1 Sayisal integral icin Polinom Yaklasim Formulleri

1.1 Polinom ifadesi

Bir $P_n(x)$ polinomu genel olarak asagidaki ifade ile temsil edilir.
$P_n(x) = c_0 + c_1x + c_2x^2 + c_3x^3 + \cdots + c_nx^n$

1.2 Ornek Noktalar

Bu ifade $x_m = x_0 + mh$ biciminde secilen her bir nokta icin duzenlenir.
$P_2(0 * h) = y_0 = c_0 + 0 * c_1 * h + 0^2 * c_2 * h^2$
$P_2(1 * h) = y_1 = c_0 + c_1 * h + c_2 * h^2$
$P_2(2 * h) = y_2 = c_0 + 2 * c_1 * h + 2^2 * c_2 * h^2$

1.3 Denklem Sistemi

Bu denklemlerden katsayi matrisi olusturulur.

$$\left[\begin{array}{ccc|c} c_0 & c_1 & c_2 & \\ 1 & 0 & 0^2 & y_0 \\ 1 & 1 & 1^2 & y_1 \\ 1 & 2 & 2^2 & y_2 \end{array}\right]$$

Denklemler, uslu ifadeler hesaplanarak asagidaki gibi yeniden duzenlenir.

$$\left[\begin{array}{ccc|c} c_0 & c_1 & c_2 & \\ 1 & 0 & 0 & y_0 \\ 1 & 1 & 1 & y_1 \\ 1 & 2 & 4 & y_2 \end{array}\right]$$

1.4 Denklem cozumu

1. satir kullanilarak asagisindaki (2,1) elemani 0 yapilir.

$$\left[\begin{array}{ccc|c} c_0 & c_1 & c_2 & \\ 1 & 0 & 0 & y_0 \\ 0 & 1 & 1 & -y_0 + y_1 \\ 1 & 2 & 4 & y_2 \end{array}\right]$$

1. satir kullanilarak asagisindaki (3,1) elemani 0 yapilir.

$$\left[\begin{array}{ccc|r} c_0 & c_1 & c_2 & \\ 1 & 0 & 0 & y_0 \\ 0 & 1 & 1 & -y_0 + y_1 \\ 0 & 2 & 4 & -y_0 + y_2 \end{array}\right]$$

2. satir kullanilarak asagisindaki (3,2) elemani 0 yapilir.

$$\left[\begin{array}{ccc|r} c_0 & c_1 & c_2 & \\ 1 & 0 & 0 & y_0 \\ 0 & 1 & 1 & -y_0 + y_1 \\ 0 & 0 & 2 & y_0 - 2y_1 + y_2 \end{array}\right]$$

$$\left[\begin{array}{ccc|r} c_0 & c_1 & c_2 & \\ 1 & 0 & 0 & y_0 \\ 0 & 1 & 0 & -\frac{3}{2}y_0 + 2y_1 - \frac{1}{2}y_2 \\ 0 & 0 & 2 & y_0 - 2y_1 + y_2 \end{array}\right]$$

$$\left[\begin{array}{ccc|r} c_0 & c_1 & c_2 & \\ 1 & 0 & 0 & y_0 \\ 0 & 1 & 0 & -\frac{3}{2}y_0 + 2y_1 - \frac{1}{2}y_2 \\ 0 & 0 & 2 & y_0 - 2y_1 + y_2 \end{array}\right]$$

$$\left[\begin{array}{ccc|r} c_0 & c_1 & c_2 & \\ 1 & 0 & 0 & y_0 \\ 0 & 1 & 0 & -\frac{3}{2}y_0 + 2y_1 - \frac{1}{2}y_2 \\ 0 & 0 & 2 & y_0 - 2y_1 + y_2 \end{array}\right]$$

Katsayi matrisi birim matrise donusturulur.

$$\left[\begin{array}{ccc|r} c_0 & c_1 & c_2 & \\ 1 & 0 & 0 & y_0 \\ 0 & 1 & 0 & -\frac{3}{2}y_0 + 2y_1 - \frac{1}{2}y_2 \\ 0 & 0 & 1 & \frac{1}{2}y_0 - y_1 + \frac{1}{2}y_2 \end{array}\right]$$

Buradan katsayi cozumleri asagidaki gibi belirlenir.

$c_0 = y_0$

$c_1 = \dfrac{1}{h}(-\dfrac{3}{2}y_0 + 2y_1 - \dfrac{1}{2}y_2)$

$c_2 = \dfrac{1}{h^2}(\dfrac{1}{2}y_0 - y_1 + \dfrac{1}{2}y_2)$

1.5 Alan Hesabi

$P_2(x)$ polinomunun integrali alinir.

$$I = \int_0^{2h} P_2(x)\,dx$$

$$I = \int_0^{2h} (c_0 + c_1 x + c_2 x^2)\,dx$$

$$I = (c_0 x + c_1 \frac{x^2}{2} + c_2 \frac{x^3}{3})\Big|_0^{2h}$$

Burada x yerine $2h$ yerlestirildiginde polinom ifadesinde bulunan $c_k * x^k$ terimlerinin tamam h ortak parantezine alinabilmektedir.

$I = c_0 * (2h) + c_1 * \frac{(2h)^2}{2} + c_2 * \frac{(2h)^3}{3}$

$I = h * ((y_0) * 2 + (-\frac{3}{2}y_0 + 2y_1 - \frac{1}{2}y_2) * \frac{2^2}{2} + (\frac{1}{2}y_0 - y_1 + \frac{1}{2}y_2) * \frac{2^3}{3})$

$I = h * (\frac{1}{3}y_0 + \frac{4}{3}y_1 + \frac{1}{3}y_2)$

$I = \frac{1}{3} * h * (y_0 + 4y_1 + y_2)$

h ve y degerleri ile integral hesaplanir.

$I = \frac{1}{3} * 0.15707963267 * (1 * 0.58778525229 + 4 * 0.70710678118 + 0.80901699436)$

$I = 0.2212324925$