

CONTENTS

Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

1 STABILITY

1.1 Second order System

1.1. Consider the following second order system with the transfer function

$$G(s) = \frac{1}{1 + 2s + s^2} \quad (1.1.1)$$

Is the system stable?

Solution: The poles of

$$G(s) = \frac{1}{1 + 2s + s^2} \quad (1.1.2)$$

are at

$$s = -1 \quad (1.1.3)$$

i.e., the left half of s-plane. Hence the system is stable.

1.2. Find and sketch the step response $c(t)$ of the system.

Solution: For step-response, we take input as unit-step function $u(t)$

$$C(s) = U(s).G(s) = \left[\frac{1}{s} \right] \left[\frac{1}{1 + 2s + s^2} \right] \quad (1.2.1)$$

$$= \frac{1}{s(1 + s)^2} \quad (1.2.2)$$

$$= \frac{1}{s} - \frac{1}{(1 + s)} - \frac{1}{(1 + s)^2} \quad (1.2.3)$$

Taking the inverse Laplace transform,

$$c(t) = L^{-1} \left[\frac{1}{s} \right] - L^{-1} \left[\frac{1}{1 + s} \right] - L^{-1} \left[\frac{1}{(1 + s)^2} \right] \quad (1.2.4)$$

$$= (1 - e^{-t} - te^{-t}) u(t) \quad (1.2.5)$$

The following code plots $c(t)$ in Fig. ??

```
codes/ee18btech11002/plot.py
```

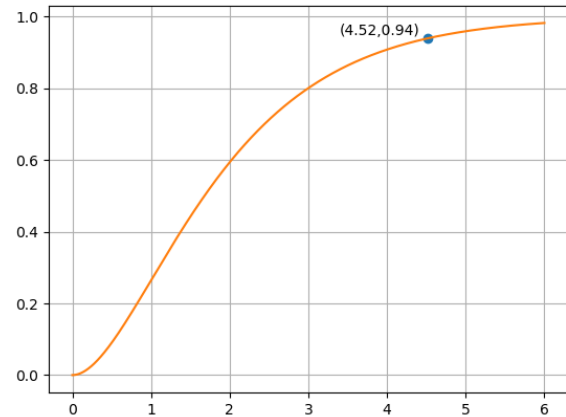


Fig. 1.2

1.3. Find the steady state response of the system using the final value theorem. Verify using ??
Solution: To know the steady response value of $c(t)$, Using Final value theorem:

$$\lim_{t \rightarrow \infty} c(t) = \lim_{s \rightarrow 0} sC(s) \quad (1.3.1)$$

We get

$$\lim_{s \rightarrow 0} s \left(\frac{1}{s} \right) \left(\frac{1}{1 + 2s + s^2} \right) = \frac{1}{1 + 0 + 0} = 1 \quad (1.3.2)$$

To verify, calculate

$$\lim_{t \rightarrow \infty} c(t) = (1 - 0 - 0) = 1 \quad (1.3.3)$$

1.4. Find the time taken for the system output $c(t)$ to reach 94% of its steady state value.

Solution: Now, 94% of 1 is 0.94, so we should now solve for a positive t such that

$$1 - e^{-t} - te^{-t} = 0.94 \quad (1.4.1)$$

The following code

```
codes/ee18btech11002/solution.py
```

provides the necessary solution as

$$t = 4.5228 \quad (1.4.2)$$

2 ROUTH HURWITZ CRITERION

3 COMPENSATORS

4 NYQUIST PLOT