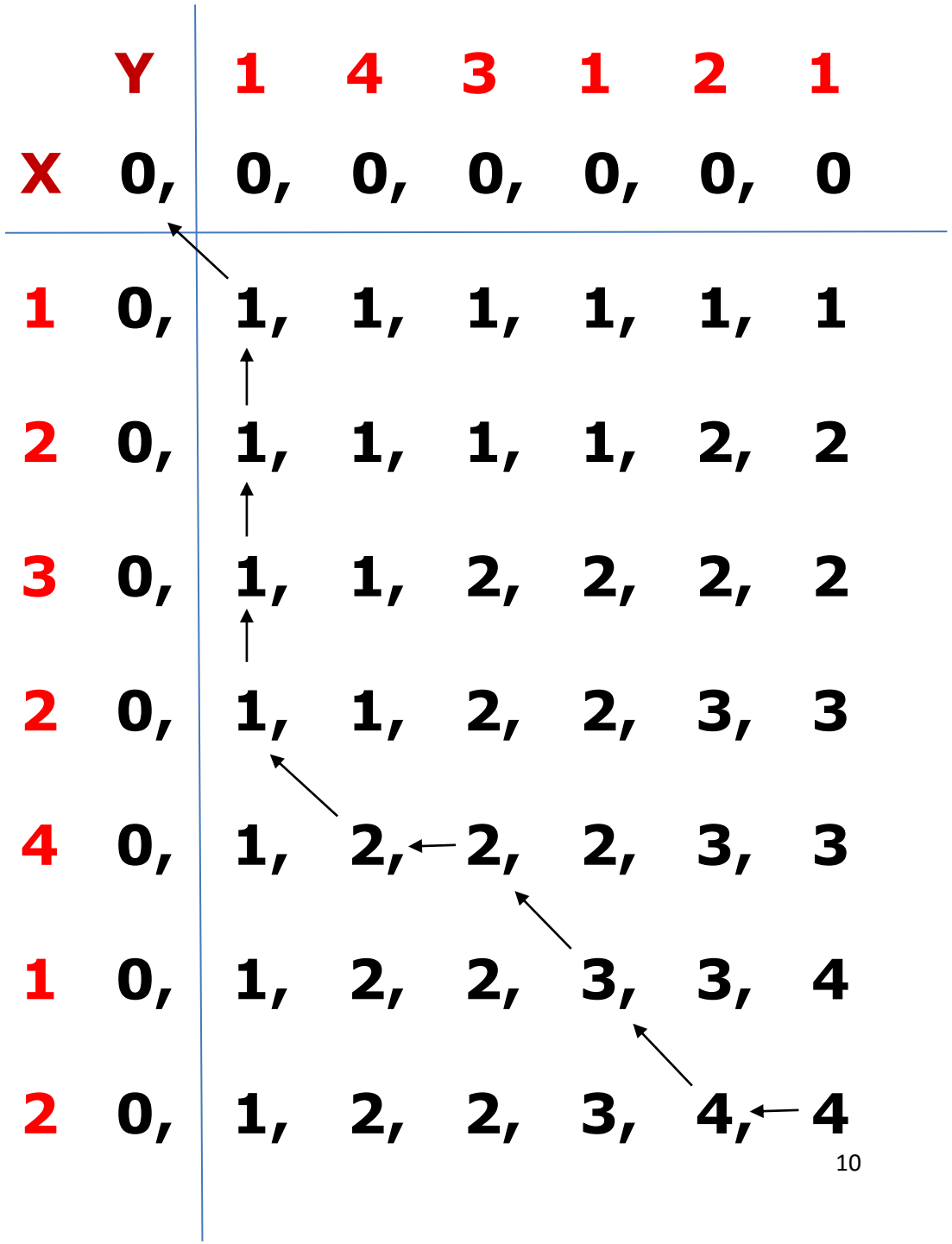
	Y	1	4	3	1	2	1
X	0,	0,	0,	0,	0,	0,	0
1	0,	1,	1,	1,	1,	1,	1
2	0,	1,	1,	1,	1,	2,	2
3	0,	1,	1,	2,	2,	2,	2
2	0,	1,	1,	2,	2,	3,	3
4	0,	1,	2,	2,	2,	3,	3
1	0,	1,	2,	2,	3,	3,	
2	0,	1,	2,	2,	3,	4,	

	Y	1	4	3	1	2	1
X	0,	0,	0,	0,	0,	0,	0
1	0,	1,	1,	1,	1,	1,	1
2	0,	1,	1,	1,	1,	2,	2
3	0,	1,	1,	2,	2,	2,	2
2	0,	1,	1,	2,	2,	3,	3
4	0,	1,	2,	2,	2,	3,	3
1	0,	1,	2,	2,	3,	3,	4
2	0,	1,	2,	2,	3,	4,	

	Y	1	4	3	1	2	1
X	0,	0,	0,	0,	0,	0,	0
1	0,	1,	1,	1,	1,	1,	1
2	0,	1,	1,	1,	1,	2,	2
3	0,	1,	1,	2,	2,	2,	2
2	0,	1,	1,	2,	2,	3,	3
4	0,	1,	2,	2,	2,	3,	3
1	0,	1,	2,	2,	3,	3,	4
2	0,	1,	2,	2,	3,	4,	4

```
i=row-1
j=col-1
count= mat[i][j]
result[] = {}

if  ( X[i-1]==Y[j-1] )
      result[count] = X[i-1]
      count--
      i--
      j--
 else
      if ( mat[i][j]==mat[i][j-1] )
            j--
      else
            i--
```


Y 1 4 3 1 2 1
X 0, 0, 0, 0, 0, 0, 0
1 0, 1, 1, 1, 1, 1, 1
2 0, 1, 1, 1, 1, 2, 2
3 0, 1, 1, 2, 2, 2, 2
2 0, 1, 1, 2, 2, 3, 3
4 0, 1, 2, 2, 2, 3, 3
1 0, 1, 2, 2, 3, 3, 4
2 0, 1, 2, 2, 3, 4, 4

[1, 4, 1, 2] → "adab"

	X	1	2	3	2	4	1	2
Y	0,	0,	0,	0,	0,	0,	0,	0
1	0,							
4	0,							
3	0,							
1	0,							
2	0,							
1	0,							

	X	1	2	3	2	4	1	2
Y	0,	0,	0,	0,	0,	0,	0,	0
1	0,	1,	1,	1,	1,	1,	1,	1
4	0,	1,						
3	0,	1,						
1	0,	1,						
2	0,	1,						

1	0,	1,
	X	1 2 3 2 4 1 2
Y	0,	0, 0, 0, 0, 0, 0, 0
1	0,	1, 1, 1, 1, 1, 1, 1
4	0,	1, 1, 1, 1, 2, 2, 2
3	0,	1, 1,
1	0,	1, 1,
2	0,	1, 2,
1	0,	1, 2,

	X	1 2 3 2 4 1 2
Y	0,	0, 0, 0, 0, 0, 0, 0
1	0,	1, 1, 1, 1, 1, 1, 1
4	0,	1, 1, 1, 1, 2, 2, 2
3	0,	1, 1, 2, 2, 2, 2, 2
1	0,	1, 1, 2,
2	0,	1, 2, 2,
1	0,	1, 2, 2,

	X	1	2	3	2	4	1	2
Y	0,	0,	0,	0,	0,	0,	0,	0
1	0,	1,	1,	1,	1,	1,	1,	1
4	0,	1,	1,	1,	1,	2,	2,	2
3	0,	1,	1,	2,	2,	2,	2,	2
1	0,	1,	1,	2,	2,	2,	3,	3
2	0,	1,	2,	2,	3			
1	0,	1,	2,	2,	3			

	X	1	2	3	2	4	1	2
Y	0,	0,	0,	0,	0,	0,	0,	0
1	0,	1,	1,	1,	1,	1,	1,	1
4	0,	1,	1,	1,	1,	2,	2,	2
3	0,	1,	1,	2,	2,	2,	2,	2
1	0,	1,	1,	2,	2,	2,	3,	3
2	0,	1,	2,	2,	3,	3,	3,	4
1	0,	1,	2,	2,	3,	3,		

	X	1	2	3	2	4	1	2
Y	0,	0,	0,	0,	0,	0,	0,	0
1	0,	1,	1,	1,	1,	1,	1,	1
4	0,	1,	1,	1,	1,	2,	2,	2
3	0,	1,	1,	2,	2,	2,	2,	2
1	0,	1,	1,	2,	2,	2,	3,	3
2	0,	1,	2,	2,	3,	3,	3,	4
1	0,	1,	2,	2,	3,	3,	4,	

	X	1	2	3	2	4	1	2
Y	0,	0,	0,	0,	0,	0,	0,	0
1	0,	1,	1,	1,	1,	1,	1,	1
4	0,	1,	1,	1,	1,	2,	2,	2
3	0,	1,	1,	2,	2,	2,	2,	2
1	0,	1,	1,	2,	2,	2,	3,	3
2	0,	1,	2,	2,	3,	3,	3,	4
1	0,	1,	2,	2,	3,	3,	4,	4


X 1 2 3 2 4 1 2
Y 0, 0, 0, 0, 0, 0, 0, 0
1 0, 1, 1, 1, 1, 1, 1, 1
4 0, 1, 1, 1, 1, 2, 2, 2
3 0, 1, 1, 2, 2, 2, 2, 2
1 0, 1, 1, 2, 2, 2, 3, 3
2 0, 1, 2, 2, 3, 3, 3, 4
1 0, 1, 2, 2, 3, 3, 4, 4

[1, 3, 2, 1] → "acba"