Innopolis University

Control Theory (Linear Control)

Lecture 7

Computed Torque Control (CTC)
Linear Control for Nonlinear Systems

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Topic of the Lecture

- Error Dynamics
- Computed Torque Controller (CTC)
- Stability of CTC
- Linearization of a Nonlinear Dynamics

Error Dynamics (1/2)

Control error:	Error dynamics:	Robot dynamics:
$e = q^* - q$	$\ddot{e} + K_d\dot{e} + K_p e = 0$	$H\ddot{q} + c = Bu$

Error Dynamics (2/2)

Error dynamics:

$$\ddot{q}^* - \ddot{q} + K_d\dot{e} + K_p e = 0$$

Since H is invertible:

$$H\ddot{q}^* - H\ddot{q} + H\left(K_d\dot{e} + K_p e\right) = 0$$

And so:

$$H\ddot{q} + c = H\ddot{q}^* + H\left(K_d\dot{e} + K_p e\right) + c$$

Computed Torque Controller (CTC) (1/3)

Since $H\ddot{q} + c = Bu$:

$$Bu = H\ddot{q}^* + H\left(K_d\dot{e} + K_pe\right) + c$$

And so:

$$u = B^+(H\ddot{q}^* + c) + B^+H(K_d\dot{e} + K_pe)$$

Computed Torque Controller (CTC) (2/3)

CTC controller:

$$u = B^{+}(H\ddot{q}^{*} + c) + B^{+}H(K_d\dot{e} + K_p e)$$

Feedback part:

$$u_{FB} = B^{+}H(K_d\dot{e} + K_p e)$$

Feedforward part:

$$u_{FF} = B^{+}(H\ddot{q}^{*} + c)$$

Computed Torque Controller (CTC) (3/3)

CTC controller:

$$\tau = (H\ddot{q}^* + c) + H(K_d\dot{e} + K_p e)$$

Feedback part:

$$\tau_{FB} = H(K_d\dot{e} + K_p e)$$

Feedforward part:

$$\tau_{FF} = H\ddot{q}^* + c$$

In research we often use:

$$\tau = Bu$$

Stability of CTC

Remember that $\ddot{e} + K_d\dot{e} + K_p e = 0$

Then K_d and K_p are diagonal, then,

if K_d and K_p are negative definite,

the **error is stable**.

How to find CTC coefficients using LQR?

Linearization of a Nonlinear Dynamics (1/2)

Nonlinear system: $\dot{x} = f(x, u)$

Simple linearization:

$$A = \left.\frac{\partial f(x,u)}{\partial x}\right|_{\substack{x=x(0)\\ u=u(0)}}$$

$$B = \left.\frac{\partial f(x,u)}{\partial u}\right|_{\substack{x=x(0)\\ u=u(0)}}$$

Linearization of a Nonlinear Dynamics (2/2)

If the dynamics is affine:

$$A = \left.\frac{\partial f(x,u)}{\partial x}\right|_{\substack{x=x(0)\\u=u(0)}}$$

$$B = \left.\frac{\partial f(x,u)}{\partial u}\right|_{\substack{x=x(0)\\u=u(0)}}$$

$$c = f(x(0),u(0)) - \left.\frac{\partial f(x,u)}{\partial u}\right|_{\substack{x=x(0)\\u=u(0)}} x(0) - \left.\frac{\partial f(x,u)}{\partial u}\right|_{\substack{x=x(0)\\u=u(0)}} u(0)$$

The End

Do you have any questions?