Signals and Systems: MoDem

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1 Choice of the Orthonormal Basis

In the process of choosing our orthonormal basis we have taken in account the final implementation of the modem which is going to work under an audio channel so the frequencies are limited by the dynamic range of a microphone and a speaker, as we don't know what that would be we wanted to generate a basis that lives in the frequency interval were both speaker and microphone perform okay.

$$\phi_k(t) = \begin{cases} \sin(2\pi[kf_0 + fMin]t) & t \in [0,5] \\ 0 \quad resto \end{cases} \tag{1}$$

2 Proof of orthogonality

We compute the inner product for any two signals ϕ_i, ϕ_j in order to check for orthogonality of all elements of our base

$$\begin{aligned} < \phi_i, \phi_j > &= \int_0^5 \sin(2\pi[if_0 + fMin]t)^* \cdot \sin(2\pi[jf_0 + fMin]t) \\ &= \frac{1}{2}\Big[\int_0^5 \cos(2\pi[if_0 + fMin] - 2\pi[if_0 + fMin]t)dt \\ &- \int_0^5 \cos(2\pi[if_0 + fMin] + 2\pi[jf_0 + fMin]t)dt\Big] \\ &= \frac{1}{2}\Big[\int_0^5 \cos(2\pi f_0[i-j]t)dt - \int_0^5 \cos(2\pi f_0[i+j]t)dt\Big] \\ &= \frac{1}{2}\Big[\frac{\sin(2\pi f_0[i-j]t)dt}{2\pi f_0[i-j]}\Big]_0^5 - \frac{1}{2}\Big[\frac{\sin(2\pi f_0[i+j]t)}{2\pi f_0[i+j]}\Big]_0^5 = 0 \end{aligned}$$

For the inner product to be 0 either the sines cancel each other out which is difficult to prove generally or both are 0 at the same time which always happens for $T_0 = 1/f_0$ which are an integer divisor of the length of the signal (T=5);

3 Modulation

The modulation process takes a bit-stream consisting in a list of 1 an 0 and modulates the signals of the basis in amplitude using)-1, 1) coefficients. As our Basis holds N = 200 elements we cannot have more than 200bits using this modulation depth. If we increase N we can expect to get more bits inside that 5 second period as our basis is larger. The modulated signal is built doing the summation of all those modulated base components.

$$x(t) = \sum_{n=0}^{N} \alpha_n \cdot \phi_n(t) \tag{2}$$

4 Demodulation

The demodulation proccess is carried out by the opposite operation, by doing the inner product of our modulated signal with all the elements of the orthonormal basis:

$$\alpha_i = \int_0^5 x(t) \cdot \phi_i(t) dt \tag{3}$$