

CONTENTS

Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

1 STABILITY

1.1 Second order System

1.1. Consider the following second order system with the transfer function

$$G(s) = \frac{1}{1 + 2s + s^2} \quad (1.1.1)$$

Is the system stable?

Solution:

$$G(s) = \frac{1}{1 + 2s + s^2} \quad (1.1.2)$$

From given expression of $G(s)$, both poles of $G(s)$ are at $(-1,0)$ which is on the left half of s -plane, therefore we can conclude that the system is stable.

1.2. Find and sketch the step response $c(t)$ of the system.

Solution: For step-response, we take input as unit-step function $u(t)$

$$C(s) = R(s).G(s) = \left[\frac{1}{s} \right] \left[\frac{1}{1 + 2s + s^2} \right] \quad (1.2.1)$$

$$C(s) = \frac{1}{s(1 + s)^2} \quad (1.2.2)$$

Using Partial Fractions,

$$C(s) = \frac{1}{s} - \frac{1}{(1 + s)} - \frac{1}{(1 + s)^2} \quad (1.2.3)$$

```
import numpy as np
import matplotlib.pyplot as plt

x = np.linspace(0,6,1000)
y = 1-(np.exp(-x))-(x*np.exp(-x))
plt.plot(4.52218,0.94,'o')
plt.text(3.4,0.95,'(4.52,0.94)')
plt.grid()
plt.plot(x,y)
plt.show()
```

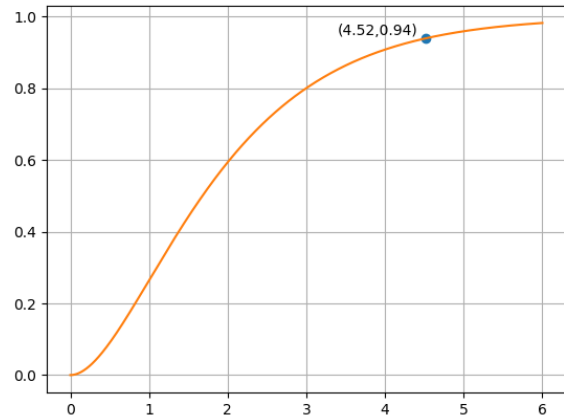


Fig. 1.2

Therefore;

$$c(t) = L^{-1} \left[\frac{1}{s} \right] - L^{-1} \left[\frac{1}{1 + s} \right] - L^{-1} \left[\frac{1}{(1 + s)^2} \right] \quad (1.2.4)$$

Using the Known inverse transforms:

$$c(t) = (1 - e^{-t} - te^{-t}).u(t) \quad (1.2.5)$$

1.3. Find the steady state response of the system.

Solution: To know the steady response value of $c(t)$, we calculate

$$\lim_{t \rightarrow \infty} c(t) = (1 + 0 + 0).(1) = 1 \quad (1.3.1)$$

1.4. Find the time system output $c(t)$ to reach 94% of its steady state value.

Solution: Now, 94% of 1 is 0.94, so we should now solve for a positive t such that

$$(1 - e^{-t} - te^{-t}) = 0.94 \quad (1.4.1)$$

```
import numpy as np
import matplotlib.pyplot as plt
```

```
t = 0
c = 0
y = 0
```

```
# defining a function whose solution of f=0
will give us the value of t for 94% of
output
```

```
def s(a):
```

```
    v = 0.06 - np.exp(-a) - a*np.exp(-
```

```

        a)
        return v

#loop starts at t = 0, checks wether function
  is +ve, if not, then increases t by 0.001
while c == 0:
    y = s(t)
    if y < 0:
        c = 0
        t = t+0.001

    else:
        c = 1

#approx. the curve as a straight line, we find
  the crossing point by weighted mean of
  the two succesive values
t1 = ((t-0.001)*(-s(t-0.001)) + t*s(t))/(-s(t
  -0.001) + s(t))

print(t1)

```

$$t = 4.5228 \quad (1.4.2)$$

2 ROUTH HURWITZ CRITERION

3 COMPENSATORS

4 NYQUIST PLOT