Benchmark for fuzzy sphere Ising model

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Model Hamiltonian

$$H = \sum_{m_1 m_2 m_3 m_4} 2U_{m_1 m_2 m_3 m_4} c^\dagger_{m_1\uparrow} c^\dagger_{m_2\downarrow} c_{m_3\downarrow} c_{m_4\uparrow} - h \sum_{m\sigma} c^\dagger_{m\sigma} c_{m\bar{\sigma}} \tag{0.1}$$

$$U_{m_1 m_2 m_3 m_4} = \sum_l U_l (4s - 2l + 1) \begin{pmatrix} s & s & 2s-l \\ m_1 & m_2 & -m_1 - m_2 \end{pmatrix} \begin{pmatrix} s & s & 2s-l \\ m_4 & m_3 & -m_4 - m_3 \end{pmatrix} \tag{0.2}$$

Parameters

$$N_{\text{orb}} = 6, \quad s = 5/2, \quad U_0 = 4.75, \quad U_1 = 1, \quad h = 3.16 \tag{0.3}$$

Table 1. The lowest spectrum for benchmark for fuzzy sphere Ising model

E_i	Δ_i	ℓ	$\mathbb{Z}_2$	$\mathscr{P}$
−10.7542	0.0000	0	+	+
−7.6594	0.5218	0	−	+
−2.5417	1.3847	0	+	+
−1.8399	1.5030	1	−	+
3.1984	2.3525	1	+	+
3.7840	2.4513	0	−	+
4.1006	2.5046	2	−	+
7.0382	3.0000	2	+	+

Notes :

1. There might be a constant shift, only the energy difference matters ;
2. The U_l have a $1/2$ factor different from the expression in the paper, as $2n_\uparrow n_\downarrow = \frac{1}{2}(n_0^2 - n_z^2)$.