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Homework 2

Bindiya Arora and Dan Wohns Quantum Mechanics

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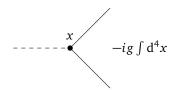
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## 1 Two real scalars

(a) We are considering here perturbative results in the quantum field theory of two interacting real massive scalars  $\varphi$ ,  $\Phi$  with respective masses m and M. The lagrangian density describing this theory is

$$\mathcal{L} = \frac{1}{2} (\partial_{\mu} \varphi)^2 - \frac{1}{2} m^2 \varphi^2 + \frac{1}{2} (\partial_{\mu} \Phi)^2 - \frac{1}{2} M^2 \Phi^2 - \frac{g}{2!1!} \Phi \varphi^2$$

where g descrives the coupling of the fields and is the parameter of our perturbative expansion. The position-space Feynman rules for perturbative computation of the interacting vacuum  $|\Omega\rangle$  n-point functions  $\langle \Omega | T \varphi(x_1) \cdots \Phi(x_k) \cdots \Phi(x_n) | \Omega \rangle$  for this theory are summed up graphically below:



- 1. Every vertex in a diagram is associated to a four-position variable x. Its contribution to the symbolic representation of the amplitude is the integral  $-ig \int d^4x$  acting on the propagators from the x vertex to other vertices.
- 3. Divide the amplitude by the symmetry factor of the diagram S = 2/W where W is the number of wick contraction producing the same symbolic diagram expression. S is computed graphically as the order of the fixed end-points automorphism group of the diagram.

- $\Delta_F^M(x-y) \qquad \Delta_F^m(x-y)$  $x \bullet - - - - \bullet y \qquad x \bullet - - \bullet y$   $1 \qquad 1 \qquad 1 \qquad 1 \qquad 1$
- 2. Each vertex is the source of two full lines and a dashed line free Klein-Gordon respectively representing a  $\varphi$  Feynman propagator  $\Delta_F^m(x-y)$  and a  $\Phi$  Feynman propagator  $\Delta_F^M(x-y)$  between points x and y (vertices or external points  $x_1 \cdots x_n$  of the expanded n-point function). Each edge of the diagram is symbolically represented as a multiplication by its associated Feynman propagator. In scalar field theory the external points contribute a trivial factor of 1 to the amplitude of the diagram.
- (b) The three-point function  $G(x,y,z) = \langle \Omega | T\Phi(x)\varphi(y)\varphi(z) | \Omega \rangle$  has no O(1) contributions because the number of contracted fields is odd at this order and no full Wick contractions can be formed: the vacuum is anihilated. At O(g), we have the diagrams
- (c)
- (d)
- (e)
- (f)
- (g)
- (h)

## 2 Acknowledgement

## References

[1] Wave packet, Wikipedia. 2023. URL: https://en.wikipedia.org/wiki/Wave\_packet.