

1 Sayisal integral icin Polinom Yaklasim Formulleri

1.1 Polinom ifadesi

Bir $P_n(x)$ polinomu genel olarak asagidaki ifade ile temsil edilir.

$$P_n(x) = c_0 + c_1x + c_2x^2 + c_3x^3 + \dots + c_nx^n$$

1.2 Ornek Noktalar

Bu ifade $x_m = x_0 + mh$ biciminde secilen her bir nokta icin duzenlenir.

$$P_2(0 * h) = y_0 = c_0 + 0 * c_1 * h + 0^2 * c_2 * h^2$$

$$P_2(1 * h) = y_1 = c_0 + c_1 * h + c_2 * h^2$$

$$P_2(2 * h) = y_2 = c_0 + 2 * c_1 * h + 2^2 * c_2 * h^2$$

1.3 Denklem Sistemi

Bu denklemlerden katsayi matrisi olusturulur.

$$\left[\begin{array}{ccc|c} c_0 & c_1 & c_2 & \\ 1 & 0 & 0^2 & y_0 \\ 1 & 1 & 1^2 & y_1 \\ 1 & 2 & 2^2 & y_2 \end{array} \right]$$

Denklemler, uslu ifadeler hesaplanarak asagidaki gibi yeniden duzenlenir.

$$\left[\begin{array}{ccc|c} c_0 & c_1 & c_2 & \\ 1 & 0 & 0 & y_0 \\ 1 & 1 & 1 & y_1 \\ 1 & 2 & 4 & y_2 \end{array} \right]$$

1.4 Denklem cozumu

1. satir kullanilarak asagisindaki (2,1) elemani 0 yapilir.

$$\left[\begin{array}{ccc|c} c_0 & c_1 & c_2 & \\ 1 & 0 & 0 & y_0 \\ 0 & 1 & 1 & -y_0 + y_1 \\ 1 & 2 & 4 & y_2 \end{array} \right]$$

1. satir kullanilarak asagisindaki (3,1) elemani 0 yapilir.

$$\left[\begin{array}{ccc|c} c_0 & c_1 & c_2 & \\ 1 & 0 & 0 & y_0 \\ 0 & 1 & 1 & -y_0 + y_1 \\ 0 & 2 & 4 & -y_0 + y_2 \end{array} \right]$$

2. satir kullanilarak asagisindaki (3,2) elemani 0 yapilir.

$$\left[\begin{array}{ccc|c} c_0 & c_1 & c_2 & \\ 1 & 0 & 0 & y_0 \\ 0 & 1 & 1 & -y_0 + y_1 \\ 0 & 0 & 2 & y_0 - 2y_1 + y_2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} c_0 & c_1 & c_2 & \\ 1 & 0 & 0 & y_0 \\ 0 & 1 & 0 & -\frac{3}{2}y_0 + 2y_1 - \frac{1}{2}y_2 \\ 0 & 0 & 2 & y_0 - 2y_1 + y_2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} c_0 & c_1 & c_2 & \\ 1 & 0 & 0 & y_0 \\ 0 & 1 & 0 & -\frac{3}{2}y_0 + 2y_1 - \frac{1}{2}y_2 \\ 0 & 0 & 2 & y_0 - 2y_1 + y_2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} c_0 & c_1 & c_2 & \\ 1 & 0 & 0 & y_0 \\ 0 & 1 & 0 & -\frac{3}{2}y_0 + 2y_1 - \frac{1}{2}y_2 \\ 0 & 0 & 2 & y_0 - 2y_1 + y_2 \end{array} \right]$$

Katsayi matrisi birim matrise donusturulur.

$$\left[\begin{array}{ccc|c} c_0 & c_1 & c_2 & \\ 1 & 0 & 0 & y_0 \\ 0 & 1 & 0 & -\frac{3}{2}y_0 + 2y_1 - \frac{1}{2}y_2 \\ 0 & 0 & 1 & \frac{1}{2}y_0 - y_1 + \frac{1}{2}y_2 \end{array} \right]$$

Buradan katsayi cozumleri asagidaki gibi belirlenir.

$$\begin{aligned} c_0 &= y_0 \\ c_1 &= \frac{1}{h} \left(-\frac{3}{2}y_0 + 2y_1 - \frac{1}{2}y_2 \right) \\ c_2 &= \frac{1}{h^2} \left(\frac{1}{2}y_0 - y_1 + \frac{1}{2}y_2 \right) \end{aligned}$$

1.5 Alan Hesabi

$P_2(x)$ polinomunun integrali alınir.

$$I = \int_0^{2h} P_2(x) dx$$

$$I = \int_0^{2h} (c_0 + c_1x + c_2x^2) dx$$

$$I = \left(c_0x + c_1 \frac{x^2}{2} + c_2 \frac{x^3}{3} \right) \Big|_0^{2h}$$

Burada x yerine $2h$ yerlestirildiginde polinom ifadesinde bulunan $c_k * x^k$ terimlerinin tamam h ortak parantezine alinabilmektedir.

$$I = c_0 * (2h) + c_1 * \frac{(2h)^2}{2} + c_2 * \frac{(2h)^3}{3}$$

$$I = h * ((y_0) * 2 + (-\frac{3}{2}y_0 + 2y_1 - \frac{1}{2}y_2) * \frac{2^2}{2} + (\frac{1}{2}y_0 - y_1 + \frac{1}{2}y_2) * \frac{2^3}{3})$$

$$I = h * (\frac{1}{3}y_0 + \frac{4}{3}y_1 + \frac{1}{3}y_2)$$

$$I = \frac{1}{3} * h * (y_0 + 4y_1 + y_2)$$

h ve y degerleri ile integral hesaplanir.

$$I = \frac{1}{3} * 0.15707963267948966 * (1 * 0.5877852522924731 + 4 * 0.7071067811865475 + 0.8090169943749475)$$

$$I = 0.2212324925$$