Ozan Argıt Önçel
1901042259

1)

a) $2^n + n^3 \leq c \cdot 4^n$ for all $n \geq k$

$k=1 \quad c=4 \Rightarrow 2^1 + 1^3 \leq 4 \cdot 4^1 \Rightarrow 3 \leq 16$ ✓
(is true)

$2^n + n^3 \in O(4^n)$

b) $\sqrt{10n^2+7n+3} \geq c \cdot n$

$c=1 \quad k=1 \Rightarrow 10n^2+7n+3 \geq n^2$

$10+7+3 \geq 1 \Rightarrow 20 \geq 1$ ✓ $\sqrt{10n^2+7n+3} \in \Omega(n)$

c) $n^2+n < cn^2$ for all $n \geq k$

$k=5 \quad c=1 \Rightarrow 25+5 < 25 \Rightarrow 30 < 25$ X

$n^2+n \notin o(n^2)$

d) $3\log_2^2 n = 6\log_2 n$

$c_1 \cdot 2 \cdot \log_2 n \leq 6\log_2 n \leq c_2 \cdot 2 \cdot \log_2 n$

$c_1 = 6 \Rightarrow 12\log_2 n \leq 6\log_2 n$ X false

$3\log_2^2 n \notin \Theta(\log_2 n^2)$

e) $(n^3+1)^6 \leq cn^3$

if we divide both side n^3, left side has $n^{15}+\ldots$

but right side has c so this is a contradiction.

CamScanner ile tarandı

2)

a/ $2n\log(n+2)^2 + (n+2)^2\log\left(\frac{n}{2}\right) = 2n \cdot 2\log(n+2) + (n+2)^2 \cdot \log(n) \ldots$

$\in \Theta(n\log n) + \Theta(n^2\log(n)) = \underline{\Theta(n^2\log n)}$

CamScanner ile tarandı

Ozan Argıt Önceken
1901042259

b)

$$C_1 n^4 \leq 0.001n^4 + 3n^3 + 1 \leq C_2 \cdot n^4$$

$C_1 = 0.0001 \wedge n_0 = 1 \Rightarrow$

$$c_1 \cdot n^4 \leq 0.001n^4 + 3n^3 + 1 \checkmark$$

$C_2 = 9 \wedge n_0 = 1$

$$0.001n^4 + 3n^3 + 1 \leq C_2 n^4 \Rightarrow 0.001n^4 + 3n^3 + 1 \in \Theta(n^4)$$

3

a)
$$\lim_{n\to\infty}\left(\frac{\ln(n)}{n^{\ln(n)}}\right) \Rightarrow \lim_{n\to\infty}\left(\frac{\frac{1}{\cancel{n}}}{\frac{2e^{\ln^2(n)}\ln(n)}{\cancel{n}}}\right)$$

$$\Rightarrow \lim_{n\to\infty}\left(\frac{1}{2e^{\ln^2(n)}\ln(n)}\right) \Rightarrow \lim_{n\to\infty}\left(\frac{1}{e^{\ln^2(n)}\ln(n)}\right)$$

$$\lim_{n\to\infty}\left(e^{\ln^2(n)} \cdot \ln(n)\right) = \infty \Rightarrow \lim_{n\to\infty}\left(\frac{1}{e^{\ln^2(n)} \cdot \ln(n)}\right) = 0$$

$$\log(n) < n^{\log(n)}$$

(L'Hopital)
$$\lim_{n\to\infty}\frac{n^{1.5}}{\log(n)} \Rightarrow \lim_{n\to\infty}\frac{1{,}5\sqrt{n}}{\frac{1}{n}} \Rightarrow \lim_{n\to\infty} 1.5\sqrt{n^3} = \infty$$

$$n^{1.5} > \log(n)$$

$$\lim_{n\to\infty}\frac{n^{\log(n)}}{n^{1.5}} = \lim_{n\to\infty}\frac{n^2\left(n^{(\log(n)-2)}\right)}{n^2\left(n^{-0.5}\right)} \Rightarrow \lim_{n\to\infty}\frac{n^{\log(n)-2}}{\frac{1}{\sqrt{n}}}$$

CamScanner ile tarandı

$$\lim_{n\to\infty}\left(n^{\log(n)-2}\cdot\sqrt{n}\right) = \infty \Rightarrow n^{\log(n)} > n^{1.5}$$ (+)

$$\Rightarrow \log(n) < n^{1.5} < n^{\log(n)}$$

b) $$\lim_{n\to\infty}\frac{n!}{2^n} = \lim_{n\to\infty}\frac{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n}{2^n} = \infty \quad \text{so } n! > 2^n$$

$$\lim_{n\to\infty}\frac{2^n}{n^2} \overset{\text{(L'Hopital)}}{=} \lim_{n\to\infty}\frac{2^n \log 2}{2\cdot n} = \infty \Rightarrow 2^n > n^2$$

$$\Rightarrow n! > 2^n > n^2$$

CamScanner ile tarandı

3 c)

$$\lim_{n\to\infty} \frac{n \log n}{\sqrt{n}} = \lim_{n\to\infty} \left(\sqrt{n} \log(n)\right) = \infty \Rightarrow n\log(n) > \sqrt{n}$$

d)

$$\lim_{n\to\infty} \left(\frac{n \cdot 2^n}{3^n}\right) = \lim_{n\to\infty} \left(n \cdot \frac{2^n}{3^n}\right) = \lim_{n\to\infty} \left(n\left(\frac{2}{3}\right)^n\right)$$

$$\overset{(\text{L'Hopital})}{=} \lim_{n\to\infty} \left(\frac{n}{\frac{1}{\left(\frac{2}{3}\right)^n}}\right) \overset{(\text{L'Hopital})}{=} \lim_{n\to\infty} \left(\frac{1}{\left(\frac{3}{2}\right)^n \ln\left(\frac{3}{2}\right)}\right) = \lim_{n\to\infty} \left(\frac{2^n}{\ln\left(\frac{3}{2}\right) \cdot 3^n}\right)$$

$$\overset{(\text{L'Hopital})}{=} \lim_{n\to\infty} \left(\frac{2^n \ln(2)}{\ln\left(\frac{3}{2}\right) \cdot 3^n \ln(3)}\right) \overset{(\text{L'Hopital})}{=} \lim_{n\to\infty} \left(\frac{\ln^2(2) \cdot 2^n}{\ln^2(3) \ln\left(\frac{3}{2}\right) \cdot 3^n}\right) \overset{(\text{L'Hopital})}{=}$$

$$\lim_{n\to\infty} \left(\frac{\ln^3(2) \cdot 2^n}{\ln^3(3) \ln\left(\frac{3}{2}\right) \cdot 3^n}\right) = \frac{\ln^3(2)}{\ln^3(3) \ln\left(\frac{3}{2}\right)} \cdot \lim_{n\to\infty} \left(\frac{2}{3}\right)^n = \frac{\ln^3(2)}{\ln^3(3) \cdot \ln\left(\frac{3}{2}\right)} \cdot 0 = \underline{0}$$

$$3^n > n \cdot 2^n$$

e)

$$\lim_{n\to\infty} \frac{\sqrt{n+10}}{n^3} \sim \lim_{n\to\infty} \frac{\sqrt{n}}{n^3} = \lim_{n\to\infty} \frac{1}{n^{2.5}} = 0$$

$$\underline{n^3 > \sqrt{n+10}}$$

CamScanner ile tarandı

4)

a) Checking the array is symetric or not.

b) $\sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2} (\underbrace{1+1+1 \dots +1}_{n-1-(i+1) \text{ times} = n-2-i \text{ times}})$

$$\sum_{i=0}^{n-2} (n-1-i) = \underbrace{(n-1)+(n-2)+(n-3)+\dots+1}_{\frac{(n-1)(n)}{2}} = \boxed{\frac{n^2-n}{2}}$$

c) $\Theta(n^2)$

CamScanner ile tarandı

5)

a) Algorithm products 2 arrays and adds it to array c.

b)

$$\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1 = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} n = \sum_{i=0}^{n-1} n^2 = n^3$$

c) $\Theta(n^3)$

CamScanner ile tarandı

b) Code:

```
for( int i=0; i<arr.lenght; i++)
    for(int J=0; J< arr.lenght; J++) //J≠i because if J=i
//output gives for example (2,3) but not (3,2)
        if (arr[J]* arr[i] == number)
            printf("( %d,%d)", arr[i], arr[J]);
```

Input: An array $A[0,\ldots,n-1]$, a decimal integer val.
Output: Print pairs.

```
for i ← 0 to n-1 do
    for J ← 0 to n-1 do
        if Arr[i] * Arr[J] = number
            print "Arr[i], Arr[J]"
        end-if
    end-for
end-for
```

First loop has $O(n)$ second loop has $O(n)$ so first loop runs $O(n)$ complexity for n times so complexity $= O(n^2)$

$$\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} 1 = \sum_{i=0}^{n-1} n = n^2$$

$\underline{\underline{\Theta(n^2)}}$

CamScanner ile tarandı