Innopolis University

Control Theory (Linear Control)

Lecture 4

Stabilizing Control

Sergei Savin, PhD

Slides design by Marko Pezer & Artem Bakhanov

Topic of the Lecture

- Stabilizing Control
- Control Error
- Proportional and Proportional-Derivative Control
- Proportional-Integral-Derivative (PID)
- Lyapunov Stability

Stabilizing Control (1/3)

$$\dot{x} = f(x, u)$$

Given desired state $x^* = const$,

find $u = u(t)$ such that solution $x = x(t)$ approaches x^*

(from all **initial conditions (IC)** or a range of IC)

Stabilizing Control (2/3: Example)

$\dot{x} = 7x + u$

For the desired state $x^* = 0$,

if $u(t) = -8x(t)$ then $\dot{x} = -x$

and $x(t) = Ce^{-t}$ which approaches 0

Stabilizing Control (3/3)

If $x^* = 0$

$\dot{x} = Ax + Bu$

$u = -Kx$

Find K such that $A - BK \prec 0$

Control Error

Control error: $e = x^* - x$

We always want $e \to 0$

Control Error and Trajectory Tracking (1/2)

Assume there is such $x^* = x^*(t)$ that

$\dot{x}^* = Ax^*$

Then: $\dot{x}^* - \dot{x} = Ax^* - Ax - Bu$

$\dot{e} = Ae - Bu$

If so, and if $u = Ke$

Find K such that $A - BK \prec 0$

Control Error and Trajectory Tracking (2/2)

$u = u^* + u_{FB}$, then

$u_{FB} = u^* - u$

Assume there is such u^* and x^* that $\dot{x}^* = Ax^* + Bu^*$

Then:

$\dot{x}^* - \dot{x} = Ax^* - Ax + Bu^* - Bu$

$\dot{e} = Ae + Bu_{FB}$

If so, and if $u_{FB} = Ke$ find K such that $A - BK \prec 0$

Affine Systems

Linear systems: $\dot{x} = Ax + Bu$

$$\boxed{\textit{Affine}\text{ systems: } \dot{x} = Ax + Bu + c}$$

Control Error and Stabilizing Control

Given $\dot{x} = Ax + Bu + c$

Assume there is such $x^* = const$ that $\dot{x}^* = Ax^* + c$

Then: $\dot{x}^* - \dot{x} = Ax^* - Ax - Bu + c - c$

$\dot{e} = Ae - Bu$

If so, and if $u = Ke$, find K such that $A - BK \prec 0$

Proportional and Proportional-Derivative Control

Proportional Control

$$u = Ke$$

(good for first order LTI: State-Space models)

Proportional-Derivative Control

$$u = K_d \dot{e} + K_p e$$

(good for second-order LTI)

Why you don't want controllers using derivatives of the same order as the equations themselves? (1/2)

$$\dot{x} = Ax + Bu$$

Let $u = K_p x + K_v \dot{x}$. Then we can pick K_v such that $K_v = B^{-1}$ and obtain:

$$\dot{x} = Ax + B\left(K_p x + B^{-1}\dot{x}\right)$$

$$\dot{x} = Ax + BK_p x + \dot{x}$$

$$0 = \left(A + BK_p\right)x$$

Which means x is in the null space of $(A + BK_p)$. However, the initial problem did not specify this, therefore the proposed control leads to an instantaneous change in x, which is not physically possible.

Why you don't want controllers using derivatives of the same order as the equations themselves? (2/2)

Alternatively, pick $u = K_p x + B^{-1}\dot{x} - c$. Then we obtain:

$$\dot{x} = Ax + BK_p x + \dot{x} - Bc$$

$$(A + BK_p)x = Bc$$

$$x = (A + BK_p)^{-1} Bc$$

Here it means that as long as $(A + BK_p)$ invertible, any x can be achieved instantaneously, which is not physical.

Proportional-Integral-Derivative (PID)

(black board)

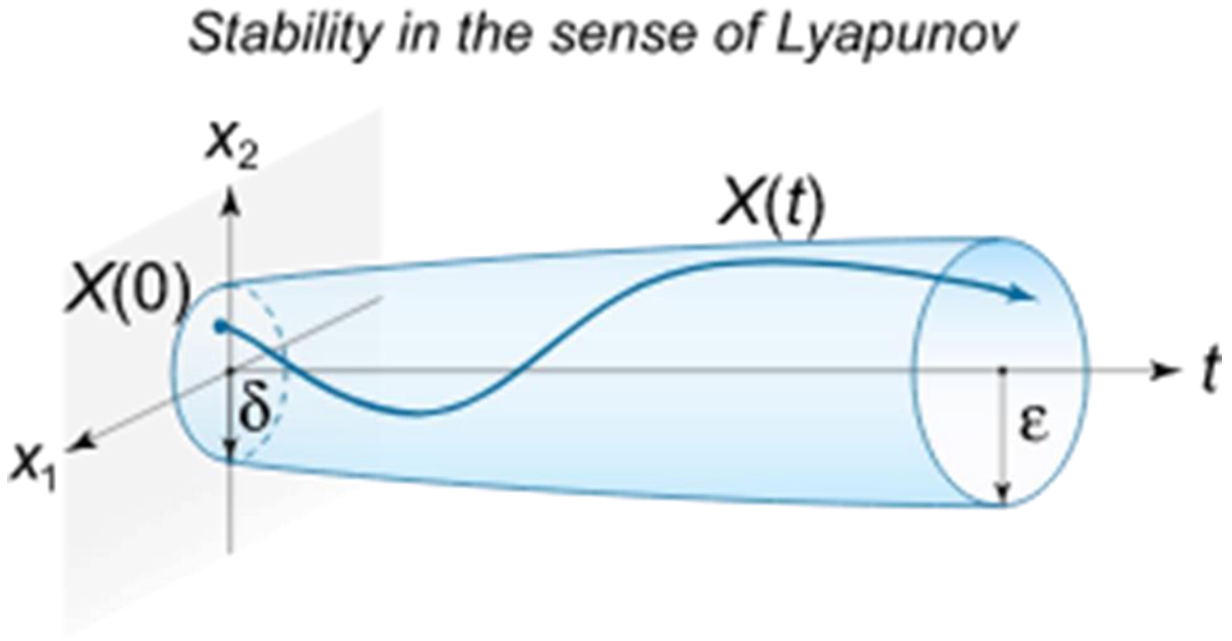
Theory:

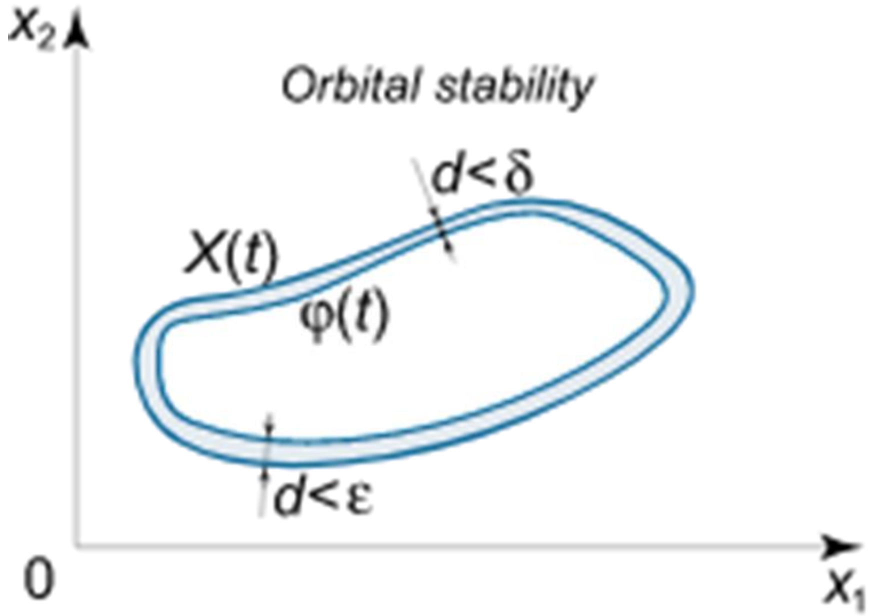
https://www.youtube.com/watch?v=wkfEZmsQqiA

Practice:

https://www.youtube.com/watch?v=FSAfFw_dqgA

Lyapunov Stability





The End

Do you have any questions?