PERIMETER SCHOLARS INTERNATIONAL

Classical Physics

PSI 2023/2024

Interview questions

30' before your interview, you will be assigned at random one of the 8 following questions. You will have 30' on your own to prepare an answer and about 10' to present it to the examiner(s). *This presentation will serve as the starting point for the interview proper*. As long as you make sure to touch on the points posed in the question, you are more than welcome to contextualize them in a bigger "story" about the subject at hand.

1. Formulate Hamilton's action principle and discuss how to derive from it the Lagrangian and Hamiltonian formulations of mechanics.

2. Formulate Noether's theorem in the Lagrangian formalism. Provide a proof of it and discuss the following two examples: angular momentum and energy conservation.

3. Define the Poisson bracket and use it to (i) formulate the Hamilton's equations of motion, (ii) discuss on-shell conserved quantities Q, and (iii) given a conserved quantity Q, find the corresponding infinitesimal symmetry transformation (δq^i, δp_i). [Note, point (iii) is a "reversed" Noether's theorem.]

4. Explain the following sentence: "in relativity the notion of simultaneity is lost". Then, derive, in your favorite way, the Lorentz-Fitzgerald time dilation, and finally discuss the twins' paradox.

5. Consider a relativistic particle in 4d Minkowski space. Define the notions of proper time, 4-velocity, 4-momentum, and (proper) mass. Introduce the relativistic second law and discuss its non-relativistic limit. Discuss one of the following two topics: the Lorentz force and its relationship to the relativistic particle action OR the formula $E = mc^2$.

6. Write down the Maxwell equations with sources (in the formalism you prefer) and explain why they imply the continuity equation for the electric charge density and its relationship to the conservation of the electric charge.

7. Introduce the relativistic electromagnetic tensor and use it to write the Maxwell equations with sources in a manifestly relativistic form. Discuss the use of the electromagnetic potential A_μ in the formulation of electromagnetism. Explain (i) your favorite way to see that the gauge transformation $A_\mu \rightarrow A_\mu + \partial_\mu \xi$ does not change any physics, and (ii) how this freedom can be used to put the Maxwell equations in the form of a wave equation with sources.

8. Derive the canonical stress energy tensor from Noether's theorem for a relativistic field theory (you can choose $L = -\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)$ or any other Lagrangian of your choice). Discuss the conservation properties of the canonical stress energy tensor as well as its physical meaning.

Extra topics

This a list of extra, advanced, topics covered in the course which are **not** part of the study material required to succeed in the interview. In **exceptional** cases, when the assigned interview question picked from the previous list is addressed in a precise, concise, and complete manner, these topics might come up as follow-up questions. I advise you to work on this list of questions only once you feel confident with your knowledge of 1-8 above, and leave it aside otherwise.

A. The Lorentz group(s) and the Lorentz Lie algebra, that is: the definitions of O(1,3), SO(1,3), and $SO^+(1,3)$, as well as the derivation of the structure constants of so(1,3), e.g. in the standard boost/rotation basis.
B. The theory of Green's functions and the advanced, retarded, and Feynman propagators.
C. The angular momentum tensor in a relativistic field theory.
D. The Hamilton-Jacobi formulation of mechanics.
E. The homomorphism between the Lie algebra of Hamiltonian vector fields and the Poisson algebra of their generators.
F. The canonical vs. Belinfante stress energy tensors in Maxwell theory.
G. The theory of generating functions.

This list is (roughly) ordered from the most to the least "urgent" topic.