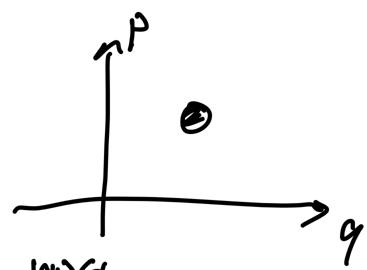
Recap: Oscillator: $H = \frac{\hat{p}^2}{2} + \frac{\hat{q}^2}{2} = a^+ a + \frac{1}{2}$ $\quad \Big| \quad a = \frac{q + ip}{\sqrt{2}}$

$$[\hat{q}, \hat{p}] = i \rightarrow [a, a^+] = I$$

$$\underset{\in \mathbb{N}_0}{|n\rangle} \xrightarrow{\hat{a}} \sqrt{n}\, |n-1\rangle$$

Wignerfunction: $W_{\hat{\rho}}(\vec{x}) = \int d\vec{y}\; e^{-i \vec{x}^T \vec{y}}\; \underbrace{Tr[D(\vec{y})\hat{\rho}]}_{\text{"Bloch vector"}}$ $\qquad \vec{x} = \begin{pmatrix} q \\ p \end{pmatrix}$

$$= W(q, p)$$



$$\int_{\mathbb{R}} dp \; W_{\hat{\rho}}(q, p) = \langle x | \underset{|\psi\rangle\langle\psi|}{\hat{\rho}} | x \rangle = |\langle \psi | x \rangle|^2$$

$$\int_{\mathbb{R}} dp \; W_{\hat{\rho}}(q, p) = |\langle p | \psi \rangle|^2$$

$$\int dp\, dq \; W(q, p) = 1 = Tr[\hat{\rho}]$$

Road to LQEC

$\mathcal{H}_c = \text{span}\{|n\rangle\}_{n\in\mathbb{N}_0}$

$$\hat\rho_L \xrightarrow[\mathcal{E}]{E} \rho \xrightarrow[\mathcal{N}]{\text{Noise}} \rho' \xrightarrow[R]{\text{Dec}} \hat\rho_L$$

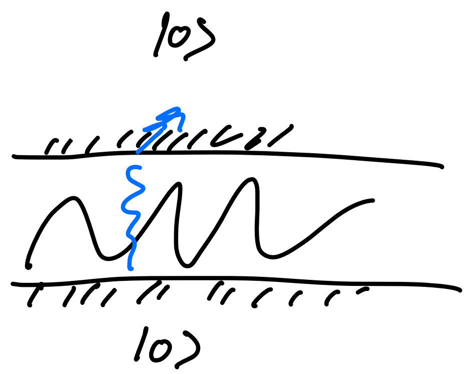
1- qubit

cleanse / QECC, enforcing symmetries

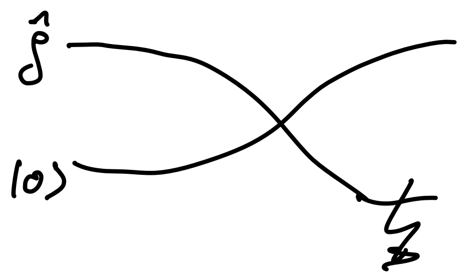
Interlude: Noise, photon loss, dephasing noise

photon loss

quantum jump



$$\hat\rho(t+dt) = \hat\rho(t) + \underset{\sim}{\kappa dt}\left\{ \hat a \rho(t) a^\dagger - \frac{1}{2}\rho a^\dagger a - \frac{1}{2} a^\dagger a \rho \right\}$$



$$\lim_{dt\to 0} \frac{\rho(t+dt)-\rho(t)}{dt}$$

$$\dot{\hat\rho}(t) = \kappa\, \underset{\sim}{D[\hat a]}(\rho)$$

$$D[A]\rho = A\rho A^\dagger - \frac{1}{2}\underline{\{A^\dagger A, \rho\}}$$

solve

$$\rho(t) = e^{t\kappa D[\hat a]}\,\rho$$

$$N_\gamma(\hat\rho) = \sum_{l=0}^{\infty} E_l\,\rho\,E_l^\dagger \quad , \quad \gamma = 1-e^{-\kappa t} \approx \kappa t$$

$$\hat E_l = \left(\frac{\gamma}{1-\gamma}\right)^{\frac{l}{2}} \frac{\hat a^l}{\sqrt{l!}}\,(1-\gamma)^{\frac{\hat n}{2}}$$

$$E_0 = \underbrace{(1-\gamma)^{\frac{\hat n}{2}}}_{\approx\, e^{-\frac{\gamma}{2}\frac{\hat q^2+\hat p^2}{2}+c.}}$$

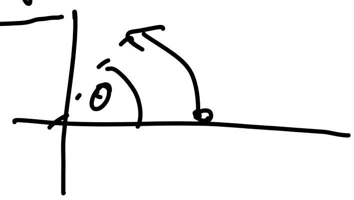
→ no photons lost

Dephasing

$$\dot{\hat\rho} = \kappa_\phi\left(\hat n\hat\rho\hat n - \frac{1}{2}\{\hat n^2,\hat\rho\}\right) = \kappa_\phi D[\hat n]\rho$$

$$N_{\langle\theta^2\rangle}(\rho) = \int_0^{2\pi} d\theta\, P(\theta)\, e^{-i\theta\hat n}\,\rho\, e^{i\theta\hat n} \qquad \langle\theta^2\rangle \approx \kappa_\phi t$$

$$= \hat\rho + \langle\theta^2\rangle\hat n\rho\hat n - \frac{\langle\theta^2\rangle}{2}\{\hat n,\rho\} + O(\langle\theta^3\rangle)$$



$$e^{-i\theta\hat n}|n\rangle = e^{-i\theta n}$$

$$e^{-i\theta\hat n}\,|n\rangle\langle n|\,e^{i\theta n} = |n\rangle\langle n|$$

Errors of type $\hat a$, $\hat n$, $a^\dagger$

$P^2 = P$, $P(\mathcal{H}_C \oplus R) = \mathcal{H}_C$

QECC: $P E_i^\dagger E_j P = c_{ij} P + x_{ij}\hat{X} + y_{ij}\hat{Y} + z_{ij}\hat{Z}$

$j \to$ "i"

QED: $\underbrace{P E_i P}_{\downarrow} = c_{ij} P$

$P + R$

• KLM (Knill-Laflamme-Milburn) (2001)

$|\bar{0}\rangle = |01\rangle$
$|\bar{1}\rangle = |10\rangle$

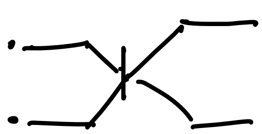
a $\longrightarrow$

b $\longrightarrow$

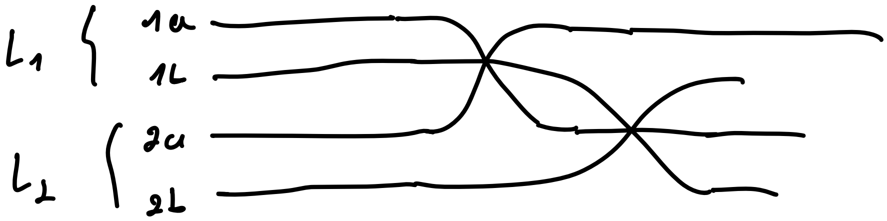
$\alpha|01\rangle + \beta|10\rangle \xrightarrow{\hat{a}_{1/2}} |00\rangle \to$ heralded erasure/error conversion.

$e^{i\pi(\hat{n}_1+\hat{n}_2)}$

$$e^{i\theta\hat{n}_a}|\bar{0}\rangle = |\bar{0}\rangle$$
$$e^{i\theta\hat{n}_a}|\bar{1}\rangle = e^{i\theta}|\bar{1}\rangle$$
} Pauli-Z rotation



$$U = e^{i\theta\underbrace{(\overset{|01\rangle\langle 10|}{a_1^\dagger a_2} + \overset{|10\rangle\langle 01|}{a_1 a_2^\dagger})}_{\hat{X}}}$$



$$e^{i\Theta \{\underbrace{a^\dagger_{1a} a_{2a} + a^\dagger_{1b} a_{2b} + h.c.}_{SWAP}\}} = e^{i\Theta SWAP}$$

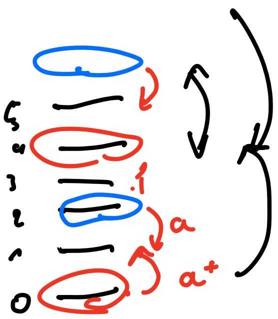
$$\downarrow \Theta = \tfrac{\pi}{4}$$

$$\sqrt{iSWAP}$$

$$P E_i^\dagger E_j P \stackrel{!}{=} c_{ij} P$$

$\langle \bar 0 | E_i^\dagger E_j | \bar 1 \rangle = 0$ errors preserve orthogonality

$\langle \bar 0 | E_i^\dagger E_j | \bar 0 \rangle = \langle \bar 1 | E_i^\dagger E_j | \bar 1 \rangle$ = normalized.



$$E = \{ I, \sqrt{\gamma}\,\hat a, (\sqrt{\gamma}\,\hat a)^2, \ldots, (\sqrt{\kappa}\, a^\dagger), \ldots, (\sqrt{\kappa}\, a^\dagger)^2 \}$$

Rotation symmetric codes:

$R_N = e^{i\frac{2\pi}{N}\hat n}$ is rotation operator

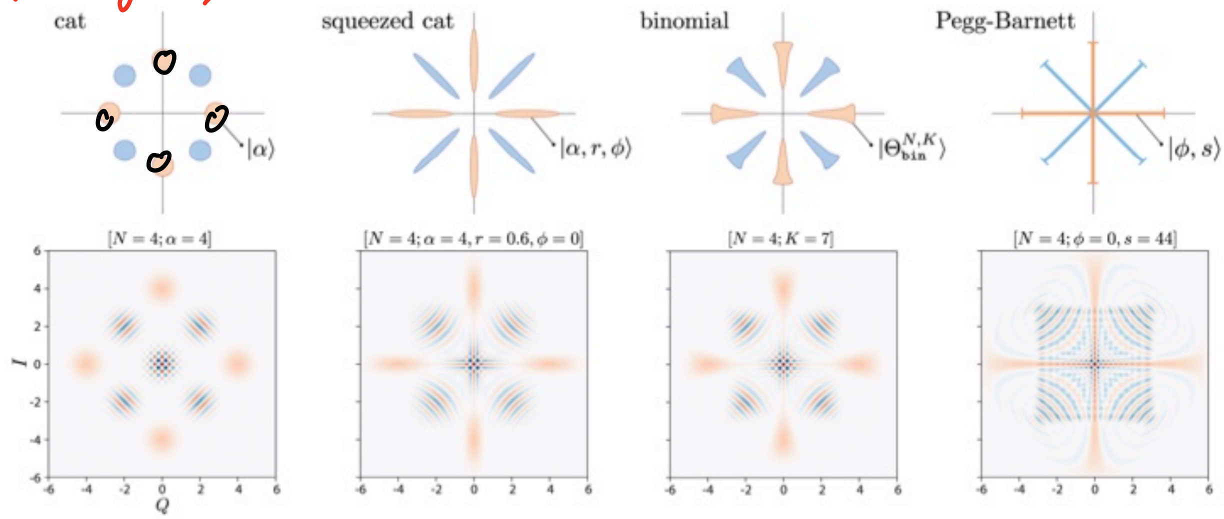
$$R_N\Big(\sum_n c_n |n\rangle\Big) = \sum_n c_n e^{i\frac{2\pi}{N} n} |n\rangle \stackrel{!}{=} |\psi\rangle$$

$|+\rangle$

$c_n \neq 0.$

$\Rightarrow \; n = 0 \mod N$

Grimsmo / Baragiola / Combes



!

$|C_\alpha^\pm\rangle := (|\alpha\rangle \pm |-\alpha\rangle)$

$H \sim \omega\, a^\dagger a + (a^\dagger a)^2$

$(a^\dagger a)(a^\dagger a)$

$\langle \alpha | -\alpha \rangle = e^{-2|\alpha|^2} \rightarrow$

"$|0\rangle$" "$|1\rangle$"

$\hat{x}$

$\hat{a}|C_\alpha^\pm\rangle = \alpha\, |C_\alpha^\mp\rangle$

$e^{i\pi\hat{n}}\, |\pm\alpha\rangle = |\mp\alpha\rangle$

$n = 0 \mod 2$

$e^{i\frac{\pi}{2}\hat{n}}$

$|\bar{0}\rangle \propto |\alpha\rangle + |-\alpha\rangle + |i\alpha\rangle + |-i\alpha\rangle$

$\hat{a}^2$

$|\bar{1}\rangle \propto |\alpha\rangle + |-\alpha\rangle - |i\alpha\rangle - |-i\alpha\rangle$

$e^{i\frac{\pi}{4}\hat{n}}\,|$

$$|0_{N,\alpha}\rangle = \frac{1}{\sqrt{N_0}} \sum_{m=0}^{2N-1} e^{i\frac{m\pi}{N}\hat n} \; |\overset{\overset{\alpha}{\shortparallel}}{\theta}\rangle$$

$$\Big\downarrow \hat a^N$$

$$|1_{N,\alpha}\rangle = \frac{1}{\sqrt{N_1}} \sum_{m=0}^{2N-1} (-1)^m e^{i\frac{m\pi}{N}\hat n} \; |\overset{\overset{\alpha}{\shortparallel}}{\theta}\rangle$$

$$W(0) = \int d^2x \; \mathrm{Tr}[D(x)\rho]$$

$$\propto e^{i\pi\hat n}$$

$$\dot{\hat\rho} = \kappa D[\hat a](\hat\rho) \quad , \quad D[A]\rho = A\rho A^\dagger - \frac{1}{2}\{A^\dagger A,\rho\}$$

$$\dot{\hat\rho} = D[\hat a^2-\alpha^2](\hat\rho)$$

$$(\hat a^2-\alpha^2)\hat\rho(\hat a^{\dagger 2}-\alpha^{*2})$$: steady state spanned by $|\pm\alpha\rangle\langle\pm\alpha|$

- Mirrahimi $\dot{\hat\rho} = -i[H_{eq},\hat\rho] + \kappa_2 D[\hat a^2]\rho$

$$\downarrow$$

$$\varepsilon a^{\dagger 2} + \varepsilon^* a^2$$

$$= -i(H_{eff}\rho - \rho H_{eff}^\dagger) + \kappa_2 a^2\rho a^{\dagger 2}$$

$$\downarrow$$

$$H_{sq} - i\frac{\kappa_2}{2} a^{\dagger 2}\hat{a}^2$$

$$-iH_{eff} = \kappa\left(\hat{a}^2 + \alpha^2 I\right)^\dagger\left(\hat{a}^2 - \alpha^2 I\right) - \frac{\kappa|\,|^2}{\kappa}$$

$$\downarrow$$

$$H_{eff}\,|\pm\alpha\rangle = 0$$

$$\mathcal{E} = 0: \lim_{\alpha\to 0} |\mathcal{C}_\alpha^\pm\rangle = |0/1\rangle$$

Alternative, Kerr: $H = -K a^{\dagger 2} a^2 + \varepsilon a^{\dagger 2} + \varepsilon^* \hat{a}^2$

$$= -K\left(\hat{a}^2 - \alpha^2\right)^\dagger\left(\hat{a}^2 - \alpha^2\right) + \text{const.}$$