1 Sayisal integral icin Polinom Yaklasim Formulleri

1.1 Polinom ifadesi

Bir $P_n(x)$ polinomu genel olarak asagidaki ifade ile temsil edilir.
$P_n(x) = c_0 + c_1x + c_2x^2 + c_3x^3 + \cdots + c_nx^n$

1.2 Ornek Noktalar

Bu ifade $x_m = x_0 + mh$ biciminde secilen her bir nokta icin duzenlenir.
$P_3(0*h) = y_0 = c_0 + 0*c_1*h + 0^2*c_2*h^2 + 0^3*c_3*h^3$
$P_3(1*h) = y_1 = c_0 + c_1*h + c_2*h^2 + c_3*h^3$
$P_3(2*h) = y_2 = c_0 + 2*c_1*h + 2^2*c_2*h^2 + 2^3*c_3*h^3$
$P_3(3*h) = y_3 = c_0 + 3*c_1*h + 3^2*c_2*h^2 + 3^3*c_3*h^3$

1.3 Denklem Sistemi

Bu denklemlerden katsayi matrisi olusturulur.

$$\left[\begin{array}{cccc|c} c_0 & c_1 & c_2 & c_3 & \\ 1 & 0 & 0^2 & 0^3 & y_0 \\ 1 & 1 & 1^2 & 1^3 & y_1 \\ 1 & 2 & 2^2 & 2^3 & y_2 \\ 1 & 3 & 3^2 & 3^3 & y_3 \end{array}\right]$$

Denklemler, uslu ifadeler hesaplanarak asagidaki gibi yeniden duzenlenir.

$$\left[\begin{array}{cccc|c} c_0 & c_1 & c_2 & c_3 & \\ 1 & 0 & 0 & 0 & y_0 \\ 1 & 1 & 1 & 1 & y_1 \\ 1 & 2 & 4 & 8 & y_2 \\ 1 & 3 & 9 & 27 & y_3 \end{array}\right]$$

1.4 Denklem cozumu

1. satir kullanilarak asagisindaki (2,1) elemani 0 yapilir.

$$\left[\begin{array}{cccc|r} c_0 & c_1 & c_2 & c_3 & \\ 1 & 0 & 0 & 0 & y_0 \\ 0 & 1 & 1 & 1 & -y_0 + y_1 \\ 1 & 2 & 4 & 8 & y_2 \\ 1 & 3 & 9 & 27 & y_3 \end{array}\right]$$

1. satir kullanilarak asagisindaki (3,1) elemani 0 yapilir.

$$\begin{array}{l} \begin{array}{cccc} c_0 & c_1 & c_2 & c_3 \end{array} \\ \left[\begin{array}{cccc|r} 1 & 0 & 0 & 0 & y_0 \\ 0 & 1 & 1 & 1 & -y_0 + y_1 \\ 0 & 2 & 4 & 8 & -y_0 + y_2 \\ 1 & 3 & 9 & 27 & y_3 \end{array}\right] \end{array}$$

1. satir kullanilarak asagisindaki (4,1) elemani 0 yapilir.

$$\begin{array}{l} \begin{array}{cccc} c_0 & c_1 & c_2 & c_3 \end{array} \\ \left[\begin{array}{cccc|r} 1 & 0 & 0 & 0 & y_0 \\ 0 & 1 & 1 & 1 & -y_0 + y_1 \\ 0 & 2 & 4 & 8 & -y_0 + y_2 \\ 0 & 3 & 9 & 27 & -y_0 + y_3 \end{array}\right] \end{array}$$

2. satir kullanilarak asagisindaki (3,2) elemani 0 yapilir.

$$\begin{array}{l} \begin{array}{cccc} c_0 & c_1 & c_2 & c_3 \end{array} \\ \left[\begin{array}{cccc|r} 1 & 0 & 0 & 0 & y_0 \\ 0 & 1 & 1 & 1 & -y_0 + y_1 \\ 0 & 0 & 2 & 6 & y_0 - 2y_1 + y_2 \\ 0 & 3 & 9 & 27 & -y_0 + y_3 \end{array}\right] \end{array}$$

2. satir kullanilarak asagisindaki (4,2) elemani 0 yapilir.

$$\begin{array}{l} \begin{array}{cccc} c_0 & c_1 & c_2 & c_3 \end{array} \\ \left[\begin{array}{cccc|r} 1 & 0 & 0 & 0 & y_0 \\ 0 & 1 & 1 & 1 & -y_0 + y_1 \\ 0 & 0 & 2 & 6 & y_0 - 2y_1 + y_2 \\ 0 & 0 & 6 & 24 & 2y_0 - 3y_1 + y_3 \end{array}\right] \end{array}$$

3. satir kullanilarak asagisindaki (4,3) elemani 0 yapilir.

$$\begin{array}{l} \begin{array}{cccc} c_0 & c_1 & c_2 & c_3 \end{array} \\ \left[\begin{array}{cccc|r} 1 & 0 & 0 & 0 & y_0 \\ 0 & 1 & 1 & 1 & -y_0 + y_1 \\ 0 & 0 & 2 & 6 & y_0 - 2y_1 + y_2 \\ 0 & 0 & 0 & 6 & -y_0 + 3y_1 - 3y_2 + y_3 \end{array}\right] \end{array}$$

$$\begin{array}{c} \begin{array}{cccc} c_0 & c_1 & c_2 & c_3 \end{array} \\ \left[\begin{array}{cccc|r} 1 & 0 & 0 & 0 & y_0 \\ 0 & 1 & 1 & 1 & -y_0 + y_1 \\ 0 & 0 & 2 & 0 & 2y_0 - 5y_1 + 4y_2 - y_3 \\ 0 & 0 & 0 & 6 & -y_0 + 3y_1 - 3y_2 + y_3 \end{array}\right] \end{array}$$

$$\begin{array}{c} \begin{array}{cccc} c_0 & c_1 & c_2 & c_3 \end{array} \\ \left[\begin{array}{cccc|r} 1 & 0 & 0 & 0 & y_0 \\ 0 & 1 & 1 & 0 & -\frac{5}{6}y_0 + \frac{1}{2}y_1 + \frac{1}{2}y_2 - \frac{1}{6}y_3 \\ 0 & 0 & 2 & 0 & 2y_0 - 5y_1 + 4y_2 - y_3 \\ 0 & 0 & 0 & 6 & -y_0 + 3y_1 - 3y_2 + y_3 \end{array}\right] \end{array}$$

$$\begin{array}{c} \begin{array}{cccc} c_0 & c_1 & c_2 & c_3 \end{array} \\ \left[\begin{array}{cccc|r} 1 & 0 & 0 & 0 & y_0 \\ 0 & 1 & 1 & 0 & -\frac{5}{6}y_0 + \frac{1}{2}y_1 + \frac{1}{2}y_2 - \frac{1}{6}y_3 \\ 0 & 0 & 2 & 0 & 2y_0 - 5y_1 + 4y_2 - y_3 \\ 0 & 0 & 0 & 6 & -y_0 + 3y_1 - 3y_2 + y_3 \end{array}\right] \end{array}$$

$$\begin{array}{c} \begin{array}{cccc} c_0 & c_1 & c_2 & c_3 \end{array} \\ \left[\begin{array}{cccc|r} 1 & 0 & 0 & 0 & y_0 \\ 0 & 1 & 0 & 0 & -\frac{11}{6}y_0 + 3y_1 - \frac{3}{2}y_2 + \frac{1}{3}y_3 \\ 0 & 0 & 2 & 0 & 2y_0 - 5y_1 + 4y_2 - y_3 \\ 0 & 0 & 0 & 6 & -y_0 + 3y_1 - 3y_2 + y_3 \end{array}\right] \end{array}$$

$$\begin{array}{c} \begin{array}{cccc} c_0 & c_1 & c_2 & c_3 \end{array} \\ \left[\begin{array}{cccc|r} 1 & 0 & 0 & 0 & y_0 \\ 0 & 1 & 0 & 0 & -\frac{11}{6}y_0 + 3y_1 - \frac{3}{2}y_2 + \frac{1}{3}y_3 \\ 0 & 0 & 2 & 0 & 2y_0 - 5y_1 + 4y_2 - y_3 \\ 0 & 0 & 0 & 6 & -y_0 + 3y_1 - 3y_2 + y_3 \end{array}\right] \end{array}$$

$$\begin{array}{c} \begin{array}{cccc} c_0 & c_1 & c_2 & c_3 \end{array} \\ \left[\begin{array}{cccc|r} 1 & 0 & 0 & 0 & y_0 \\ 0 & 1 & 0 & 0 & -\frac{11}{6}y_0 + 3y_1 - \frac{3}{2}y_2 + \frac{1}{3}y_3 \\ 0 & 0 & 2 & 0 & 2y_0 - 5y_1 + 4y_2 - y_3 \\ 0 & 0 & 0 & 6 & -y_0 + 3y_1 - 3y_2 + y_3 \end{array}\right] \end{array}$$

Katsayi matrisi birim matrise donusturulur.

$$
\left[\begin{array}{cccc|r}
c_0 & c_1 & c_2 & c_3 & \\
1 & 0 & 0 & 0 & y_0 \\
0 & 1 & 0 & 0 & -\frac{11}{6}y_0 + 3y_1 - \frac{3}{2}y_2 + \frac{1}{3}y_3 \\
0 & 0 & 1 & 0 & y_0 - \frac{5}{2}y_1 + 2y_2 - \frac{1}{2}y_3 \\
0 & 0 & 0 & 1 & -\frac{1}{6}y_0 + \frac{1}{2}y_1 - \frac{1}{2}y_2 + \frac{1}{6}y_3
\end{array}\right]
$$

Buradan katsayi cozumleri asagidaki gibi belirlenir.

$c_0 = y_0$
$c_1 = \dfrac{1}{h}(-\dfrac{11}{6}y_0 + 3y_1 - \dfrac{3}{2}y_2 + \dfrac{1}{3}y_3)$
$c_2 = \dfrac{1}{h^2}(y_0 - \dfrac{5}{2}y_1 + 2y_2 - \dfrac{1}{2}y_3)$
$c_3 = \dfrac{1}{h^3}(-\dfrac{1}{6}y_0 + \dfrac{1}{2}y_1 - \dfrac{1}{2}y_2 + \dfrac{1}{6}y_3)$

1.5 Alan Hesabi

$P_3(x)$ polinomunun integrali alinir.

$$I = \int_0^{3h} P_3(x)\,dx$$

$$I = \int_0^{3h} (c_0 + c_1x + c_2x^2 + c_3x^3)\,dx$$

$$I = (c_0x + c_1\frac{x^2}{2} + c_2\frac{x^3}{3} + c_3\frac{x^4}{4})\Big|_0^{3h}$$

Burada x yerine $3h$ yerlestirildiginde polinom ifadesinde bulunan $c_k * x^k$ terimlerinin tamam h ortak parantezine alinabilmektedir.

$I = c_0 * (3h) + c_1 * \dfrac{(3h)^2}{2} + c_2 * \dfrac{(3h)^3}{3} + c_3 * \dfrac{(3h)^4}{4}$

$I = h * ((y_0) * 3 + (-\dfrac{11}{6}y_0 + 3y_1 - \dfrac{3}{2}y_2 + \dfrac{1}{3}y_3) * \dfrac{3^2}{2} + (y_0 - \dfrac{5}{2}y_1 + 2y_2 - \dfrac{1}{2}y_3) * \dfrac{3^3}{3} + (-\dfrac{1}{6}y_0 + \dfrac{1}{2}y_1 - \dfrac{1}{2}y_2 + \dfrac{1}{6}y_3) * \dfrac{3^4}{4})$

$I = h * (\dfrac{3}{8}y_0 + \dfrac{9}{8}y_1 + \dfrac{9}{8}y_2 + \dfrac{3}{8}y_3)$

$I = \dfrac{3}{8} * h * (y_0 + 3y_1 + 3y_2 + y_3)$

h ve y degerleri ile integral hesaplanir.

$I = \dfrac{3}{8} * 0.01 * (1 * 0.0 + 3 * 0.00999983333 + 3 * 0.01999866669 + 0.0299955002)$

$I = 0.0004499662$