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HOMework 2

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Contents

1	Two real scalars	2
2	Acknowledgement	2

1 Two real scalars

- (a) We are considering here perturbative results in the quantum field theory of two interacting real massive scalars φ , Φ with respective masses m and M . The lagrangian density describing this theory is

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2}m^2 \varphi^2 + \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2}M^2 \Phi^2 - \frac{g}{2!1!} \Phi \varphi^2$$

where g describes the coupling of the fields and is the parameter of our perturbative expansion. The position-space Feynman rules for perturbative computation of the interacting vacuum $|\Omega\rangle$ n -point functions $\langle \Omega | T \varphi(x_1) \cdots \Phi(x_k) \cdots \Phi(x_n) | \Omega \rangle$ for this theory are summed up graphically below:



1. Every vertex in a diagram is associated to a four-position variable x . Its contribution to the symbolic representation of the amplitude is the integral $-ig \int d^4x$ acting on the propagators from the x vertex to other vertices.
 2. Each vertex is the source of two full lines and a dashed line free Klein-Gordon respectively representing a φ Feynman propagator $\Delta_F^m(x-y)$ and a Φ Feynman propagator $\Delta_F^M(x-y)$ between points x and y (vertices or external points $x_1 \cdots x_n$ of the expanded n -point function). Each edge of the diagram is symbolically represented as a multiplication by its associated Feynman propagator. In scalar field theory the external points contribute a trivial factor of 1 to the amplitude of the diagram.
 3. Divide the amplitude by the symmetry factor of the diagram $S = 2/W$ where W is the number of wick contraction producing the same symbolic diagram expression. S is computed graphically as the order of the fixed end-points automorphism group of the diagram.
- (b) The three-point function $G(x, y, z) = \langle \Omega | T \Phi(x) \varphi(y) \varphi(z) | \Omega \rangle$ has no $O(1)$ contributions because the number of contracted fields is odd at this order and no full Wick contractions can be formed: the vacuum is annihilated. At $O(g)$, we have the diagrams

- (c)
(d)
(e)
(f)
(g)
(h)

2 Acknowledgement

References

[1] *Wave packet*, *Wikipedia*. 2023. URL: https://en.wikipedia.org/wiki/Wave_packet.