Innopolis University

Control Theory (Linear Control)

Lecture 5

MIMO, LTI, LTV Systems

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Topic of the Lecture

- Multi-Input Multi-Output (MIMO) Systems
- Linear Time-Invariant (LTI) Systems
- Linear Time-Variant (LTV) Systems
- Equilibrium Points

Multi-Input Multi-Output (MIMO) Systems (1/2)

Multi-input multi-output (MIMO) is an antenna technology for wireless communications in which multiple antennas are used at both the source and the destination.

MIMO is one of several forms of smart antenna technology, the others being **multi-input single-output (MISO)** and **single-input multiple-output (SIMO)**.

Multi-Input Multi-Output (MIMO) Systems (2/2)

$$\begin{cases} \dot{x}_1 = 3x_1 + 4x_2 - u_1 \\ \dot{x}_2 = 1.2x_1 + 2x_2 - 2u_1 + u_2 \end{cases}$$

$$\begin{cases} y_1 = x_1 \\ y_2 = x_1 + 2x_2 \end{cases}$$

$$\Rightarrow$$

$$\dot{\mathbf{x}} = \begin{bmatrix} 3 & 4 \\ 1.2 & 2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} -1 & 0 \\ -2 & 1 \end{bmatrix}\mathbf{u}$$

$$\mathbf{y} = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}\mathbf{x}$$

Single-Input Multiple-Output (SIMO): Example

$$\begin{cases} \dot{x}_1 = -2x_1 - u \\ \dot{x}_2 = -x_1 + 20x_2 - 2u \end{cases}$$

$$\begin{cases} y_1 = x_2 \\ y_2 = 2x_1 \\ y_3 = x_1 + x_2 \end{cases}$$

Multiple-Input Single-Output (MISO): Example

$$\begin{cases} \dot{x}_1 = 2x_1 - u_1 \\ 2\dot{x}_2 = 3x_3 + x_2 \\ \dot{x}_3 = x_1 + x_2 + 10u_2 \end{cases}$$

$$y = x_2$$

Linear Time-Invariant (LTI) Systems (1/2)

Linear time-invariant (LTI) systems are a class of systems used in signals and systems that are both linear and time-invariant.

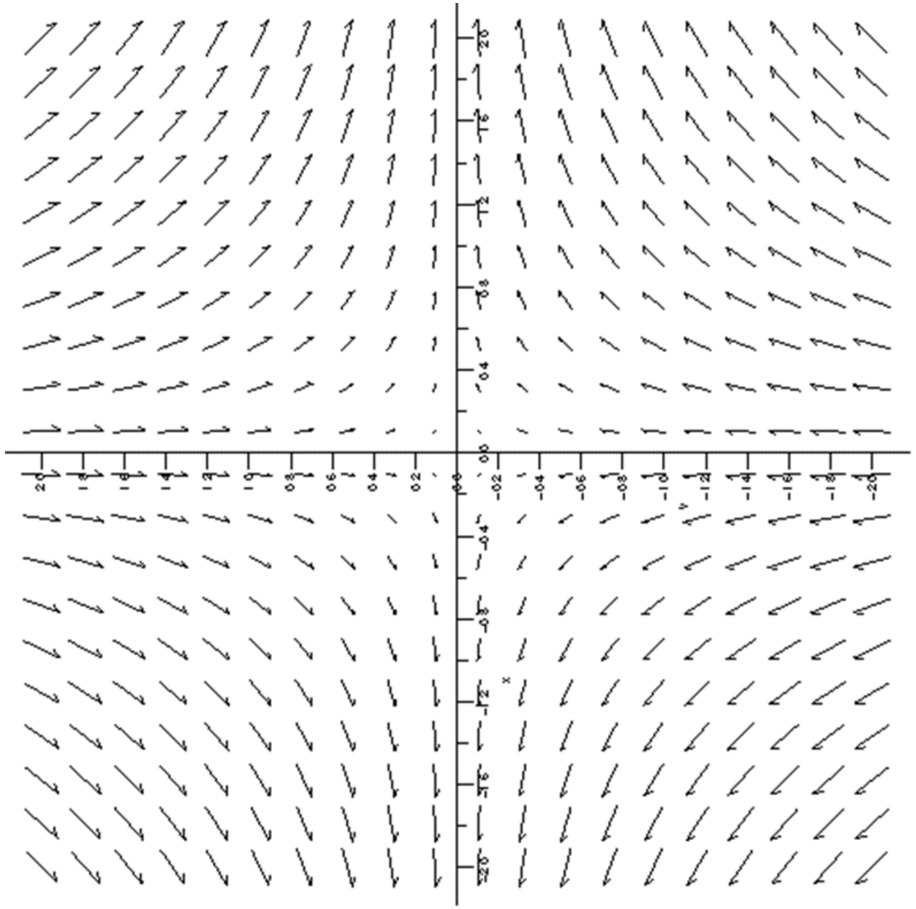
- **Linear systems** are systems whose outputs for a linear combination of inputs are the same as a linear combination of individual responses to those inputs.
- **Time-invariant systems** are systems where the output does not depend on when an input was applied.

Linear Time-Invariant (LTI) Systems (2/2)

What topics will we consider?

- Stability criteria
- Phase portraits (state space trajectories/orbits)
- What control does to phase portraits

LTI Systems Phase Portrait (Flow Diagram)



Linear Time-Variant (LTV) Systems (1/2)

Linear time-variant (LTV) systems are a class of systems used in signals and systems that are both linear and time-variant.

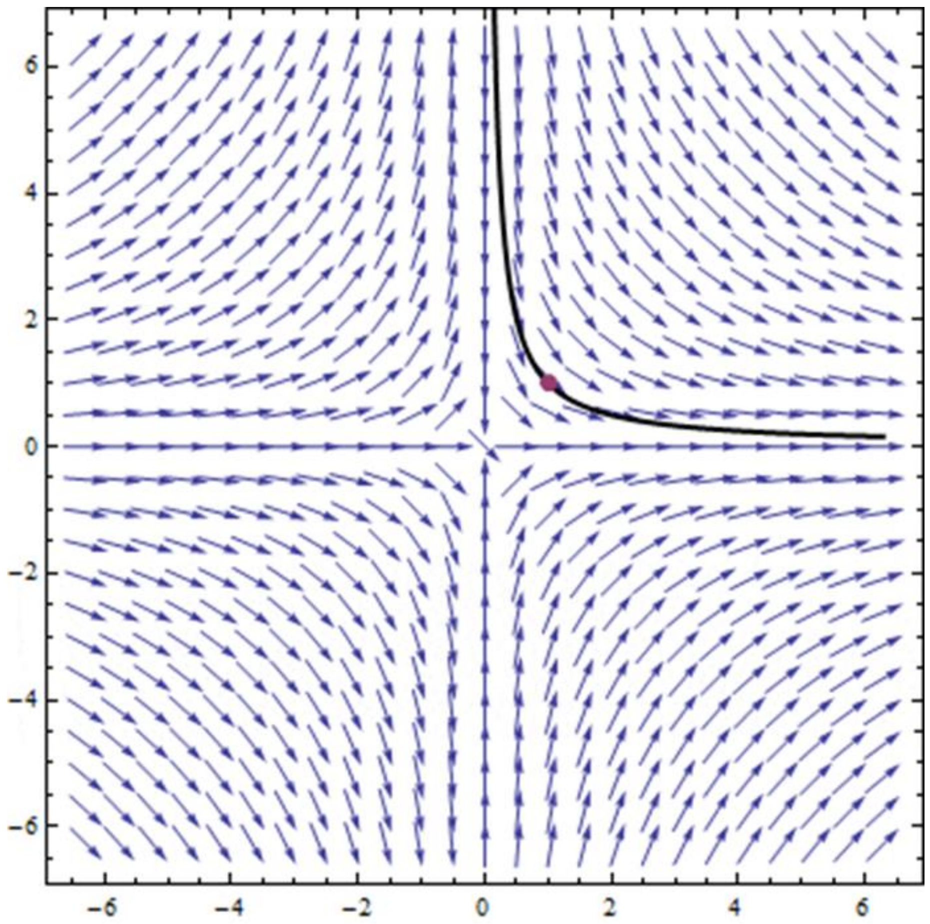
- **Linear systems** are systems whose outputs for a linear combination of inputs are the same as a linear combination of individual responses to those inputs.
- **Time-variant systems** are systems where the output depends on when an input was applied.

Linear Time-Variant (LTV) Systems (2/2)

What topics will we consider?

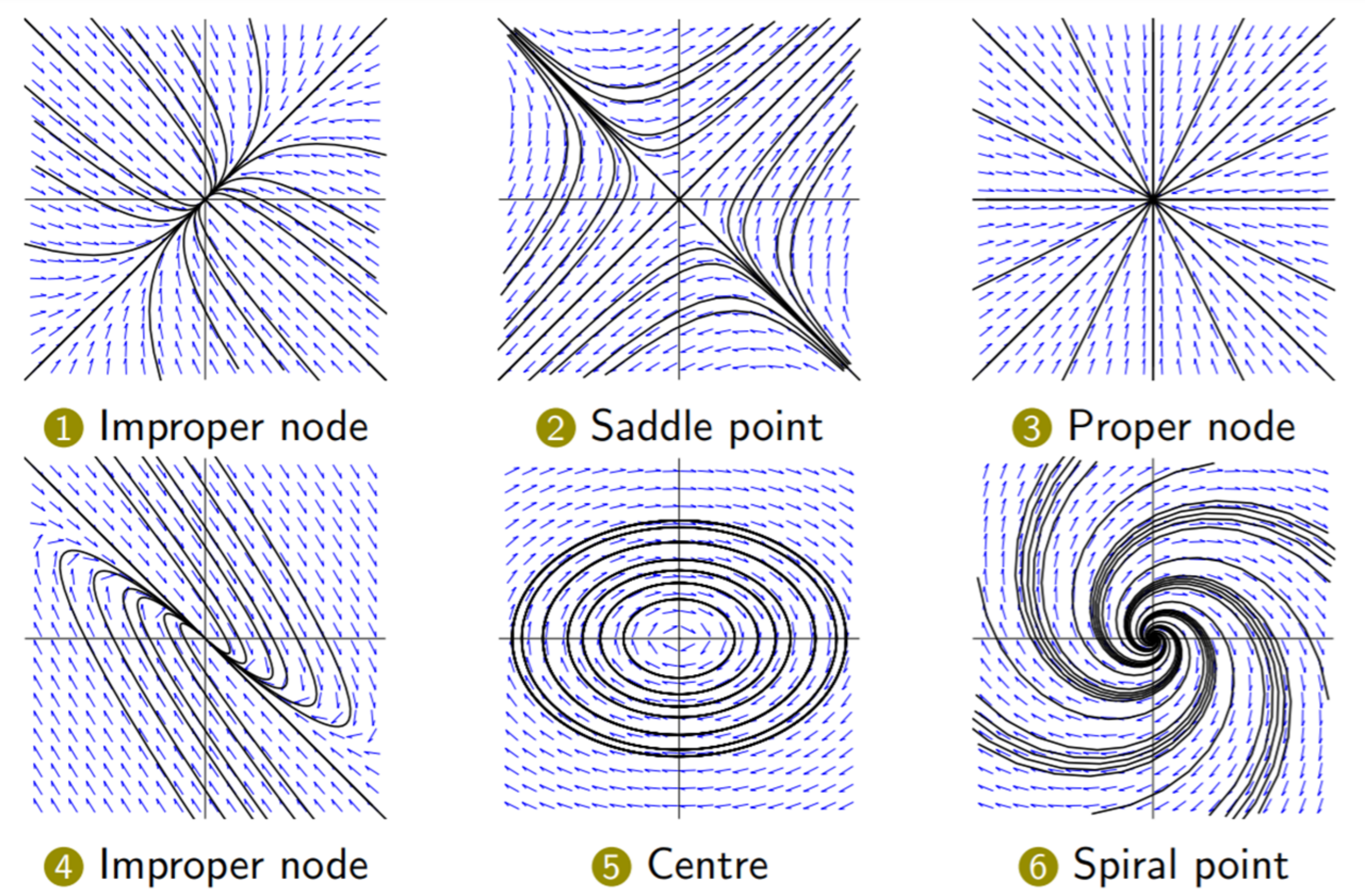
- How is stability criteria different?
- What do phase portraits mean?

LTV Systems Phase Portrait (Flow Diagram)



Equilibrium Points

Phase Portraits of Linear Systems



Types of Equilibrium Points (1/6)

Let a second order linear homogeneous system with constant coefficients be given:

$$\begin{cases} \dfrac{dx}{dt} = a_{11}x + a_{12}y \\ \dfrac{dy}{dt} = a_{21}x + a_{22}y \end{cases}.$$

This system of equations is autonomous since the right hand sides of the equations do not explicitly contain the independent variable t.

In matrix form, the system of equations can be written as

$$\mathbf{X}' = A\mathbf{X}, \quad \text{where} \quad \mathbf{X} = \begin{bmatrix} x \\ y \end{bmatrix}, \; A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}.$$

Source:

https://www.math24.net/linear-autonomous-systems-equilibrium-points/

Types of Equilibrium Points (2/6)

In general, when the matrix A is nonsingular, there are 4 different types of equilibrium points:

#	Equilibrium Point	Eigenvalues λ_1, λ_2
1	Node	λ_1, λ_2 are real numbers of the same sign $(\lambda_1 \cdot \lambda_2 > 0)$
2	Saddle	λ_1, λ_2 are real numbers and non-zero of opposite sign $(\lambda_1 \cdot \lambda_2 < 0)$
3	Focus	λ_1, λ_2 are complex numbers, the real parts are equal and non-zero $(\mathrm{Re}\,\lambda_1 = \mathrm{Re}\,\lambda_2 \neq 0)$
4	Center	λ_1, λ_2 are purely imaginary numbers $(\mathrm{Re}\,\lambda_1 = \mathrm{Re}\,\lambda_2 = 0)$

Types of Equilibrium Points (3/6)

The roots λ_1, λ_2 are distinct $(\lambda_1 \neq \lambda_2)$ and negative $(\lambda_1 < 0, \lambda_2 < 0)$.

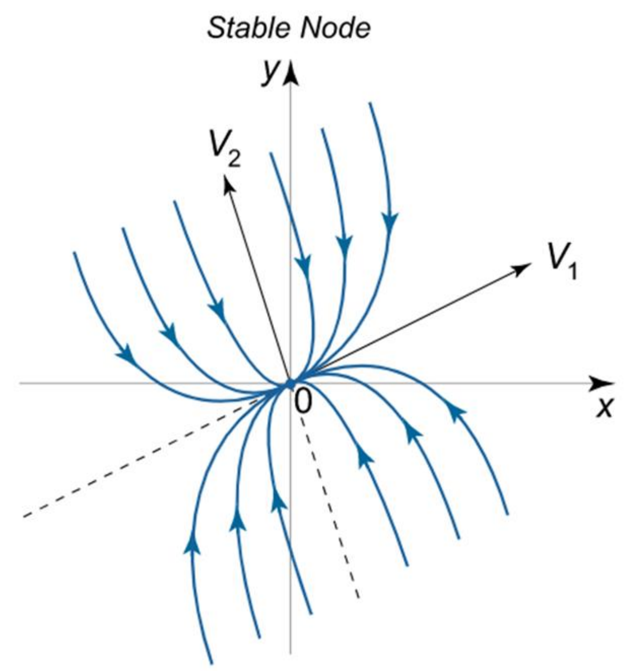


Figure 2.

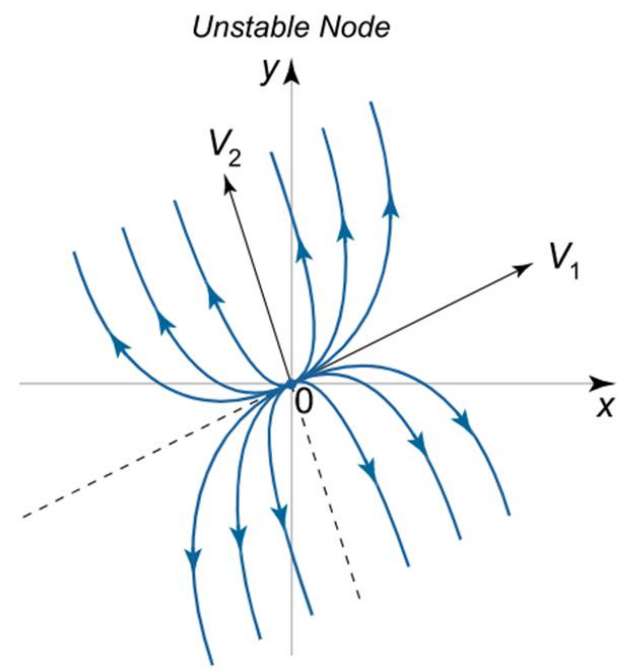


Figure 3.

Types of Equilibrium Points (4/6)

Case $\lambda_1 = \lambda_2 = \lambda < 0.$

Such an equilibrium position is called a stable dicritical node (Figure 4).

Case $\lambda_1 = \lambda_2 = \lambda > 0.$

This combination of eigenvalues corresponds to an unstable dicritical node (Figure 5).

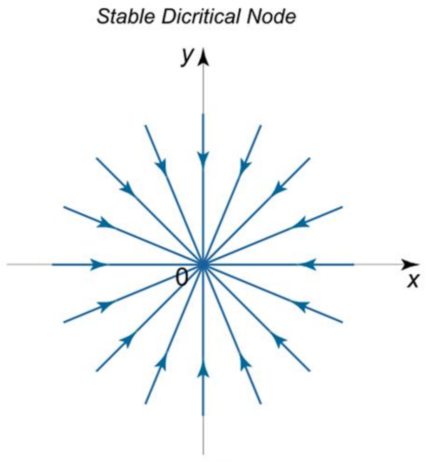


Figure 4.

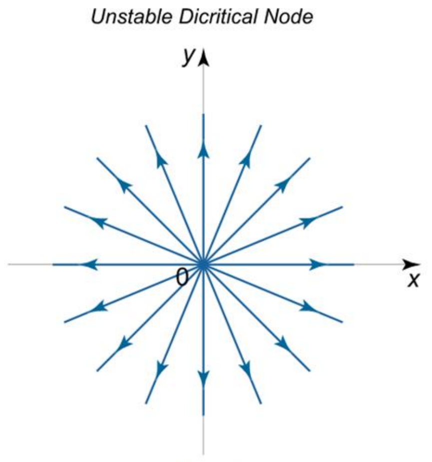


Figure 5.

The system has a basis of two eigenvectors

Types of Equilibrium Points (5/6)

Case $\lambda_1 = \lambda_2 = \lambda < 0.$

The equilibrium point is called stable singular node (Figure 6).

Case $\lambda_1 = \lambda_2 = \lambda > 0.$

The equilibrium position is called unstable singular node (Figure 7).

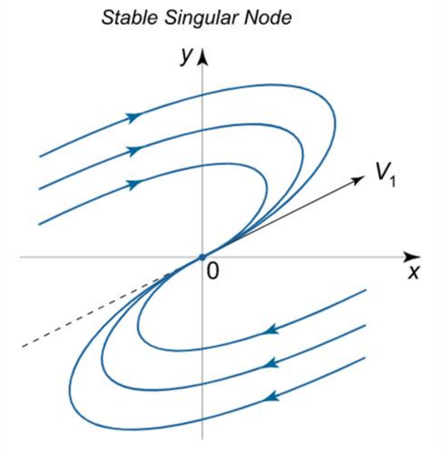


Figure 6.

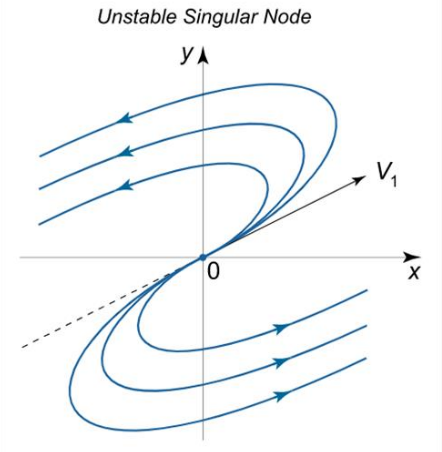


Figure 7.

matrix A has only one eigenvector

Types of Equilibrium Points (6/6)

Saddle

The equilibrium point is a saddle under the following condition:

$$\lambda_1, \lambda_2 \in \Re, \quad \lambda_1 \cdot \lambda_2 < 0.$$

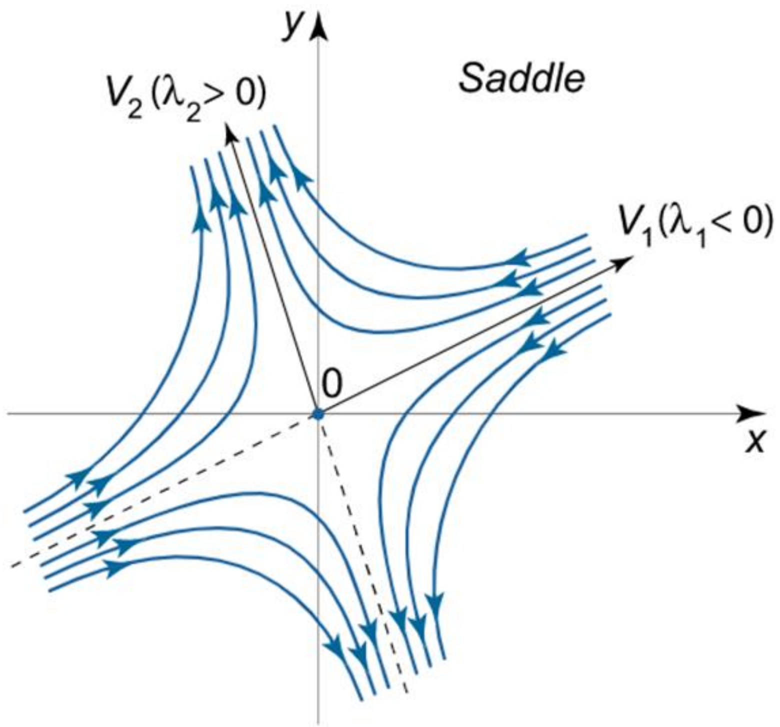


Figure 8.

The End

Do you have any questions?