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Homework 2

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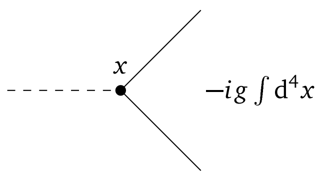
Contents

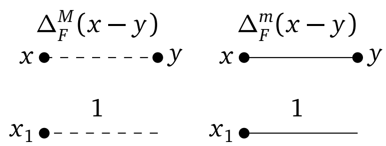
1 Two real scalars

(a) We are considering here perturbative results in the quantum field theory of two interacting real massive scalars φ, Φ with respective masses m and M. The lagrangian density describing this theory is

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2}m^2\varphi^2 + \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2}M^2\Phi^2 - \frac{g}{2!1!}\Phi\varphi^2$$

where g descrives the coupling of the fields and is the parameter of our perturbative expansion. The position-space Feynman rules for perturbative computation of the interacting vacuum $|\Omega\rangle$ n-point functions $\langle\Omega| T\varphi(x_1)\cdots\Phi(x_k)\cdots\Phi(x_n)|\Omega\rangle$ for this theory are summed up graphically below:



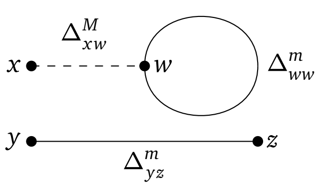


1. Every vertex in a diagram is associated to a four-position variable x. Its contribution to the symbolic representation of the amplitude is the integral $-ig\int \mathrm{d}^4x$ acting on the propagators from the x vertex to other vertices.

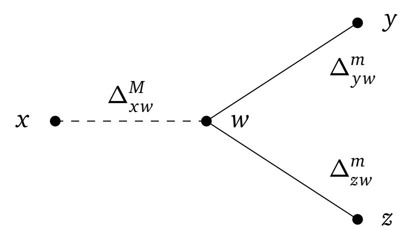
3. Divide the amplitude by the symmetry factor of the diagram $S = 2/W$ where W is the number of wick contraction producing the same symbolic diagram expression. S is computed graphically as the order of the fixed end-points automorphism group of the diagram.

2. Each vertex is the source of two full lines and a dashed line free Klein-Gordon respectively representing a φ Feynman propagator $\Delta_F^m(x-y)$ and a Φ Feynman propagator $\Delta_F^M(x-y)$ between points x and y (vertices or external points $x_1\cdots x_n$ of the expanded n-point function). Each edge of the diagram is symbolically represented as a multiplication by its associated Feynman propagator. In scalar field theory the external points contribute a trivial factor of 1 to the amplitude of the diagram.

(b) The three-point function $G(x,y,z) = \langle\Omega| T\Phi(x)\varphi(y)\varphi(z)|\Omega\rangle$ has no $O(1)$ contributions because the number of contracted fields is odd at this order and no full Wick contractions can be formed: the vacuum is anihilated. At $O(g)$, we have the diagrams



$A = -\frac{ig}{2}\int \mathrm{d}^4w\Delta^M(x-w)\Delta^m(w-w)\Delta^m(y-z)$



$B = -ig\int \mathrm{d}^4w\Delta^M(x-w)\Delta^m(w-y)\Delta^m(w-z)$

Where the symmetry factor for the left diagram gains a factor of 2 from exchanging the endpoints of the internal loop. We have the amplitude

$$G(x,y,z) = -ig\int \mathrm{d}^4w\Delta^M(x-w)\Delta^m(w-y)\Delta^m(w-z) - \frac{ig}{2}\int \mathrm{d}^4w\Delta^M(x-w)\Delta^m(w-w)\Delta^m(y-z) + O(g^2).$$

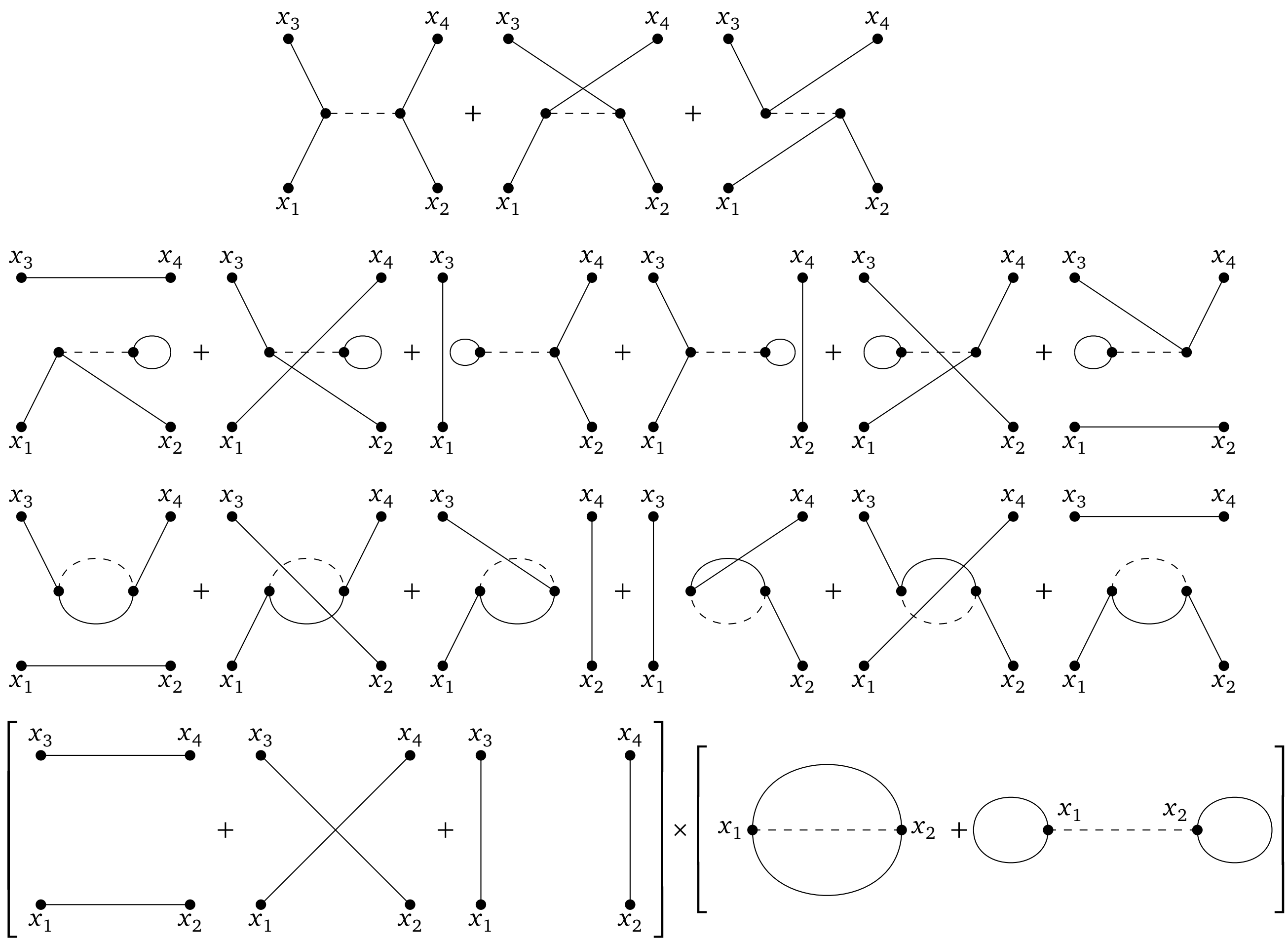
(c) i. The fourier transform of the previously computed A, B contribution to the position-space n-point function is

$$\tilde{G}_A(p_1,p_2,p_3) =$$
$$\tilde{G}_B(p_1,p_2,p_3) =$$

ii.

(d)

(e)



(f)

(g)

(h)

2 Acknowledgement

Work in progress, but thanks to Thiago and ChatGPT for help drawing the diagrams