Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```


4.1 Polar plot

4.1. Sketch direct and inverse polar plots for a unity feedback system with open loop transfer function

$$G(s) = \frac{1}{s(1+s)^2} \tag{4.1.1}$$

Solution: For Unity feedback system, given the open loop transfer function

$$G(s) = \frac{1}{s(1+s)^2} \tag{4.1.2}$$

Now, Polar plot is defined as: The plot of points(represented as $r.e^{\jmath\phi}$) obtained by varying w from 0 to ∞ where

$$r = |H(\jmath\omega)||G(\jmath\omega)| \tag{4.1.3}$$

$$\phi = \angle H(\jmath\omega)G(\jmath\omega) \tag{4.1.4}$$

Inverse Polar plot is similar, in this we have;

$$r = \frac{1}{|H(\jmath\omega)||G(\jmath\omega)|} \tag{4.1.5}$$

$$\phi = -\angle H(\jmath\omega)G(\jmath\omega) \tag{4.1.6}$$

The system we're analysing is unity feedback which means $H(\jmath\omega) = 1$ Therefore ;

$$|H(\jmath\omega)||G(\jmath\omega)| = \frac{1}{\omega(1+\omega^2)} \tag{4.1.7}$$

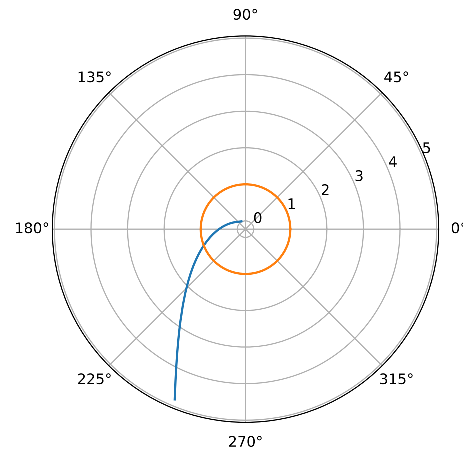

Fig. 4.1: Polar Plot

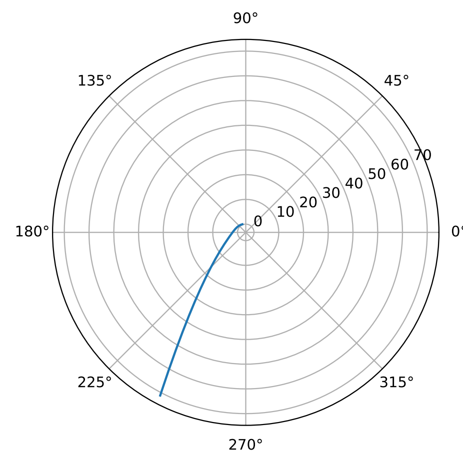

Fig. 4.1: Inverse Polar Plot

and Phase;

$$\angle H(\jmath\omega)G(\jmath\omega) = \frac{-\pi}{2} - 2tan^{-1}(\omega) \tag{4.1.8}$$

The following code plots the polar and inverse polar plots

```
codes/ee18btech11002/polarplot.py
```

4.2. Explain the utility of Polar plot.
Solution: We saw earlier that the Polar Plot is the Plot of $G(\jmath\omega)$ on a complex plane by varying ω. The Polar plot does not give any information about the number of poles lying in positive half of s-plane. However, the two things which can be seen easily on the Polar Plot are Gain Margin and Phase margin. The Gain Margin is the value of $\frac{1}{|G(\jmath\omega)|}$ at the point

where the plot crosses the x-axis. The Phase margin is the phase of the point which intersects the unit circle, measured anticlockwise from the negative x-axis. From the values of Gain and Phase Margin we can conclude:

- Greater values of Gain and Phase margin indicate greater stability.
- If Gain margin is greater than 1 **and** Phase margin is greater than 0, system is stable.
- If Gain margin = 1 **and** Phase margin = 0, system is marginally stable.
- if Gain margin is lesser than 1 **or** Phase Margin is lesser than 0, system is unstable

From our Plot, we can conclude that Gain Margin is greater than 1 **and** Phase margin is greater than 0, therefore our system is stable.

4.3. Find the frequency at which $|G(\jmath\omega)| = 1$ and corresponding phase angle $\angle G(\jmath\omega)$

Solution: For $|G(\jmath\omega)| = 1$

$$\frac{1}{\omega(1+\omega^2)} = 1 \tag{4.3.1}$$

$$\omega + \omega^3 - 1 = 0 \tag{4.3.2}$$

and for the corresponding phase

$$\angle G(\jmath\omega) = \frac{-\pi}{2} - 2tan^{-1}(\omega) \tag{4.3.3}$$

The following code calculates the value of w and $\angle G(\jmath\omega)$

```
codes/ee18btech11002/solution.py
```

and we get

$$\omega = 0.682 \tag{4.3.4}$$

$$\angle G(\jmath\omega) = -\frac{52}{59}\pi \tag{4.3.5}$$