Problem Set 1: Mean Field Theory, Variational Wavefunctions, Tensor Networks

Main deadline: Friday, March 15

*Submit **a single file** (.pdf or zip) online using the PSI Portal.*
Acknowledge any references you use as well as any other students with whom you collaborate.

1 Mean Field Theory 50

Consider the 1d transverse field Ising model with Hamiltonian

$$H = -J \sum_{i=1}^{L} Z_i Z_{i+1} - g \sum_{i=1}^{L} X_i,$$

and periodic boundary conditions, $Z_{L+1} = Z_1$.

For $g = 0$, the ground states can be chosen to be two product states polarized in Z direction, whereas for $J = 0$, the ground state is polarized in X direction.

(a) Write down a product state wavefunction with a variational parameter that can interpolate between the two extremes above. 20

(b) For such a variational product state, find the variational parameter which minimizes the expected value of the energy with respect to the Hamiltonian H, as a function of g/J. 20

(c) Plot how $\langle Z \rangle$ varies with g/J, for the optimized variational wavefunction. Interpret the results. 10

2 Matrix Product States 50

The product state ansatz in problem 1 is simplest variational wavefunction, with zero entanglement. To approximate ground states more precisely, we can consider a more powerful ansatz called a tensor network or in one dimension, a matrix product state (MPS). We will consider a translationally invariant wavefunction

$$\psi(s_1, ..., s_L) = \mathrm{Tr}[M(s_1)...M(s_L)],$$

where $s_1, \ldots, s_L$ is a configuration of the physical spins in a given on-site basis and $M(s)$ are D by D matrices (one matrix for every different value of s). The parameter D is called the *bond dimension* of the MPS representation.

Note: There are many valid MPS representations for a given state $|\psi\rangle$.

(a) Draw a picture representing an MPS wavefunction. 10

(b) Write down the matrices for an MPS representing a GHZ state $|00\cdots 0\rangle + |11\cdots 1\rangle$. 10

(c) Write down the matrices for an MPS representing the ground state for $J = 0$. 10

(d) Write down an MPS which approximates the ground state for small but finite J to lowest order in J/g. 10

(e) Show that any constant bond dimension MPS has area law scaling of entanglement. 10

3 (Optional) The Practical Relevance of Domain Walls

Consider a game in which 20 people stand in a line from left to right, and everyone faces toward the right. The game host randomly puts either a black or red hat on each person, and each person can see the colors of the hats of everyone in front of them, but not their own hat. The game host goes one by one from the leftmost person to the rightmost person asking what color hat the person is wearing. Everyone can hear the answers given. If the person answers correctly, he/she wins, otherwise he/she loses. What is the greatest number of people that can be guaranteed to win under a collective strategy?