Van Roosbroeck equations for a WSM subjected to a strong magnetic field and a RCP light pulse in the quantum limit

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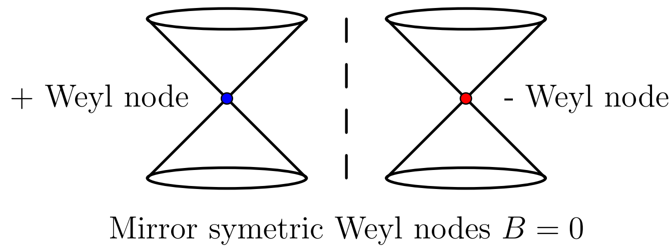

FIG. 1. Representation of the linear dispersion of a mirror symmetric WSM ommiting one of the k axis

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I. INTRODUCTION

II. MODEL

A. Unknows

We are interested in a unidimensional (for constraints associated to unidimensionnality, see sec.II F) system where a local charge density depending on time t and position z is distributed in **four** bands. These unknown densities are combined with a **fifth** unknown: the static electric field $\mathbf{E} = E\hat{z}$. This field contains an internal part $E^{int}\hat{z}$ (generated by charges) and an externaly applied part $E^{app}\hat{z}$. Technically $\mathbf{E}$ would depend on motion of

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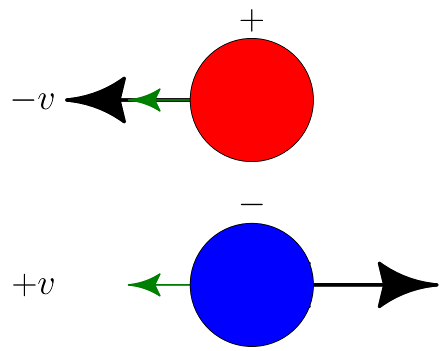

FIG. 2. Definition of chirality

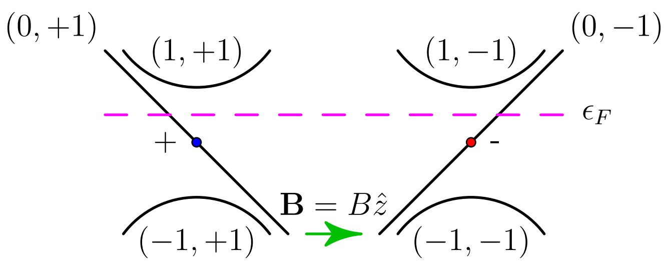

FIG. 3. Band structure of a miror symetric WSM submitted to a uniform magnetic field $\mathbf{B} = B\hat{z}$

the charges but we neglect such dependencies here. The link between the charge densities and the electric field is provided by the first unidimensional Maxwell equation which involves the permittivity ϵ that we will treat as a constant troughout all calculations.

The bands where the charges evolve correspond to the first two Landau levels $n = \{0, 1\}$ doubly represented by the chiralities $\chi = \{+, -\}$ of a Weyl semimetal (WSM) submitted to a strong uniform magnetic field $B\hat{z}$. The band structure of a WSM in such conditions is represented in fig.3. The $n = -1$ part of the band structure is ignored as a result of optical considerations mentionned in sec.II F. We denote ρ_n^χ the charge carrier density in the band associated with the χ chirality of the n Landau level (the band labeled (n, χ) in fig.3). Keep in mind that ρ_n^χ is a number of particle per units of volume. The associated *charge density* is $q\rho_n^\chi$ where q is the charge of the electron. Our model focuses on the quantum limit so we suppose that the Fermi level (represented in magenta) is located in the $n = 0$ Landau level. The Fermi group velocity is given by the slope of the band structure for each chirality at $n = 0$. Since the bandstructure is linear there, we have a Fermi group velocity of $-\chi v$ no matter where the Fermi level is located in the $n = 0$ Landau level.

Each function ρ_n^χ can be separated in three parts : an equillibrium part $\rho_{n,\text{eq}}^\chi$, a steady part $\rho_{n,\text{st}}^\chi$ and a transient part $\rho_{n,\text{tr}}^\chi$. The equilibrium part is obtained using the fermi-dirac distribution and corresponds to the solution when the WSM is at thermal equilibrium (they are time independant). The equillibrium part is time independant and we suppose it is uniform in space. The steady part describes the deviations from equlibrium observed when the steady uniform electric field $E^{app}\hat{z}$ is added to the system. Finally, the transient part corresponds to the deviation frome the steady and equillibrium parts generated by an excitation with finite temporal entension. Here, this transient excitation is due to the exposition of the system to a transient light pulse.

For each of the unknown carrier densities ρ_n^χ, the corresponding current density is denoted j_n^χ. These current densities are **additionnal** unknowns but the can be quickly related to the carrier densities (see sec.II C). They describe the motion of charge carriers in a given band and can be separated in an equillibrium, steady and transient part just like the carrier densities generating them. If there were no exchanges between the bands, we could write the following set of continuity equations to ensure charge conservation in the system:

$$\frac{\partial j_n^\chi}{\partial z} + q\frac{\partial \rho_n^\chi}{\partial t} = 0.$$

Since current densities only model **intraband** realisations of the conservation of charge, terms are added to the right hand side (see sec.II B) to model **interband** exchanges. The modified continuity equations then read:

$$\frac{\partial j_n^\chi}{\partial z} + q\frac{\partial \rho_n^\chi}{\partial t} = q\left(\frac{\partial \rho_n^\chi}{\partial t}\right)_{\text{inter}}. \tag{1}$$

To sumarize, we have to solve for the following unknowns

$$\rho_n^\chi = \underbrace{\rho_{n,\text{eq}}^\chi}_{\text{equilibrium}} + \underbrace{\underbrace{\rho_{n,\text{st}}^\chi}_{\text{steady}} + \underbrace{\rho_{n,\text{tr}}^\chi}_{\text{transient}}}_{\delta_n^\chi}, \tag{2}$$

$$j_n^\chi = \underbrace{j_{n,\text{eq}}^\chi}_{\text{equilibrium}} + \underbrace{j_{n,\text{st}}^\chi}_{\text{steady}} + \underbrace{j_{n,\text{tr}}^\chi}_{\text{transient}}, \tag{3}$$

$$E = E^{\text{app}} + E^{\text{int}} \tag{4}$$

where we have introduced the quantity δ_n^χ which represents an excess carrier density from equillibrium. Since the equillibrium distributions are known from the start, the real unknow is δ_n^χ. Furthermore, the there are no currents at equillibrium in the bands associated to $n = 1$ so $j_{1,\text{eq}}^\chi = 0$. Regarding the $n = 0$ bands, there is a current at equillibrium since all the carriers of a given chirality move in the same direction. However the combination of the + and − parts of the $n = 0$ Landau level lead to a 0 net current (because of equillibrium mirror symmetry).

To simplify our analysis we ignore $j_{0,\mathrm{eq}}^{\chi}$ without loosing any feature of the dynamics. In the end we simply take

$$j_n^{\chi} = j_{n,\mathrm{st}}^{\chi} + j_{n,\mathrm{tr}}^{\chi}. \tag{5}$$

B. Interband motion

The possible exchanges between the bands are due to four phenomena: optical excitation, relaxation, elastic collisions with impurities and chiral anomaly. As it was establish in sec.II A with equation 1, each of these exchange terms are responsible fot a part of the carrier density time derivative that isn't modeled by the intraband current density. The **optical excitation** is due to the exposure of the system to a monochromatic light source. The way it moves electrons across the band structure is represented in fig.4. The *frequency* of light is such that it excites electrons from the Fermi level to the $n = 1$ Landau level and its *intensity* is directly given by the function $G(z,t)$ which has finite spacial and temporal extension (the excitation is *transient*). Since the band structure displays mirror symmetry between the two chiralities, the light source will divide its intensity equally (in first approximation) to affect each chiral part of the $n = 0$ Landau level in the same way. This is only true when the excess populations are not big enough to generate non-linear effects (see sec.II F). The time derivative of the carrier densities associated to this phenomena are given by

$$\left(\frac{\partial \rho_{\mathbf{0}}^{\chi}}{\partial t}\right)_{\mathrm{optic}} = \left(\frac{\partial \delta_{\mathbf{0}}^{\chi}}{\partial t}\right)_{\mathrm{optic}} = -\frac{G}{2} \tag{6}$$

$$\left(\frac{\partial \rho_{\mathbf{1}}^{\chi}}{\partial t}\right)_{\mathrm{optic}} = \left(\frac{\partial \delta_{\mathbf{1}}^{\chi}}{\partial t}\right)_{\mathrm{optic}} = +\frac{G}{2} \tag{7}$$

where we have used the fact that $\rho_{n,eq}^{\chi}$ is time independant. If an electron were to be excited to high in the $n = 1$ Landau level, it would quickly drop to the bottom of this Landau level.

Once accumulated in the Landau level $n = 1$ the charges can emit a photon and return to the lower Landau level. The characteristic time of this relaxation is denoted τ_r. This process is represented schematically in fig.5. In our linear treatement, the rate at wich the carrier densities change with relaxation depend only on the excess carrier density of the $n = 1$ Landau level. In principle, the amount of excess electron in the $n = 0$ level would affect the capacity of electrons to relax from the $n = 1$ level but we neglect this effect here as it is described by non-linear combinations of the excess densities. Following the notation of sec.II A, we write

$$\left(\frac{\partial \delta_{\mathbf{0}}^{\chi}}{\partial t}\right)_{\mathrm{relax}} = +\frac{\delta_1^{\chi}}{\tau_r} \tag{8}$$

$$\left(\frac{\partial \delta_{\mathbf{1}}^{\chi}}{\partial t}\right)_{\mathrm{relax}} = -\frac{\delta_1^{\chi}}{\tau_r}. \tag{9}$$

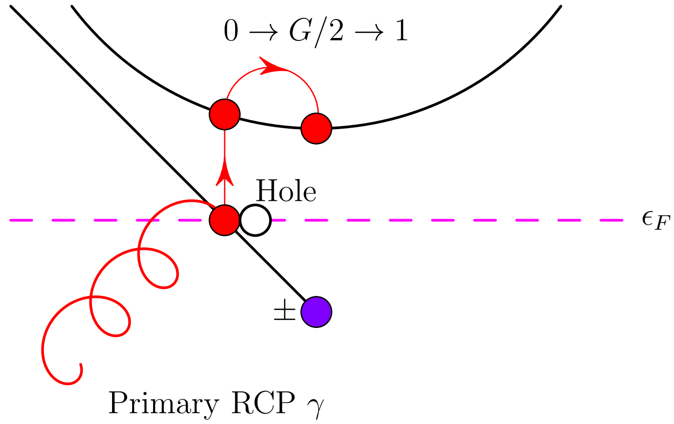

FIG. 4. Schematic representation of the optical excitation of electrons from the Fermi level to the non-chiral Landau level

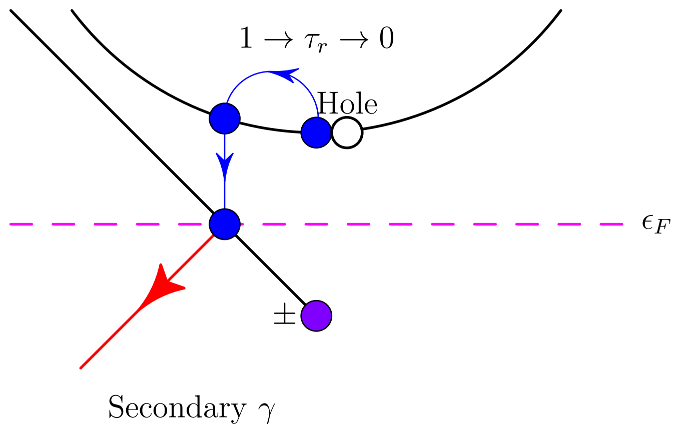

FIG. 5. Schematic representation of the relaxation of electrons from the non-chiral Landau level to the Fermi level

As the emission/absorption of a photon does not affect much the momentum of the electron, the relaxation/excitation transfers charges between $(1,\chi)$ and $(0,\chi)$ (only changing its energy). In parallel, elastic collisions with static impurities transfer charges between the $(n,+)$ and $(n,-)$ bands. The effect of such collisions is represented schematically in fig.6. Their energy transfer is negligible, but they reverse the group velocity of the $n = 0$ electrons by transfering enough momentum to them (recall that the Fermi group velocity depends on chirality in the $n = 0$ Landau level). Similar jumps in momentum are possible for the $n = 1$ Landau level but they don't affect the group velocity in a special way. This is due to the fact that the $n = 1$ Landau level doesn't change shape with chirality. The characteristic time of these collisions is noted τ_v. The considered diffusion is associated to a random walk where $1/\tau_v$ gives the probability of a change of direction and $(1 - 1/\tau_v)$ the probability of a conservation of motion at each infinitesimal time step. This kind of random walk is different from the one ordinary diffusion processes: it leads to relativistic

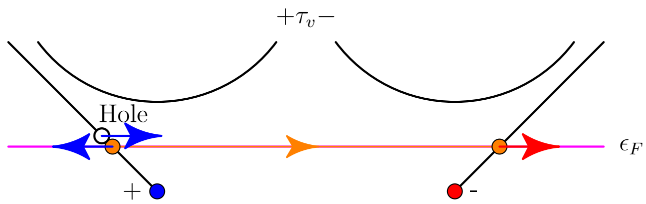

FIG. 6. Schematic representation of the elastic collisions of electrons with static impurities

diffusion which reduces to normal diffusion if τ_v goes to 0. ajouter source ici In a given Landau level, the rate at which the collisions happen is proportionnal to the imbalance between the excess carriers for two chiralities. We have

$$\left(\frac{\partial \delta_n^\chi}{\partial t}\right)_{\text{coll}} = -\frac{\delta_n^\chi - \delta_n^{-\chi}}{\tau_v}. \tag{10}$$

Finally, as it is represented in fig.7. there is a link betwen the two parts of the $n = 0$ Landau level. This link allows charges to be transfered between chiralities when an electric field is applied parallel to the magnetic field:this is the chiral anomaly effect. Both the internal and external electric fields lead to to an imbalance in chiral carriers in the $n = 0$ Landau level. Since we condider parellel fields, the rate at which electrons switch chirality with the chiral anomaly is porportionnal to the electric and magnetic field components in the z direction. The associated excess carrier times derivatives are given by

$$\left(\frac{\partial \delta_0^\chi}{\partial t}\right)_{\text{anomaly}} = -\chi\frac{q^2}{4\pi\hbar^2}EB = \frac{\chi\epsilon}{2vq}\frac{1}{\tau_A^2}E \tag{11}$$

where we have defined a characteristic time $\tau_A \equiv -\frac{vq^3}{2\pi\hbar^2\epsilon}B > 0$ for the effect of chiral anomaly. This generation rate has a stabilizing effect on the distribution of charges. Imagine there are two point charges on the z axis generating a uniform electric field directed from left to right between them. We a looking at a one dimensionnal dipole which is indistinguishable from a capicitor (see sec.II D). Because of this field, the positive chirality band will loose electrons and the negative one will gain them. Indeed we have $E > 0$, $B > 0$ and $\left(\frac{\partial \delta_0^+}{\partial t}\right)_{\text{anomaly}} = -\left(\frac{\partial \delta_0^-}{\partial t}\right)_{\text{anomaly}} > 0$ There will be more electrons moving with speed $-v$ (to the left) than electrons moving with speed $+v$ (to the right) and a net current will form. The resulting charge separation takes the form of a dipole with electric field going from right to left and we see from this that the chiral anomaly tends to cancel the initial electric field. The different interband exchanges phenomenas are sumerized with the graph of fig.8.

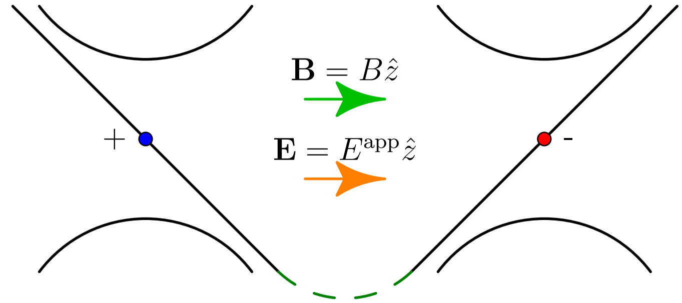

FIG. 7. Schematic representation of effect of chiral anomaly

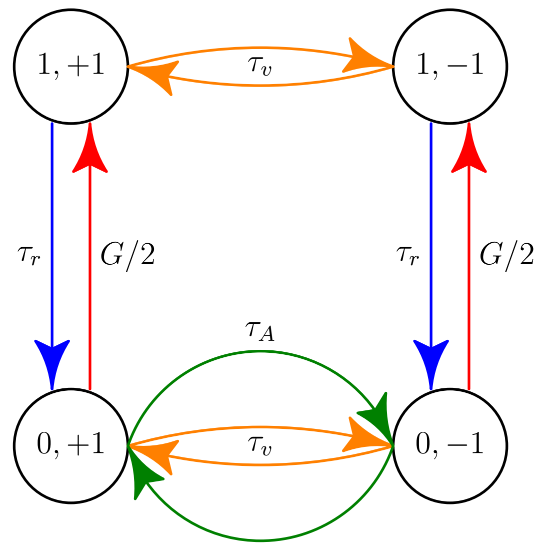

FIG. 8. Schematic representation of effect of chiral anomaly

C. Intraband motion

In the bands associated to $n = 1$, the electron motion is described by a drift-diffusion current like the one used to describe motion in usual conduction bands. These two phenomena are represented schematicaly on fig.9. While diffusion occurs with the random motion of concentrated carriers around a center with no net motion, drift happens when charge carriers follow the local electric field. The diffusion part of the current is given by ficks law: reference ici

$$j^\chi_{\text{diff},1} = -qD_1\frac{\partial \rho_1^\chi}{\partial z} = -qD_1\frac{\partial \delta_1^\chi}{\partial z} \tag{12}$$

where D_1 is the diffusion coefficient. The current density can be expresses in term of the derivative of the excess carrier density because $\rho^\chi_{\text{eq},n}$ independent of z. The drift part of the current has the following form:

$$j^\chi_{\text{drift},1} = -q\mu_1\rho_1^\chi E \tag{13}$$

with μ_1 the electronic mobility. We can define a drift speed $v_1 = -\mu_1 E$ to write the current density in the more familiar form $j^\chi_{\text{drift},1} = v_1\rho_1^\chi$.

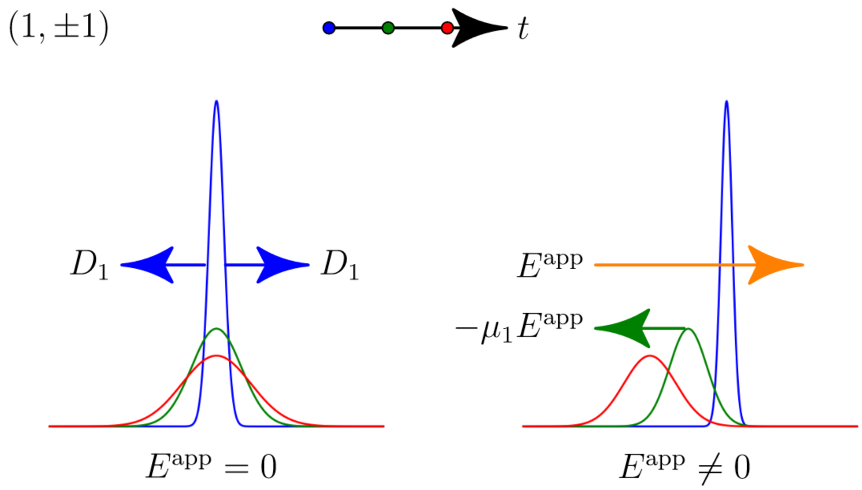

FIG. 9. Schematic representation of the diffusion and drift/diffusion of charge cariers from a localised distribution at $t = 0$

Since it contains the product of two unknown functions this current expression is non-linear. We wish to obtain linear equations and we can get there by linearising this current and its first derivative with respect to z. This linearisation relies on two facts. First, it is required that temperature is sufficiently low to ensure $\rho^\chi_{\text{eq},1}$ is small compared to the value of δ^χ_1 during most of the evolution of the system (of course, $\delta^\chi_0 < \rho^\chi_{\text{eq},1}$ for a short time after the impulse but we neglect the associated effects). Second, the intensity of the light source must be small so the maximum value of E^{int} and of $\left(\delta^\chi_{\text{eq},1}\right)^2$ are small compared to E^{app} and δ^χ_1 respectively. With these restrictions, we can perform the approximations

$$\begin{aligned}
j^\chi_{\text{drift},1} &= -q\mu_1\rho^\chi_1 E \\
&= -q\mu_1(\rho^\chi_{\text{eq},1} + \delta^\chi_1)(E^{\text{app}} + E^{\text{int}}) \\
&\approx -q\mu_1\delta^\chi_1 E^{\text{app}} - q\mu_1\rho^\chi_{1,\text{eq}}(E^{\text{app}} + E^{\text{tr}}) \\
&\approx -q\mu_1\delta^\chi_1 E^{\text{app}}
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial j^\chi_{\text{drift},1}}{\partial z} &= -q\mu_1\frac{\partial\rho^\chi_1}{\partial z}E - q\mu_1\rho^\chi_1\frac{\partial E}{\partial z} \\
&= -q\mu_1\frac{\partial(\rho^\chi_{\text{eq},1} + \delta^\chi_1)}{\partial z}(E^{\text{app}} + E^{\text{int}}) - q\mu_1(\rho^\chi_{1,\text{eq}} + \delta^\chi_1)\delta^\chi_1 \\
&\approx -q\mu_1\frac{\partial\delta^\chi_1}{\partial z}E^{\text{app}}
\end{aligned}$$

where we have neglected terms proportionnal to $\rho^\chi_{\text{eq},1}$, $\left(\delta^\chi_{\text{eq},1}\right)^2$ and/or E^{int}. In the second approximation, we also used the first Maxwell equation (see sec.II D). In the scope of our linear treatement, the drift speed is $v_1 = -\mu_1 E^{\text{app}}$ and the total current density in the $n = 1$ Landau level for the chirality χ reads

$$j^\chi_1 = qv_1\delta^\chi_1 - qD_1\frac{\partial\delta^\chi_1}{\partial z}. \tag{14}$$

Note that the expression for j^χ_1 is independent of χ because the $n = 1$ bands are identical up to a momentum shift (see fig.3).

For the bands associated to $n = 0$, the motion is light-like in the sens that it follows a relativistic dispersion at the speed $-\chi v$. Since current density is the product of the charge density with the speed at which it is moving, we have

$$j^\chi_0 = -\chi qv\rho^\chi_0. \tag{15}$$

D. Maxwell equations

To describe the electric fields at play in our system, we use one dimensionnal electrodynamics (see the discussion on dimensionnality in sec.II F). This dynamics arises from the fact that each slice of the system perpendicular to the z axis behaves like a mobile infinite capacitor plate. This means that the slices are uniformly charged and have a minimal side length large compared to the distance on which charges evolve on the z axis. Electrodynamics with cpacitors plates as fundamental objects is called one dimensionnal because it has translationnal symmetry in the x and y axis leaving only one axis where the dynamics can occur. Further more, this symmetry forces the field lines of the electric field to be parallel to the z axis : no field is lost in orther direction and E has to stay constant outside of the plate. Equivalently the laplace equation in one dimension states that outside a charged plate (in free space), the only non-zero electric field component satisfies $\frac{\partial E}{\partial z} = 0$. So the field is uniform except at the location z' of the plate add ref 1D electrodynamics where it changes abruptly. Because of mirror symmetry about the plate, it only changes sign. Thus, the field generated at z by an infinite plane with uniform charge density $\sigma\ \delta(z-z')$ (σ being the surface charge density) located at z' is

$$E = \sigma \mathrm{sign}(z-z')\frac{1}{2\epsilon}$$

which is the free space (boundary conditions at infinity) solution to the one dimensionnal Maxwell equation:

$$\frac{\partial E}{\partial z} = \sigma\delta(z-z')\frac{1}{\epsilon}$$

The factor 1/2 comes from the fact that exactly half of the field lines go on each side of the plate. For our unknown densities, we have

$$q\rho_n^\chi(z) = \int_{-\infty}^{\infty} \underbrace{q\rho_n^\chi(z')dz'}_{\sigma}\ \delta(z-z')$$

which tells us that to construt the field of such distribution, we have to superpose the fields generated by surface charge densities $\sigma = q\rho_n^\chi(z')dz'$ everywhere on the z axis. The field reads:

$$E_n^\chi = \int_{-\infty}^{\infty} q\rho_n^\chi(z')dz'\ \mathrm{sign}(z-z')\frac{1}{2\epsilon} \tag{16}$$

and is the infinite boundary condition solution to the first Maxwell equation

$$\frac{\partial E_n^\chi}{\partial z} = \rho_n^\chi.$$

The total charge density cancels at equillibrium because the ionic distribution compensates the electronic densities. For the total electric field E, we have

$$\frac{\partial E}{\partial z} = \frac{q}{\epsilon}\underbrace{\sum_{n,\chi}\rho_n^\chi}_{\text{electrons}} - \frac{q}{\epsilon}\underbrace{\sum_{n,\chi}\rho_{eq,n}^\chi}_{\text{ions}} = \frac{q}{\epsilon}\sum_{n,\chi}\delta_n^\chi \tag{17}$$

We now obtain a one-dimensionnal relation between the current and the time derivative of the electric field. It comes from a combination of the continuity equation and the first Maxwell equation (17). We denote the total carrier density $\rho = \Sigma_{n,\chi}\rho_n^\chi$ and the total current $j = \Sigma_{n,\chi} j_n^\chi$. We have

$$\frac{\partial E}{\partial z} = \frac{q}{\epsilon}\rho.$$

Taking a time derivative and changing the order of derivation (we suppose the field satisfies the Clairaut relation hypothesis), we finds

$$\frac{\partial^2 E}{\partial z\partial t} = \frac{\partial^2 E}{\partial t\partial z} = \frac{q}{\epsilon}\frac{\partial \rho}{\partial t} = -\frac{1}{\epsilon}\frac{\partial j}{\partial z}.$$

With an indefinite integration with respect to z we find

$$\frac{\partial E}{\partial t} = -\frac{1}{\epsilon}j - \frac{1}{\epsilon}J(t).$$

In general $J(t)$ is adjusted to fit the boundary conditions and we obtain the result for a boundary at infinity here. The quantity $J(t)$ is independent of z and it is therefore a uniform current over the entire z axis. Since we don't consider z uniform sources with time dependencies (E^{app} is static), J is also time indpendant and can be associated with the total steady current $j_{\mathrm{st}} = \Sigma_{n,\chi} j_{\mathrm{st},n}^\chi$ (see (5)) because the transient current has z dependency. More precisely, in the steady regime, we have

$$0 = \frac{\partial E^{\mathrm{app}}}{\partial t} = -\frac{1}{\epsilon}j_{\mathrm{st}} - \frac{1}{\epsilon}J \iff J = -j_{\mathrm{st}}$$

and it is the only regime where J can be non-zero. With this result, we can write

$$\frac{\partial E}{\partial t} = -\left(\frac{1}{\epsilon}\sum_{n,\chi} j_n^\chi - \frac{1}{\epsilon}\sum_{n,\chi} j_{n,\mathrm{st}}^\chi\right) = -\frac{1}{\epsilon}\sum_{n,\chi} j_{n,\mathrm{tr}}^\chi. \tag{18}$$

Note that, the time derivative of the electric field corresponds to the usual displacement current of three-dimensionnal electrodynamics and it makes sens to call this equation an equivalent of the fourth Maxwell law.

E. Differential equations

We are now ready to obtain the main equations governing the dynamics of the charge densities defined in sec.II A. They satisfy eq.(1) and we can make it more explicit using the interband exchange rates from sec.II B and relating currents densities to carrier densities with the expressions of sec.II C. The resulting set reads

$$v_1\frac{\partial \delta_1^\chi}{\partial z} - D_1\frac{\partial^2 \delta_1^\chi}{\partial z^2} + \frac{\partial \delta_1^\chi}{\partial t} = -\frac{\delta_1^\chi}{\tau_r} + \frac{\delta_1^{-\chi} - \delta_1^\chi}{\tau_v} + \frac{G}{2} \tag{19}$$

$$-\chi v\frac{\partial \delta_0^\chi}{\partial z} + \frac{\partial \delta_0^\chi}{\partial t} = \frac{\delta_1^\chi}{\tau_r} + \frac{\delta_0^{-\chi} - \delta_0^\chi}{\tau_v} - \frac{G}{2} - \frac{\chi\epsilon}{2vq}\frac{1}{\tau_A^2}E \tag{20}$$

where we have simplified a q factor apearing in every term. Like before, the left hand side corresponds to the usual continuity equation and the right hand side takes care of interband exchanges. The four equations are coupled and solving them starts with decoupling them (see sec.III A).

F. Scope

In order for the unknown densities to be well behaved functions, the size of the system must be larger than the temporal (transition time) and spatial (mean free path) scale of the collisions. This allows to describe the charge density function sought with an averaging over a sufficiently large portion of space and time to make the fast fluctuations negligible. When it comes to drift and diffusion phenomenas, collisions send electrons on random walks with very sharp edges both in space and in time. Replacing the sharp features of their motion by a local average leads to density functions that respect differential equations such as the diffusion equation.

For the equations used to describe the behavior of the charges, the magnetic field must be strong enough in z to dominate all the induction effects generated by the charge displacements. It is assumed that the charges are not fast and accelerated enough to produce induced magnetic and electric fields comparable to static and applied fields. Our inclusion of light in the model is such that

1. we only treat the effects of Right-Circularly-Polarised light on the system. This type of light only excites electrons from the chiral Landau level to the higher energy non-chiral Landau level. To treat the effects of arbitrary polarisations of light we would have to add unknows charge densities in the -1 Landau level.

2. we neglect the effect of the electric and magnetic components of light. Since they play a role in the chiral anomaly we could upgrade our model by including its effects.

3. we neglect higher order interactions involving more than one photon and/or impurity. These interactions would allow more complex interband motion and make our equations harder to decouple.

4. we neglect non-linear effects. In general, the effect of light intensity depends on the occupation of the Landau levels. Electrons or holes can't be moved by a photon if it brings them to a state already occupied by another electron or hole. This kind of effect would be modeled by non-linear terms and our linear treatement is therefore limited to low excess populations systems. This sets a limit to the maximum intensity of the light source. We also neglect the shift in effective chemical potential for different chiralities. This shift would modify the effect of light on the two chiralities and in term introduce a population difference for the two chiralities in the non-chiral landau level. As a first approximation we can consider a shift in effective chiral chemical potenital given by the chirality population imbalence in the setady regime (itself proportionnal to the applied field).

For the system to be one-dimensional, it must be sufficiently thin so that the effective light intensity over the cross-sectionnal area is approximatively independent of x and y.The penetration length δ of the incident light must be large compared to the depth of the material. A large penetration length compared to the size of the material is accompanied by a small decrease in current density with distance from the surface due to the skin effect. That said, there is also a lower bound on the sample thickness : it must be thick enough for the surface effects observed in Weyl semimetals to be negligible (since we are ignoring them in our model).

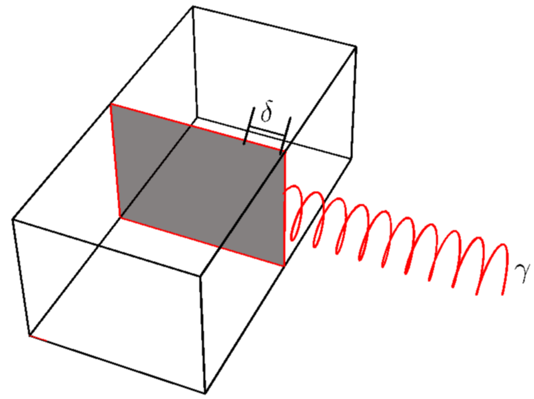

FIG. 10. Skin depth of the sample

For the system to behave as a one-dimensional system, the x, y slice for a given z must be uniformly charged and is therefore similar to a capacitor plate. Consider two such plates. If they are close enough, they behave like capacitors. However, if they are too far apart, they behave more like two point charges (a dipole). We say that plates are "close enough" if the distance between them is smaller than their smallest side. When a light pulse in dirac delta $\delta(t)\delta(z)$ hits the system, it generates two pulses propagating on the z axis at speeds $\pm v$. These charge pulses initially play the role of plane capacitors with a certain surface charge density given by the pulse, but they quickly transform into effective point charges. Let w represent the smallest side length of the cross-section of the sample exposed to light. We can say that the capacitor (one-dimensionnal) behavior exists for times smaller than $w/2v$. It is the time taken by the pulses to reach a seperation of w by moving at relative speed $2v$.

Finaly, we suppose that the length of the sample perpendicular to the illuminated cross-section is infinite. This comes with boundary conditions for the unknown densities and their derivatives at ∞: their transiant part must all vanish and their steady parts must be uniform. And upgraded treatement of our model would include an analysis of more complex boundary conditions.

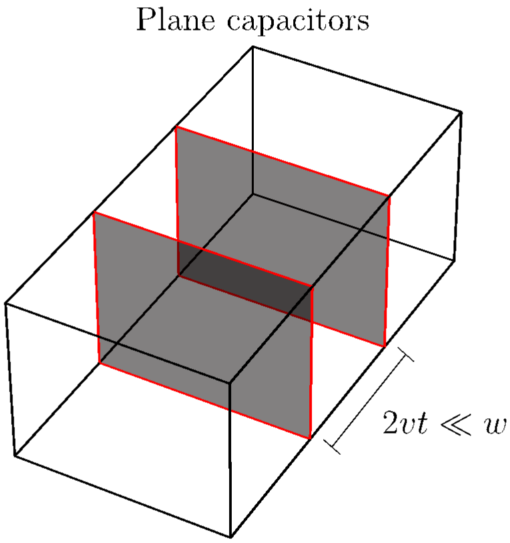

FIG. 11. Capacitor limit of charged sections

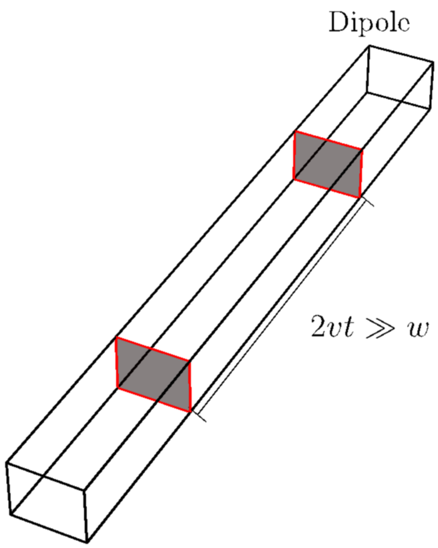

FIG. 12. Dipole limit of charged sections

III. SOLVING

A. Decoupling

To decouple eq. 19 and 20, we start by writting them in terms of the new unknown functions

$$\Sigma^n = \delta_n^+ + \delta_n^-$$
$$\Delta^n = \delta_n^+ - \delta_n^-$$

which inheritates the steady and transiant parts of the δ_n^χ carrier densities. These parts are again denoted using subscripts (ex. $\Delta_{\rm tr}^n = \delta_{{\rm tr},n}^+ - \delta_{{\rm tr},n}^-$ denotes the transiant part of Δ^n). To make these functions appear explicitly, we simply need to *add* and *substract* the equations for the two chiralities at the same Landau level. Doing this, we obtain the following *decoupled* equations for the $n = 1$ Landau level:

$$v_1\Delta_z^1 - D_1\Delta_{zz}^1 + \Delta_t^1 = -\left(\frac{1}{\tau_r} + \frac{2}{\tau_v}\right)\Delta^1 \tag{21}$$

$$v_1\Sigma_z^1 - D_1\Sigma_{zz}^1 + \Sigma_t^1 = -\frac{1}{\tau_r}\Sigma^1 + G \tag{22}$$

In these equation we have introduced a new notation for partial derivatives: the subscripts now represent variables with respect to which the subscripted function is differenciated and the number of times the appear gives the order of the associated differenciation. Since the bahavior in the $n = 1$ landau level is completely decoupled from the dynamics of the $n = 0$ Landau level, solving (21) and (22) makes Σ^1 and Δ^1 known functions that can appear in the equations for Σ^0 and Δ^0 without problem. Because of this, we only need to decouple Σ^0 and Δ^0 from each other and we can forget about their coupling to the $n = 1$ Landau Level.

We the repeat the procedure for the $n = 0$ Landau level to find the *coupled* equations:

$$v\Delta_z^0 - \Sigma_t^0 = -\frac{1}{\tau_r}\Sigma^1 + G \tag{23}$$

$$v\Sigma_z^0 - \Delta_t^0 = \frac{2}{\tau_v}\Delta^0 - \frac{1}{\tau_r}\Delta^1 + \frac{\epsilon}{vq}\frac{1}{\tau_A^2}E \tag{24}$$

On the basis of the begining of sec.III B we take $\Delta^1 = 0$. Next, we write the equations in matrix form to get

$$\begin{pmatrix} v\partial_z & -\partial_t \\ -\partial_t & v\partial_z \end{pmatrix}\begin{pmatrix}\Delta^0 \\ \Sigma^0\end{pmatrix} = \begin{pmatrix} -\frac{1}{\tau_r}\Sigma^1 + G \\ \frac{2}{\tau_v}\Delta^0 + \frac{\epsilon}{vq}\frac{1}{\tau_A^2}E \end{pmatrix} \tag{25}$$

We then multiply by the matrix

$$\begin{pmatrix} v\partial_z & \partial_t \\ \partial_t & v\partial_z \end{pmatrix} \tag{26}$$

in order to make the left hand side diagonal and get:

$$\begin{pmatrix} v^2\partial_{zz} - \partial_{tt} & 0 \\ 0 & v^2\partial_{zz} - \partial_{tt} \end{pmatrix}\begin{pmatrix}\Delta^0 \\ \Sigma^0\end{pmatrix} \tag{27}$$

$$= \begin{pmatrix} v\partial_z & \partial_t \\ \partial_t & v\partial_z \end{pmatrix}\begin{pmatrix} -\frac{1}{\tau_r}\Sigma^1 + G \\ \frac{2}{\tau_v}\Delta^0 + \frac{\epsilon}{vq}\frac{1}{\tau_A^2}E \end{pmatrix} \tag{28}$$

$$= \begin{pmatrix} -\frac{v}{\tau_r}\Sigma_z^1 + vG_z + \frac{2}{\tau_v}\Delta_t^0 + \frac{\epsilon}{vq}\frac{1}{\tau_A^2}E_t \\ -\frac{1}{\tau_r}\Sigma_t^1 + G_t + \frac{2v}{\tau_v}\Delta_z^0 + v\frac{\epsilon}{vq}\frac{1}{\tau_A^2}E_z \end{pmatrix} \tag{29}$$

To remove the electric field dependencies from (29), we use results (17) and (18) from section sec.II D. We can write

$$\begin{pmatrix} \frac{\epsilon}{vq}\frac{1}{\tau_A^2}E_t \\ v\frac{\epsilon}{vq}\frac{1}{\tau_A^2}E_z \end{pmatrix} = \frac{1}{\tau_A^2}\begin{pmatrix} \frac{1}{v}(-v_1\Sigma_{\rm tr}^1 + D_1\Sigma_{z,{\rm tr}}^1 + v\Delta_{\rm tr}^0) \\ \Sigma^1 + \Sigma^0 \end{pmatrix}$$

$$= \frac{1}{\tau_A^2}\begin{pmatrix} -\frac{v_1}{v}\Sigma_{\rm tr}^1 + \frac{D_1}{v}\Sigma_{z,{\rm tr}}^1 \\ \Sigma^1 \end{pmatrix} + \frac{1}{\tau_A^2}\begin{pmatrix} \Delta_{\rm tr}^0 \\ \Sigma^0 \end{pmatrix}$$

where we used the fact that

$$q \sum_{n,\chi} \rho_n^\chi = q\left(\Sigma^1 + \Sigma^0\right),$$

$$q \sum_{n,\chi} j_{n,\mathrm{tr}}^\chi = -qv_1 \Sigma_{\mathrm{tr}}^1 + qD_1 \Sigma_{z,\mathrm{tr}}^1 + qv\Delta_{\mathrm{tr}}^0.$$

We now have the following set of equations

$$(v^2\partial_{zz} - \partial_{tt})\begin{pmatrix}\Delta^0\\ \Sigma^0\end{pmatrix} = \frac{1}{\tau_A^2}\begin{pmatrix} -\frac{v_1}{v}\Sigma_{\mathrm{tr}}^1 + \frac{D_1}{v}\Sigma_{z,\mathrm{tr}}^1 \\ \Sigma^1 \end{pmatrix} + \frac{1}{\tau_A^2}\begin{pmatrix}\Delta_{\mathrm{tr}}^0\\ \Sigma^0\end{pmatrix} + \begin{pmatrix} -\frac{v}{\tau_r}\Sigma_z^1 + vG_z + \frac{2}{\tau_v}\Delta_t^0 \\ -\frac{1}{\tau_r}\Sigma_t^1 + G_t + \frac{2v}{\tau_v}\Delta_z^0 \end{pmatrix}$$

The remaining coupled part is highlighted in red. To replace the Δ_z^0 dependency by a Σ_t^0 dependency, we use (23) and we finally obtain

$$(v^2\partial_{zz} - \partial_{tt})\begin{pmatrix}\Delta^0\\ \Sigma^0\end{pmatrix} = \frac{1}{\tau_A^2}\begin{pmatrix} -\frac{v_1}{v}\Sigma_{\mathrm{tr}}^1 + \frac{D_1}{v}\Sigma_{z,\mathrm{tr}}^1 \\ \Sigma^1 \end{pmatrix} + \frac{1}{\tau_A^2}\begin{pmatrix}\Delta_{\mathrm{tr}}^0\\ \Sigma^0\end{pmatrix} + \begin{pmatrix} -\frac{v}{\tau_r}\Sigma_z^1 + vG_z + \frac{2}{\tau_v}\Delta_t^0 \\ -\frac{1}{\tau_r}\Sigma_t^1 + G_t + \frac{2v}{\tau_v}\left(\Sigma_t^0 - \frac{1}{\tau_r}\Sigma^1 + G\right) \end{pmatrix}$$

Reorganizing the equations to make all Δ^0 dependencies appear on the left hand side we get

$$v^2\Delta_{zz}^0 - \Delta_{tt}^0 - \frac{2}{\tau_v}\Delta_t^0 - \frac{1}{\tau_A^2}\Delta_{\mathrm{tr}}^0 = \frac{1}{\tau_A^2}\left(-\frac{v_1}{v}\Sigma_{\mathrm{tr}}^1 + \frac{D_1}{v}\Sigma_{z,\mathrm{tr}}^1\right) - \frac{v}{\tau_r}\Sigma_z^1 + vG_z.$$

Since the steady part of Δ^0 is time and space independant, we can replace the derivatives of Δ^0 in the preceding equation by the derivatives of its transient part. In the end, we obtain the main equations (30) and (31).

$$v^2\Delta_{\mathrm{tr},zz}^0 - \Delta_{\mathrm{tr},tt}^0 - \frac{2}{\tau_v}\Delta_{\mathrm{tr},t}^0 - \frac{1}{\tau_A^2}\Delta_{\mathrm{tr}}^0 = \frac{1}{\tau_A^2}\left(-\frac{v_1}{v}\Sigma_{\mathrm{tr}}^1 + \frac{D_1}{v}\Sigma_{z,\mathrm{tr}}^1\right) - \frac{v}{\tau_r}\Sigma_z^1 + vG_z \tag{30}$$

$$v^2\Sigma_{zz}^0 - \Sigma_{tt}^0 - \frac{2}{\tau_v}\Sigma_t^0 - \frac{1}{\tau_A^2}\Sigma^0 = \Sigma_1 - \frac{1}{\tau_r}\Sigma_t^1 + G_t + \frac{2v}{\tau_v}\left(-\frac{1}{\tau_r}\Sigma^1 + G\right) \tag{31}$$

B. Steady solution

Since the unknowns functions are have steady and transient parts (see sec.II A), we must solve for these two parts. While the study of the steady and transient parts of Σ^1, Σ^0 and Δ^0 require different treatements, the solution for Δ^1 works for both parts. Using the equation (21) we can formulate the following initial value/boundary condition problem:

$$\begin{cases} v_1\Delta_z^1 - D_1\Delta_{zz}^1 + \Delta_t^1 + \left(\frac{1}{\tau_r} + \frac{2}{\tau_v}\right)\Delta^1 = 0, \\ \Delta^1(0^-, z) = 0 \quad \text{(light is recieved at } t = 0\text{)}, \\ \Delta_z^1(t, z \to \pm\infty) = 0 \quad \text{(nothing special happens at } \infty\text{)} \end{cases}$$

The imediate solution to this problem is $\Delta^1 = 0$ and it describes the system both in the steady ($\Delta_{\mathrm{st}}^1 = 0$)) and transient regime ($\Delta_{\mathrm{tr}}^1 = 0$)). This is a consequence of the fact that (21) doesn't change if light is present ($G \neq 0$) or absent ($G = 0$). Physically, Δ^1 cancels out because of the symmetry between the $\chi = +$ and $\chi = -$ parts of the $n = 1$. Since we neglect the chiral shift in chemical potential (see sec.II F), light affects the parts with the same intensity. In general, the effects of the excess carriers in the $n = 0$ Landau level are neglected from the point of view of the $n = 1$ Landau level. Thus any asymmetry between $n = 0$ chiralities doesn't generate such an asymetry in the $n = 1$ level and the initial condition $\Delta^1(0^-, z) = 0$ never evolves into something else.

We now consider the steady part of Σ^1 and Σ^0 and Δ^0. In this regime we set $G = 0$ and only a time and space independant electric field is affects the system. Translationnal symmetry in both space and time of the associated dynamics leads our three steady unknows to be independant of t and z. This tells us that their derivatives all cancel and we obtain immediate solutions for Σ_{st}^1 and Σ_{st}^0 from equations (22) and (31) respectivly. In the steady regime, these equations reduce to $\Sigma_{\mathrm{st}}^1 = \Sigma_{\mathrm{st}}^0 = 0$.

The only unknown that doesn't cancel in the steady regime is $\Delta^0_{\rm st}$. It's constant value is given by (24). In the steady regive, this equation reduces to

$$0 = \frac{2}{\tau_v}\Delta^0_{\rm st} + \frac{\epsilon}{vq}\frac{1}{\tau_A^2}E \iff \Delta^0_{\rm st} = -\frac{\epsilon\tau_v}{2vq}\frac{1}{\tau_A^2}E_{\rm st}$$

where $E_{\rm st}$ is electric field generated by the steady charge distribution combined with the applied electric field. This field can be obtained by solving equation (17) but we can see $E_{\rm st} = E^{\rm app}$ from the fact that there is no net charge in the steady regime ($\Sigma^0_{\rm st} + \Sigma^1_{\rm st} = 0$). With this in mind, we have $\Delta^0_{\rm st} = -\frac{\epsilon\tau_v}{2vq}\frac{1}{\tau_A^2}E^{\rm app}$. Physically, this steady chiral carrier density imbalence arises as a result of chiral anamaly generated by the applied electric field. However the effect of this chiral anamaly generation is limited by the rate of collisions with impurities which tend to send electrons to their original chiralities and cancel the anomaly. The solution found corresponds to the imbalence between the $n = 0$ chiralities at which the amount of electrons switching chirality in the $n = 0$ Landau level with the anomaly is exactly the same as the amount of electrons sent back by collisions.

C. Green's functions

From the results of sec.III B, we know $\Sigma^0 = \Sigma^0_{\rm tr}$ and $\Sigma^1 = \Sigma^1_{\rm tr}$. It is now possible to formulate the core of the partial differential equation (PDE) system as

$$\Sigma^1_{\rm tr}\begin{cases} v_1\Sigma^1_{{\rm tr},z} - D_1\Sigma^1_{{\rm tr},zz} + \Sigma^1_{{\rm tr},t} + \dfrac{1}{\tau_r}\Sigma^1_{\rm tr} = S, \\ S = G, \\ \Sigma^1_{\rm st}(0^-, z) = 0, \\ \Sigma^1_{{\rm st},z}(t, z \to \pm\infty) = 0, \end{cases} \tag{32}$$

$$\Sigma^0_{\rm tr}\begin{cases} v^2\Sigma^0_{{\rm tr},zz} - \Sigma^0_{{\rm tr},tt} - \dfrac{2}{\tau_v}\Sigma^0_{{\rm tr},t} - \dfrac{1}{\tau_A^2}\Sigma^0_{\rm tr} = S(\Sigma^1_{\rm tr}, G), \\ S = \Sigma_1 - \frac{1}{\tau_r}\Sigma^1_t + G_t + \frac{2v}{\tau_v}\left(-\dfrac{1}{\tau_r}\Sigma^1 + G\right), \\ \Sigma^0(0^-, z) = 0, \\ \Sigma^0_z(t, z \to \pm\infty) = 0, \end{cases} \tag{33}$$

$$\Delta^0_{\rm tr}\begin{cases} v^2\Delta^0_{{\rm tr},zz} - \Delta^0_{{\rm tr},tt} - \dfrac{2}{\tau_v}\Delta^0_{{\rm tr},t} - \dfrac{1}{\tau_A^2}\Delta^0_{\rm tr} = S(\Sigma^1_{\rm tr}, G), \\ S = \dfrac{1}{\tau_A^2}\left(-\frac{v_1}{v}\Sigma^1_{\rm tr} + \frac{D_1}{v}\Sigma^1_{z,{\rm tr}}\right) - \frac{v}{\tau_r}\Sigma^1_z + vG_z, \\ \Delta^0(0^-, z) = 0, \\ \Delta^0_z(t, z \to \pm\infty) = 0. \end{cases} \tag{34}$$

In each case we have an inhomogeneous partial differential equation with no initial excitation and boundary conditions at infinity. The standart procedure to solve these equations is to use the Green's function g_D associated to their differential opperators D. This Green's function is the solution of a simplified version of the equations where the source term S has been replaced by an elementary impulse $\delta(t)\delta(z)$. More precisely, we say g_D is the *free space Green's function* of a differencial opperator D if it satifies $Dg_D = \delta(t)\delta(z)$ and *respects boundary conditions at infinity* with *no initial impulse*. When g_D is known, the solution of the equation $Df = S$ is known for any sufficiently well behaved source S as long as f satisfies the same boundary/initial conditions as g_D. The solution for f is given explicitly as the two dimensionnal convolution product:

$$f = g_D \star S = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} {\rm d}z'{\rm d}t'\ \underbrace{S(z',t')}_{\text{Source}}\underbrace{g_D(z-z',t-t')}_{\text{effect at }(z,t)}. \tag{35}$$

This kind of expression can be tought of as the sum over effects at (z,t) (given by g_d) of all parts of the source. The solution for g_D is not unique but we can make it unique by imposing causality. We chose the Greens's function to ensure that all parts of the source only affect the future of the system (to reach physical solutions). Such Green's functions are called retarded Green's functions add source

Up to a change of variable (see sec.III D), (22) is a diffusion PDE. In parallel, (31) and (30) are both damped Klein-Gordon equations with damping coefficient $1/\tau_v$. It follows that we will be using the retarded free space Green's functions of these two types of PDE. They are derived in source!! and read

$$g_d \equiv \frac{1}{\sqrt{4\pi D_1 t}} \exp\left(-\frac{z^2}{4D_1 t} - \frac{t}{\tau_r}\right)\theta(t) \quad \text{solves} \quad g_{d,t} - D_1 g_{d,zz} = \delta(z)\delta(t) \text{ (Diffusion)} \tag{36}$$

$$g_{kg} \equiv -e^{-t/\tau_v}\frac{1}{2v} J_0\left(\frac{\omega}{v}s\right)\theta(vt - |z|) \quad \text{solves} \quad v^2 g_{kg,zz} - g_{kg,tt} - \frac{2}{\tau_v} g_{kg,t} - \frac{1}{\tau_A^2} g_{kg} = \delta(z)\delta(t) \text{ (Klein-Gordon)} \tag{37}$$

where J_0 is a bessel function of the first kind, θ is the heaviside function, $s = \sqrt{v^2t^2 - z^2}$ and $\omega = \sqrt{1/\tau_A^2 - 1/\tau_v^2}$ is the analog of the characteristic frequency of an harmonic oscillator in the context of the Klein-Gordon equation. The general shape of g_{kg} is represented on fig.13. Note that this Greens function has equipotentials in the z, t plane on hyperbolas where s is constant (see fig.14). The quantity s is the analog of the space time interval from special relativity where the speed of light has been replaced with the magnitude v of the group velocity in the $n = 0$ Landau level. Thus the $s = 0$ equipotential is the future light-cone centered at $0, 0$ (position of the the $\delta(z)\delta(t)$ impulse). The Green's function vanished outside of this light cone because $n = 0$ electrons can't propagate the effect of the impulse faster than v or before the impulse (this is precesly the role of the absolute value in the heaviside function which cancels for $t < 0$).

The Green's functions of the diffusion equation is represented by the left curves of fig.9. At all times the distribution is centered around the pulse (located at $0, 0$) and at $t = 0$ the distribution reduces to $\delta(z)$. More generaly, for a given time t, the distribution looks like a normal distribution with standart deviation $\sqrt{D_1 t}$ and average at $z = 0$. The total charge, given by the integral of density over all the z axis, is 1 at all times (this is because the normal distribution is normalised and the impulse $\delta(z)\delta(t)$ has a weight of 1).

D. Elementary light pulse

In this section, we solve the boundary/initial conditon problems (32), (33) and (34) proposed in sec.III C for a light pulse $G = \delta(t)\delta(z)$. We first solve for Σ^1, we use the solution to obtain the source terms in the equations for Σ^0 and Δ^0. There is no loss of generality when we consider an elementary pulse with unit weight because the solution for other weights is obtained by rescaling our results with the new weight. Note that this is only true because our system is linear. In principle high enough weights would deplete the pool of excitable electrons and the solution would be very different from the one from our linearised treatement. For more details, see sec.II F.

1. Σ^1

To solve (32) for $G = \delta(t)\delta(z)$, we use the Greens's function approach from section III C. Since (32) is not directly a diffusion equation problem, we start by making it one. Since the two phenomena at play are drift and diffusion, reducing the problem to a diffusion problem requires a change of variable canceling the drift term $v_1 \frac{\partial \Sigma^1_{\text{tr}}}{\partial z}$. The drift speed v_1 being constant, it makes sens to change the z variable so that it *follows* the distribution at speed v_1. Explicitly, we have

$$z' = z + \mu_1 E^{app} t.$$

This change of variable is such that

$$\Sigma^1_{\text{tr},z} = z'_z \Sigma^1_{\text{tr},z'} + t'_z \Sigma^1_{\text{tr},t'} = \Sigma^1_{\text{tr},z'},$$
$$\Sigma^1_{\text{tr},t} = z'_t \Sigma^1_{\text{tr},z'} + t'_t \Sigma^1_{\text{tr},t'} = -v_1 \Sigma^1_{\text{tr},z'} + \Sigma^1_{\text{tr},t'}.$$

and it transform (32) to

$$-D_1 \Sigma^1_{\text{tr},z'z'} + \Sigma^1_{\text{tr},t} + \frac{1}{\tau_r}\Sigma^1_{\text{tr}} = \delta(z' + vt)\delta(t) = \delta(z')\delta(t).$$

Note that the impulse terme stays elementary because $(0, 0)$ is a fixed point of the change of variable. Having removed the drift term from (32), we still need to remove the effect of relaxation from to the $n = 0$ Landau level to reach the diffusion equation. Supposing relaxation has a damping effect, we use the ansatz $\Sigma^1 = e^{-t/\tau_r}\Sigma^{1'}$ that leads to the equation

$$\Sigma^{1'}_{\text{tr},t} - D_1 \Sigma^{1'}_{\text{tr},z'z'} = e^{t/\tau_r}\delta(z')\delta(t) = \delta(z')\delta(t)$$

which is the elementary impulse diffusion equation. Using the Green's function $d_d(z', t)$ for this equation and rverting to the initial variables and functions we get the solution

$$\boxed{\Sigma^1_{\text{tr}} = \frac{1}{\sqrt{4\pi D_1 t}} \exp\left(-\frac{(z - v_1 t)^2}{4D_1 t} - \frac{t}{\tau_r}\right)\theta(t).} \tag{38}$$

This function looks like the right curves of fig.9 that are essentially the retarded free-space diffusion Green's function moving at speed $v_1 = -\mu_1 E^{\text{app}}$ and depleting with the exponential factor e^{-t/τ_r}

2. $\Sigma^0 \& \Delta^0$

From the result of sec.III D 1, the source terms of problems (33) and (34) are enterily known since they are functions of Σ^1_{tr} and $G = \delta(t)\delta(z)$. On the basis of sec.III C,

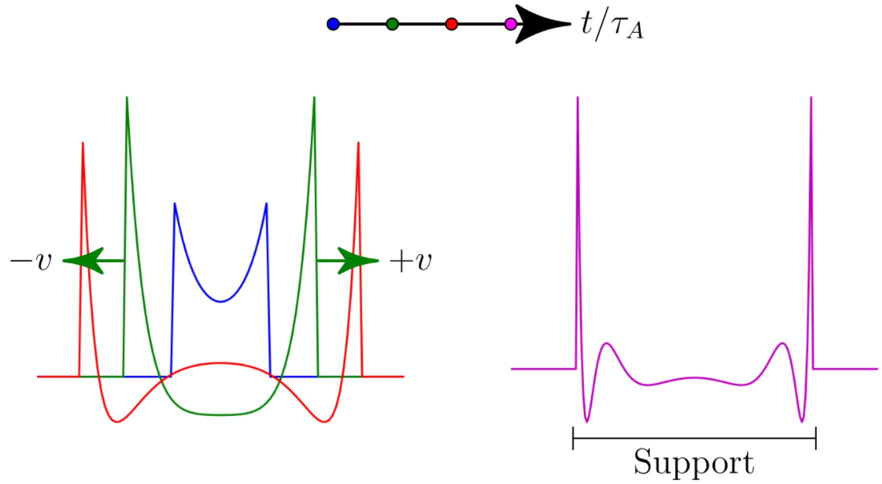

FIG. 13. General shape of the damped Klein-Gordon Green's function when $\tau_A \ll \tau_v$ (this order relation is satisfied when the applied magnetic field is high enough since $1/\tau_A$ grows faster with B than $1/\tau_v$)

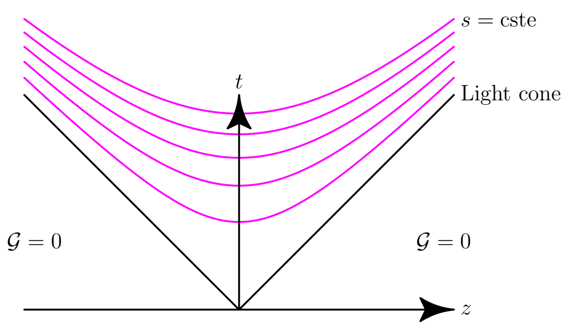

FIG. 14. Equipotentials of g_{kg} are equal *space time interval* (s) lines

the solution to both problems (which are identical up to the source terms) can be expressed with the convolutions

$$\Sigma^0_{\text{tr}} = g_{kg} \star \left(-\frac{2}{\tau_r \tau_v} \Sigma^1_{\text{tr}} + \frac{2}{\tau_v} G - \frac{1}{\tau_r} \Sigma^1_{\text{tr},t} + G_t + \frac{1}{\tau_A^2} \Sigma^1_{\text{tr}} \right)$$

and

$$\Delta^0_{\text{tr}} = g_{kg} \star \left(-\frac{v}{\tau_r} \Sigma^1_{z,\text{tr}} + vG_z + \frac{1}{v\tau_A^2} \left(v_1 \Sigma^1_{\text{tr}} + D_1 \Sigma^1_{\text{tr},z} \right) \right)$$

Taking advantage of properties of the convolution product, these expressions can be refined. For functions, f, g and h, the convolution product is such that

1. $f \star (g + h) = f \star g + f \star h$ (Linearity),
2. $f_t \star g = f \star g_t$ (Also valid for z derivatives).
3. $f \star \delta(t)\delta(z) = f$ (Neutral element of the convolution).

The second property follows from integration by parts (IBP) and it holds if f and g cancel at infinity. Explicitly

$$\begin{aligned}(f_t \star g)(z,t) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}z'\mathrm{d}t'\ f_{t'}(z',t')g(z-z',t-t') \\ &= -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}z'\mathrm{d}t'\ f(z',t')g_{t'}(z-z',t-t') \quad \text{IBP} \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}z'\mathrm{d}t'\ f(z',t')g_t(z-z',t-t') \\ &= (f \star g_t)(z,t)\end{aligned}$$

The last property follows from

$$f \star \delta(t)\delta(z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}z'\mathrm{d}t'\ f(t',z')\delta(z-z')\delta(t-t') = f.$$

Using the convolution properties, we end up with

$$\boxed{\Sigma^0_{\text{tr}} = \left(\left[\frac{1}{\tau_A^2} - \frac{2}{\tau_r \tau_v} \right] g_{kg} - \frac{1}{\tau_r} g_{kg,t} \right) {\color{red}\star \Sigma^1_{\text{tr}}} + \frac{2}{\tau_v} g_{kg} + g_{kg,t}} \tag{39}$$

and

$$\boxed{\Delta^0_{\text{tr}} = \left(\frac{1}{\tau_A^2} \frac{v_1}{v} g_{kg} + \left[\frac{D_1}{v\tau_A^2} - \frac{v}{\tau_r} \right] g_{kg,z} \right) {\color{red}\star \Sigma^1_{\text{tr}}} + v g_{kg,z}.} \tag{40}$$

The red part of the expression is the only non-analytic part since it involves the convolution of a bessel function with a gaussian spreading over time. We can identify the $\star\Sigma^1$ term with the effect of the $n = 1$ charge carriers on the $n = 0$ charge carriers. On the other hand, the remaining terms represent the direct effect of the light pulse on the $n = 0$ level.

To highlight some caracteristics of the solution we look at the derivatives of g_{kg}:

$$g_{kg,t} = e^{-t/\tau_v}\frac{\omega t}{2s}J_1\left(\frac{\omega}{v}s\right)\theta(vt-|z|) - e^{-t/\tau_v}\frac{1}{2}\theta(t)(\delta(vt-z)+\delta(vt+z)) - \frac{1}{\tau_v}g_{kg}$$

$$g_{kg,z} = -e^{-t/\tau_v}\frac{\omega z}{2v^2 s}J_1\left(\frac{\omega}{v}s\right)\theta(vt-|z|) - e^{-t/\tau_v}\frac{1}{2v}\theta(t)(-\delta(vt-z)+\delta(vt+z))$$

The Dirac deltas come from the derivatives of the heaviside function. Physically, they are direct consequences of the Dirac delta of the light pulse. To make everything more intuitive, we go back to the δ_n^χ variables and ignore the effect of Σ^1_{tr} for now. The pure effect of light is given by $\Sigma^0_{\mathrm{tr}} = \frac{2g_{kg}}{\tau_v} + g_{kg,t}$ and $\Delta^0_{\mathrm{tr}} = vg_{kg,z}$ which leads to

$$\delta^+ = \frac{1}{2}\left(\frac{2g}{\tau_v} + g_t + vg_z\right) = \frac{1}{2}e^{-t/\tau_v}\left(J_1\left(\frac{\omega s}{v}\right)\frac{\omega}{vs}(vt-z) - J_0\left(\frac{\omega s}{v}\right)\frac{1}{2\tau_v v}\right)\theta(vt-|z|) - \frac{1}{2}e^{-t/\tau_v}\theta(t)\delta(vt-z)$$

$$\delta^- = \frac{1}{2}\left(\frac{2g}{\tau_v} + g_t - vg_z\right) = \frac{1}{2}e^{-t/\tau_v}\left(J_1\left(\frac{\omega s}{v}\right)\frac{\omega}{vs}(vt+z) - J_0\left(\frac{\omega s}{v}\right)\frac{1}{2\tau_v v}\right)\theta(vt-|z|) - \frac{1}{2}e^{-t/\tau_v}\theta(t)\delta(vt+z)$$

When light hits the electrons of the $n=0$ landau level, it moves them to the $n=0$ Landau level, creating negative carrier densities shaped like the pulse but splitted evenly between the chiralities. Since they are created in the $n=0$ Landau level these pulses travel at speeds $\pm v$.

3. Perturbative expansion

4. Chiral charge

E. Gaussian light pulse

IV. CONCLUSION