F85: Optik Grundpraktikum

Jonathan Böcker, Vincent Mader

08.07.-10.07.2019

Heidelberg University
Tutor: Keerthan Subramanian

Abstract
This FP lab course serves as an introduction to three important properties of light waves: Polarization, phase and frequency are observed using commonly used optical components such as wave-plates, polarizing beam-splitters, electro-optical modulators and acousto-optical modulators.

The material we use as an optical medium in this course is $LiNbO_3$.

Contents

List of Figures

1 Introduction

2 Theory

2.1 Polarization

A beam of light can be described as a plane electromagnetic wave, like so:

$$\vec{E}(z,t) = E_0 \cdot \hat{e}_x \cdot e^{i(\omega t - kz)} \tag{1}$$

$$\vec{B}(z,t) = B_0 \cdot \hat{e}_y \cdot e^{i(\omega t - kz)} \tag{2}$$

The propagation vector $\hat{e}_z$ together with $\vec{E}$ and $\vec{B}$ create an orthogonal system.

The direction of $\vec{E}$ defines the polarization, which can be distinguished into three types:

1. linear polarization
2. elliptical polarization
3. circular polarization

In the case of elliptical and circular polarisation, the wave can be thought of as a superposition of two orthogonal plane waves with a phase difference $\Delta\varphi = \frac{\pi}{2}$: What happens for $\Delta\varphi \neq \frac{\pi}{2}$?

$$\vec{E}_{\pm} = (E_x \hat{e}_x \mp i E_y \hat{e}_y) \cdot e^{i(\omega t - kx)} \tag{3}$$

If, in addition, both waves carry the same amplitude ($E_x = E_y$), we speak of circular polarization.

2.1.1 Snell's law

If a beam of light reaches the surface between two optical media with refractive indices n_1 and n_2, the beam splits into a reflected and a refracted one. The refraction angle α_2 can be determined by

$$n_1 \cdot sin(\alpha_1) = n_2 \cdot sin(\alpha_2) \tag{4}$$

The angles α_1 and α_2 are taken between the incoming/outgoing beam of light and the surface.

For reflection, the condition $\alpha_{in} = \alpha_{out}$ must hold.

2.1.2 Malus law

The Malus law states that when a perfect polarizer is placed in a polarized beam of light, the intensities before and after tranversing the polarizer are related by

$$I_f = I_i \cdot cos^2(\theta) \tag{5}$$

Here, θ is the angle between the light's initial polarization direction and the axis of the polarizer.

2.1.3 Brewster's angle

Brewster's angle is a specific angle of incidence at which light with a particular polarization is perfectly transmitted through a transparent dielectric surface, without any reflection.

When unpolarized light is incident at this angle, the light that is reflected from the surface is therefore perfectly polarized. Meaning? Sources?

2.1.4 Fresnel equations

The Fresnel equations describe the reflection and transmission of light when incident on an interface between different optical media. equations

2.1.5 Reflection and polarization

Consider again a beam of light traversing from one medium into another. The beam splits into a reflectd and a refracted part. From Maxwell's equation the amplitudes and intensities of the two beams can be derived. The amplitude coefficients corresponding to the reflected and the transmitted beams are labeled r and t. The intensity coefficients R and T can easily be calculated by taking the square of r and t. If the polarization is perpendicular to the plane of incidence, the effect is called transversal-electric polarization (German: S-Polarisation).
The coefficients are given by

$$r_{TE} = -\frac{sin(\alpha_1 - \alpha_2)}{sin(\alpha_1 + \alpha_2)} \tag{6}$$

and

$$t_{TE} = \frac{2 \cdot sin(\alpha_1) \cdot cos(\alpha_2)}{sin(\alpha_1 + \alpha_2)} \tag{7}$$

If the wave is polarized parallel to the plane of incidence, we speak of transversal-magnetic polarization (German: P-Polarisation), the coefficients are given by

$$r_{TM} = \frac{tan(\alpha_1 - \alpha_2)}{tan(\alpha_1 + \alpha_2)} \tag{8}$$

and

$$t_{TM} = \frac{2 \cdot sin(\alpha_1) \cdot cos(\alpha_2)}{sin(\alpha_1 + \alpha_2) \cdot cos(\alpha_1 - \alpha_2)} \tag{9}$$

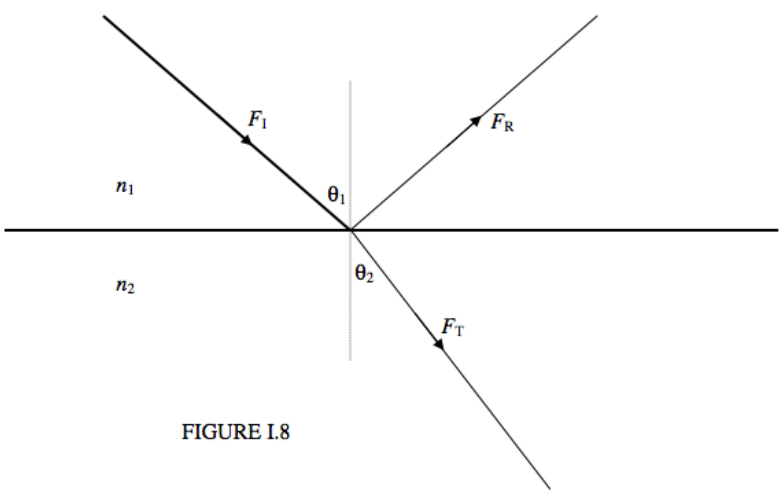


Figure 1: refraction and reflection

plot?

2.1.6 Birefringence

If the medium is perfectly homogenous and isotropic, the refraction index is the same in all directions. Here, there is the simple relation $\vec{D} = \varepsilon \vec{E}$ with permittivity $\varepsilon = n^2$. In general, a medium is likely to be anisotropic. The permittivity is best described by a tensor ε_{ij}.
uniaxial, biaxial media

2.1.7 Wave plates

Wave plates are birefringent crystals used to shift the phase of a traversing wave by a certain fraction of a whole period.

Quarter-wave plates can be used to turn a light beam's polarization from elliptical to linear and the other way round. For this its width ideally is $(m + \frac{1}{4}) \cdot \lambda, m \in \mathbb{N}$.

Half-wave plates can be used to change the polarization state of linearily polarized light. The optimal width for this is $(m + \frac{1}{2}) \cdot \lambda$.

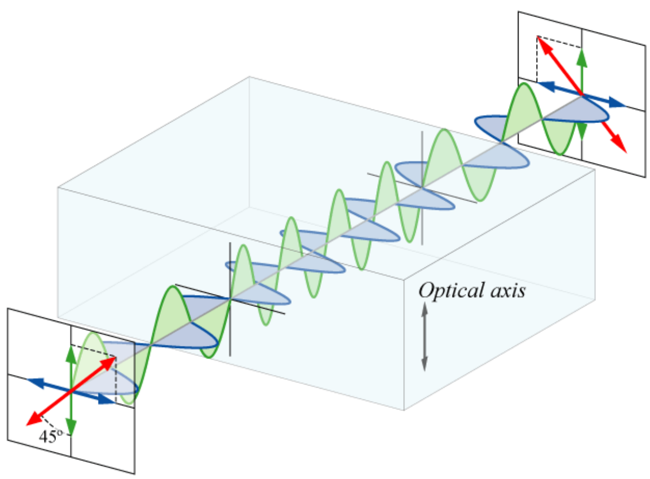


Figure 2: effect of a half-wave plate on polarization

2.2 Electro-optical effect

Some materials experience a change in their optical properties when they are brought into an external electric field. If the refractive index n is a function of the applied field E, we speak of an electro-optical modulator (EOM).

2.2.1 Pockels effect (linear electro-optical effect)

The refractive index $n = n(E)$ can be expanded around $E = 0$ for small field strengths.

With $r = -\frac{2}{n^3}\left(\frac{dn}{dE}\right)\Big|_{E=0}$ and $s = -\frac{1}{n^3}\left(\frac{d^2n}{dE^2}\right)\Big|_{E=0}$, this leads to the relation

$$n(E) = n_0 - \frac{1}{2}rn^3E - \frac{1}{2}sn^3E^2 \tag{10}$$

The linear electro-optical effect, also called Pockels effect, occurs for $r \gg s$:

$$n(E) \approx n_0 - \frac{1}{2}rn^3E \tag{11}$$

Here, r is called the Pockels coefficient. If, on the other hand, $r \ll s$, the quadratic dependence of n on E is known as the Kerr effect. This effect will not be studied in this lab course.

2.2.2 Pockels effect in a non-isotropic crystal

Since the electro-optical crystals are in general birefringent, the Pockels coefficient is not a scalar, but a tensor. For the material $LiNbO_3$, the entries of this coefficient-tensor are given by

$$\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{pmatrix} = \begin{pmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -3.4 & 8.6 \\ 0 & 3.4 & 8.6 \\ 0 & 0 & 30.8 \\ 0 & 28 & 0 \\ 28 & 0 & 0 \\ -3.4 & 0 & 0 \end{pmatrix} \cdot 10^{-12}\ \frac{\mathrm{m}}{\mathrm{V}}$$

If no external field is applied to the crystal, the indicatrix is given by

$$\frac{1}{n_o^2}x^2 + \frac{1}{n_o^2}y^2 + \frac{1}{n_e^2}z^2 = 1 \tag{12}$$

In this lab course, the electric field is applied along the extraordinary axis of the Pockels cell, the light propagates along one of the ordinary axes. With $\vec{E} = E \cdot \hat{e}_z$ the indicatrix can be written as

$$\left(\frac{1}{n_0^2} + r_{13}E_z\right)x^2 + \left(\frac{1}{n_0^2} + r_{13}E_z\right)y^2 + \left(\frac{1}{n_e^2} + r_{33}E_z\right)z^2 \tag{13}$$

For small applied field strengths this leads to the relations

$$n_o'(E) \approx n_o - \frac{1}{2}r_{13}n_o^3E_z \tag{14}$$

$$n_e'(E) \approx n_e - \frac{1}{2}r_{33}n_e^3E_z \tag{15}$$

2.2.3 Pockels cell

An electro-optical crystal between two capacitor plates is called an electro-optical modulator, abbreviated as EOM. A Pockels cell is an EOM which exhibits the Pockels effect, meaning that the refractive index is approximately proportional to the strength of an applied electric field $\vec{E}$ (for small values of $|\vec{E}|$)

If light with a wavelength λ travels through a medium with refractive index n for a distance L, the accumulated phase is given by the relation

$$\Phi = 2\pi n \frac{L}{\lambda} \tag{16}$$

The phase difference between a beam of light traveling along the ordinary vs the exraordinary axis is thus:

$$\Delta\Phi = 2\pi(n_e - n_o)\frac{L}{\lambda} \tag{17}$$

With a Pockels cell, we can manipulate the magnitude of both n_o and n_e. Plugging equation 14 and 15 into this last equation, we get the following:

$$\Delta\Phi(E) = 2\pi\frac{L}{\lambda}\left(n_e - n_o - \frac{1}{2}(r_{33}n_e^3 - r_{13}n_e^3 - r_{13}n_o^3)E_z\right) \tag{18}$$

This can be written as

$$\Delta\Phi(V) = \Phi_0 - \pi\frac{V}{V_\pi} \tag{19}$$

with $V = Ed$, $\Phi_0 = 2\pi\frac{L}{\lambda}(n_e - N_o)$ and $V_\pi = \frac{d}{L}\frac{}{\lambda}r_{33}n_e^3 - r_{13}n_o^3$ A Pockels cell can be used to modulate the intensity of light. For this, the cell has to be positioned between two crossed polarizers, each at an angle of 45° relative to the crystal's optical axis. The transmittance of this setup is

$$T(V) = sin^2\left(\frac{\Phi_0}{2} - \frac{\pi}{2}\frac{V}{V_\pi}\right) \tag{20}$$

2.2.4 Faraday effect

Some media become optically active when an axial magnetic field is applied. Optically active means that the plane of polarization of linear polarized light is rotated. The angle of rotation α_{rot} depends on the length of the medium L, the magnetic field strength $|\vec{B}|$ and the so-called Verdet constant v:

$$\alpha_{rot} = vL|\vec{B}| \tag{21}$$

The Verdet constant is a function of the wavelength λ:

$$v = -\frac{\pi\gamma}{\lambda n} \tag{22}$$

Here, γ is a material constant of the medium, the so-called magnetogyration coefficient.

2.2.5 Optical isolator

The Faraday effect can be used to build an optical isolator, also called an optical diode.

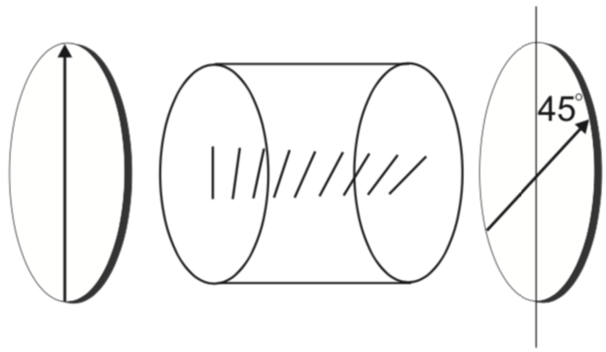


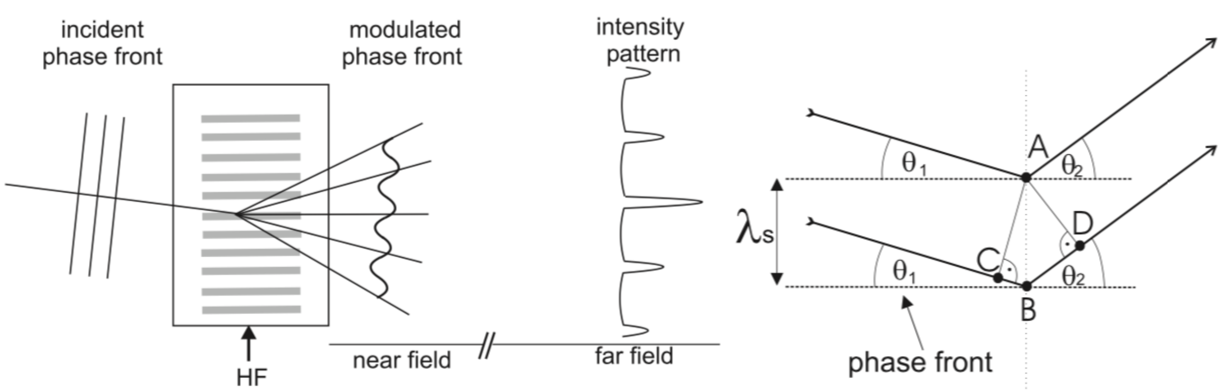
Figure 3: optical diode

2.3 Acousto-optical effect

If a sound wave passes through a crystal, its density varies periodically, which also leads to a periodic variation in the refractive index. A plane sound wave with wavelength λ_s in a crystal with initial refractive index n_0 can be described by

$$n(x,t) = n_0 - \Delta n \cdot cos\left(\omega t - \frac{2\pi}{\lambda_s} x\right)$$

Here, the amplitude $\Delta n = \frac{1}{2} p n^3 s_0$ depends on the photo-elastic constant p and the amplitude of the strain s_0. In this course the main interaction between sound and light will be the so-called Debye-Sears effect which occurs for short interaction lengths, i.e. for a thin crystal or thin sound beam. Parts of the light beam that are travelling through the denser regions experience a phase shift. Maxima in the far field can be observed, if these parts of the beam interfere constructively with each other, as can be seen in the next figure.



(a) deformation of a plane wavefront by a sound wave (b) setup 2

Figure 4: acousto-optical effect

Because the light is interacting with a moving sound wave, the diffracted light is Doppler-shifted:

$$\omega_{out} = \omega_{in} + m\Delta\omega = \omega_{in} + m\omega_s$$

A different approach to explain the effect of an AOM on a light beam is to describe the system as two scattering quasi-particles, namely a phonon and a photon. Their momentum vectors are $\hbar\vec{k}_l$ and $\hbar\vec{k}_s$, respectively. Conservation of momentum and energy leads to the relations

$$\vec{k}_{l,f} = \vec{k}_{l,i} \pm m\vec{k}_s$$

and

$$\nu_{l,f} = \nu_{l,i} \pm m\nu_s$$

Here, m is the diffraction order, i.e. the number of phonons that interacted with the photon. Constructive interference occurs when

$$sin\theta_1 + sin\theta_2 = m\frac{\lambda}{\lambda_s}$$

Here, θ_1 is the angle under which light enters the crystal and θ_2 the diffraction angle. efficiency

3 Experiment

3.1 Polarizers and wave plates

Date of carrying out experiment:

3.1.1 Brewster's angle

1. find Brester's angle
   only light polarized perpendicular to plane of incidence is reflected
   $\Rightarrow$ polarization of reflected light is then linear
2. take glass plate
3. vary angle of incidence
4. observations
5. second glass plate (perpendicular planes of incidence)

3.1.2 Polarizers

1. calibrate (for that: find polarization axis)
2. set reference for calibrating polarizers and PBS

3.1.3 Polarizing beam splitters

1. how is transmitted/reflected light polarized?
2. calibrate

3.1.4 Wave-plates

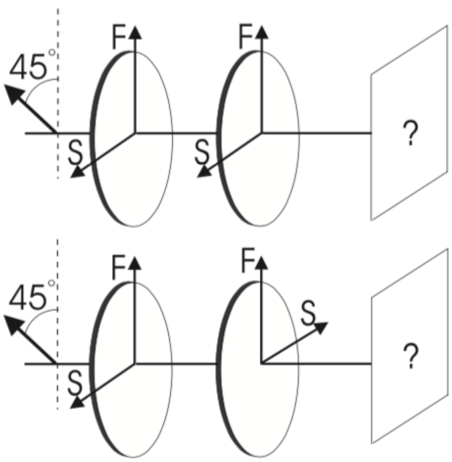
1. find angle: half-waveplates rotate polarization by $\frac{\pi}{2}$ ($\alpha = 45°$)
2. find angle: quarter-waveplates produce circular polarized light ($\alpha = 45°$)
3. experiments from figure 5a and 5b

3.1.5 Mirror

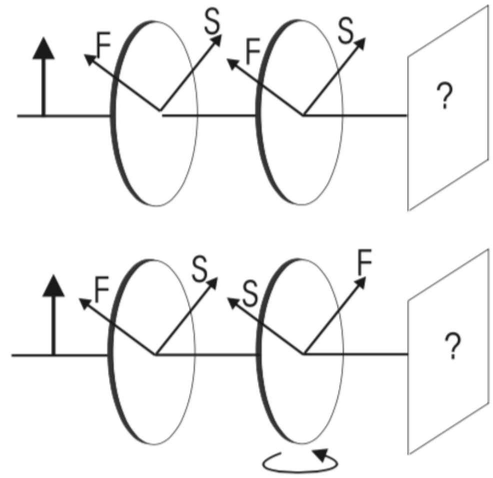
1. measure polarization of reflected light
2. send linear through two quarter-waveplates
3. ...

3.1.6 Lamp light

...



(a) setup 1



(b) setup 2

Figure 5: setups for wave-plate experiment

3.2 Electro-optical effect

Date of carrying out experiment:
Material: $LiNbO_3$ (uniaxial crystal, $3m$-Symmetrie, $n_1 = n_2 = n_o, n_3 = n_e$)
Pockels coefficients are very hard to measure, can be affected by crystal impurities

In the first part of this experiment, the change of the index of refraction will be measured directly. In the second part, the effect will be observed by measuring the polarization state of light before and after traversing the crystal.

3.2.1 Phase shift due to electro-optical modulation

1. $\vec{E}$-field along optical axis
2. laser beam is incident perpendicular to the field
3. HV supply: output should be between 0 V and 1.8 kV
4. measure amplification factor of HV amplifier
5. evaluate measurements

3.2.2 Mach-Zehnder interferometer

To quantify the extent by which the phase is shifted, we use a Mach-Zehnder interferometer.

1. why non-polarizing beam splitters
2. direction of optical axes
3. good overlap of arms
4. setup measurement
5. output intensity as function of applied voltage (for both axes)
6. compare results
7. just for fun: speaker

3.2.3 Manipulation of polarization and intensity

1. why Pockels cell at 45°
2. measure transmitted power as function of HV for ±45°

3.2.4 Linear amplitude modulation

1. ...

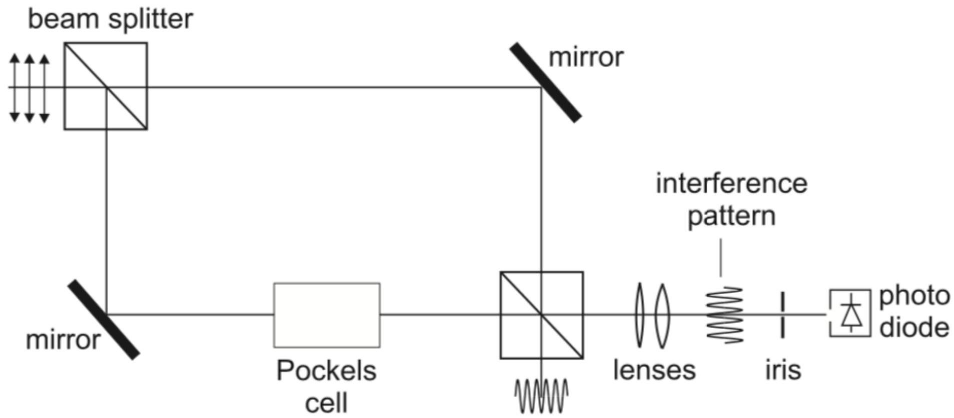


Figure 6: setup of Mach-Zehnder interferometer

3.3 Acousto-optical effect

Date of carrying out experiment:

3.3.1 Experiments with a single AOM

- adjust AOM so that amplitude of diffraction pattern is approximately symmetric in both ± 1 orders
- measure diffraction angles for order 1 and 2 for various frequencies
- measure power of order 1 maximum relative to power of undiffracted beam as a function of frequency (same steps as above)
- optimize power in order 1 max, ...

3.3.2 Two perpendicular AOMs

- expectation of diffraction pattern? compare with results
- what happes when offset voltage(s) is/are changed?

4 Results

5 Analysis

6 Summary

7 Discussion