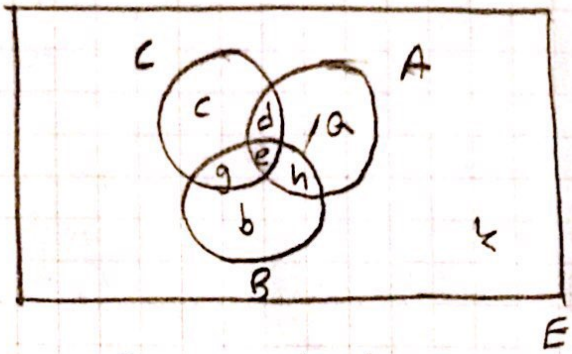
COMP 341 - HW 3

Orçun Özdemir
54020

1. $P(A) = 0{,}4$ $\quad P(A \cup B) = 0{,}8$

$P(B) = 0{,}7$ $\quad P(B \cap C) = 0{,}2$

$P(C) = 0{,}3$ $\quad P(C \cap (A \cup B)) = 0{,}2$



$a + d + e + h = 0{,}4$

$b + h + e + g = 0{,}7$

$c + g + d + e = 0{,}3$

$a + b + h + d + e + g = 0{,}8$

$g + e = 0{,}2$

$d + e + g = 0{,}2$

$g + e + h = 0{,}4$

$a = 0{,}1 \quad b = 0{,}3 \quad c = 0{,}1 \quad h = 0{,}2$

$g = 0{,}1 \quad d = 0{,}0 \quad e = 0{,}1 \quad k = 0{,}1$

A) $d + g + h = 0{,}3$ $\qquad$ B) $k = 0{,}1$

2. According to the given table, $P(X_1 = T) = \frac{277}{1000}$ then $P(X_2 = T) = \frac{110}{1000}$, $P(X_3 = T) = \frac{296}{1000}$.

Let's Assume:

$X_1 = A \quad X_2 = B$	$X_3 = A \quad X_2 = B$	$X_3 = A \quad X_1 = B$	$X_1 = C$
$P(B \mid A) = P(B)$	$P(B \mid A) = P(B)$	$P(B \mid A) = P(B)$	$X_2 = B$
$\frac{110}{1000} \overset{?}{=} \frac{P(A \cap B)}{P(B)} = \frac{91}{277}$	$\frac{110}{1000} \overset{?}{=} \frac{P(A \cap B)}{P(B)} = \frac{34}{296}$	$\frac{110}{1000} \overset{?}{=} \frac{P(A \cap B)}{P(B)} \quad \frac{212}{296}$	$X_3 = A$
$= 0{,}11 \neq = 0{,}33$	$= 0{,}11 \qquad = 0{,}114$	$= 0{,}11 \qquad = 0{,}71$	
	$\rightarrow 0{,}11 \cong 0{,}114$	$\rightarrow 0{,}11 \neq 0{,}71$	

3.) 1: $S \perp\!\!\!\perp P \mid H$: T   2. $A \perp\!\!\!\perp P \mid S$: F   3. $C \perp\!\!\!\perp P \mid H$: F

4: $H \perp\!\!\!\perp A \mid M$: T   5. $S \perp\!\!\!\perp A \mid H, P$: F   6. $S \perp\!\!\!\perp C \mid P$: T

4.) a) $P(C).P(A).P(P).P(J \mid C, A).P(M \mid C, A).P(W \mid A, P).P(S \mid J, W).P(H \mid M, S, P)$

b.) $2^0 + 2^0 + 2^0 + 2^2 + 2^2 + 2^2 + 2^2 + 2^3 = \boxed{27}$

5.) a.) $P(G).P(M).P(B).P(I \mid B).P(S \mid M, I, G)$ b. $P(S = \text{true}) = \boxed{14/39}$ c.) $P(S = \text{true} \mid C = \text{false}, a = \text{true}) = \boxed{6/13}$

d.) We will assume 1 to non-evidence probabilities and evidences will keep their probabilities, since they do not depend other states in this case.

$$\frac{0{,}42 . 31 + 0{,}42 . 1 + 0{,}42 . 66 + 0{,}42 . 0}{(0{,}42). 200} = \boxed{\frac{49}{100}}$$

6) Solution for 6th Question is on the other page.

4.) c.) $P(+c), P(+a), P(P), P(J \mid +c, +a), P(M \mid +c, +a), P(W \mid +a, P), P(S \mid J, W), P(H \mid S, M, P)$

d.) $P(C).P(A).P(+p).P(J \mid C, A).P(M \mid C, A) P(+w \mid A, +p). P(S \mid J, +w).P(H \mid S, M, +p)$

→ The biggest factors are $P(H \mid S, M, +p)$, $P(J \mid C, A)$ and $P(M \mid C, A)$ each contains three non-evidence variables. Therefore size of largest factor is 8.

e.) Initial factors: $f_1(P)$, $f_2(M, C=c, A=a)$, $f_3(S, J=j, W=w)$, $f_4(H, S, M, P)$

Step 1: Join $f_2(M, C=c, A=a)$, $f_3(S, J=j, W=w)$ to obtain $f_5(M, C=c, A=a, S, J=j, W=w)$

Step 2: Sum out M from $f_5(M, C=c, A=a, S, J=j, W=w)$ to obtain $f_6(C=c, A=a, S, J=j, W=w)$

Step 3: Join $f_3(P)$ and $f_4(H, S, M, P)$ to obtain $f_7(H, S, M, P)$

Step 4: Sum out P to obtain $f_8(H, S, M)$.

Step 5: Join $f_6(C=c, A=a, S, J=j, W=w)$ and $f_8(H, S, M)$ to obtain $f_9(C=c, A=a, S, J=j, W=w, H, M)$

Step 6: Sum out S to get $f_{10}(C=c, A=a, J=j, W=w, H, M)$

Step 7: Sum out M to get $f_{11}(C=c, A=a, J=j, W=w, H)$

Step 8: Normalize f_{11} to get $P(H)$ which is our target probability.

6.) a) $P(S \mid B=full, M=true, I=true, G=low) =$

$$= \frac{P(S, B=full, M=true, I=true, G=low)}{P(B=full, M=true, I=true, G=low)}$$

$$= \frac{\cancel{P(B=full)} \cdot \cancel{P(M=true)} \cdot \cancel{P(I=true \mid B=full)} \cdot P(G=low) \cdot P(S \mid G=low, I=true, M=true)}{\cancel{P(B=full)} \, \cancel{P(M=true)} \, \cancel{P(I=true \mid B=full)} \, P(G=low) \sum_S P(S \mid G=low, I=true, M=true)}$$

$$= \frac{P(S \mid G=low, I=true, M=true)}{0{,}2+0{,}8} = P(S \mid G=low, I=true, M=true)$$

b) $P(M \mid B=full, I=true, G=low, S=false) = \frac{P(M, B=full, I=true, G=low, S=false)}{P(B=full, I=true, G=low, S=false)}$

$$= \frac{\cancel{P(B=full)} P(M) \cdot \cancel{P(I=true \mid B=full)} \cancel{P(G=low)} \cdot P(S=false \mid G=low, I=true, M)}{\cancel{P(B=full)} \cdot \cancel{P(I=true \mid B=full)} \cancel{P(G=low)} \sum_M P(M) \, P(S=false \mid G=low, I=true, M)}$$

$$= \frac{P(M) \cdot P(S=false \mid G=low, I=true, M)}{\sum_M P(M) \cdot P(S=false \mid G=low, I=true, M)} = \frac{P(M) \cdot P(S=false \mid G=low, I=true, M)}{0{,}6 \cdot 0{,}8 + 0{,}4 \cdot 0{,}95}$$

$$= \frac{P(M) \cdot P(S=false \mid G=low, I=true, M)}{0{,}86}$$

c) $P(M=true \mid B=full, I=true, G=low, S=false)$

$$= \frac{P(M=true) \cdot P(S=false \mid G=low, I=true, M=true)}{0{,}86} = \frac{0{,}6 \cdot 0{,}8}{0{,}86} \approx \boxed{0{,}558}$$

therefore $P(M=false \mid B=full, I=true, G=low, S=false) = 1 - 0{,}558 = 0{,}442$

and $0{,}4 \leq 0{,}442$

so we need to choose M=false