PERIMETER SCHOLARS INTERNATIONAL

Quantum Information

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Eduardo Martín-Martínez, Bindiya Arora and Eirini Telali

Homework 2: Quantum circuits

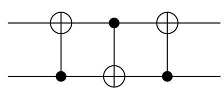
Deadline I: March 29, 2024, Dropbox link

Deadline II: April 12, 2024 Dropbox link

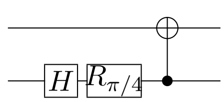
1 Back to basics: quantum circuits

Evaluate the following unitary operators, giving each answer as a matrix in the computational basis. H is the Hadamard gate and $R_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$ is the phase rotation gate.

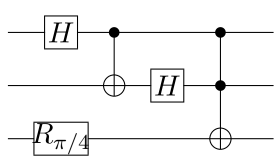
a)



b)



c)



Hint: For part (c), it is perhaps easier to work with kets (if you use pen and paper) and then find its matrix representation after you are done with all the gates. It is useful to make use of $(-1)^{\alpha}$ for some α, to keep or eliminate some terms.

2 Quantum Adder

In this question, you will design a circuit that adds numbers (in binary).

(a) Draw the circuit for an n-qubit generalization to the Toffoli gate.

(b) Use the CNOT gates and Toffoli gates to construct a circuit that adds two binary numbers.

Hint: you need will *three* qubit inputs to record the result of the addition (to "carry over" the result: $1+1=2$, when written in binary is $1+1=10$). This third qubit should be initialized in the constant $|0\rangle$ state.

(c) Now generalize this to 3 qubits: use the CNOT gates, Toffoli gates and their generalizations, to construct a circuit that adds a third number to the output of the circuit in part (c).

Hint: how many qubits do you need now?

(d) **(Bonus)** can you see what happens if you need to sum n numbers?

3 Grover's algorithm on IBM composer

There are various implementations of Grover's algorithm and we have discussed during the lecture a 2-qubit and a 3-qubit implementation with single-solution oracles. In this question we will implement the Grover's algorithm using the **IBM composer**. We will do this for the search space of $N = 8$ and $N = 16$ elements. Since we can write $N = 2^n$ where n is the number of qubits (excluding the oracle qubit), these correspond to 3-qubit and 4-qubit implementations of the algorithm. Here we will practice with single-solution oracle and also multi-solution oracles.

How to use the IBM composer

(i) Go to https://quantum-computing.ibm.com/ and click on $\boxed{\text{Launch Composer}}$. First time users may be asked to register, or you may choose to login using your existing Google account.

(ii) An "Untitled circuit" project page will open with a menu bar on the left containing different kinds of gate operation.

- On your circuit, you can click on the qubits (q[0], q[1], etc.) to delete or add more qubits.
- The gates can be dragged and dropped to create a circuit on the right. The information on each gate that we can use on the IBM composer can be found in the guide here. A good starting point for practicing with the IBM composer is to use the various single-qubit gates and two-qubit gates and see what they do and verify that they work as they should.

(iii) The two panels below the circuit shows some bars for the "Probabilities" and "Statevector" of states in the computational basis.

As a practice, try implementing 2-qubit Grover's search algorithm (search space $N = 4$) to search for the binary number 11. You may choose to use an extra qubit as the oracle qubit or implement the search without an extra oracle qubit. A CNOT gate can be used as an oracle in the former case, whereas a controlled-Z gate can be used as an oracle in the latter case. The circuits are shown in Figure 1 and 2. It is also a good practice to see if you can modify the search for the other three binaries $(00, 01, 10)$.

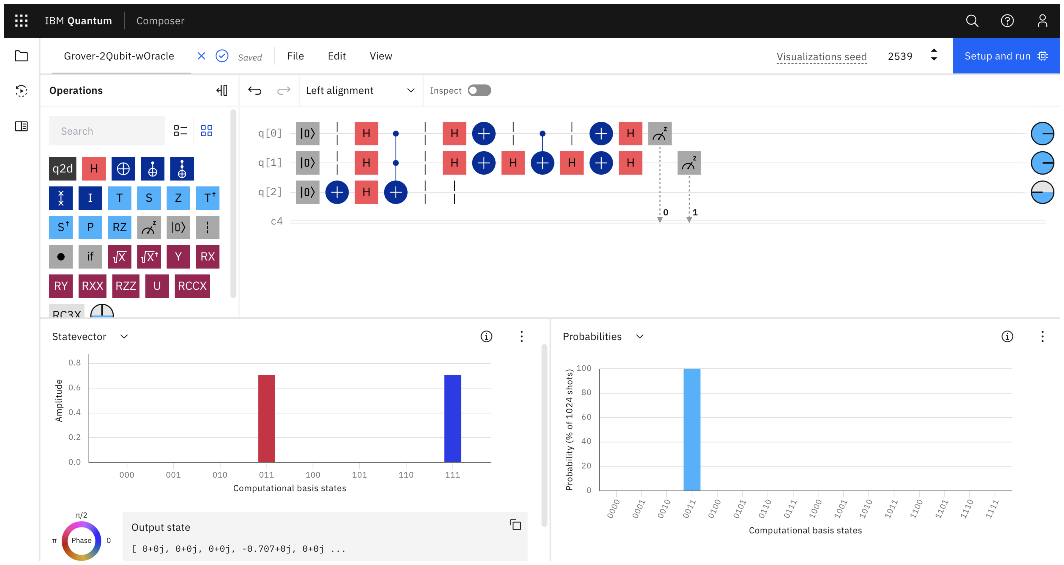


Figure 1: 2-qubit Grover search with an extra oracle qubit q[2].

Figure 2: 2-qubit Grover search without an extra oracle qubit.

Now, use the Inspect button above the circuit panel to visualize the implementation stages of the algorithm step-by-step. This will give you an idea that:

(1) In the first stage of the algorithm, all qubits are set to be in a superposition. This is done by applying the Hadamard gate H to each qubit.

(2) The oracle function O performs a phase flip (i.e., inverts the amplitude) on the desired state — in the above example, the state 11 gets a phase flip. Note that there can be more than one way to build the gate corresponding to the desired oracle.

(3) The amplification stage performs an inversion about the average of the amplitudes.

(4) The final stage is to measure the qubits.

Step (2) and (3) together performs a rotation of the uniform superposition state closer to the desired state (since two reflections is a rotation).

Tasks:

(1) On the IBM composer, implement a 3-qubit Grover's search for the binary 000. How many implementations of oracle + amplitude amplification will maximise the probability for the desired states? Verify that your circuit works for other binary searches — try 101, 001 or other binaries by adding X gates

(2) On the IBM composer, implement a 3-qubit Grover's search for the binaries 011 and 101.

(3) On the IBM composer, implement a 4-qubit Grover's search for the binaries 0110, 1000, 1010 and 1100.

Step-by-step guide to submit your IBM circuit solutions:

- Name your circuit conveniently - for example, YourName-binarystring would work.
- Click on the Composer file icon on the top left corner (just beside the title of your circuit).

- Click on the three-dot ⋮ button menu beside the filename of your circuit and choose View details.
- Share your circuit by copying the sharing link. Please make sure the link *can be copied* as an actual link (i.e., do not send a screenshot image of the link because it is too long for the TA to manually type it). For example, you can copy-paste the link into a Word document, Google Doc, Notepad, e-mail, etc.

For any questions related to using the IBM composer or submitting your answers feel free to contact Bindiya at barora@perimeterinstitute.ca.