F85: Optik Grundpraktikum

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Abstract
This FP lab course serves as an introduction to three important properties of light waves: Polarization, phase and frequency are observed using commonly used optical components such as wave-plates, polarizing beam-splitters, electro-optical modulators and acousto-optical modulators.

The material we use as an optical medium in this course is $LiNbO_3$.

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2.1 Polarization

A beam of light can be described as a plane electromagnetic wave, like so:

$$\vec{E}(z,t) = E_0 \cdot \hat{e}_x \cdot e^{i(\omega t - kz)} \tag{1}$$

$$\vec{B}(z,t) = B_0 \cdot \hat{e}_y \cdot e^{i(\omega t - kz)} \tag{2}$$

The propagation vector $\hat{e}_z$ together with $\vec{E}$ and $\vec{B}$ create an orthogonal system.

The direction of $\vec{E}$ defines the polarization, which can be distinguished into three types:

1. linear polarization
2. elliptical polarization
3. circular polarization

In the case of elliptical and circular polarisation, the wave can be thought of as a superposition of two orthogonal plane waves with a phase difference $\Delta\varphi = \frac{\pi}{2}$: What happens for $\Delta\varphi \neq \frac{\pi}{2}$?

$$\vec{E}_{\pm} = (E_x \hat{e}_x \mp i E_y \hat{e}_y) \cdot e^{i(\omega t - kx)} \tag{3}$$

If, in addition, both waves carry the same amplitude ($E_x = E_y$), we speak of circular polarization.

2.1.1 Snell's law

If a beam of light reaches the surface between two optical media with refractive indices n_1 and n_2, the beam splits into a reflected and a refracted one. The refraction angle α_2 can be determined by

$$n_1 \cdot sin(\alpha_1) = n_2 \cdot sin(\alpha_2) \tag{4}$$

The angles α_1 and α_2 are taken between the incoming/outgoing beam of light and the surface.

For reflection, the condition $\alpha_{in} = \alpha_{out}$ must hold.

2.1.2 Malus law

The Malus law states that when a perfect polarizer is placed in a polarized beam of light, the intensities before and after tranversing the polarizer are related by

$$I_f = I_i \cdot cos^2(\theta) \tag{5}$$

Here, θ is the angle between the light's initial polarization direction and the axis of the polarizer.

2.1.3 Brewster's angle

Brewster's angle is a specific angle of incidence at which light with a particular polarization is perfectly transmitted through a transparent dielectric surface, without any reflection.

When unpolarized light is incident at this angle, the light that is reflected from the surface is therefore perfectly polarized. Meaning? Sources?

2.1.4 Fresnel equations

The Fresnel equations describe the reflection and transmission of light when incident on an interface between different optical media. equations

2.1.5 Reflection and polarization

Consider again a beam of light traversing from one medium into another. The beam splits into a reflectd and a refracted part. From Maxwell's equation the amplitudes and intensities of the two beams can be derived. The amplitude coefficients corresponding to the reflected and the transmitted beams are labeled r and t. The intensity coefficients R and T can easily be calculated by taking the square of r and t. If the polarization is perpendicular to the plane of incidence, the effect is called transversal-electric polarization (German: S-Polarisation).
The coefficients are given by

$$r_{TE} = -\frac{sin(\alpha_1 - \alpha_2)}{sin(\alpha_1 + \alpha_2)} \tag{6}$$

and

$$t_{TE} = \frac{2 \cdot sin(\alpha_1) \cdot cos(\alpha_2)}{sin(\alpha_1 + \alpha_2)} \tag{7}$$

If the wave is polarized parallel to the plane of incidence, we speak of transversal-magnetic polarization (German: P-Polarisation), the coefficients are given by

$$r_{TM} = \frac{tan(\alpha_1 - \alpha_2)}{tan(\alpha_1 + \alpha_2)} \tag{8}$$

and

$$t_{TM} = \frac{2 \cdot sin(\alpha_1) \cdot cos(\alpha_2)}{sin(\alpha_1 + \alpha_2) \cdot cos(\alpha_1 - \alpha_2)} \tag{9}$$

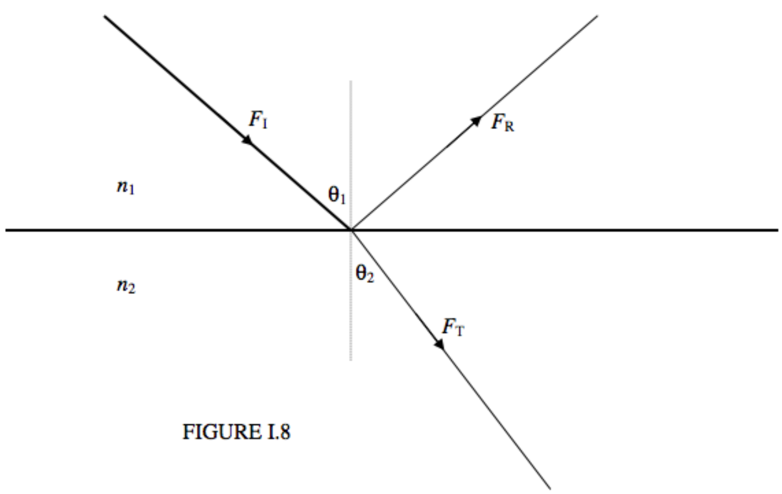


Figure 1: refraction and reflection

plot?

2.1.6 Birefringence

If the medium is perfectly homogenous and isotropic, the refraction index is the same in all directions. Here, there is the simple relation $\vec{D} = \varepsilon \vec{E}$ with permittivity $\varepsilon = n^2$. In general, a medium is likely to be anisotropic. The permittivity is best described by a tensor ε_{ij}.
uniaxial, biaxial media

2.1.7 Wave plates

Wave plates are birefringent crystals used to shift the phase of a traversing wave by a certain fraction of a whole period.

Quarter-wave plates can be used to turn a light beam's polarization from elliptical to linear and the other way round. For this its width ideally is $(m + \frac{1}{4}) \cdot \lambda, m \in \mathbb{N}$.

Half-wave plates can be used to change the polarization state of linearily polarized light. The optimal width for this is $(m + \frac{1}{2}) \cdot \lambda$.

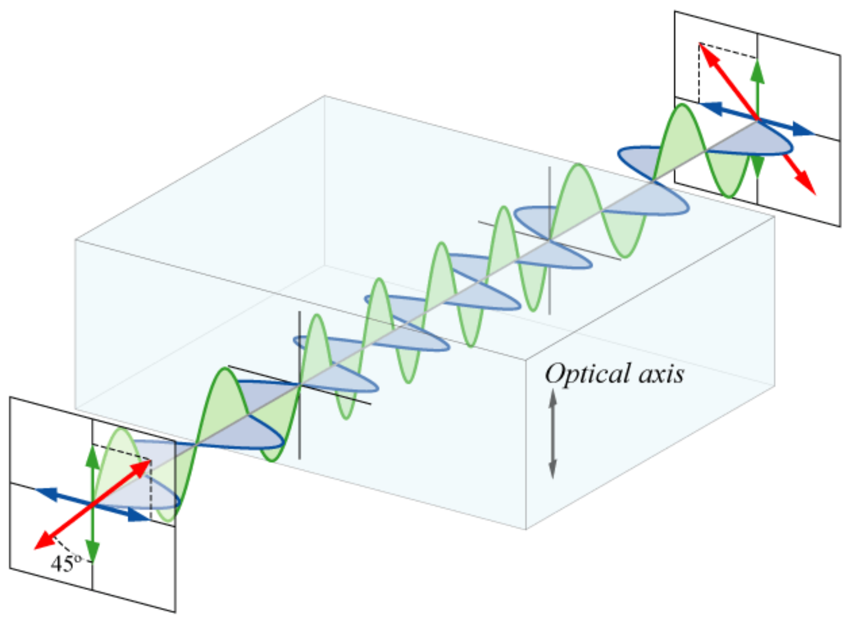


Figure 2: effect of a half-wave plate on polarization

2.2 Electro-optical effect

Some materials experience a change in their optical properties when they are brought into an external electric field. If the refractive index n is a function of the applied field E, we speak of an electro-optical modulator (EOM).

2.2.1 Pockels effect (linear electro-optical effect)

The refractive index $n = n(E)$ can be expanded around $E = 0$ for small field strengths.

With $r = -\frac{2}{n^3}\left(\frac{dn}{dE}\right)\bigg|_{E=0}$ and $s = -\frac{1}{n^3}\left(\frac{d^2n}{dE^2}\right)\bigg|_{E=0}$, this leads to the relation

$$n(E) = n_0 - \frac{1}{2}rn^3E - \frac{1}{2}sn^3E^2 \tag{10}$$

The linear electro-optical effect, also called Pockels effect, occurs for $r \gg s$:

$$n(E) \approx n_0 - \frac{1}{2}rn^3E \tag{11}$$

Here, r is called the Pockels coefficient. If, on the other hand, $r \ll s$, the quadratic dependence of n on E is known as the Kerr effect. This effect will not be studied in this lab course.

2.2.2 Pockels effect in a non-isotropic crystal

Since the electro-optical crystals are in general birefringent, the Pockels coefficient is not a scalar, but a tensor. For the material $LiNbO_3$, the entries of this coefficient-tensor are given by

$$\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{pmatrix} = \begin{pmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -3.4 & 8.6 \\ 0 & 3.4 & 8.6 \\ 0 & 0 & 30.8 \\ 0 & 28 & 0 \\ 28 & 0 & 0 \\ -3.4 & 0 & 0 \end{pmatrix} \cdot 10^{-12}\,\frac{\mathrm{m}}{\mathrm{V}}$$

If no external field is applied to the crystal, the indicatrix is given by

$$\frac{1}{n_o^2}x^2 + \frac{1}{n_o^2}y^2 + \frac{1}{n_e^2}z^2 = 1 \tag{12}$$

In this lab course, the electric field is applied along the extraordinary axis of the Pockels cell, the light propagates along one of the ordinary axes. With $\vec{E} = E \cdot \hat{e}_z$ the indicatrix can be written as

$$\left(\frac{1}{n_0^2} + r_{13}E_z\right)x^2 + \left(\frac{1}{n_0^2} + r_{13}E_z\right)y^2 + \left(\frac{1}{n_e^2} + r_{33}E_z\right)z^2 \tag{13}$$

For small applied field strengths this leads to the relations

$$n_o'(E) \approx n_o - \frac{1}{2}r_{13}n_o^3E_z \tag{14}$$

$$n_e'(E) \approx n_e - \frac{1}{2}r_{33}n_e^3E_z \tag{15}$$

2.2.3 Pockels cell

An electro-optical crystal between two capacitor plates is called an electro-optical modulator, abbreviated as EOM. A Pockels cell is an EOM which exhibits the Pockels effect, meaning that the refractive index is approximately proportional to the strength of an applied electric field $\vec{E}$ (for small values of $|\vec{E}|$)

If light with a wavelength λ travels through a medium with refractive index n for a distance L, the accumulated phase is given by the relation

$$\Phi = 2\pi n \frac{L}{\lambda} \tag{16}$$

The phase difference between a beam of light traveling along the ordinary vs the exraordinary axis is thus:

$$\Delta\Phi = 2\pi(n_e - n_o)\frac{L}{\lambda} \tag{17}$$

With a Pockels cell, we can manipulate the magnitude of both n_o and n_e. Plugging equation 14 and 15 into this last equation, we get the following:

$$\Delta\Phi(E) = 2\pi\frac{L}{\lambda}\left(n_e - n_o - \frac{1}{2}(r_{33}n_e^3 - r_{13}n_e^3 - r_{13}n_o^3)E_z\right) \tag{18}$$

This can be written as

$$\Delta\Phi(V) = \Phi_0 - \pi\frac{V}{V_\pi} \tag{19}$$

with $V = Ed$, $\Phi_0 = 2\pi\frac{L}{\lambda}(n_e - N_o)$ and $V_\pi = \frac{d}{L}\frac{}{\lambda}r_{33}n_e^3 - r_{13}n_o^3$ A Pockels cell can be used to modulate the intensity of light. For this, the cell has to be positioned between two crossed polarizers, each at an angle of 45° relative to the crystal's optical axis. The transmittance of this setup is

$$T(V) = sin^2\left(\frac{\Phi_0}{2} - \frac{\pi}{2}\frac{V}{V_\pi}\right) \tag{20}$$

2.2.4 Faraday effect

Some media become optically active when an axial magnetic field is applied. Optically active means that the plane of polarization of linear polarized light is rotated. The angle of rotation α_{rot} depends on the length of the medium L, the magnetic field strength $|\vec{B}|$ and the so-called Verdet constant v:

$$\alpha_{rot} = vL|\vec{B}| \tag{21}$$

The Verdet constant is a function of the wavelength λ:

$$v = -\frac{\pi\gamma}{\lambda n} \tag{22}$$

Here, γ is a material constant of the medium, the so-called magnetogyration coefficient.

2.2.5 Optical isolator

The Faraday effect can be used to build an optical isolator, also called an optical diode.

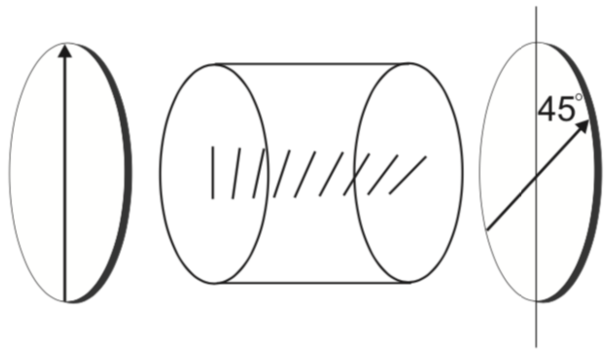


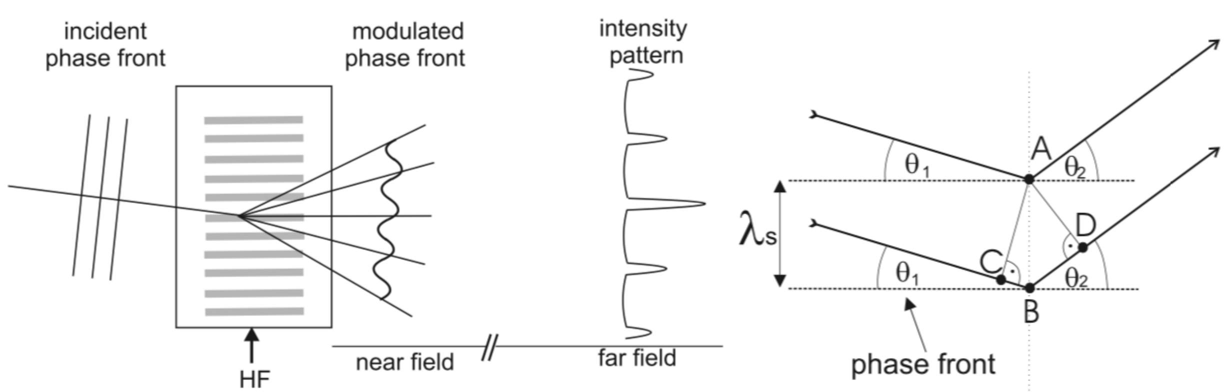
Figure 3: optical diode

2.3 Acousto-optical effect

If a sound wave passes through a crystal, its density varies periodically, which also leads to a periodic variation in the refractive index. A plane sound wave with wavelength λ_s in a crystal with initial refractive index n_0 can be described by

$$n(x,t) = n_0 - \Delta n \cdot cos\left(\omega t - \frac{2\pi}{\lambda_s}x\right)$$

Here, the amplitude $\Delta n = \frac{1}{2}pn^3 s_0$ depends on the photo-elastic constant p and the amplitude of the strain s_0. In this course the main interaction between sound and light will be the so-called Debye-Sears effect which occurs for short interaction lengths, i.e. for a thin crystal or thin sound beam. Parts of the light beam that are travelling through the denser regions experience a phase shift. Maxima in the far field can be observed, if these parts of the beam interfere constructively with each other, as can be seen in the next figure.



(a) deformation of a plane wavefront by a sound wave (b) setup 2

Figure 4: acousto-optical effect

Because the light is interacting with a moving sound wave, the diffracted light is Doppler-shifted:

$$\omega_{out} = \omega_{in} + m\Delta\omega = \omega_{in} + m\omega_s$$

A different approach to explain the effect of an AOM on a light beam is to describe the system as two scattering quasi-particles, namely a phonon and a photon. Their momentum vectors are $\hbar\vec{k}_l$ and $\hbar\vec{k}_s$, respectively. Conservation of momentum and energy leads to the relations

$$\vec{k}_{l,f} = \vec{k}_{l,i} \pm m\vec{k}_s$$

and

$$\nu_{l,f} = \nu_{l,i} \pm m\nu_s$$

Here, m is the diffraction order, i.e. the number of phonons that interacted with the photon. Constructive interference occurs when

$$sin\theta_1 + sin\theta_2 = m\frac{\lambda}{\lambda_s}$$

Here, θ_1 is the angle under which light enters the crystal and θ_2 the diffraction angle. efficiency

3 Experiment

3.1 Polarizers and wave plates

Date of carrying out experiment: 8th of July, 2019

This first part of the experiment has the purpose of getting used to the optical devices (polarizers, beam splitters and waveplates). The laser used in this part of the experiment has a wavelength of 632.8 nm and a power output of less than 1 mW.

3.1.1 Brewster's angle

First, we would like to find Brewster's angle so we can later on calibrate the optical devices. For this, we take a glass plate and mount it in front of the laser at an angle α. This angle can now be varied. If the angle between the incoming beam and the surface of the glass plate equals Brewster's angle, then the reflected beam contains only photons that are polarized perpendicular to the plane of incidence.

This can in principle be verified easily by mounting a polarizer in the path of the reflected beam, but the polarizers are not yet calibrated. This means that we have to turn the polarizers by 360° for each angle α. We're looking for the setup which lets us filter out the maximum amount of light. A minimum in intensity tells us that the polarizer filters out all or almost all of the light, which means the beam contains only a single polarization component. This is exactly what one would expect at Brewster's angle.

Unfortunately, it's not easy to do this in a precise way, since two quantities have two be varied at the same time: the angle α and the angle of the polarizer. Also, the laser beam is polarized linearily. This means, that both the reflected and the refracted/transmitted beam are polarized. It is always possible to block the reflected ray using a polarizer. Because of this, we mount a quarter-waveplate between laser and glass plate to make sure both components are present.

Our result:

$$\alpha_{Brewster} \approx (60 \pm 10)^\circ$$

Now we want to observe what happens when we use other optical devices after the beam has been reflected at Brewster's angle. As expected, the beam splitter splits the beam into one beam with very high intensity and another beam whose intensity is almost zero.
A polarizer either blocks the beam or has no effect, depending on orientation.

3.1.2 Second glass plate

A second glass plate is mounted perpendicular to the plane of incidence of the first glass plate, so that the beam reflected on the first glass plate is reflected off of it. The incoming beam is once again split into two. In contrast to the previous observation, it is now possible to use a polarizer to block both beams for any orientation of the glass plate. This is to be expected, since the beam that was reflected on plate 1 is polarized, so both beams after plate 2 are polarized as well.

3.1.3 Polarizers

Now we want to calibrate the polarizers, i.e. we want to know for which angles linearily polarized light is blocked. For this, we use a similar setup as in the last observation. The laser is reflected off the glass plate at Brewster's angle through the polarizer. By varying the orientation of the polarizer and looking for a minimum of the intensity, we find:

blocking angle for polarizer 130000: $\alpha_1 = (100 \pm 5)^\circ$
blocking angle for polarizer 130200: $\alpha_2 = (101 \pm 5)^\circ$

Because we know that the beam reflected from the glass plate is polarized vertically, we can conclude that the polarizers are calibrated in such a way that the vertically polarized laser light can pass through the polarizer if it is set to $\approx 10^\circ$,

3.1.4 Beam splitters

We want to find out the polarization of the two beams exiting a beam splitter. After mounting a polarizer behind the beam splitter and looking for which angles the beams can pass through, we observe that both beams show maximum intensity for an angle of 0° and minimum for 90°.

3.1.5 Half-waveplates

Now we'd like to observe how a half-waveplate or a quarter-waveplate influences the polarization state of the light beam.

First, we take a look at the half-waveplate. It is mounted between the glass plate and a polarizer. Minima are observed for a the angles 2°, 91°, 182° and 270° with uncertainties of about 5°. These are the positions where the incident polarization is rotated by 90°.

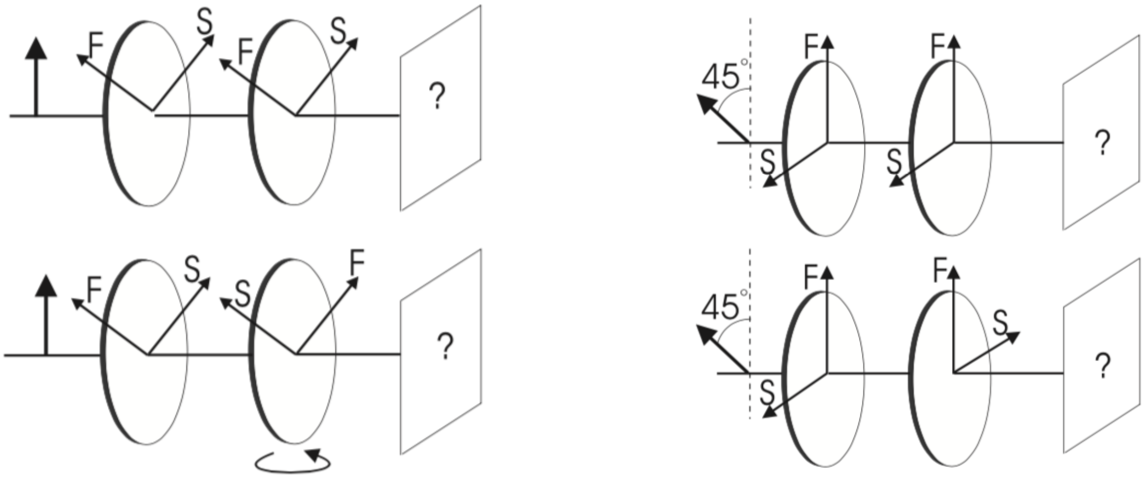
3.1.6 Quarter-waveplates

Herfore, a quarter-waveplate is mounted between two polarizers. For waveplate a, the minima are observed at the angles 8°, 98°, 188° and 279°. For waveplate b, the minima are observed for the angles, 59°, 151°, 241° and 330° with an uncertainty of about 5°

3.1.7 Setup from fig. 5

Next, we realize the experimental setup shown in figure 5a. Waveplate a is set to the angle $8^\circ + 45^\circ = 53^\circ$ and waveplate b to $59^\circ + 45^\circ = 104^\circ$ We find minima for the polarizer angles 180° and 360°. If the second waveplate is turned by 180° around the axis on which it is mounted, we get minima at 88° and 266°.

To realize the setup from figure 5b, we need to rotate the initial polarization by 45°. This can be done with a half-waveplate that is set to an angle that corresponds to halfway from a minimum to the next maximum. That is about 22.5°. The waveplate angle for a is now 59°, the angle for b is 8°. The minima are at angles of 25° and 205°. After turning waveplate b on its axis, the minima are at roughly the same positions as before. This is two be expected, because...



(a) waveplate setup 1

(b) waveplate setup 2

Figure 5: setups for waveplate experiment

3.1.8 Mirror between quarter-waveplates

Now, a mirror is positioned between the two quarter-waveplates in a way so that the angle of reflection is bigger than 45° (total angle bigger than 90°). After this is done, we do not not observe any dependance of the intensity on the polarizer angle. We conclude that the light is circularily polarized. maybe both quarter-waveplates were switched, wrong angles

3.1.9 Reflection back into quarter-waveplate

The laser beam is lead through a quarter-waveplate and onto a mirror. The mirror is positioned perpendicular to the optical axis, so that the beam goes back through the quarter-waveplate.
If the quarter-waveplate is set to an angle of 8°, the mimima are found for the polarizer angles 165° and 350°. If it is instead set to 53°, the minima are at 85° and 265°

3.1.10 Reflection back into half-waveplate

The same as above is done again, this time with a half-waveplate. At a waveplate angle of 2°, no minima could be found. Then we set the waveplate angle to 47°. Here, we did find minima at 75° and 255°.

3.1.11 Mirror

A mirror is set up in a way such that the laser beam is reflected from the glass onto the mirror in an angle bigger than 45°. Minima are detected for the polarizer angles of 90° and 270°. Now we do the same for perpendicular polarization. Now the minima are at 186° and 6°.

3.1.12 Lamp light through optical isolator

Looking through the optical isolator at the ceiling lamp, the lamp appears either orange or green, depending on the orientation of the isolator. Why?

3.2 Electro-optical effect

Date of carrying out experiment: 9th of July, 2019

The laser used in this part of the experiment produces light with a wavelength of $\lambda = 635$ nm and outputs a power of less than 1 mW. The material in the Pockels cell is $LiNbO_3$, which is a uniaxial crystal with $3m$-symmetry ($n_1 = n_2 = n_o, n_3 = n_e$)
Pockels coefficients are very hard to measure, can be affected by crystal impurities

3.2.1 Measurement of amplification

First, we want to determine the amplification factor of the high voltage power supply. The DC power supply is connected to the amplifier. For several DC voltages U_0 we record the corresponding high voltage value U_{amp}.

U_0 [V]	6.50	6.00	5.50	5.00	4.50	4.00	3.50	3.00	2.50	2.00	1.50	1.00	0.50
U_{amp} [kV]	1.95	1.80	1.65	1.50	1.35	1.20	1.05	0.90	0.75	0.60	0.45	0.30	0.15

We take care not to let the amplified voltage get higher than 2.00 kV, this is the maximum voltage the Pockels cell can endure. The uncertainty on the display of the power supply devices is 0.005 V for U_0 and 0.005 kV for U_{amp}. other error?

3.2.2 Calibration of Pockels cell

To determine the directions of the optical axes in the Pockels cell, it is placed between two polarizers. The polarizers are oriented perpendicular to each other, so that without the cell no light can pass through.

polarizer serial number	polarizer angle
130000	100°
130200	10°

The laser light passes first through the polarizer 130200, then through the Pockels cell and 130000.

The Pockels cell angle scale goes from $-90°$ to $+90°$ and the minima are observed for the Pockel cell angles $10°$ and $-80°$. Minima are observed when the laser beam is parallel to either the fast or the slow axis of the crystal.

3.2.3 Mach-Zehnder interferometer

We would like to quantify the effect that the Pockels cell has on the phase of a laser beam. For this, a Mach-Zehnder interferometer is set up. The setup can be seen in figure 6.

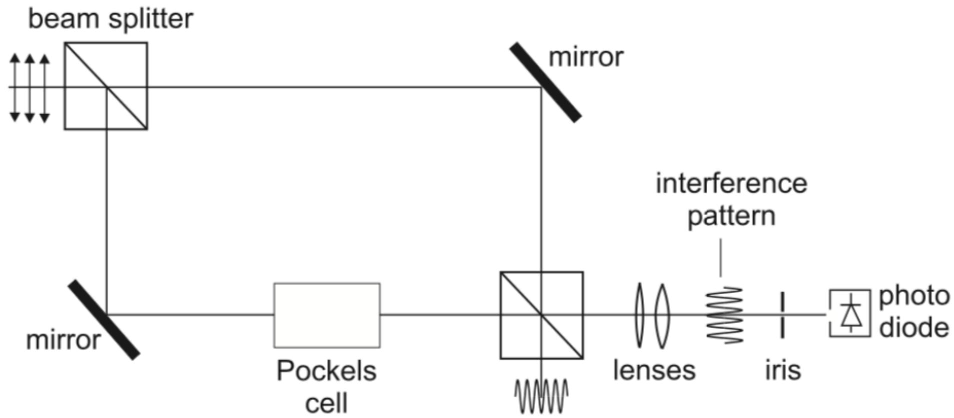


Figure 6: setup of Mach-Zehnder interferometer

After the separated laser beams are joined back together, we want to make sure they are as close to being parallel as possible. A piece of paper is held into the beam directly behind the beam splitter. The mirrors are oriented in such a way that both beams hit the paper in a single spot. Then the distance between the piece of paper and the beam splitter is increased and the mirror positions once again adjusted.

A triangular waveform is chosen on the function generator. The frequency of this signal is ≈ 3.5 Hz !!!. The generator is connected to the amplifier, the amplifier to the Pockels cell. The output of the function generator as well as the photo diode are connected to the oscilloscope. The oscilloscope output can be saved to .csv and used for later analysis.
This is done for the Pockels cell angles $-80°$, $10°$, $-90°$ and $90°$

3.2.4 Manipulation of polarization and intensity

The setup is shown in figure 7. The angle between the polarization axes of the filters and the Pockels cell need to be 45°, because...

The angle of the first filter (set to vertical polarization) is 10°, the second one 100°. For the Pockels cell, we know from the calibration that an angle of $+45°/-45°$ relative to the vertical polarization axis corresponds to an actual angle setting of $55°/-35°$ at the Pockels cell.

The transmitted power on the photodiode is recorded as a function of the applied high voltage by choosing again a triangular waveform on the the function generator. This is done for the angles $-45°$ and $45°$.

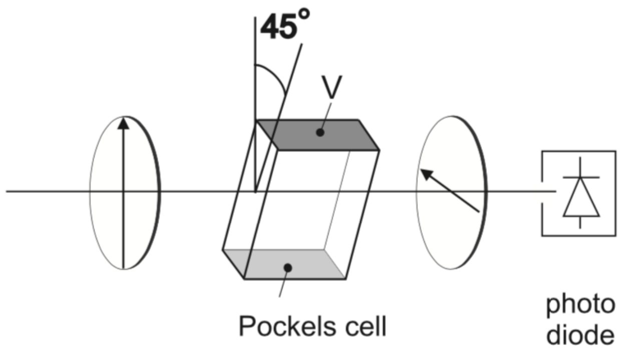


Figure 7: manipulation of polarization and intensity

3.2.5 Linear amplitude modulation

Now, the Pockels cell is connected to a DC high voltage. The amplifier is set to intern. Multiple measurements will be made, the DC voltage will be varied from one measurement to the next. A modulated signal is added to the DC voltage and the resulting signal is given to the Pockels cell. The frequency of the modulated (triangular) signal is about 1 ± 0.0005 kV. For our setup the half-wave voltage is given as $V_\pi = 380$ V in the lab course script[1]. Our first measurement is done at a voltage of 330 V, then we increase the voltage in steps of 10 V until we reach 430 V.

3.2.6 Phase shift due to electro-optical modulation

1. $\vec{E}$-field along optical axis
2. laser beam is incident perpendicular to the field
3. HV supply: output should be between 0 V and 1.8 kV
4. measure amplification factor of HV amplifier
5. evaluate measurements

3.2.7 Mach-Zehnder interferometer

To quantify the extent by which the phase is shifted, we use a Mach-Zehnder interferometer.

1. why non-polarizing beam splitters
2. direction of optical axes
3. good overlap of arms
4. setup measurement
5. output intensity as function of applied voltage (for both axes)
6. compare results
7. just for fun: speaker

3.2.8 Manipulation of polarization and intensity

1. why Pockels cell at 45°
2. measure transmitted power as function of HV for ±45°

3.2.9 Linear amplitude modulation

1. ...

3.3 Acousto-optical effect

Date of carrying out experiment:

3.3.1 Experiments with a single AOM

- adjust AOM so that amplitude of diffraction pattern is approximately symmetric in both ± 1 orders
- measure diffraction angles for order 1 and 2 for various frequencies
- measure power of order 1 maximum relative to power of undiffracted beam as a function of frequency (same steps as above)
- optimize power in order 1 max, ...

3.3.2 Two perpendicular AOMs

- expectation of diffraction pattern? compare with results
- what happes when offset voltage(s) is/are changed?

4 Results

5 Analysis

6 Summary

7 Discussion

References

[1] lab course script for F85
https://www.physi.uni-heidelberg.de/Einrichtungen/FP/anleitungen/F85.pdf