FOURIER METHOD FOR NUMERICAL SIMULATIONS OF GROSS-PITAEVSKII EQUATION

N. A. Taylanov

Jizzakh State Pedagogical Institute, Jizzakh, Uzbekistan

S. X. Urinov

Samarkand State University, Samarkand, Uzbekistan

E-mail: taylanov@yandex.ru

Abstract: *This article presents a numerical approach to splitting into physical processes using the Fourier transform (SSFT) for solving the one dimensional nonlinear Schrödinger equation (NSE). Based on our MATLAB numerical simulations, we have extrapolated that this numerical approach consistently demonstrates the most robust capability for solving this equation.*

Keywords: *non-linear Schrödinger equation (NLSE), step splitting method, optical fibers, pseudospectral Fourier method*

Аннотация: *В этой статье представлен численный подход к разбиению на физические процессы с использованием преобразования Фурье (SSFT) для решения одномерного нелинейного уравнения Шредингера (NSE). Основываясь на нашем численном моделировании в MATLAB, мы экстраполировали, что этот численный подход последовательно демонстрирует наиболее надежные возможности для решения этого уравнения.*

Ключевые слова: *нелинейное уравнение Шредингера (NLS), метод ступенчатого расщепления, оптические волокна, псевдоспектральный метод Фурье*

Introduction

Partial differential equations are a widely used mathematical apparatus in the development of models in various fields of science and technology. Unfortunately, the explicit solution of these equations in an analytical form is possible only in special simple cases, and, as a result, the possibility of analyzing mathematical models is provided by solving these equations by approximate numerical methods. In recent years, non-linear evolution equations have become a very active field for describing various areas of non-linear sciences. One-dimensional nonlinear Schrödinger equation (1D NLSE) is a classical field equation. Its most prominent applications are related to the propagation of light waves in optical fibers and planar waveguides along with many others [1]. In particular, 1D NLSE is a non-linear second-order partial differential equation applicable to both classical and quantum mechanics. The nonlinear Schrödinger equation has an extremely high universality and is used to describe wave processes in many areas of physics: in the theory of surface waves [1], in models of the evolution of plasma oscillation distributions [2], nonlinear optics [3], biophysics, etc. . The non-linear Schrödinger equation describes the propagation of non-linear Langmuir waves, waves in deep water; waves in transmission lines, acoustic waves in liquids with bubbles and, above all, the propagation of optical radiation in nonlinear media. A typical application of the nonlinear Schrödinger equation is the dynamics of optical pulses in an optical fiber. The time evolution of the envelope of an optical pulse in a fiber is well approximated by the nonlinear Schrödinger equation, including the description of very long transoceanic optical communication lines, see, for example, [4, 5].

The nonlinear Schrödinger equation under consideration is a nonlinear differential equation with partial derivatives, which in the general case cannot be solved analytically. Therefore, numerical simulation methods are used to solve this problem. The numerical methods used to solve the propagation equations can be divided into two classes: pseudospectral methods and finite difference schemes. In the general case, pseudospectral methods turn out to be an order of magnitude faster than difference schemes, with the same calculation accuracy [6]. The most common method for solving equations is the method of splitting into physical processes using the Fourier transform at a linear step (Split - Step Fourier Method , SSFM) [7, 8]. This method is easy to implement, fast, and has high accuracy with respect to the time variable. The high counting rate of the splitting method is achieved through the use of the fast Fourier transform algorithm [9] .

Methodology.

This article presents a numerical approach with a split Fourier transform step for solving the one-dimensional nonlinear Schrödinger equation. This impressive numerical method is essential to understanding the nonlinearity of fiber optics, as both dispersion and nonlinear effects are introduced in this process. It is it that can be effectively used to simulate the propagation of light pulses in an optical fiber over a short distance h. In addition, it is useful to consider its advantage in being a faster approach, especially when compared to the finite difference approach. In particular, 1D NLSE is a non-linear second-order partial differential equation applicable to both classical and quantum mechanics. This can be written as follows [2]:

$$i\frac{\partial\psi}{\partial t}=\frac{\partial^2\psi}{\partial x^2}+2|\psi|^2\psi, \tag{1}$$

where x , t are spatial and temporal variables, i is an imaginary number, ψ complex amplitude. More importantly, Equation (1) models wave propagation in a lossless optical fiber, which is a non-linear medium; the unknown function $\partial\psi/\partial t$ is a wave. The second order derivative $\partial^2\psi/\partial x^2$ represents the variance and the non-linear term $k|\psi|^2\psi$ represents the non-linearity of the problem. In our case, the time advance due to the nonlinear part can be written as

$$i\frac{\partial\psi}{\partial t} = 2|\psi|^2\psi, \tag{2}$$

which can be exactly solved

$$\tilde{\psi}(x, t_0 + \Delta t) = \exp\left[-2i|\psi(x, t_0)|^2 \Delta t\right]\psi(x, t_0), \tag{3}$$

where is Δt the time step. We write the linear part of Eq. (1) as

$$i\frac{\partial\psi}{\partial t} = \frac{\partial^2\psi}{\partial x^2}. \tag{4}$$

Using the Fourier series, one can imagine that [4]

$$\tilde{\psi}(x, t_0 + \Delta t) = F^{-1}\left[\exp\left(ik^2\Delta t\right)F|\tilde{\psi}(x, t_0 + \Delta t)|\right]. \tag{5}$$

Therefore, by combining (3) and (5), the full splitting form can be written as

$$\tilde{\psi}(x, t_0 + \Delta t) = F^{-1}\left[\exp\left(ik^2\Delta t\right)F\left[\exp\left(-2i|\tilde{\psi}(x, t_0)|^2\right)\psi(x, t_0)\right]\right].$$

Based on the initial conditions, this expression can be solved explicitly.

Results and discussions

It was found that this equation gives a solution if and only if the nonlinear effect is balanced by dispersion phenomena. When this balance takes place, the one-soliton solution, the multi-soliton solution, and the boundary soliton solution [5]

$$\psi(x, t) = \exp(i(2x - 3t)) \cdot \operatorname{sech}(x - 4t). \tag{6}$$

For numerical estimates, we choose the initial condition:

$$\psi(x, 0) = \exp(2ix) \cdot \operatorname{sech}(x). \tag{7}$$

We plotted the solution obtained with our numerical methods over the spatial domain x from −10 to 10 and in the time domain t from 0 to 1 using different spatial steps $\Delta x = 0.1 \div 1$ at the time step $\Delta t = 0.001$, computed at time $t = 0.1 \div 1$. The calculation results are presented in Figures 1-3, which show the graphs of the solution of the nonlinear Schrödinger equation.

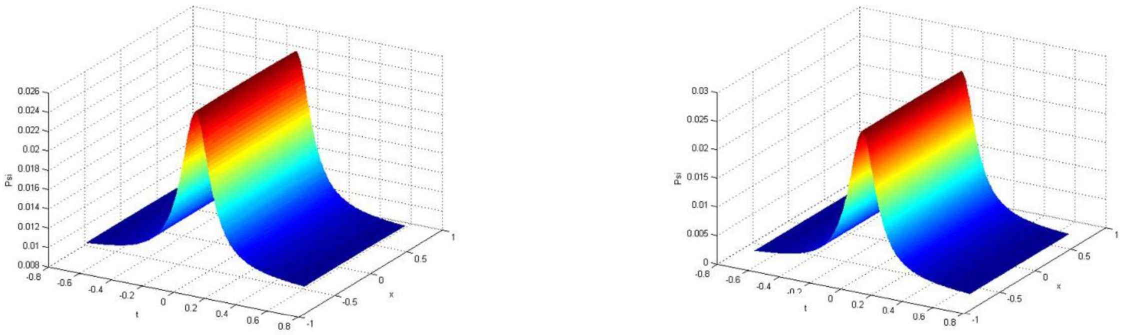

Fig.1. A 3D graph of approximate numerical solution of 1D NLSE using the split-step Fourier transform (SSFT) approach (a) $t = 0.1$ and (b) $t = 0.3$.

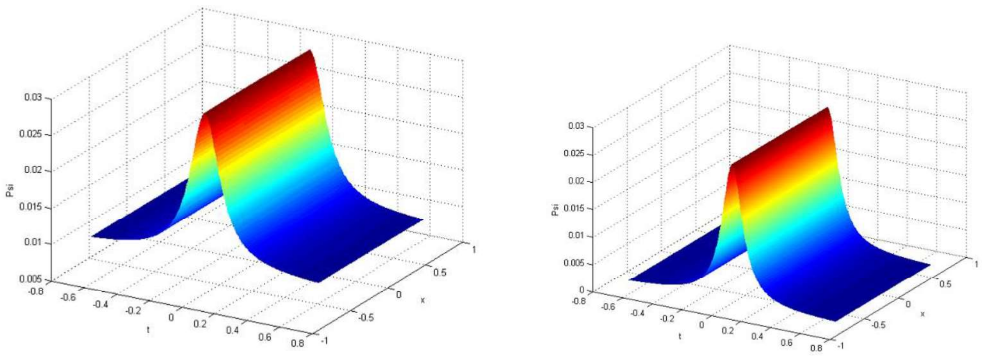

Fig.2.. A 3D graph of approximate numerical solution of 1D NLSE using the split-step Fourier transform (SSFT) approach (a) $t = 0.4$ and (b) $t = 0.6$.

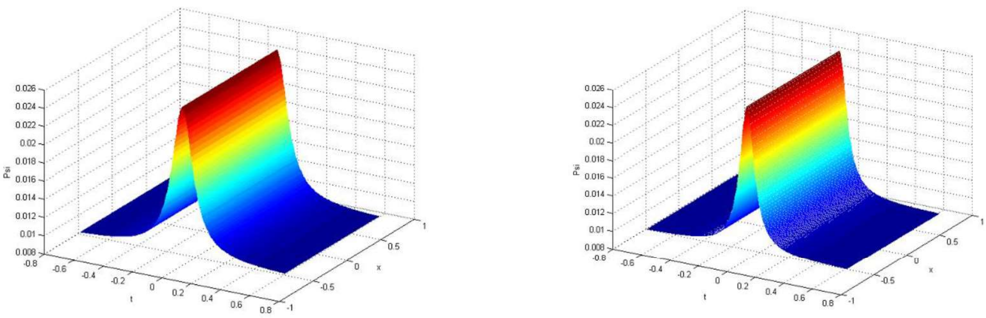

Fig.3. Fig.1. A 3D graph of approximate numerical solution of 1D NLSE using the split-step Fourier transform (SSFT) approach (a) $t = 0.8$ and (b) $t = 1$.

According to the results presented in these figures, the method of splitting into physical processes using the Fourier transform ensures high accuracy of the numerical solutions of the nonlinear Schrödinger equation. On the other hand, as can be seen from the figures, the result obtained by the implicit exponential difference scheme has better results than the results obtained by other numerical schemes. These calculations show that the accuracy of the solutions is quite high even in the case of a small number of grid nodes.

Conclusion

In this study, we consider the method of splitting into physical processes using the Fourier transform for the numerical simulation of the nonlinear Schrödinger equation. Approximate solutions of the nonlinear Schrödinger equation were obtained using the Matlab program. It is shown that the proposed method significantly increases the computational costs. This improvement becomes more significant, especially for large time evolutions.

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