Innopolis University

Control Theory (Linear Control)

Lecture 1

Introduction

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Topic of the Lecture

- Instructors
- Grading
- Introduction to Control
- Single Input Single Output (SISO) Systems
- Block Diagrams
- From Linear DE to State Space Models

Instructors

Lecture Instructor:

Sergei Savin

Tutorial Instructor:

Sergei Savin

Labs Instructors:

Shamil Mamedov

Mike Ivanov

Oleg Balakhnov

Grading

Item	Percentage
Lab Quizzes	**10%** (5x2%)
Homework	**50%** (5x10%)
Mid-term Exam	**16%**
Final Exam	**20%**
Bonus Points	**4%** (can be more)

Letter Grade	Percentage
A	**85-100**
B	**70-84**
C	**50-69**
D	**0-49**

Process, System & Disturbance

A process is any operation to be controlled. It can be chemical, biological, economical…

A system is a combination of components that act together and perform a certain objective.

A disturbance is a signal that adversely affects the value of the output of a system. It can be internal or external.

Control & Control System

What is Control?

- Control is a science.
- Control is an engineering field.
- Control is a process.

What is Control System?

- Control System is an object.
- Control system can be a model, a program, a circuit, or an electro-mechanical system.

Control System Examples

Industrial arms

Chemical plants

Self-driving cars

Single-Input Single-Output (SISO) Systems

Single-Input Single-Output (SISO) System is a simple single variable control system with one input and one output.

Examples of SISO systems:

- Fan speed control
- Air conditioner temperature control

Single-Input Single-Output (SISO) Systems

Consider the following ODE:

$$\dot{x} + ax + b = u$$

If we call u an input, x an output, then it is a SISO system.

Consider the following ODE:

$$\ddot{x} + a_1\dot{x} + a_2x = u$$

Let us introduce output variable y. Possibilities to choose it:

$$y = x$$

$$y = \dot{x}$$

$$y = 7\dot{x} - x + 2$$

Open-Loop System vs. Closed-Loop System

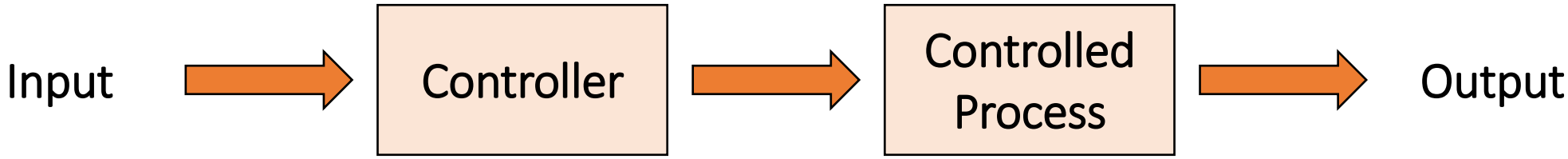
Open-Loop System

- no feedback
- difficult to control output with accuracy
- most of automates has open loop principle

Closed-Loop System

- must have feedback
- must have sensor on output
- almost always negative

SISO Systems in Block Diagrams



Open Loop System

State-Space Models

State-Space Representation is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations or difference equations.

General form of State-Space representation:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Single-Input Single-Output (SISO) Systems

Classic example (second order mass-spring system):

$$m\ddot{p} + c\dot{p} + kp = F$$

State-space model:

$$\begin{bmatrix}\dot{p}\\ \ddot{p}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -k/m & -c/m\end{bmatrix}\begin{bmatrix}p\\ \dot{p}\end{bmatrix} + \begin{bmatrix}0\\ 1/m\end{bmatrix}F$$

Let us set:

$$x_1 = p$$
$$x_2 = \dot{p} = \dot{x}_1$$
$$\dot{x}_2 = \ddot{p} = (F - c\dot{p} - kp)/m = (F - cx_2 - kx_1)/m$$
$$u = F$$

Now we will have:

$$\mathrm{x} = \begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}p\\ \dot{p}\end{bmatrix} \Rightarrow \dot{\mathrm{x}} = A\mathrm{x} + Bu$$

Additional Materials about State-Space Models

From ODE to State Space

https://www.youtube.com/watch?v=HGFZ_fl3C0c

Who wants to learn more?

https://www.youtube.com/playlist?list=PLn8PRpmsu08podBgFw66-IavqU2SqPg_w

Linearity

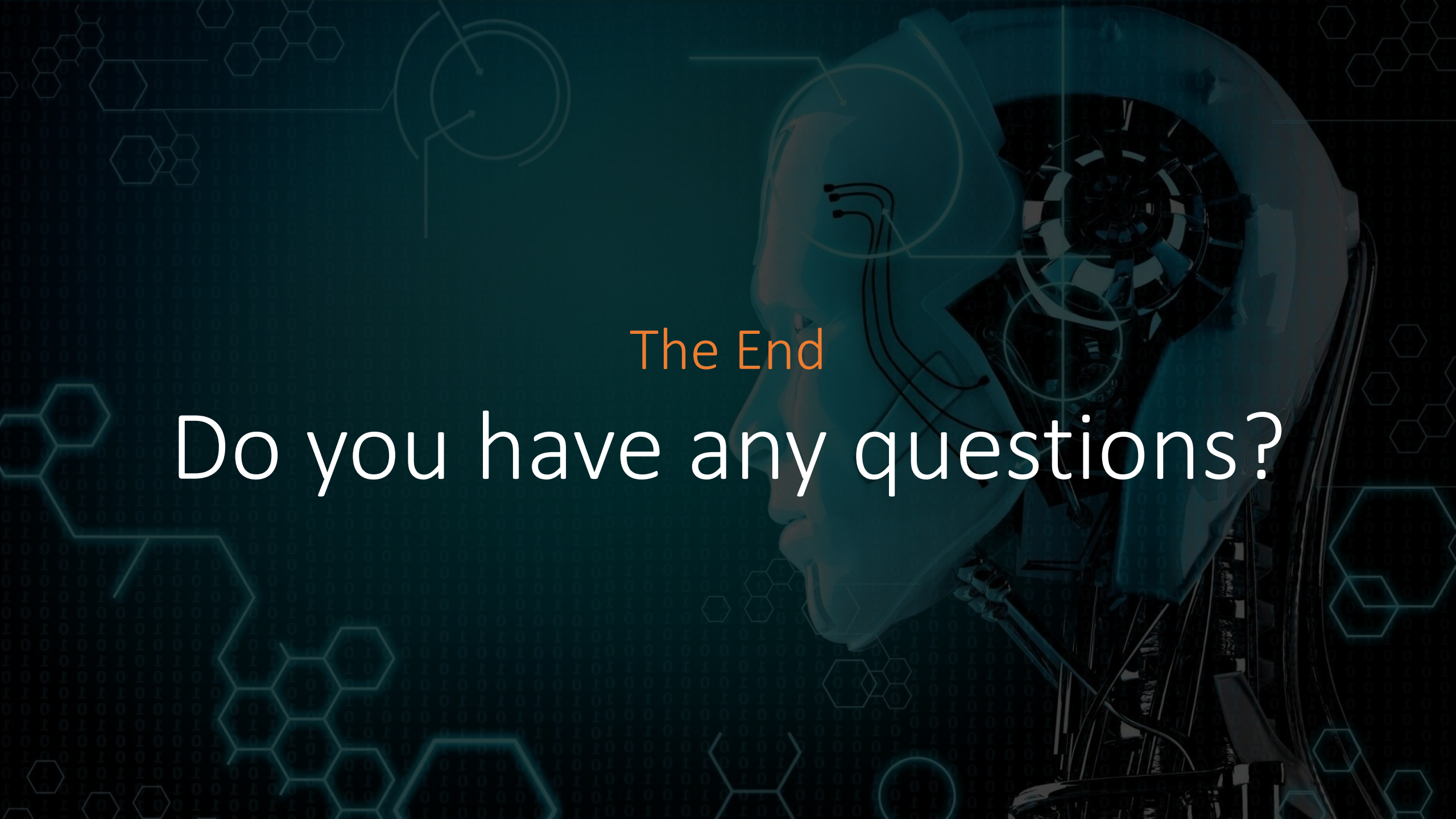
A system G is linear with respect to its inputs and output

$$\mathrm{u}(t) \rightarrow \boxed{G(s)} \rightarrow \mathrm{y}(t)$$

if and only if superposition holds:

$$G(\alpha_1 \mathrm{u}_1 + \alpha_2 \mathrm{u}_2) = \alpha_1 G\mathrm{u}_1 + \alpha_2 G\mathrm{u}_2$$

So, if y_1 is the response of G to u_1 ($\mathrm{y}_1 = G\mathrm{u}_1$), and y_2 is the response of G to u_2 ($y_2 = Gu_2$), then the response to $\alpha_1 \mathrm{u}_1 + \alpha_2 \mathrm{u}_2$ is $\alpha_1 \mathrm{y}_1 + \alpha_2 \mathrm{y}_2$



The End

Do you have any questions?