

CONJUNTO DE EJERCICIOS

- 1) Dados los puntos $(0,1)$, $(1,5)$, $(2,3)$, determine el *spline* cúbico.
- 2) Dados los puntos $(-1,1)$, $(1,3)$, determine el *spline* cúbico sabiendo que $f'(x_0) = 1$, $f'(x_n) = 2$.
- 3) Diríjase al pseudocódigo del *spline* cúbico con frontera natural provisto en clase, en base a ese pseudocódigo complete la siguiente función:

```
#####
6 def cubic_spline(xs: list[float], ys: list[float]) -> list[sym.Symbol]:
7     """
8     ... Cubic spline interpolation ``S``. Every two points are interpolated by a cubic polynomial
9     ... ``S_j`` of the form ``S_j(x) = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3``
10
11     ... xs must be different but not necessarily ordered nor equally spaced.
12
13     ... ## Parameters
14     ... - xs, ys: points to be interpolated
15
16     ... ## Return
17     ... - List of symbolic expressions for the cubic spline interpolation.
18     ... """
19
20     ... points = sorted(zip(xs, ys), key=lambda x: x[0]) # sort points by x
21
22     ... xs = [x for x, _ in points]
23     ... ys = [y for _, y in points]
24
25     ... n = len(points) - 1 # number of splines
26
27     ... h = [xs[i + 1] - xs[i] for i in range(n)] # distances between contiguous xs
28
29     ... # alpha = # completar
30     ... for i in range(1, n):
31         ... alpha[i] = 3 / h[i] * (ys[i + 1] - ys[i]) - 3 / h[i - 1] * (ys[i] - ys[i - 1])
32
33     ... = [1]
```

https://github.com/ztjona/EPN-numerical-analysis/blob/main/cubic_splines.ipynb

- 4) Usando la función anterior, encuentre el *spline* cúbico para:
 $xs = [1, 2, 3]$
 $ys = [2, 3, 5]$
- 5) Usando la función anterior, encuentre el *spline* cúbico para:
 $xs = [0, 1, 2, 3]$
 $ys = [-1, 1, 5, 2]$
- 6) Use la función `cubic_spline_clamped`, provista en el enlace de Github, para graficar los datos de la siguiente tabla.

Curva 1				Curva 2				Curva 3			
i	x_i	$f(x_i)$	$f'(x_i)$	i	x_i	$f(x_i)$	$f'(x_i)$	i	x_i	$f(x_i)$	$f'(x_i)$
0	1	3.0	1.0	0	17	4.5	3.0	0	27.7	4.1	0.33
1	2	3.7		1	20	7.0		1	28	4.3	
2	5	3.9		2	23	6.1		2	29	4.1	
3	6	4.2		3	24	5.6		3	30	3.0	-1.5
4	7	5.7		4	25	5.8					
5	8	6.6		5	27	5.2					
6	10	7.1		6	27.7	4.1	-4.0				
7	13	6.7									
8	17	4.5	-0.67								