Bindiya Arora & Sercan Husnugil

Homework I

Due date: Friday September 22nd @ 9PM
https://www.dropbox.com/request/xK2pgWT8YKo2z2p4CpoK
Late deadline: Friday October 6th @ 9PM
https://www.dropbox.com/request/uXwjzZJoTH5vaTcILQfs
Pass/fail deadline: formal request needed by Friday October 20th

Kindly acknowledge the individuals or references that contributed to your successful completion of the assignment.

Q1. In quantum mechanics, the quantum revival is a periodic resurrection of the quantum wave function or a quantum property from its original form during the time evolution. One of the ways to see if the system as revived is to study correlation function. A correlation function gives the statistical correlation between variables at different points in space or time. It quantifies how microscopic variables co-vary with one another across space or time. The following form of correlation function provides a statistical description of the time-evolution of position for an ensemble

$$C(t) = \langle x(t)x(0)\rangle,$$

where $x(t)$ is the position operator in the Heisenberg picture.
(a) Find the position operator at time t in Heisenberg picture for a simple harmonic oscillator. The Hamiltonian for a one-dimensional harmonic oscillator is

$$H = \frac{p^2(t)}{2m} + \tfrac{1}{2}m\omega^2 x^2(t).$$

(b) Evaluate the correlation function explicitly for the ground state of a one-dimensional simple harmonic oscillator.

Q2. Suppose we have a composite system consisting of n parts, with $\Omega^{(u)}$ being the operator of u^{th} part. The operator for the composite system is defined as

$$\begin{aligned}\Omega &= \sum_{u=1}^{n} \Omega^{(u)} \\ &= \Omega^{(1)} + \Omega^{(2)} + \Omega^{(3)} + \cdots \Omega^{(n)} \\ &= \Omega^{(1)} \otimes \mathbb{I}^{(2)} \otimes \mathbb{I}^{(3)} + \cdots \otimes \mathbb{I}^{(n)} + \mathbb{I}^{(1)} \otimes \Omega(2) \otimes \mathbb{I}^{(3)} + \cdots \otimes \mathbb{I}^{(n)} + \cdots\end{aligned}$$

Here, $\mathbb{I}^{(u)}$ is the unit operator defined for the u^{th} part of the composite system. Thus, u^{th} term of the operator is defined as $\mathbb{I}^{(1)} \otimes \mathbb{I}^{(1)} \otimes \mathbb{I}^{(3)} + \cdots \otimes \Omega^{(u)} \otimes \cdots \mathbb{I}^{(n)}$.

Find the composite $\sigma_x, \sigma_y, \sigma_z$ spin matrices for a system consisting of 2 particles with S=$\frac{1}{2}$.

Q3. Determine the probability amplitude by evaluating

$$A = \int Dq(t)e^{\iota S[q(t)]}$$

using path integral approach for a free particle with the Lagrangian $L = \frac{1}{2}m\dot{q}^2$ and check if the results agree with what Quantum Mechanics would predict.

Chapter 3 in 'Quantum Mechanics and Path Integrals' by Feynman and Hibbs can serve as a reference.

Q4. Throughout our course, we've explored various methods to delve into quantum interference phenomena. Given that quantum computers are constructed using quantum systems, they possess the capability to manifest quantum phenomena, including interference. In this exercise you are asked to utilize the IBM Quantum Composer to construct a Mach-Zehnder interferometer and observe the resulting interference pattern. Remember during our tutorial 4 we saw that each component of Mach-Zehnder interferometer can be written as a unitary operator which can also be represented as quantum gates. Apply appropriate quantum gate to your system (qubit) to see the resulting probabilities.

How to use the IBM composer

a) Go to https://quantum-computing.ibm.com/ and click on Launch Composer. First time users may be asked to register, or you may choose to login using your existing Google account.

b) An "Untitled circuit" project page will open with a menu bar on the left containing different kinds of gate operation.
   - On your circuit, you can click on the qubits (q[0], q[1], etc.) to delete or add more qubits.
   - The gates can be dragged and dropped to create a circuit on the right. The information on each gate that we can use on the IBM composer can be found in the guide here. A good starting point for practicing with the IBM composer is to use the various single-qubit gates and two-qubit gates and see what they do and verify that they work as they should.

c) The two panels below the circuit shows some bars for the "Probabilities" and "Statevector" of states in the computational basis.

Step-by-step guide to submit your IBM circuit solutions:

- Name your circuit conveniently - for example, YourName-PSI2023 would work.
- Click on the Composer file icon on the top left corner (just beside the title of your circuit).
- Click on the three-dot ⋮ button menu beside the filename of your circuit and choose View details.
- Share your circuit by copying the sharing link. Please make sure the link *can be copied* as an actual link (i.e., do not send a screenshot image of the link because it is too long for the TA to manually type it). For example, you can copy-paste the link into a Word document, Google Doc, Notepad, e-mail, etc.

For any questions related to using the IBM composer or submitting your answers feel free to contact Bindiya at barora@perimeterinstitute.ca.

Optional question with no deadline

Create an animation, interactive simulation, or visualization focusing on a quantum phenomenon or concept that interests you the most. You have the flexibility to utilize any software or tool of your choice, such as Desmos, GeoGebra, Mathematica, Python, Blender, After Effects, Procreate, and more.

Please note the following:

- The creation of this animation is optional and will not affect your certificate eligibility.
- Developing an animation can provide you with a deeper understanding of the chosen concept.
- You can also submit a brief document or video explaining your animation.
- To submit your animation, use this link and ensure that you rename the file as Yourname-PSI2023.fileformat.
- By submitting this assignment, you grant permission for the animation to be used for teaching and learning purposes in the future. Proper acknowledgment will be given to contributors if their animations are utilized.