Control Systems (EE2227)

Abhishek

IIT Hyderabad

April 2, 2020

Question

Consider the following second order system with the transfer function:

$$G(s) = \frac{1}{1 + 2s + s^2}$$

with input unit step

$$R(s) = \frac{1}{s}$$

Let C(s) be the corresponding output. The time taken by the system output c(t) to reach 94% of its steady state value, rounded off to two decimal places is

(A)5.25 (B)4.50 (C)3.89 (D)2.81

Answer

The approach for finding the solution is as follows:

- finding C(s)
- finding c(t)
- finding the time at which c(t) attains 94% of its steady state value

Finding C(s)

We are given G(s) and R(s), to find C(s), we can simply multiply these two

$$C(s) = R(s).G(s) = (\frac{1}{s})(\frac{1}{1+2s+s^2})$$

$$C(s) = \frac{1}{s(1+s)^2}$$

Finding c(t)

To find c(t), we have to do inverse Laplace transform on C(s)

$$c(t) \longleftrightarrow C(s)$$

Inverse Laplace transform can be calculated by the formula:

$$f(t) = \frac{1}{2\pi j} \int_{a-j\infty}^{a+j\infty} F(s) e^{st} ds$$

From the above formula, the inverse Laplace for some common expressions are:

$$u(t) \longleftrightarrow \frac{1}{s}$$

$$e^{-at} u(t) \longleftrightarrow \frac{1}{s+a}$$

$$te^{-at} u(t) \longleftrightarrow \frac{1}{(s+a)^2}$$

Finding c(t)

We found C(s) as:

$$C(s) = \frac{1}{s(1+s)^2}$$

Now, we will use partial fractions to make applying Inverse Laplace easy.

$$C(s) = \frac{1}{s(1+s)^2} = \frac{A}{s} + \frac{B}{(1+s)} + \frac{C}{(1+s)^2}$$

We get,

$$A = 1 \qquad A + B = 0 \qquad 2A + B + C = 0$$
$$A = 1 \qquad B = -1 \qquad C = -1$$

Therefore,

$$C(s) = \frac{1}{s} - \frac{1}{(1+s)} - \frac{1}{(1+s)^2}$$

Inverse Laplace

$$c(t) = L^{-1}(\frac{1}{s} - \frac{1}{(1+s)} - \frac{1}{(1+s)^2})$$

From the properties of inverse Laplace transform,

$$L^{-1}(F_1(s) + F_2(s) + F_3(s)) = L^{-1}(F_1(s)) + L^{-1}(F_2(s)) + L^{-1}(F_3(s))$$

Therefore;

$$c(t) = L^{-1}(\frac{1}{s}) - L^{-1}(\frac{1}{(1+s)}) - L^{-1}(\frac{1}{(1+s)^2})$$

Using the Known inverse transforms:

$$c(t) = (1 - e^{-t} - te^{-t}).u(t)$$

Finding the time for reaching 94%

To know the steady state value of c(t), we calculate

$$\lim_{t \to \infty} c(t) = (1 + 0 + 0).(1) = 1$$

Now, 94% of 1 is 0.94, so we should now solve for a positive t such that

$$(1 - e^{-t} - te^{-t}) = 0.94$$

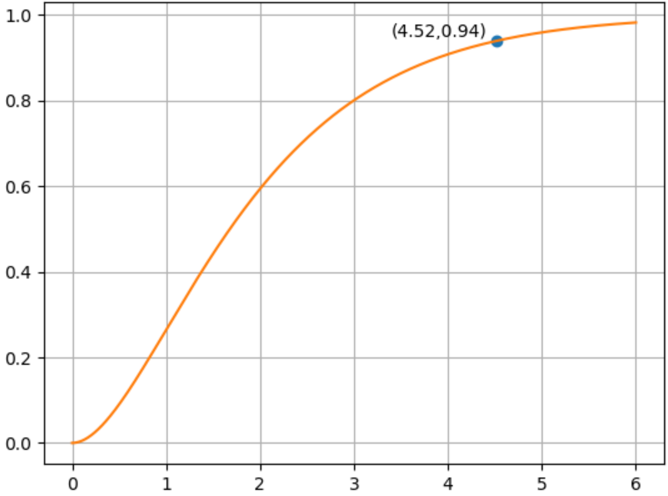
the attached code gives us the solution for the equation and t turns out to be

$$t = 4.5228$$

Therefore, answer is option (b)

Plot

We can verify the solution by plotting c(t):



THANK YOU