A review of modern which-way and delayed quantum erasing experiments: demystifying retro-causality and the point-particle myth

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A review of modern which-way and delayed quantum erasing experiments: demystifying retro-causality and the point-particle myth.

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Abstract

There has been much confusion regarding some experimental tests, such as the which-way experiments and the working of the delayed quantum erasers. Apparently paradoxical results emerge which seem to suggest retro-causal action from the future back into the past. We review some of the most fundamental experiments of the last decades and show how no paradox arises, in particular regarding supposed quantum retro-causal effects, if we abandon any reference to a local point-particle ontology and examine the physics from the standpoint that considers quantum superposition and entanglement not as mere formal representations but accepts it as non-local ontological statements about reality that must be taken seriously.

***Note**: This chapter is an excerpt from the second volume of my book on quantum physics.* [1] *Therefrom the semi-didactical approach. The frequent references in the text to "Vol. I" direct to the first published volume.* [2]

Contents

I. Introduction

This section will deal with some of the most notorious and odd quantum optics experiments that have been performed in the last few decades and that further investigate the foundations and ontology of QP. These experiments can be considered as the continuation of Wheeler's delayed choice experiment and the interaction-free Mach Zehnder Interferometer (MZI) experiments presented in Vol. I. In principle, they could complement it as a concluding part. However, the higher level of theoretical and formal sophistication makes it more appropriate to present this information in the present volume, which addresses the advanced reader.

Apart from furnishing an overview of modern state-of-the-art quantum optics experiments, the aim of the following chapters is to further highlight the non-local aspect of quantum mechanics (QM), inviting the reader to abandon the naïve standpoint of a differentiating 'which-way' (or 'which-path') particle perspective, still frequently invoked by professional physicists, and to embrace the more holistic non-separable point of view which takes entanglement and state superposition as quantum phenomena that must be regarded seriously, not just as formal expedients. Along the way, we will also demystify the myth of temporal quantum retro-causality, according to which some experiments supposedly prove that the effect can precede the cause. While, indeed, physics does not explicitly disallow the existence of retro-causal effects, we will show that, at least so far, the delayed choice quantum eraser (DCQE) experiments that seem to suggest 'back from the future' actions can be explained without invoking alternative cause-and-effect orders other than the conventional one.

1. The double crystal experiment of Zou, Wang and Mandel

Let us begin with a quantum optics experiment that is somewhat less known to the public but that is still quite mind-boggling. It was performed by a group of physicists from the University of Rochester in 1991. We will call it the Zou, Wang, Mandl (ZWM) experiment. [3] It sets the stage as an introductory experiment which, apart from being interesting per se, will acquaint us with the ambiguities involved in a which-way ontology that imagines individual particles on deterministic paths that are supposed to be localized in space and time.

Fig. 1 shows the experimental setup. The light source is a *'single-photon source'* (or *'one-photon source'*) coming from an argon ultraviolet laser – that is, only one photon is heading for spontaneous parametric down-conversion (SPDC) during a time interval no shorter, or eventually longer, than the time of flight through the device of the two entangled photons. Therefore, the device always contains only a couple of photons. This source sends a photon to beamsplitter BS_1, which splits it in a superposition state along two paths. One path leads (after a reflection in a mirror) the photon to a nonlinear crystal (NL_1), after which the SPDC transforms it into two entangled photons with half the wavelength of the original one, called the '*signal photon'* and *'idler photon'*, labeled s_1 and i_1, respectively.

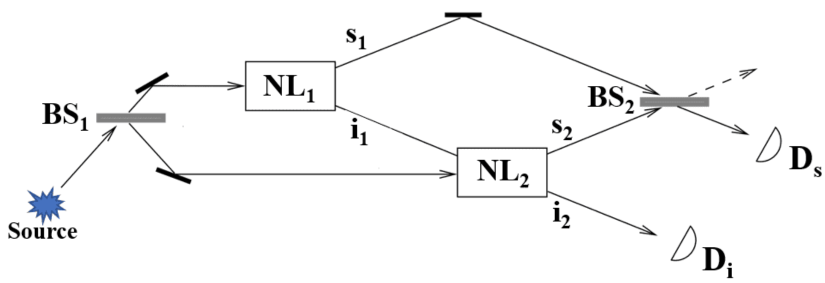


Fig. 1 The experimental setup of the ZWM experiment [3]

Without going into too many details, it may simply be said that by *'nonlinear crystals'*, one indicates a more general class of optical media that can produce entangled photons (one example that we mentioned so far was the beta-barium-borate (BBO) crystals, though there exist several other types of crystals capable of producing entangled photons). They are nonlinear insofar as their physical and optical properties respond non-linearly to the intensity of the electric field of the stimulating light beam.

The other path after BS_1 leads the photon to another nonlinear crystal (NL_2), which transforms it into two entangled signal and idler photons, s_2 and i_2. One of the most important details to fix in our minds is how the idler photon i_1 of the first crystal is sent through the second crystal NL_2. In fact, if a light beam is sent at an appropriate angle, no entangled photons are produced. The crystal behaves only as a transparent medium, just like a piece of glass with low absorption. This allows for interference between the two idler photons i_1 and i_2, which can be measured at the 'idler detector' D_i. Meanwhile, on the upper stage of this optical device, the two signal photons, s_1 and s_2, are led to converge onto the second beamsplitter BS_2, where they will interfere as well. This latter interference can be measured at one side of the beamsplitter by a 'signal-detector' D_s by slightly displacing beamsplitter BS_2 from its position or inclination and changing the relative optical lengths of the two optical paths involved. (This requires very precise mechanical control on the order of less than a micrometer.) A coincidence counter (not shown in the figure) measures when detectors D_i and D_s click almost jointly. 'Almost' means that they both click during a time interval no longer than that which a photon requires to traverse the device to make certain that they, indeed, measured the signal and idler photon generated by the one and the same source photon.

In fact, in this configuration, neither detector D_s nor detector D_i can determine the path of the signal and idler photons, respectively. This is because if D_s clicks, the so-measured signal photon could have been photon s_1 coming from crystal NL_1 and transmitted through beamsplitter BS_2 or photon s_2 coming from crystal NL_2 and reflected at the same beamsplitter BS_2. Similarly, if D_i clicks, the so-measured idler photon could have been photon i_1 coming from the down-conversion at crystal NL_1 and transmitted through crystal NL_2, or photon i_2 coming directly from the down-conversion of crystal NL_2. Therefore, at first glance, it does not seem surprising that interference fringes appear, as expected, as the which-path information isn't available.

However, notice that while two entangled photons are propagating through the device, there could be only one signal photon (s_1 or s_2) and one idler photon (i_1 or i_2). Think about this carefully, and you will realize that something weird is at work. There can't exist two down-converted photons at once, one at crystal NL_1 and the other at NL_2, because that would be contrary to the fact that the source is a single-photon source. There can be only one pair of photons travelling inside the device, and these must be either photons s_1 and i_1 coming from the SPDC of crystal NL_1 or photons s_2 and i_2 coming from the SPDC of crystal NL_2. There can't be only photons s_1 and i_2 or only photons s_2 and i_1, nor could there be two signal photons (idler photons) without their idler twins (signal twins). Otherwise, that would imply that both crystals have down-converted the pump laser photon to only one signal (idler) photon. This is something that has never been observed for the crystals in isolation. Moreover, what would be entangled?

Looking at things from the particle which-way perspective, the question is: How can a signal photon interfere with another signal photon at beamsplitter BS_2 if only one of them is allowed to exist? Similarly, how can the idler photon interfere with another idler photon at crystal NL_2 if only one of them is allowed to exist? One might argue that it is like the situation in the double-slit experiment. There, also, we think of only one photon going through two slits. However, here we are speaking of the interference between two different photons which are supposed to be created by an SPDC into different places at different times, namely, one generated in crystal NL_1 and another in crystal NL_2.

The fact, however, is that one observes an oscillating interference phenomenon. Curve A of *Fig. 2* shows the counting rate (per second) of photons measured at detector D_s by slightly displacing beamsplitter BS_2. A sin/cos wave clearly appears, testifying to the fact that interference indeed occurs. The interference at detector D_s is manifested independently of whether coincident signals are recorded at detector D_i. Similarly, one could show the existence of interference 'fringes' at detector D_i by slightly changing the optical path between crystals NL_1 and NL_2.

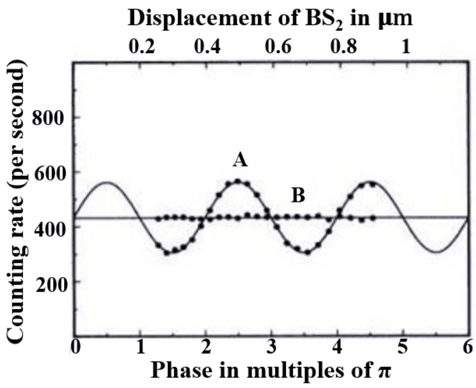


Fig. 2 Measured photon counting rate at D_s as a function of BS_2 displacement.

All this is even weirder if we keep in mind that the two processes – those corresponding to the two signal photon emissions s_1 and s_2 – are emitted at random times. SPDC is 'spontaneous' in the sense that the time of the energy transition involved at a microscopic level responsible for the creation of the entangled photons is determined by probabilistic quantum rules and there is no way to synchronize the down-conversion process of two nonlinear crystals.

To further test the behavior of this device, ZWM wondered what would happen if one interrupted the first idler path from crystal NL_1 to NL_2 – for example, by placing in between an object that fully absorbs idler photon i_1? In this case, one obvious thing is that there can be no interference at detector D_i, as idler photon i_1 which could be interfered with, is blocked physically. However, we can't say the same about the interference of the idler photon(s?) s_1 and/or(?) s_2 at beamsplitter BS_2. Blocking the idler photon i_1 should have no influence whatsoever on what happens to the signal photon and we should expect the same interference pattern emerging at detector D_s of the previous case – that is, curve A in *Fig. 2*. And yet, when the idler photon i_1 is blocked, the interference fringes disappear not only at detector D_i but also at detector D_s. The flat line B of measurement points in *Fig. 2* appears, clearly testifying to the fact that the previous interference pattern has disappeared.

In some sense, this is not too surprising. That is because now we can distinguish the path the photons take. If detector D_i clicks and detector D_s does not, that could be due only to the absorption of the idler photon i_2 generated by the SPDC of crystal NL_2 and coming from the lower path under beamsplitter BS_1. If detector D_s clicks and detector D_i does not, that could be due only to the absorption of the signal photon s_1 generated by the SPDC of crystal NL_1 and coming from the upper path above beamsplitter BS_1. If both detectors D_i and D_s click at the same time (that is, during the time interval allowed by the coincidence counter to rule out the possibility that other photons are propagating inside the device), the signal photon s_2 at detector D_s and the idler photon i_2 triggering D_i must have been generated by the SPDC at crystal NL_2. So, complete which-way information is present and no interference is expected, as observed.

On the other hand, if we insist on maintaining an ontology which imagines particles traveling along separate deterministic paths, this behavior is quite difficult to explain. Why does the spatial interruption between crystals NL_1 and NL_2 influence signal photons s_1 or s_2 traveling along completely different paths and making interference fringes disappear at detector D_s? We are no longer allowed to sweep the question under the carpet by saying that this is due to the fact that only one photon is arriving at BS_2 and that it could not interfere with any other photon. In the previous configuration, that without blocking the idler photon i_1, we saw how interference phenomena appear even if, according to our which-way conception, only one photon is present.

The only way out is to accept the fact that the physical object interrupting the idler photon path between crystal NL_1 and NL_2 also instantly collapses the wavefunction throughout the entire device. Don't forget that the signal and idler photons, s_1 (s_2) and i_1 (i_2), are entangled. While in the first configuration with no interruption, the collapse takes place only later, at the instant of detection of one of the two detectors D_s or D_i, in the second configuration with the interrupting object inserted along the path of idler photon i_1, the wavefunction collapses from two entangled particles, which we could naively see as being in two places at the same time, to a quantum state of two distinct and individualized particles being in two separate regions of space. At that stage of the process, there are 'really' two individualized particles, each on different paths and unable to interfere.

As we have amply discussed in Vol. I, this means they are one and the same until the wavefunction collapses. Entanglement implies indistinguishability in a more radical and fundamental sense that we should never mistake for the classical indistinguishability. Entangled – that is, indistinguishable – particles are one and the same object until observed and can't be described by the sum of subsystems that one would like to analyze separately.

Moreover, recognize how not only entanglement is at work but quantum superposition as well. What we must conceive of being 'superimposed' is the SPDC of the two crystals. We know that particles can be in a superposition of quantum states, such as the spin-up AND spin-down states of the electron. Here, the down-conversion process in both crystals is 'on' AND 'off' at the same time. (Of course, that happens at a microphysical sub-atomic level, not for the entire crystal.) As long as the time of triggering one or both detectors hasn't come, there is a state of superposition of the two entangled photons emitted from crystal NL_1 and the two entangled photons emitted from crystal NL_2. However, once a detector collapses the state function, it is not possible to detect a photon from each crystal and, finally, only two (not four) photons will be measured. By the way, this also shows that the physical process of SPDC does not collapse the source photon that traverses the crystals and does not spatially collapse the wavefunction but, rather, transforms it from the state of a single photon to that of an entangled photon.

In some of the following experiments, we will see additional examples of the coexistence of quantum entanglement and superposition occurring at the same time. The point is, we must take these seriously, not just as an abstract representation of facts. As long as instantaneous state reduction does not come into play, only one object is propagating along all paths, namely, that from a point of emission in the source to one of the detectors. In between, and during, the emission-detection time interval, we can describe the 'state of being' of this entity only with an abstract state vector, though any reasoning based on a counterfactual definiteness pointing at some separate object with definite properties in a dividing and separative space-time conception is misplaced. In a certain sense, speaking of 'entangled particles' is a self-contradiction in terms, because we still picture, in our minds, two separated particles somehow interweaved throughout space, though one can't do otherwise due to the limitedness of human language. A more appropriate understanding might furnish us Feynman's path integrals approach (see path integrals and Feynman's diagrams in Vol. I), in which one considers the final trajectory of a particle as resulting from the interference of all the possible paths, that is, a sum over 'histories'. Feynman's calculation technique determines the probability of a particle traveling from a space-time point to another (the propagator) and assumes that it travels along all the possible paths allowed at once. It does not work with anything being a corpuscle; it simply adds all the wavefunctions describing the possible histories that we, by counterfactual definiteness, imagine to be the paths of a single particle.

What we should evince from this paradigmatic experiment is that the most sensible way to interpret the facts is to give up a particle model and embrace a more integral and holistic perspective. What spans the experimental set-up of this quantum optics experiment is never a particle, and not even a wave, but a wavefunction, a probability wave, a not-better-defined 'physical entity' that takes all the possible paths through the device at once and, at the instant of measurement or absorption, collapses. Or, to put it in other words, if you prefer, one might complement this view by that which is contrary, namely, by conceiving of this entity as taking neither 'this path or that path', nor that of traveling 'this path and that path', because there is only ONE path.

At any rate, the experiment of ZWM should instill some doubt, to say the least, as to whether it still makes sense to insist on a which-way interpretation of QM, where one imagines corpuscles (photons or material particles) possessing definite properties, localized in space and time and traveling along well-defined deterministic paths, as our classical Newtonian mindset would like to believe. If, instead, we give up this model and begin to realize that there are neither particles nor waves, this might lead us a step further.

In this chapter, we considered ZWM's experiment as an 'appetizer' that paves the way to other experiments which will further highlight the subtleties and deep conceptual implications of modern QT. The following chapter will shed more light on the extent to which the which-path perspective is appropriate and, at best, only complementary to a more integrating and non-dual perspective.

2. The which-way reconsidered.

Even among several physicists and philosophers of science, it is considered a common wisdom that whenever an experiment is performed that attempts to gain insight into a particle's path, that is, its 'which-way' information, then automatically all interference phenomena must disappear. While this interpretation isn't incorrect, it carries with itself some ambiguities that are potentially misleading, especially when it comes to the celebrated DCQE experiments that the next chapters will illustrate. In this chapter, we will clarify the deeper meaning of the 'which-way' (or 'which-path') rule and how far we are allowed to effectively think in terms of deterministic paths resorting to local realism. This will prepare us to tackle the supposed effects of temporal quantum retro-causality which are sometimes invoked incorrectly. There are several methods, interpretations, and approaches that one can use to explain the DCQE experiments in a more appropriate context. Here, we will adopt that taken by David Ellerman [4]. (Other, equivalent approaches that demystify retro-causality exist as well [5] [6], [7].)

Let us go back to the very basics, which told us how the interference fringes of the classical double-slit experiment come into being. (Recall Young’s double-slit experiment in Vol. I.) Please keep in mind how that formulation, which is also what one finds in most textbooks, holds only when the two interfering beams from the slits have the same polarization – that is, when the two field vectors are co-directional on the polarization plane at every instant. If one studies the conventional Young double-slit experiment, these are only subtleties that one can ignore, as the two slit-beams emerge from the very same incident beam and, therefore, always have the same path difference and polarization. However, in double-slit experiments involving polarizers, these aspects can no longer be neglected. Moreover, by inserting polarizer filters in front of one or the other slits, one must also consider the extra phase shift that these could eventually induce on the path of the respective slit. If these are taken into account, the considerations involving interference patterns require further attention and its form requires a slightly more complex representation, which, however, will allow us to gain much deeper insight into the meaning and correct interpretation of the 'which-way' experiments from the quantum mechanical perspective.

Let us illustrate this in detail beginning with *Fig. 3*. Suppose the light coming from a light source is polarized along the horizontal direction (0° polarization, by convention) with a polarizer in front of both slits. Then, if one were to place a linear polarizer after one of the two slits (say, S_1), this would change the polarization state of the beam from S_1 relative to that of the beam coming from slit S_2 (still in the 0° polarization state) by an angle $\delta\theta$. (The symbol δ will always imply a relative difference between two quantities – here, the relative angular direction difference of the polarization vector between the two beams.)

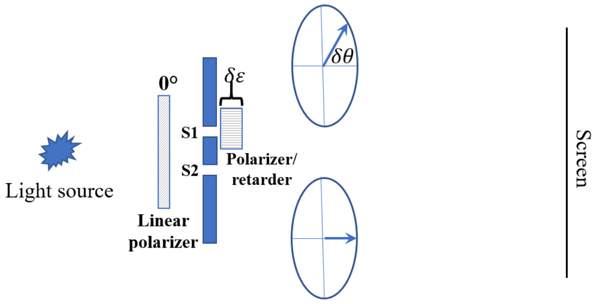


Fig. 3 Polarization angle difference $\delta\theta$ and phase shift $\delta\varepsilon$ between light beams coming from slits S_1 and S_2.

Moreover, a polarizer, as any transparent plate with some optical refractive index (that is, light is slowed down), will also cause a change in the optical path – here, a retarding relative path-phase shift of the beam from slit S_1 relative to that of slit S_2 by an amount $\delta\epsilon$.

The latter quantity is usually expressed in degrees (or radians) as a phase shift proportional to the wavelength. For example, a half wavelength $\frac{\lambda}{2}$-shift is identical to a $\delta\varepsilon = 180°$ (or $\delta\varepsilon = \pi$ radians) shift. It can be shown (see Appendix A I) that this change in phase and polarization will also lead to a different double silt interference pattern than that to which we were accustomed in the conventional Young experiment and that can be captured by modifying the intensity function of the double-slit beams interfering on the screen (as a reminder, see the Appendix of Vol. I on trigonometric functions and waves and complex numbers) as follows (for the sake of simplicity, the dependence on the x-coordinate is omitted):

$$
\begin{aligned}
I(\delta\phi, \delta\varepsilon, \delta\theta) &= I_1 + I_2 + 2\sqrt{I_1 \cdot I_2} \cdot cos(\delta\phi + \delta\varepsilon) \cdot cos(\delta\theta) \\
&= 2I_0[1 + cos(\delta\phi + \delta\varepsilon) \cdot cos(\delta\theta)]\,, \qquad \textit{Eq. 1}
\end{aligned}
$$

where, as usual, $\delta\phi$ is the angular phase path difference of the beams that determines the angular dependence along the vertical screen direction while the last passage simplifies the expression assuming $I_1 = I_2 = I_0$, that is, with the two identical slits transmitting the same amount of light with intensity I_0. Strictly speaking, the real intensity behavior we show in the graphs is obtained by multiplying Eq. 1 with an exponential damping factor (such as, for example, $e^{(\delta\phi/10)^2}$). As we know (see the many slits interference and diffraction in the appendix of Vol. I), the realistic intensity function of the fringes is damped out by an angular term, the diffraction envelope. We will maintain this in the graphs but, to keep things simple, will not consider it in the text equations because it isn't relevant to the aspects we will point out.

Let us inspect Eq. 1 and understand how it works. Note that both cosine functions still exclusively modulate the interference term. If one or both of these is zero (say, for example, for $\delta\theta = 90°$), only the first two terms are left ($I = I_1 + I_2 = 2I_0$). This aligns with the case in which the two slits' intensities are added without interference, corresponding to the normal distribution curve, as we shall see next. Meanwhile, when one of the cosine functions equals -1 and the other +1, for there is full intensity subtraction at some point on the screen: $I = 0$, it is a 'no fringe' minimum. When both are +1 or -1, there is full intensity summation: $I = 4I_0$, the fringe peak. If you wonder how it could be that two slits can produce an intensity peak of four slits, just consider how energy must be conserved. The energy is not lost but only shifted from the Gaussian distribution minima locations, concentrating it due to interference on the peaks and, so to speak, 'piling' it up on the maxima.

To visualize this, we can play around with phase shifts and polarizations for different experimental double-slit setups that involve polarizers.

If we change the path phase difference at one of the slits ($\delta\varepsilon$ in Eq. 1 varies) by introducing a 'retarder' plate, for example, a polarizer that does however leave the polarization of the beam unaffected ($\delta\theta = 0$), or just several retarding transparent media with different thicknesses, then one obtains the interference patterns as shown in Fig. 4.

The first interference pattern ($\delta\varepsilon = 0$) represents the conventional double-slit fringes interference with no retardation and no polarization difference. Varying the phase shift between the two slits causes the interference fringes to shift as well. The central most intense fringe is no longer centered with the geometric horizontal axis of the two slits.

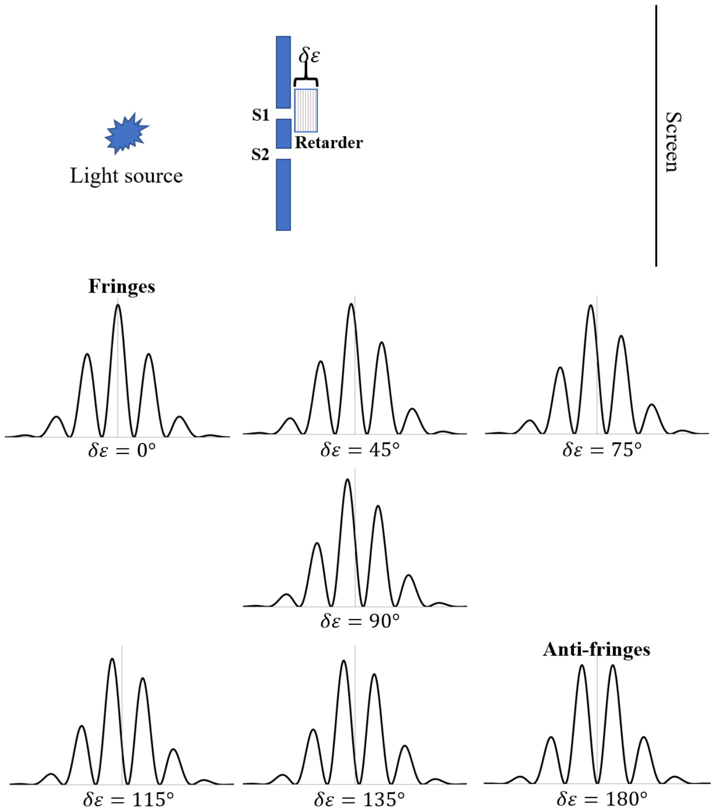


Fig. 4 How the double slit interference pattern changes due to a phase shift difference $\delta\varepsilon$ imparted by a 'retarder' plate.

The most interesting situation for us is that of a phase shift of 180° (half a wavelength path phase shift). In such a case, in opposition to the conventional fringes, the maxima become minima and the minima become maxima – that is, the white fringes are replaced by the black ones and vice-versa. These fringes are therefore called '*anti-fringes*'.

Compare the above result, obtained exclusively from the perspective of an optical electromagnetic (EM) wave, to the which-way perspective. As long as the polarization of the two beams is the same, and whatever phase shift one applies to the wave/photon traveling through the slits, the interference never disappears. This aligns with the idea that we can't gain any information about which path a photon takes only from the phase. The phase difference can't be used as a marking method for one single photon going through one or the other slit. The concept of the phase is a relative, not an absolute, one. It makes no sense to speak of a phase of one photon or a wave. Rather, only a phase difference between two waves is a meaningful physical quantity.

Note, however, one interesting and decisive fact: If the first fringe pattern is added to the last anti-fringe pattern of Fig. 4, the resulting curve is the usual Gaussian bell-shaped normal distribution (Fig. 5), which we know to be the curve that appears when we attempt to obtain the particle's which-way information or the diffraction pattern which appears for the 'single-slit' or the pinhole with an aperture size comparable to the wavelength of the incident light. (See the chapter on Heisenberg's uncertainty principle in Vol. I.)

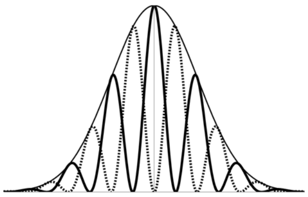


Fig. 5 The normal distribution as a sum of fringes and anti-fringes.

In all the previous experiments and considerations, we have always taken the disappearance of the interference fringes into the bell-shaped curve as a sure sign that we have switched from a wave-like behavior to a particle-like behavior. And, according to the which-way interpretation, the existence of fringes signals the wave-like character of a particle but does not allow for any information about which path it went along (i.e., through which slits or through which arm of an MZI). Meanwhile, the clumpy curve, which is the diffraction envelope itself, signals that a logical inference about the path is allowed, but the interference is inevitably lost.

Here, however, things look much more subtle. It turns out that the pattern that lacks the interference fringes, which manifests due to a lack of knowledge of the path the particle takes, hides the information by overlapping the complementary fringe- and anti-fringe interference patterns that appear when this knowledge is available. We might say that the particle behavior resulting from the manifestation of the diffraction envelope is a combination of wave-like phenomena in disguise. Is this simply a mathematical coincidence or does it have a deeper meaning?

Let us analyze the opposite case, that with a difference in the polarization of light between the two slits ($\delta\theta$ in Eq. 1 varies) but no phase shift ($\delta\varepsilon = 0$). One possible way to obtain this could be, for example, by inserting before the slits a linear polarizer and in front of both slits two identical '*half-wavelength plates*' (HWP). HWPs also shift (retard) the path phase by $\frac{1}{2}\lambda$ but their main function is to rotate the linearized light by twice the angle between the fast axis and the polarization vector. Waveplates are characterized by a '*fast axis*' and a '*slow axis*' along which the polarization component travels faster or slower, respectively. For example, if a polarized light beam enters an HWP with its polarization vector tilted by 45° relative to the fast axis, the outcoming beam will be tilted by 90° – that is, it will transform from 45° to -45° polarized light (see Fig. 6).

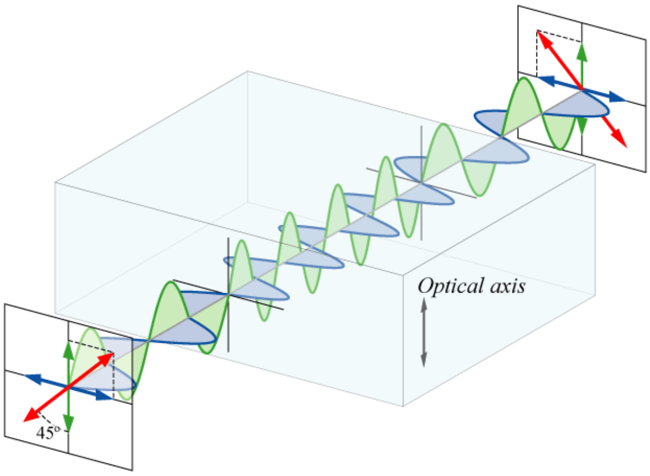


Fig. 6 90° Rotation of the polarization vector with a half-wave-plate.

Please keep in mind that, from a quantum physical perspective, polarizers and wave plates are ***not*** measuring devices. We should not associate an operator with them, at least not in the sense of an observable that causes state reduction. They do not 'measure', 'collapse', or 'reduce' anything. They 'select', 'convert', or 'change' the evolving quantum state of a particle and its related state function before a measurement takes place. This is an important distinction to which we will need to pay further attention later.

Let us apply the HWP for our purposes here. The linear polarizer's function is to fix one polarization vector of the incoming light, say, at 0° relative to the fast axis of the HWP. Two HWPs are needed in front of each slit to compensate for the retarding optical path phase shift leading to a net phase difference of $\delta\varepsilon = 0$. One HWP is left fixed at 0° while a physical rotation of the second HWP by an angle α causes a polarization angle difference between the first and second slits of $\delta\theta = 2\alpha$. The resulting different interference patterns are shown in Fig. 7.

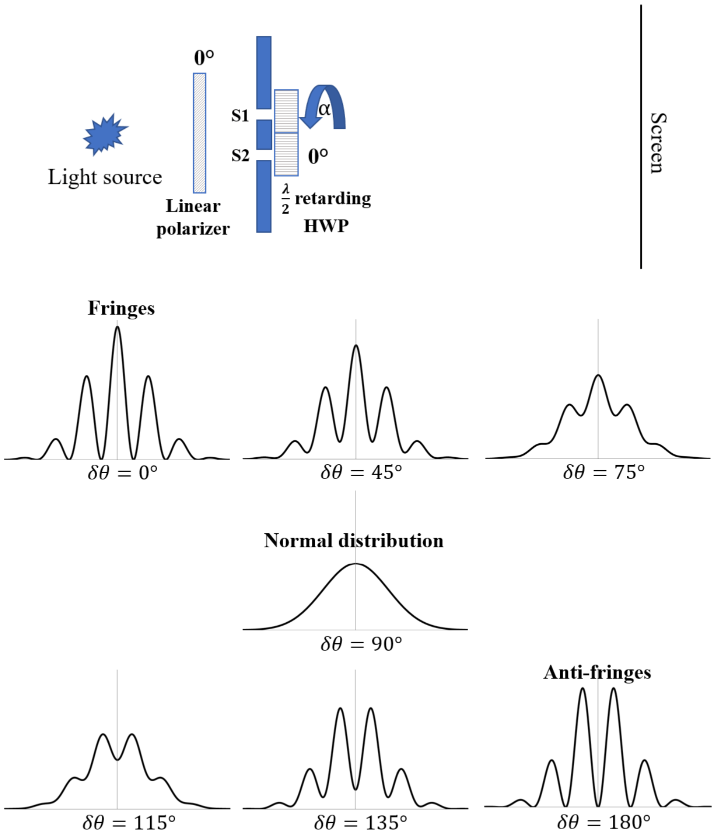


Fig. 7 How the double slit interference pattern changes due to a polarization shift difference $\delta\theta$ imparted by rotating the WHP.

One can observe a different behavior than the phase-shift fringes patterns of Fig. 4. The peaks are not shifted due to polarization but their amplitude is modulated. Again, $\delta\theta$ =0 is the conventional double slit situation (the same as $\delta\varepsilon$=0 of Fig. 4). For the orthogonal polarization ($\delta\theta$ =90°), one obtains the normal distribution density probability function, which was not (explicitly) present in the phase-shift case.

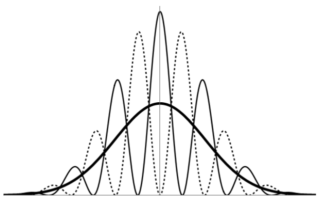


Fig. 8 Fringes and anti-fringes overlay, and the normal distribution for $\delta\theta$ =90° of Fig. 7.

The normal distribution resulting for the orthogonal polarization is half the height of the fringes' central peak. This is obvious if you consider that the area under the interference curves (the integral, that is, the total photon count) for either the fringes, anti-fringes, or normal distribution case must always be the same for each due to energy conservation.

This time, a special situation arises in which the interference disappears for the orthogonal polarization. This aligns with the which-way perspective. In fact, in contrast to phase shifting, from what we have seen in the which-way experiments of Vol. I, we well know that polarization can be used as a means of 'marking' a photon to gain information about which slit (or MZI arm) it went through and, with this, to resort to retro-ductive reasoning about the path we imagine it has taken. Again, the wave-perspective aligns one to one with the corpuscular which-way perspective.

To investigate this equivalence further, let us consider the special case illustrated in Fig. 9.

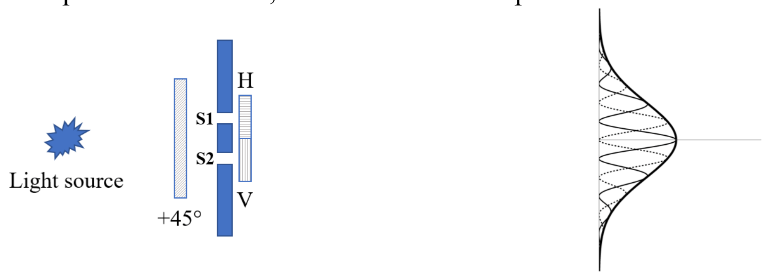


Fig. 9 Double slit with photon-marking horizontal and vertical polarizers.

A linear polarizer with diagonal 45° polarization (say, relative to the horizontal x-axis) is inserted before the two slits, while a horizontal polarizer and a vertical polarizer are inserted in front of slits S_1 and S_2, respectively. From the wave perspective, this is simply another example of the aforementioned orthogonal polarization case ($\delta\varepsilon = 0$; $\delta\theta = 90°$). From the which-way perspective, because photons are therefore marked, the interference fringes disappear and the bell-shaped curve forms on the screen. According to the quantum mechanical formalism, the state of a single photon going through this arrangement of polarizers and slits must be in a superposition state of a photon going through slit S_1 with horizontal polarization H and slit S_2 with vertical polarization, namely:

$$|\Psi\rangle = \frac{|H\rangle_{S_1} + |V\rangle_{S_2}}{\sqrt{2}}. \qquad \textit{Eq. 2}$$

The vital detail we must always keep in mind is how the which-way perspective must be interpreted correctly. One could take two approaches.

The first one thinks of the particle as going through both slits and interfering with itself only as long as we do not try to find the slit it went through, that is, its which-way. This which-way conception, however, insists on the particle-like idea and implicitly suggests that what the polarizers do is reveal to us through which slit the photon went once we measure its polarization. It seems so obvious to us that, if the outcoming photon has a horizontal (vertical) polarization, it must have gone through the first (second) slit. That is, one imagines, again by a retro-ductive cognitive act of counterfactual definiteness, that the wavefunction may have been collapsed into a particle state earlier, at the stage in which it went through the +45° polarizer, and that then the photon has gone through one – and only one – slit, finally manifesting this information to us in case we measure its polarization with the H/V polarizers. Even if we would not have placed the polarizers after the slits, by a mental projection we nevertheless imagine the photon going through one or the other slit, though we would then never know which.

The second interpretation of this state of affairs, which is less prone to accepting such a naive quantum ontology, thinks of the same particle going through both slits. It doesn't forget that, according to QP, **slits *do not separate*** a quantum object along different paths but superimpose its quantum state and that **polarizers are *not* measurement devices** that project or collapse but instead only select or change the evolving state vector. As long as there is no measurement, that is, the collapse of the state function, due to an interaction with a sensitive measurement device that allows for a readout, the quantum system is still in the state described by the wavefunction (or state vector). In fact, a polarizer reveals nothing (no operator, no observable is acting on the wavefunction) unless we do not place in front of it a detector (such as a photomultiplier, a CCD camera, a photodiode, etc.) which absorbs the photon and by which, then and only then, state reduction as signaled by a 'click' or a readout of a value becomes possible (the eigenvalue). Before that instant of the collapse, we can't conceive of any particles flying separately along a path with separate polarizations. Rather, we must still think of the wavefunction describing the system as being in a state of real superposition as a whole inseparable and unique entity being both here and there and both having one and the other polarization.

It is easy to show that the second conception must be taken seriously. Simply place, in front of the experiment of Fig. 9, another diagonal 45° polarizer, as shown in Fig. 10 top. Interference fringes reappear. When the quantum erasing polarizer is rotated by 90°, that is, by orienting it along the -45° direction, as in Fig. 10 bottom, interference is still present, but the anti-fringes will appear instead (for any polarization angle between these perpendicular directions, the intermediate cases of Fig. 7 appear, something we won't dwell on any longer here).

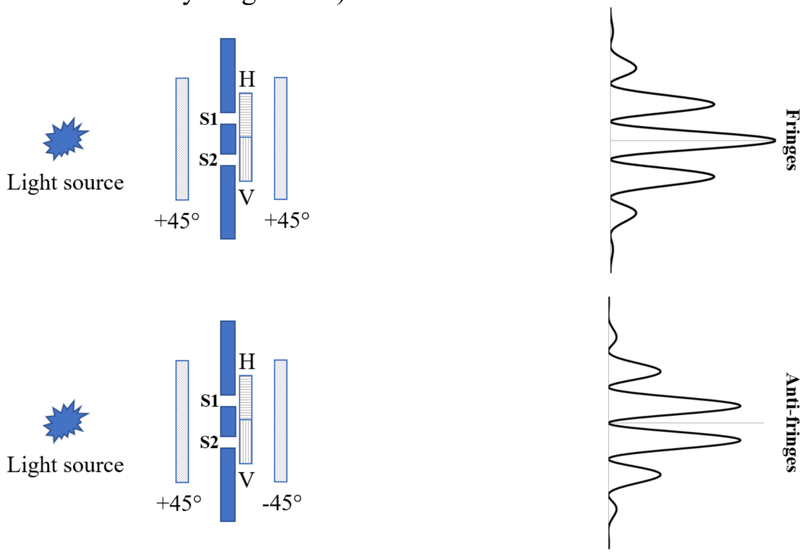


Fig. 10 A diagonal polarizer acting as quantum eraser in the experiment of Fig. 9.

From the wave optics perspective, this is a quite obvious fact because the difference in polarization is cancelled ($\delta\theta = 0°$): All photons will emerge with the same $\pm$ 45° polarization. Also, from the which-way perspective, everything looks fine because the photon's marking has been lifted – that is, the last polarizer acts as a quantum information eraser that makes the two slits' paths indistinguishable; the fringes reappear, as expected. (Recall that we had a similar situation with the reappearance of the spin superposition states in the MSG experiment; see the chapter in Vol. I where we questioned whether information is fundamental.)

This should give us food for thought with regards to the above two interpretations. First, note that if there were truly a photon taking a definite path through one or the other slit, then, when it traveled farther until it encountered the $\pm$45° polarizer acting as a quantum eraser, it must, from that point on, 'forget' where it came from, and the quantum eraser must have acted retro-causally in the past!

This is because, if the first conception which imagines particles going through one or the other slit with a definite polarization caused by a state projection of the two orthogonal polarizers were correct, there could be no recovery of the fringe or anti-fringe pattern later. If a photon truly is going through a slit with some polarization 'out there', this would have been the end of the story. The second diagonal polarizer cannot recover any interference pattern or reproduce it out of the blue.

For example, say the photon went through the slit with the horizontal polarizer; then, in the short time interval during which it traveled from the H polarizer to the $\pm$45° polarizer, we would conceive that it had gone through one and only one slit. It 'says': *"I went through only slit S_1 and will not be able to interfere with a copy of myself coming from slit S_2. I will have to hit the screen according to a normal distribution"*. However, shortly after, it encounters the $\pm$45° polarizer, and this information about which slit it went through will be erased. At that point, how can the single photon 'change its mind' and distribute itself on the detection screen according to a fringe (or anti-fringe) probability wave if it has no copy of itself from the other slit with which to interfere? This seemingly paradoxical state of affairs forces those who do not abdicate from a particle conception to assume that there must be some sort of temporal quantum retro-causal effect according to which the particle that encounters the quantum eraser in the present sends some information back into the past to itself, before it traversed the slit, 'telling' it to go through both slits instead of only one, in order to recover the interference pattern.

Retro-causality is not forbidden, in principle, according to the current known laws of physics. However, this should, at a minimum, lead us to some eyebrow-raising. And, for those who abhor nondeterministic interpretations of QM without hidden variables, not all hope is lost: It is possible to cerebrate elaborate models such as the 'De Broglie-Bohm pilot wave theory' (also called 'Bohmian mechanics' (BM)), which could, in principle, save the appearances and explain all this without quirky 'backward-in-time influences' that restore interference fringes, and, nevertheless, maintain a deterministic particle-like ontology. We will take a look at this and other interpretations of QM in the dedicated chapter later.

However, the question is: Is it really necessary to resort to retro-causation and/or preserve determinism to explain the observed facts? The answer is simple: It is not at all necessary if we give up the idea of an ontology describing point-like particles traveling definite paths and being localized in space and time.

Such an idea arises due to our unaware misunderstanding of how things work, which we might call the '*separation and measurement fallacy*' ('separation fallacy' being terminology suggested by David Ellerman [4]). In this fallacy, we imagine something individualized and separated into two or more paths and/or measured when it is not. We must simply accept that the two slits do not 'separate' anything; they only create a superposition of states. Otherwise, this would presuppose the collapse of the wavefunction to an eigenstate, whereas, at this stage, there is still none. The same applies to the polarizers: They do not collapse the wavefunction and they are not measurement devices. One measures with a detector that provides a readable eigenvalue, which a polarizer does not do. If we conceive of a wavefunction as describing the system as a whole, not separable into subsystems, describing a physical entity propagating towards the screen in a state of 'real' superposition (that is, 'real' in the sense that Eq. 2 expresses an ontology, not just a state of ignorance), then the paradox dissolves naturally.

Moreover, note the analogy with the experiment of ZWM in the previous chapter 0.1. On that occasion, we also dealt with a physical situation in which a single photon stream at a beamsplitter (the signal photons s_1 or s_2 at beamsplitter BS_2 of *Fig. 1*) nevertheless produced a wavy interference pattern. We wondered how one particle could interfere if it did not have another particle with which to interfere. We showed that if we wanted signal photon s_1 to interfere with a second signal photon s_2 at beamsplitter BS_2, then the SPDC of both nonlinear crystals NL_1 and NL_2 must come into play. However, this is impossible because only one source photon at a time is produced, which cannot lead to four down-converted photons (two signal and two idler photons) due to simple considerations of energy conservation. Retro-causality didn't even enter our minds because it wouldn't explain anything. The simplest and most natural conclusion was to give up the separative space-time conception.

In conclusion to this chapter, it is instructive to see how this is also manifested in the formal description of QM, of which we will take extensive advantage in the coming discussions. Recall how we defined the vertical and horizontal polarization vectors (see the chapter on the quantum superposition principle of Vol. I):

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}}|\nearrow\rangle + \frac{1}{\sqrt{2}}|\searrow\rangle,$$
$$|\uparrow\rangle = \frac{1}{\sqrt{2}}|\nearrow\rangle + \frac{1}{\sqrt{2}}|\nwarrow\rangle\ .$$

Notice how, also, other equivalent geometrical representations are possible. For example, because vectors can be represented equivalently by their reflections, one can equate $|\nwarrow\rangle = -|\searrow\rangle$ and we can rewrite the vertical polarization as:

$$|\uparrow\rangle = \frac{1}{\sqrt{2}}|\nearrow\rangle - \frac{1}{\sqrt{2}}|\searrow\rangle.$$

Adopting this alternative representation, one can highlight the anti-symmetric components. Let us use degrees instead of arrows and define the angles relative to the horizontal x-reference axis. Then we can rewrite the two polarization vectors as:

$$|H\rangle = \frac{1}{\sqrt{2}}|45°\rangle + \frac{1}{\sqrt{2}}|-45°\rangle, \quad \textit{Eq. 3}$$

$$|V\rangle = \frac{1}{\sqrt{2}}|45°\rangle - \frac{1}{\sqrt{2}}|-45°\rangle. \quad \textit{Eq. 4}$$

Expressing the same quantum state of the system in the 45° diagonal basis, that is, inserting these into Eq. 2, it 'splits' into the sum of two terms:

$$|\Psi\rangle = \frac{|45°\rangle_{S_1} + |45°\rangle_{S_2}}{2} + \frac{|-45°\rangle_{S_1} - |-45°\rangle_{S_2}}{2} \quad \textit{Eq. 5}$$

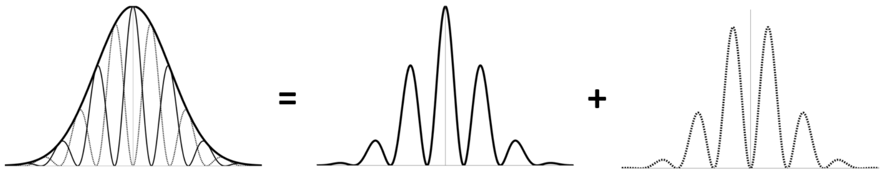


Fig. 11 Graphical representation of Eq. 5.

The first right-hand side term of Eq. *5* represents the symmetric wavefunction while the second one represents the anti-symmetric. We know now the significance of that negative signature of the second term as indicating an anti-symmetric wavefunction (see also Bosons, Fermions, and Pauli's exclusion principle in Vol. I). The $\frac{1}{2}$ coefficients tell us that there is a 25% probability of obtaining one or the other outcome instead of 50%. This is because a diagonal polarizer filtering horizontal or vertical polarized photons will block 50% of them.

What we have done is create a change of the eigenvectors basis from the H/V to the 45°/-45° basis. (There are much more rigorous and formally precise quantum algebraic methods with matrix and group representations by which to do this than what we have tentatively done here, but this should not come as entirely new information; it is, in essence, the same operation we completed in Vol. I in the chapters on spinors or the superposition principle.) One represents the very same quantum state in a different eigenbasis – or, to put it in a more intuitive language, we are 'looking' at the state of the quantum system no longer along the horizontal and vertical directions but along the two diagonal ones. However, the physical state of the system remains unaltered. By doing so, one discovers that the quantum state of a photon emerging from the polarization plates in the setup of Fig. 9, which led to the normal distribution diffraction pattern, is a quantum superposition state of the fringe and anti-fringe interference patterns. What the insertion of the 45° or -45° diagonal quantum erasing polarizers do in Fig. 10 is 'select' and 'filter out' from the bell-shaped distribution the anti-fringe or fringe, respectively. They do not 'collapse' anything. This, obviously, recovers the interference pattern characteristic of a wave.

So, finally, we can summarize the last two chapters as follows. The separation and measurement fallacy rests on the (more or less unaware) assumption that state reduction occurs earlier than the measurement of a device and that the detector reveals only what was already present. Instead, what all this must tell us is that the photons that make it through in Fig. 9 and that are 'marked' with the H/V polarizers, are set into a $\pm 45°$ superposition state only along the diagonal polarization orientations where the single photon is still, so to speak, a 'fringe-photon' and 'anti-fringe photon' at the same time. The polarizers modify the system's quantum state function while it is evolving towards a measurement device, but they do nothing that can be compared to a measurement on the incident beam. They don't even provide any information, as this is something that arises only at the time of the act of measurement – that is, when state reduction occurs. Prior to that detection, nothing exists in one or the other eigenstate. Projection or collapse happens only at the very end of the chain, when the detector 'clicks'. The $\pm 45°$ superposition evolves until it hits a detector and some distinction is made – that is, it selects the fringe or anti-fringe state and then the single photon hits correspondingly the fringe or anti-fringe. This is a distinction we call 'measurement' or 'detection'. A measurement can be defined as an 'act of distinction'. In a certain sense, we might even say that quantum erasers do not 'erase' anything. They only 'change', 'filter', or 'select'. The so-called 'which-way' information cannot be 'erased' because no particles were traveling along one or another path in the first place. And even less do they lead to any retro-causal actions into the past. After all, speaking of 'which-way' experiments and information 'erasing' devices is bad terminology.

3. The complementary principle and the Scully-Englert-Walther quantum eraser

It is time to look in more detail at the so-called '*complementarity principle*' of quantum mechanics. Let us also use this as a chance to recollect some of the facts we have learned so far, in this volume as well as in the former one.

Complementarity refers to the fact that QM is contextual. As discussed previously (and also highlighted with the MZI experiments in Vol. I), the result of an experiment that tests specific properties of a particle, or a quantum system, depends on the context – that is, the arrangement of the experimental setup. Therefore, in QM, only the whole set of possible arrangements and observations will form a complete description of the quantum object under measurement. Each aspect is not exclusive but is complementary to the others. Bohr called it the '*principle of complementarity*'.

The typical example we know well is that, according to a specific arrangement, you will reveal the wave nature of a quantum system, whereas in a different experimental context, it may behave as a particle, but you cannot see both at the same time.

The wave-particle duality is not the only example. Think of the spin property of particles and the SG experiment. We saw that the spin along one axis does not commute with that along another spin axis, which means you cannot measure, at the same time, the spin along the x-axis and the spin along the y- or z-axes. Only one spin component can be measured, leaving the others completely undetermined. Once again, it is the experimental context that determines, by measurement, in which eigenstate the system will be projected, leaving the other observable in state superposition. In this sense, the spin components of a particle are 'complementary' to each other, just as the wave-particle aspect can't appear at once – and nor can the two spins be definite at the same time.

Heisenberg's uncertainty principle is another important example. In the same way, you can't determine the momentum and position of a particle at the same time, in what is expressed formally by the non-commutation relation of the space and momentum operator observables. In this sense, precise measurements of the position and momentum aren't possible because these are 'complementary' properties of a particle.

The question, however, is: Can complementarity be explained away by the uncertainty principle itself? Here, we are again confronted by a problem that is similar to – if not the same as – the problem we already analyzed. We saw that the appearances of the wave or particle nature of photons cannot be ascribed to the interaction or perturbation of the measurement system with the measured object. And we could ask ourselves again whether the fact that we cannot determine, at the same time, the intrinsic angular momentum of a particle might be due to a physical interaction and perturbation caused by the measurement. Could it be the case that the attempt to measure the spin of a particle causes, somewhere and somehow, a small perturbation that flips its spin value along another axis? If so, the entire principle of complementarity would rest on Heisenberg's uncertainty principle; that is, the uncertainty principle would be more fundamental than the complementarity principle, as the latter would be simply a consequence of the former. This was, and still is, a belief also held (more or less implicitly and subconsciously) by many physicists who are not trained in the foundations of QP.

However, with the SG experiments discussed in Vol. I, we showed that this is now a difficult – if not impossible – conjecture to defend, and that it does not stand up to the proof of facts. We saw that, with the application of the MSG apparatus, which is essentially a quantum eraser system, it is possible to 'restore' the spin state along an axis if we build the experimental measurement set-up, that is, if we frame a particular experimental context that does not allow for a which-way information retrieval or, more precisely, the evolving state function is maintained in superposition, avoiding the existence of separate particle paths in the first place. We concluded that this implies that we cannot think of the spin commutation relations being a consequence of interaction. There is no interaction or perturbation of the H/V filters on the photons emerging from the two slits that could explain it. We reached the same conclusion with the interaction-free which-way and Wheeler's delayed choice experiments: The way in which Nature manifests the properties of a quantum system depends on how we ask the question, not on the fact that we weren't gentle enough and supposedly perturbed the system by interacting with it. We found this again in its photonic version in the previous section. The addition of the $\pm 45°$ polarizers in Fig. 10 restored the interference patterns.

However, most of the experiments realized in practice were conducted with photons or particles which are believed to be elementary, like electrons. One might legitimately suspect that all these strange quantum paradoxes arise due to the fact that photons – that is, objects we imagine to be sort of evanescent light waves, or light particles with zero mass – might possibly have some ghostly property whereby they can act non-locally and have sufficient 'plasticity' to transform themselves, displaying first a particle-behaviour and then a wave-behaviour, and apparently even looking into the future to see what the experimenter's choice will be. However, we can hardly imagine that to be a property of a composed material object like an atom or an even bigger material object, can we?

At this point, then, we might wonder whether systems composed of material particles like atoms would display different behavior. Well, you should already know the answer. This is because, according to the de Broglie relation $\lambda = \frac{h}{p}$, there is a correspondence between the wavelength λ and the momentum p of anything with mass, like atoms, or even a large molecule; there is no physical or logical restriction to consider the momentum p of a composite and large material object. We know that electrons – particles with mass – could be diffracted by a lattice, giving rise to the Bragg-refraction and the corresponding interference pattern. We also mentioned how this has been done with macromolecules as large as 2000 atoms. [8] We should therefore not expect that so-called 'material' objects will be likely to act differently than photons. However, on the other hand, can we imagine a chunk of matter as tiny as an atom to be diffracted, going through two or many slits at the same time, and apparently changing from a particle to wave-like behavior shortly before hitting a detector or a screen, according to our delayed choice? As if this wasn't difficult enough to grasp with photons, it all enters even more conflict with our intuitive notion of what matter is.

So, to be sure that we have it right, we now analyze an experiment performed in 1991 by Scully, Englert, and Walther, subsequently known as the SEW experiment. This experiment is a synthesis of several experiments we have seen so far, but it goes beyond them; it combines, in a fascinating manner, the wave-particle duality, the delayed choice experiment, and quantum erasing for atoms. It uses atoms to show that they are also subject to particle-wave duality and interference as well, and that the interference pattern can also disappear without disturbing the atom's path in any way. Furthermore, this will show that quantum erasure and delayed choices have the same effect on the system as predicted by QM for photons.

SEW built atom interferometers with detectors that were constructed with the aid of quantum optics devices, which emerged from new technological advances of that time. They published their experiment in the renowned journal Nature under the title "Quantum optical test of complementarity". [9]

Fig. 12 shows a plane wave of incoming atoms from the left. They are collimated to produce a couple of well-defined beams of caesium atoms. These can be excited to a higher energy level by a laser beam, as seen in the figure. The laser's photons carry energy that is absorbed in energy packets, which then allow the caesium atoms to acquire a higher energetic configuration.

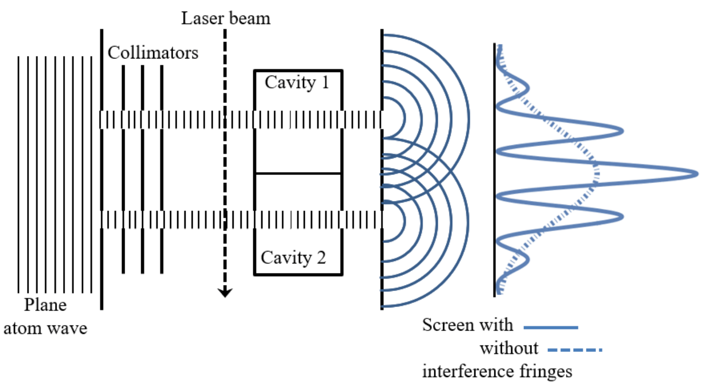


Fig. 12 Maser cavities for a which-way experiment with atoms.

The two streams of excited atoms are then sent into what are called '*micromaser cavities*'—that is, a couple of microwave cavities where EM radiation of a specific wavelength can be stored. The interesting point here is that the two micromaser cavities are capable of storing single photons. If the

cavities are properly designed, they resonate for specific frequencies (that is, wavelengths or energies), they can store the photon emitted by a specific atom transition, and, if they are long enough, can even ensure that when an atom enters in an excited state it will emit its photon and leave the cavity in the ground state, with certainty. Therefore, the which-way information can be obtained through the act of reading out cavity 1 and cavity 2, to see which of the them contains the photon. Finally, immediately after leaving the two micromaser cavities, the atoms encounter the usual double slit and the detection screen.
Note that, contrary to what Feynman claimed, this is yet another example of how one can circumvent difficulties related to the uncertainty principle. While it is true that we impart a little kick to the atoms by illuminating them with the laser, this is really negligible if they absorb photons of low energy. And, anyway, the point is that the which-way information is not obtained by the scattering between the laser photons and the atoms but, rather, by controlling which cavity has stored the photon.

So, what will be observed when the laser is turned off? As in the case of the single photon counts in the double-slit experiment, here it is also possible to tune the atom flux in such a way that we send only one single atom at a time. When there is no laser beam, the atom is not excited, the micromaser cavities can't store the photon because the atom is in the ground state, and it can't release any photons when it traverses the cavities. Therefore, the cavities play the role of simply a further collimating device and we have no information to read out about the which-way the atom took. This implies that interference fringes will appear (the continuous line in the graph in Fig. 12) if we count a sufficient number of atoms, one after the other, hitting the screen. Effectively, this means that we are simply using a standard double slit device, but using atoms instead of photons.

What will happen when we turn on the laser? Then the interference fringes will be destroyed. This is because if an atom's energy level is lifted up by a laser photon, it will release the photon with certainty in one of the two maser cavities, allowing us to determine which way it travelled along. Thus, the characteristic interference fringes will be replaced by the bell-shaped intensity curve (the discontinuous line in the graph of Fig. 12). This happens with almost no interaction with the atom beam, but only because of the contextual nature of QM. Recognize the analogy with the experiment of Fig. 9. There, the 'slit-superposition' of two orthogonally polarized single photon states led to the lack of interference fringes. Here it is the 'cavity-superposition' of the single atom symmetric and anti-symmetric wavefunction states (more on that next).

In a second experimental configuration (see Fig. 13), the two resonant cavities are no longer physically divided from each other, but are separated only by a pair of electro-optical shutters placed in front of a detector wall.

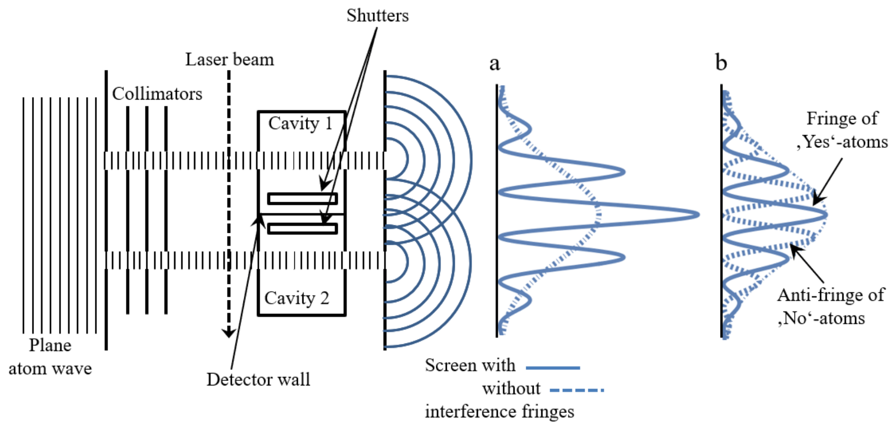


Fig. 13 Experimental setup of the SEW experiment.

These can be closed or open—that is, let the photon through, or not, towards the common internal wall of ***both*** micromaser cavities. The latter is covered by a thin-film semiconductor which absorbs microwave photons and acts as a photodetector when the shutters are open. Let us consider what happens for the passage of a single atom in the two cases with the closed or the opened shutters.
We can send one atom through the system of cavities and the slits. Observe the individual spot appearing on the screen. Then, ***after*** the atom hits the screen, interrogate the micromaser cavities. That is, only after we see the spot on the screen will we make the delayed choice of whether we want to keep the

shutters closed or open. This procedure is then repeated several times until an intelligible pattern can be recognized on the screen.
Note how we have built a delayed-choice device which furnishes the atom's which-way information but does not at all disturb the atoms themselves. We can let the atom hit the screen and then, only after that, control the state of the cavities that allow us to determine its path. Therefore, there could be no sort of physical interaction at all that could have destroyed the interference fringes. This is, again, another demonstration of Heisenberg's microscope fallacy.

This situation is analogous, but not entirely, to Feynman's microscope we discussed in the which-way experiments of Vol. I. There an electron passing through the slits was entangled with a 'test-photon' shining on one slit in order to retrieve its whereabouts. Here a caesium atom is entangled with the photon in both cavities. Labeling the atom's path-state through cavity 1 and cavity 2 as $|\Psi_1\rangle_a$ and $|\Psi_2\rangle_a$ and the photon being stored in cavity 1 and 2 as $|1{,}0\rangle_\gamma$ and $|0{,}1\rangle_\gamma$, then the entangled state between the atom and the cavities is:

$$|\Psi\rangle = \frac{|\Psi_1\rangle_a\,|1{,}0\rangle_\gamma \;+\; |\Psi_2\rangle_a\,|0{,}1\rangle_\gamma}{\sqrt{2}}.$$

If we chose to keep the shutters closed, the photon emitted by the atom in whatever cavity is not absorbed by the detector wall, and we can read out whether the atom left its photon in cavity 1 or cavity 2. Because the two cavities are placed in front of each slit, the apparatus with closed shutters is capable of telling us where an atom has gone through, by controlling in which cavity the photon has been stored. Therefore, we can extract information about the atom's which-way, which implies that no interference fringes can appear on the screen. (One again obtains the dashed line in graph (a) of Fig. 13).
If, instead, we chose to open the shutters and let the photon that the atom emitted during its passage in one of the cavities be absorbed by the detector walls, or simply be 'removed', then the 'memory of passage' (the which-way information) could be said to be 'erased'. That is, we have built a quantum eraser.
Consider that in this case (for quantum mechanical reasons too long to be discussed here), for an ideal photon detector having 100% efficiency, the probability that the detector wall will absorb the photon in both cavities is only 50%. (In the remaining cases, the photon remains unchanged and bounces back and forth in the cavity.) Let us label the atoms for the case of open shutters where the photodetector in the micromaser cavity clicked as a 'yes-atom', while those atoms where no photocount is observed in the cavities are called the 'no-atoms' and, respectively, the 'yes-' or 'no-eraser' photons.

If the choice of opening the shutters is made ***after*** the atoms hit the screen, the device acts like a delayed quantum erasure, because the atom leaves its photon in the cavity but without anyone reading out which of the two cavities, and then travels towards the screen and forms a spot. Only then do we decide whether, as in the previous case, we are going to keep the shutters closed and read out where the photon is, or open the shutters, whereby the photon is absorbed by the detector wall, erasing the information about its whereabouts. However, there is a 50% chance that the detector wall in the cavity will respond to the presence of the photon (the 'yes' eraser photon) and a 50% chance that it will not (the 'no' eraser photon). In both cases, we lose the which-way information because the no-photon is absorbed by the cavity walls after bouncing back and forth, and we will not be able to read out in which cavity the atom left it.

It turns out that if the which-way information is erased ***after*** the atoms hit the screen the interference fringes don't appear. This is because the atoms already hit the screen before the choice was made to erase the which-way information. To observe interference fringes, we must perform the quantum erasure before the atoms hit the screen. Otherwise, it is too late. It is like waiting to insert the $\pm 45°$ polarizers in Fig. 10 until after the photon hits the screen and still hope for the interference fringes to reappear. It is hard to believe that this could be the case, as that would imply a retro-causal action into the past. And, in fact, it isn't. Otherwise, it would appear that the particles or atoms detected in the present (future) time must inform their own ‘selves’ in the past (present) about whether they should take a path that forms a bell-shaped or fringy pattern. Another analogy that comes to mind is the MZI version of Wheeler's delayed choice experiment. (See the delayed choice experiment in Vol. I.) We showed that a delayed choice is possible while the photons are 'in flight' before the detection, between the first and second beam splitters in the MZI. If the quantum erasure process was performed during that short time period, you recover the interference fringes. However, if it was performed after it went through the second beam splitter, then it is too late; we will accordingly obtain the lump of particles on the screen.

However, this is only part of the story. As usual, Nature is tremendously subtle and is able to mix things up when the human mind can see only mutually exclusive, logical options. It turns out that one can recover the interference fringes from the collected data. After a suitable amount of time, during which you have collected a sufficient number of events (that is, several atoms hitting the screen), the spots on the screen will build up the classical Gaussian bell-shaped probability function. However, when you look only at the spots left behind by the 'yes-atoms', those where the detector clicked, or, alternatively, the spots correlating with the 'no-atoms', when the detector did not click (one can separate the two sets of spots on the screen, as one knows when the photodetector clicked for the yes-atoms and did not click for the no-atoms), then the good-old interference fringes are recovered. The continuous and the dashed interference fringes in the graph (b) of Fig. 13 show the two cases. The fringes are equal but shifted interference patterns comprising the fringes for the 'yes-atom' and the anti-fringes for the 'no-atoms'. When we sum them up, the bell-shaped curve appears and the interference pattern disappears—or, more precisely, remains hidden.

So, is there a contradiction in the temporal order of events? Fringes and anti-fringes are there, aren't they? The delayed quantum erasure process was performed after the atoms hit the screen. We can somehow recover the interference fringes by correlating the 'yes-' and 'no-atoms' with the spots on the screen. This seems to suggest that the atoms chose to displace themselves on the screen according to a choice that still had to be made at the time they hit the screen! How can an interference pattern appear if the physical process that is supposed to determine it (our choice of opening the shutters erasing the which-way information) is performed after it came into existence?! How could the 'yes-atoms' and 'no-atoms' know that we would have erased the which-way information and distribute themselves on the screen according to an interference pattern when this choice had yet to be made at that time? We are apparently again confronted with quantum retro-causal effects: The choices we make in the present determine the atoms' past behavior. Or, if you prefer, present physical events are influenced by the future. The behavior of a quantum system today seems to depend on events that will materialize tomorrow.

The author would love for that to be true, as this would, in fact, tend to confirm a symmetry in which past, present, and future are one. This is a vision of things with which he is sympathetic. However, a healthy skepticism combined with factual objectivity—and not personal or ideological preferences—should lead our thoughts.

Everything becomes intelligible when we realize that the 'yes-' or 'no-atom' correlation is decided at the time when the atom hits the screen, ***not*** when we open the shutters. What we do is no 'delayed choice' at all; rather, it is just a control, a readout of the whereabouts of a photon that had already come into existence before the choice. The point is that the state vector represented by a superposition of states (the atom taking both ways with the photon being stored in both cavities at once) undergoes a state reduction, that is 'collapses', only at the instant when the atom hits the screen. It is only at this point that the state vector is projected onto one of the two possible eigenstates (the atom taking only one or the other path and the photon stored in one or the other cavity, but not both). Then the 'game is over'.

In fact, when the atom hits the screen, it sets the wavefunction of the photon inside the cavity to a symmetric or anti-symmetric state by a non-local correlation. Then, this radiation remains inside one or the other cavity (but not both) and couples to the photon-counter detector wall of the micromaser cavity only later, when one opens the shutters (but not before; we do that after the atom hits the screen). It is the former or the latter wavefunction, the symmetric or anti-symmetric wavefunction, which determines whether or not the detector will fire. Therefore, before our decision regarding whether to open the shutters, the photon's destiny is determined at the instant of the atom's detection on the screen, not by our choice. So, there is no problem with retro-causations here.

As a side note, you might have noticed a similarity between the two interference fringes of the Aharonov-Bohm effect (see Vol. I), those in which the magnetic field in the solenoid behind the slit screen is turned on or off, the neutron interferometer experiment and the interference fringes of the yes- and no-atoms in this SEW experiment. In the former case, the shift of the peaks of the fringes was due to a phase difference that the magnetic potential vector imparted on the wavefunctions corresponding to the two states with the magnetic field on or off. Here, the same happens but with the phase difference due to the 'yes' or 'no' states.

The bottom line of this experiment is the following. Composite material systems like atoms (or, eventually, even molecules, as shown in other experiments not discussed here), and not merely photons or elementary particles, are also subject to the wave-particle duality. The SEW experiment shows that these quantum phenomena are not due to the interaction or perturbation with the atoms; it is an intrinsic property, a law of nature, that the wave or particle character realizes in accordance with the specifics of the situation. This also leads to the conclusion that complementarity must be a universal and very fundamental feature for all particles and systems of the quantum world. Heisenberg's uncertainty principle is only one aspect of it, and is not the cause or source of complementarity. Therefore, complementarity is more fundamental than the uncertainty principle. Finally, no mystical retro-causation is needed to explain the facts. We must always keep in mind the difference between the evolution of the wavefunction and its collapse. Once the latter has occurred, the state reduction reflects itself non-locally throughout the system.

4. The delayed quantum erasure experiment of Walborn et al.

About then years later, in 2001, yet another overall combination of a double-slit delayed quantum eraser experiment was performed by a Brazilian group (Walborn et al. [10]). It might be instructive to dwell further on these experiments to overcome doubts about any supposed retro-causality in QP. It can also be considered the photonic replica and continuation of the SEW experiment.

Fig. 14 illustrates how photons from an argon laser (at 351 nm wavelength) are focused by a lens and sent through a BBO crystal to create, via a SPDC, a couple of entangled photons.

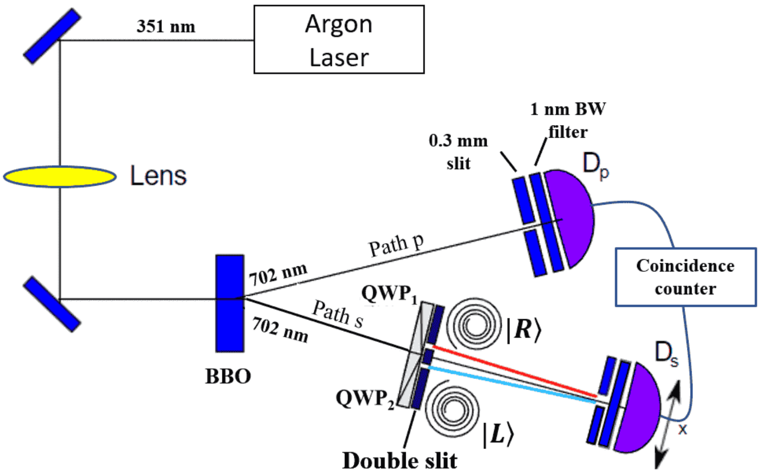


Fig. 14 The which-way experiment with entangled photons: version I.

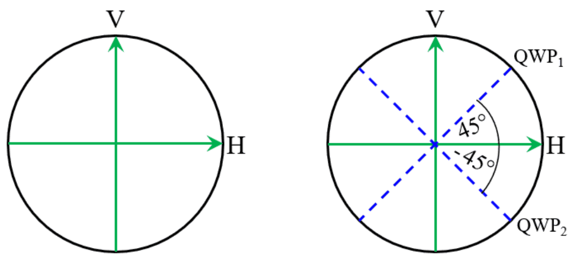


Fig. 15 Polarization diagrams for experiment version I.
Left: for path p. Right: for path s.

Because the total energy must be conserved, the wavelength of the two down-converted photons must be twice as much as the original one (or equivalently, they have half the frequency of the incoming photon, of course, due to Planck's relation between the energy and frequency). Therefore, the two photons have a wavelength of 702 nm (deep red color). They are orthogonally polarized—that is, they are type-II entangled photons (in contrast to the type-I having the same polarization, see the discussion in Vol. I on SPDC entangled photons). This means that, as long as we do not measure it, we must conceive of it as being in both horizontal (H) and vertical (V) polarizations at the same time, as shown by the solid arrows in the polarization diagram of Fig. 15. (More generally, an orthogonal polarization considers any 90° basis, not only horizontal-vertical orientations. However, to keep things simple, we restrict ourselves to the 45° and -45° basis.) The photon travelling the upper path (let us label it path ‘p’) is sent towards a small slit of 0.3 mm, behind which a 1 nm band width (BW) filter is placed (to ensure that no other photons from the environment will be counted). It is then detected by a photodetector D_p.

In a brief interlude, without going too much into the technical details, let us also mention how circular polarized light can be obtained from unpolarized light. This can be done through us of a ‘*quarter-wave plate*’ (QWP), as shown in *Fig. 16*. From the right to the left: Unpolarized light is first rendered linearly polarized with a linear polarizer and then filtered again with a QWP, which converts it into circularly polarized light. A QWP is a retardation sheet such that horizontally and vertically polarized light entering in phase will emerge from the retardation plate at 1/4 of a wavelength ($\frac{1}{4}\lambda \sim 90°$), out of phase. If linearly polarized light enters at an angle of $+45°$ between the fast and slow axis, then the x and y components of the electric field will be phase shifted with one component lagging behind, resulting in a rotating electric field vector, that is, a left-handed circular polarization. Whereas, a $-45°$ linearly polarized light (not shown in the figure) results in right-handed circularly polarized light.

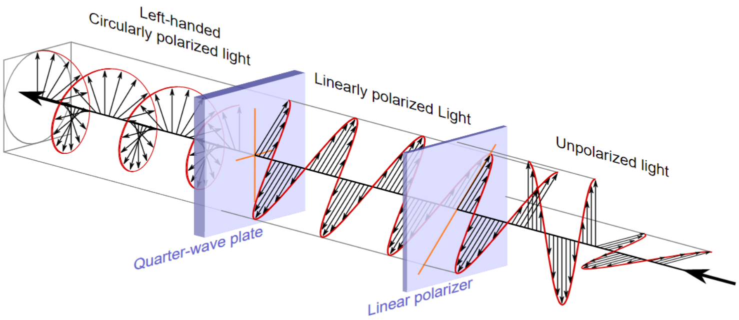


Fig. 16 Circular polarization of light.

From the quantum mechanical perspective, this amounts to saying that the photons emerging from this polarization device possess a specific spin, that is, a positive or negative helicity. (See also the chapter on the photon’s polarization and spin in Vol. I.) QWPs are not polarizers which select photons with a specific polarization; rather, they are devices that change the polarization of all the incoming photons, ideally without absorption.

Now apply QWPs in this context: The entangled photon following the lower path, labeled ‘s’, which is also in the superposition of the horizontal and vertical polarizations, before going through a double-slit device, will encounter two circular quarter-wave plates, QWP_1 and QWP_2, placed in front of each slit— say, QWP_1 in front of slit S_1 and QWP_2 in front of slit S_2. (By analogy, compare this to the experiment of Fig. 9.) What differentiates the two QWPs is their orientation (relative to H and V of their fast and slow axes), which is shown by the dashed lines in the polarization diagram of Fig. 15 right. A +45° or -45° difference between the photons’ polarization and the QWPs orientation always leads to an opposite circular polarization. Therefore, QWP_1 in front of slit 1, will impart to the photon a left-handed (clockwise) circular polarization $|L\rangle$ or, equivalently, a negative helicity —that is, the photon has $S_\gamma = -\hbar$ spin. Meanwhile, QWP_2 in front of slit 2 will impart it a right-handed (counter-clockwise) circular polarization $|R\rangle$ or, equivalently, a positive helicity—that is, the photon has $S_\gamma = +\hbar$ spin.
Photons will then also traverse a small slit behind which another 1 nm filter is placed and will then be detected by photodetector D_s. A stepping motor moves detector D_s, scanning along the x-direction in order to read out the intensity curve with a photocounter. Overall, only those photons are counted when a coincidence counter confirms the detection of both photons s and p.

With this experimental setup, we have built a which-way tester. In fact, the role of the two QWPs inducing circular polarization in front of the two slits should not have gone unnoticed: The QWPs ‘mark’ the photon. No matter what happens to the photon on path p, this allows us to determine the which-way. It shouldn’t come as a surprise that no interference fringes will be measured at D_s and that one observes only the usual bell-shaped intensity curve as shown in *Fig. 17*. (The vertical bars represent the measurement error).

From the wave perspective one must consider how the clockwise and counter-clockwise polarization vectors rotate. Their angular difference $\delta\theta$ varies very rapidly (with the frequency of a 702 nm light wave!) between 0° and 180°. That is, the interference patterns of Fig. 7 change and mix together extremely fast so that the net resulting interference pattern is the sum of all these, leading to the Gaussian probability function.

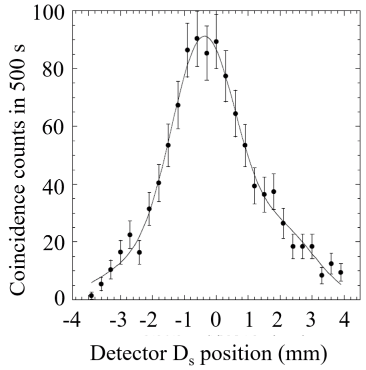


Fig. 17 Coincidence counts at D_s with QWPs on path s but without $POL_{1/2}$ on path p.[8]

In a second version of this which-way experiment (see Fig. 18), one inserts in the upper path, that of photon p, a linear polarizer $POL_{1/2}$ in front of detector D_p. If it is oriented along the 45° direction we label it POL_1, while for the −45° tilt it is labeled POL_2. Keep in mind that, again, a polarizer selects photons only. There is a 50% probability that POL_1 (POL_2) will allow the photon in a vertical and horizontal polarization superposition to slip through as a 45° (−45°) photon, while the others are blocked. That is, only half of the photons on path p will be detected by detector D_p, it will see only those photons. Meanwhile, the entangled photon on paths s will always hit detector D_s, which will observe, instead, a stream of photons with both possible polarizations. However, this is of no concern because what we are looking for are the coincidental counts where both photons are detected and correlated to each other. We are not looking for the interference pattern they produce individually at each detector.

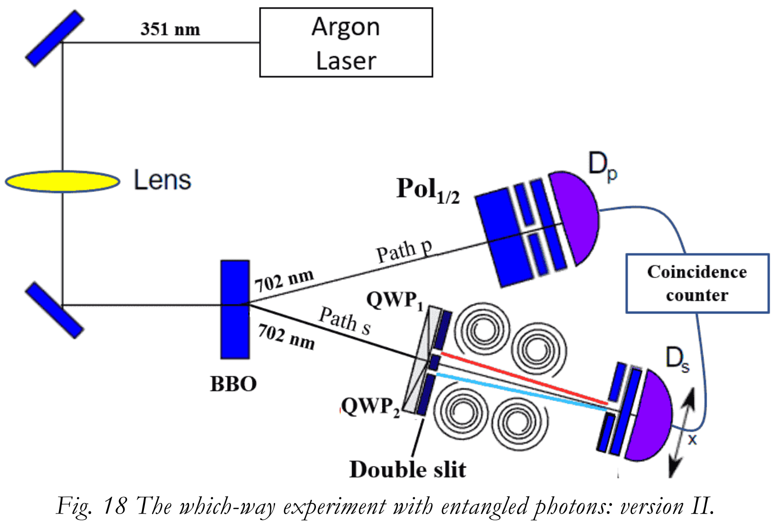


Fig. 18 The which-way experiment with entangled photons: version II.

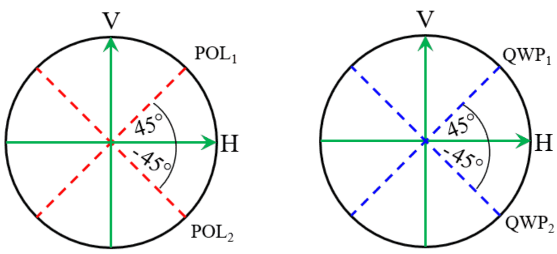


Fig. 19 Polarization diagrams for experiment version II.
Left: for path p. Right: for path s.

Another important aspect to notice is that $POL_{1/2}$ is placed, from the BBO, at a distance about half as long as the distance of detector D_s, but farther than the slits. This implies that the first detection will always take place at D_p when the photon on path s has already traversed the slits but is still on its way to detector D_s.

Let us analyze this second case in more detail. Recall how the measurement of entangled photons works. For type-II entangled photons, if on path p a linear polarizer selects a photon along the H (V) polarization direction, the photon on path s must anti-correlate and be in polarization state V (H). When the evolving state function is before the two slits, it must be represented by the entangled state, which we know to be:

$$|\Psi\rangle = \frac{|H\rangle_s|V\rangle_p + |V\rangle_s|H\rangle_p}{\sqrt{2}}, \quad Eq.\ 6$$

with the obvious labeling 'p' and 's' being the photon on path p and s respectively.

Once the photon on path s has traversed the two slits, it is set into 'slit-superposition' S_1 and S_2 and its linear polarization is transformed into the circular polarization. Remember that this is not a measurement and therefore there is no state collapse. It is the state function that has been changed by the slits and circular polarizers without any interaction or information readout.

Each photon on path s emerging from slits S_1 ***and*** S_2, having either vertical or horizontal polarization, is also entangled with photon p. (One might say that there are three beams but only two particles!). There is polarization and slit superposition intertwined with entanglement. Therefore, the overall system can be described with the state vector $|\Psi\rangle$ as:

$$|\Psi\rangle = \frac{|\Psi\rangle_{S_1} + |\Psi\rangle_{S_2}}{\sqrt{2}}, \quad Eq.\ 7$$

where

$$|\Psi\rangle_{s_1} = \frac{|H\rangle_{s_1}|V\rangle_p + |V\rangle_{s_1}|H\rangle_p}{\sqrt{2}} \quad \text{and} \quad |\Psi\rangle_{s_2} = \frac{|H\rangle_{s_2}|V\rangle_p + |V\rangle_{s_2}|H\rangle_p}{\sqrt{2}}.$$

Expressing it in words, it means that the state of the system is in a superposition of four potentialities: Photon s goes through slit 1 with horizontal polarization, with photon p having vertical polarization AND also with a vertical polarization with photon p having horizontal polarization AND the same photon s also goes through slit 2 with horizontal polarization, with photon p having vertical polarization AND also with vertical polarization, with photon p having horizontal polarization. Insertion of the two QWPs in front of the two slits, as in the second experimental version, would replace the horizontal and vertical polarizations with the left- and right-hand circular polarizations (with the addition of some subtleties that we do not discuss further here). Maybe this clarifies why mathematical formalism is so much more useful than employing annoying, lengthy, and clumsy sentences!

At this point, we can proceed on the same line of discussion that led us to Eq. *5*. There, a photon before entering the slits is in a diagonal polarization state and is then filtered by a horizontal and vertical polarizer after the two slits (see Fig. 10). The outgoing evolving quantum state could be expressed in a diagonal polarization base as a superposition of a two-slits-state with a symmetric and anti-symmetric wavefunction. In this experiment things are pushed further than that, as to the two-slits-superposition of the photon on path s one must also add the entanglement with the photon on path p. Here also, the orthogonal states $|H\rangle$ and $|V\rangle$ on paths s or p and the rotating polarization vectors $|R\rangle$ and $|L\rangle$ after slits S_1 or S_2 can be expressed in a $\pm 45°$ basis as a superposition of a symmetric and anti-symmetric wavefunction. The principle is similar to that which furnished Eq. *5* (though algebraically more involved and we won't develop it here, the interested reader is referred to the original article [25]) and leads to the following sate function:

$$|\Psi\rangle = \frac{|45°\rangle_{S_1} - i|45°\rangle_{S_2}}{2}|45°\rangle_p + i\frac{|-45°\rangle_{S_1} + i|-45°\rangle_{S_2}}{2}|-45°\rangle_p. \quad Eq.\ 8$$

The analogy with Eq. *5* is manifest except for the imaginary numbers which account for a phase difference of 90° induced by the QWPs fast and slow axis on the two slit paths and by exchange of the fringes with anti-fringes. It contains the two slits-superposition states on S_1 and S_2 of the photon on path s times the entanglement with its twin photon on path p. The two terms express an anti-symmetric and symmetric state, respectively. On the line of Eq. 7, these represent not just the state of one or the other photon, or the superposition of one photon, or the entanglement of two photons, but all that put together as an overall quantum state describing a unique and inseparable whole as an entanglement between paths p and s plus a slit/polarization-superposition on path s. Any inference about 'two individualized photons on two paths, one of which is going through one or the other slit' exists only in our fantasy and has nothing to do with reality—at least, as long as state projection does not occur.

Note that the very same state function is equivalent to place polarizer $POL_{1/2}$ after the QWPs instead on path p. In doing so, one would insert a quantum eraser, as a $\pm 45°$ filter would pick out all the photons along that angular direction and render the two circular polarizations indistinguishable—that is, it erases the marking circular polarization which previously allowed for which-way information and the reappearance of the interference fringes. Filtering out the wavefunction after the slits and the QWPs with a 45° polarizer selects the left-hand side anti-symmetric state function of *Eq. 8* and displays an interference anti-fringe pattern. Doing so with a -45° polarizer would select the right-hand side symmetric part of *Eq. 8*, and the fringes would appear along the x-direction of detector D_s.

However, no diagonal polarizer is inserted along path s but, rather, on path p! It is by orienting the polarizer along path p, as POL_1 or POL_2, that one can select the symmetric or anti-symmetric part of the wavefunction emerging from the slits. We might say that entanglement allows not only for 'spooky actions at a distance' but also for a sort of non-local instantaneous 'insertion at a distance' of a polarizer along a path without its physical presence there.

In fact, if the polarizer on path p is set into a 45° orientation and the photon in the $|H\rangle_s$ and $|V\rangle_s$ superposition will get through (50% chance), then, once it reaches detector D_p, state collapse occurs and by a 'click' we know that it must have been a $|45°\rangle_p$ state photon. And because, before the collapse occurring at D_p, it was entangled with the photon on path s, which has already traversed the silts, the photon will be set on path s into the anti-symmetric quantum state and the 45° polarization state. This implies that the quantum erasure is performed when the photon is in flight between the slits and D_s. Once the photon on path s hits detector D_s it will displace itself on one of the anti-fringes. Because only one photon at a time travels through the entire experimental setup, if the photon on path p made it through POL_1 with a 50% chance, then the coincidence counter will register how both detectors clicked (even though not at the same time) and correlate the photon on path p with polarization $|45°\rangle_p$ with the spot on the anti-fringe pattern along the scan of detector D_s. Similarly, if the polarizer on path p is set into a -45° orientation and the entangled photon will get through (a 50% chance as well), detector D_p will also click but we know that it must have been a $|-45°\rangle_p$ state photon. And, again, once it collapses at D_p, it will set photon on path s into the symmetric -45° polarization quantum state which will displace itself on one of the fringes of detector D_s. The coincidence counter will correlate the photon on path p with polarization $|-45°\rangle_p$ with a spot on one of the fringes. These two cases are summarized in the coincidence count of *Fig. 20*.

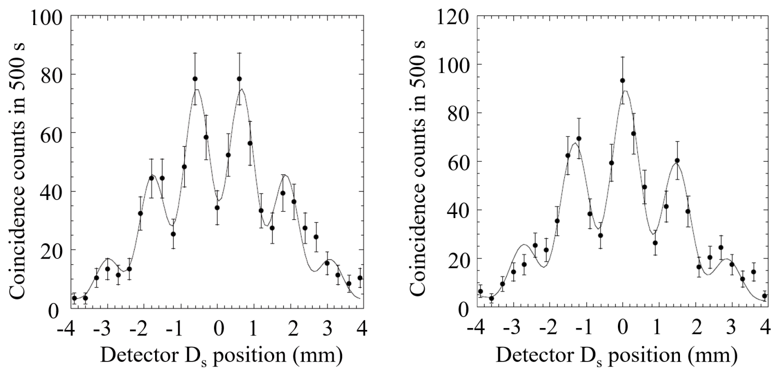


Fig. 20 Coincidence counts at D_s with POL1 or POL2 on path p.[8]

So, this is a three-step process: First, the $POL_{1/2}$ selects one of the two sides of *Eq. 8* but does not collapse or 'lift' either the entanglement between the two photons or the 'slit-superposition state' of the photon on path s. However, at this stage, it has already been decided whether the photon on path s is in a symmetric or anti-symmetric quantum state. Secondly, the first state reduction will occur only once the photon on path p reaches detector D_p. Once this has occurred, the photon on path s is no longer entangled but is still evolving in a (symmetric or anti-symmetric) slit- superposition state. It was the polarization and subsequent measurement of the photon on path p that 'steered' the quantum state of the photon on path s. One speaks of *'quantum steering'* (concept introduced by Schrödinger) when for two entangled systems, the quantum state of one system can be prepared by a measurement on the other. Third, the photon on path s also reaches detector D_s. The second state reduction occurs, and the game is over.

Then, the overlying of the two fringe and anti-fringe interference patterns will result in the Gaussian bell-shaped curve. This is in analogous to what we have elucidated at length with respect to the experiment of Fig. 10, in which the two orientations of the polarizer led to a fringe and anti-fringe pattern respectively and their sum to Fig. 11. In fact, if we would look directly at the pattern emerging on D_s without separating the fringe from anti-fringe photons by means of the coincidence counter correlation, we would observe only the Gaussian bell-shaped curve without a sign of any interference. It is only by noting the 'coincidence click'—that is, by correlating the $|45°\rangle_p$ or $|-45°\rangle_p$ photons on path p with the fringe or anti-fringe photons on path s respectively—that we can filter out the fringe from the anti-fringe pattern in a figure that otherwise would result in a normal distribution. And how could it be otherwise? Locally, there is no polarizer $POL_{1/2}$ in place on path s and from the point of view of the detector D_s, the photons going through the two slits and the two marking circular polarizers are still distinguishable. It is only with the entanglement between the two photons and the quantum steering action at a distance of $POL_{1/2}$ that one can act 'as if' the same polarizer is placed on path s and transform the evolving state function such that the photons traveling towards D_s will 'lose their marking' and behave like waves. But, at D_s, we have no knowledge of whether the entangled photon on path p collapsed into a $|45°\rangle_p$ or $|-45°\rangle_p$ state. It is only the correlation (which must be communicated via a classical communication channel) that reveals which was in which state.

So, the bottom line is that the quantum erasure induced by a distant polarizer via entanglement with a non-local action on a which-way experiment indeed allows for the reappearance of the interference fringes, as well as when photons on path s have already passed the silts and the marking of circular polarizers. But this 'recovery' comes at a price: These fringes and anti-fringes are subsumed and hidden in the bell-shaped curve and can be filtered out only by a count that correlates the photons' state on the two paths. This is perfectly analogous with the SEW experiment in which the (anti-)symmetric (no-)yes-atom wavefunction led to the (anti-)fringe patterns.

The natural question that arises at this point is: What would happen if polarizer $POL_{1/2}$ is placed farther along path p (eventually even light years away)? Say that the distance of D_p from the BBO crystal is much greater than that of D_s, as illustrated in Fig. 21.

This implies that the quantum erasure is delayed. One speaks of a '*delayed quantum erasure*' (not to be confused with a which-way delayed choice!) because photon p is set into a linear polarization state and absorbed by detector D_p only after (eventually years after) photon s has already reached detector D_s and has been absorbed by the measurement process. Moreover, here, the 'choice' is not made by a human observer manipulating polarizer $POL_{1/2}$. Rather, it is decided by Nature's random determination of which of the two possible states arises after the entanglement is gone due to the measurement at D_s first.

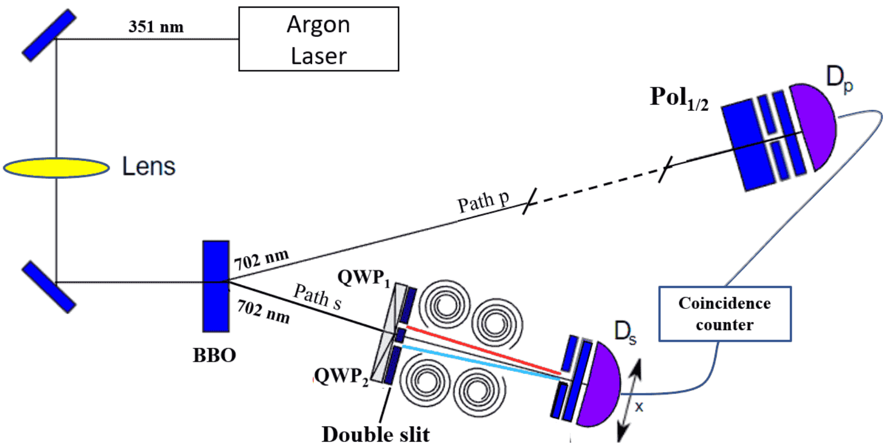


Fig. 21 The which-way experiment with entangled photons: version III. The delayed erasure: as in version II but with D_p shifted along path p.

The answer that might sound initially surprising is that the order of detection—that is, whether detector D_s clicks first or detector D_p clicks first— is not relevant. As long we correlate the photons on the two paths by the coincidence counter, the interference fringes and anti-fringes remain there even if polarizer $POL_{1/2}$ is far away and the quantum erasure is delayed until photon s hits detector D_s. This might suggest, again, that a sort of retro-causality exists whereby the $|45°\rangle_p$ or $|-45°\rangle_p$ state of photon p in the future determines the symmetric or anti-symmetric state of photon s in the present. Or, equivalently, that 'lifting' the entanglement state of photon p in the present retro-causes the fringe or anti-fringe displacement of photon s in the past. This sounds extremely weird and is reminiscent of the time machines of sci-fi fantasy movies.

Again, the analogy with the SEW experiment is straightforward. The delayed quantum erasure in this third experiment version, in which detector D_p is much farther from the BBO than detector D_s, corresponds to the situation of the SEW experiment with the opening of the shutters at a later time than the time of absorption of the caesium atom on the screen. A clumpy curve of particles appeared.

Things are similar here, too, and much more simple and down to earth than any 'back to the past actions'. There is no need to invoke any retro-causality if we remember where and how the collapse of the state function occurs and especially where it does not. In the first case, with detector D_p clicking first, the state collapse occurs only for the entanglement of the system, with the two photons acquiring a separate existence. However, the two-slit superposition of the photon on path s remains unaffected. On the other hand, once the photon on path s hits detector D_s a state reduction is caused that involves the superposition as well. At the instant of detection on detector D_s, the photon on path p will be set into a corresponding diagonal definite quantum state $|45°\rangle_p$ or $|-45°\rangle_p$ already before it hits the (light years away) detector D_p. As in the previous experimental version, polarizer $POL_{1/2}$ doesn't collapse anything, it simply filters out the photon in a $|\pm 45°\rangle_p$ state. Remember that only those photon correlations between those on path p and paths s are counted when the coincidence counter registers the correlation. If a photon on path p is blocked by polarizer $POL_{1/2}$ (as in 50% of the cases), detector D_p does not measure its presence and these entangled photon pairs are discarded from the data count. Finally, we 'uncover' only a correlation. The click of detector D_p tells us only which photon to pick out and ascribes it to the fringe or anti-fringe photons. However, it does not retro-cause anything. This is one of the few instances in which QP is less weird than it might appear. There is nothing 'mystical' about the delayed quantum eraser and the classical temporal order is safe. To be more precise, in this second configuration of the experiment, there is no 'delayed erasure' at all. The 'game was already over' when the photon on path s was absorbed by detector D_s. It would be misleading to suggest that quantum retro-causality exists, as some have done.

In a certain sense, the first part of the experiment is much more interesting, when the selection of polarizer $POL_{1/2}$ and the collapse at detector D_p takes place before the photon on path s hits detector D_s. In that configuration, QM reminds us, again, that as long as the state vector projection has not taken place, we must not regard a quantum system as being made of different separate and independent parts. Rather, we must regard it as a unique and undifferentiated non-locally connected whole. Eq. 7 must be taken seriously, not just as an abstraction without reality. It is only by adopting this perspective that we can understand how, once a measurement is taken of one element of the system, this reverberates on the whole system via an instantaneous non-local 'action'. The state of a quantum system must be conceived of as a potentiality that realizes itself with some probability and that describes the system as a whole, never as a local subset of parts put together but, rather, as a non-local oneness that only upon the act of measurement (state collapse, reduction, projection) distinguishes among the several possible potentialities. If we stick with the idea that a separation exists between beams and particles going through slits or polarizers collapsing wavefunctions, we will be dangerously prone to the separation and measurement fallacy.

5. Putting it all together: the delayed choice quantum eraser of Kim et al.

Almost simultaneous with the experiment of Walborn et al., another fascinating DCQE experiment was performed by a group of scientists from the University of Baltimore and the Texas A&M University (Kim et al. [11]). This experiment did a good job of putting everything together: the double slits, quantum entanglement, a delayed choice quantum erasure (well, not really as we will see), and an interaction free which-way measurement. A sketch of the experimental setup is shown in Fig. 22.

An argon pump laser beam (351 nm wavelength, ultraviolet light) shines on a double slit behind which a BBO crystal generates a pair of entangled type-II orthogonally polarized photons (702 nm wavelength each) by SPDC at regions A and B (0.7 mm center separation). As usual, the light beam is a single-photon source. The signal-photon, that which goes through the lens focusing on detector D_0, and the idler-photon that which travels towards the prism. Detector D_0 scans with a step motor along the perpendicular axis of the path of the incoming signal photon (as shown by the arrows at D_0 in Fig. 22) counting each photon cumulatively in order to reconstruct the interference pattern (that is, the fringes or the bell-shaped intensity curves).

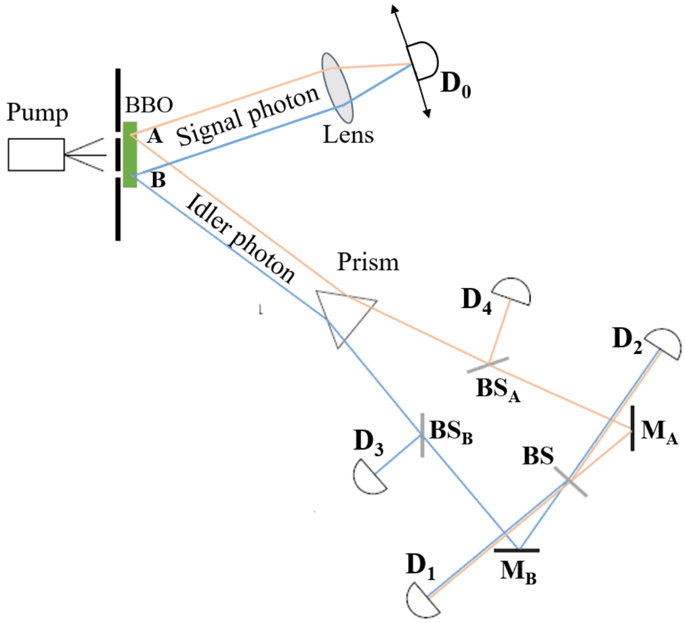


Fig. 22 The delayed choice quantum eraser of Kim et al. [11]

The prism (a *'Glan-Thompson prism'*) has the peculiarity of splitting orthogonally polarized beams. Because regions A and B determine two mutually orthogonal polarizations, the splitting prism sends the idler-photon towards beamsplitter BS_A if it has a horizontal (vertical) polarization or towards beamsplitter BS_B if it has a vertical (horizontal) polarization. Therefore, in the event (50% chance) that this photon, according to its polarization, is deflected towards detectors D_3 or D_4, one can determine the which-path from region A or B, respectively. However, if, instead, it is transmitted through beamsplitters BS_A or BS_B, it will travel farther and be reflected at mirrors M_A and M_B and, later on, will encounter a third beamsplitter (BS). Here, again, it can be either reflected or transmitted with a 50% chance. If it is coming from (or, more precisely, we imagine that it is coming from) mirror M_A (M_b), a reflection at beamsplitter BS sends it to detector D_2 (D_1), whereas, in the case of transmission, it will travel towards D_1 (D_2). This means that whenever detector D1 or detector D2 clicks, we can no longer determine whether it came from region A or B, because for both cases there is a 50% chance that one or the other paths has been taken. In other words, beamsplitter BS works as a (passive) quantum eraser of the which-way information.

Finally, a coincidence circuit (not shown in Fig. 22.) correlates each photon measured along the stepping-axis of detector D_0 by a 'joint detection' on one of the four detectors that its twin photon has triggered.

An important aspect of this experimental configuration is that it has been built in such a way that the optical path of each idler-photon—whichever path it will take from the BBO to whichever detector $D_{1\text{-}4}$— is at least 2.5 m longer than the optical path of the signal-photon from the BBO to detector D_0. This means that detector D_0 is always triggered first and that only later (by a delay of at least 8 ns) will one of the four other detectors click. This space-like separation is a necessary condition for ensuring that there is no potentially unknown physical effect that might 'inform' detector D_0 about what the other detectors will do in the future. If it does, then only FTL effects could be responsible, but no causal correlation that the theory of relativity allows for. Of course, an extremely fast electronic readout at the detectors is necessary and must occur in a time-lapse no longer than a few billionths of a second. This is, however, no issue for modern electro-optical devices.

Once you have this experimental configuration clearly in mind, it should not be difficult to see that:

a) If detector D_3 (D_4) clicks, the idler- and signal-photons could have come from only slit B (slit A)—that is, we have delayed which-way information.

b) If detector D1 or detector D2 clicks, the idler- and signal-photons could have come from either A or B—that is, we have a delayed which-way information erasure at beamsplitter BS.

The measurements are performed by collecting the data of the 'joint detection rate', R_{0j}, between detector D_0 and $D_{1\text{-}4}$. The detection time at D_0 and $D_{1\text{-}4}$ is not exactly simultaneous because of the mentioned minimum 8 ns delay that temporally separates the incidence of the signal-photon at D_0 and the idler-photon on one of the other four detectors. However, if the coincidence monitor registers a common detection at D_0 and $D_{1\text{-}4}$ within a time interval no longer than that which light needs to traverse the entire experimental setup (the single-photon source provides that no other photons are 'in-flight' during that time lapse), this confirms the coincidence as a joint detection, correlating the individual signal-photon at D_0 with the idler-photon at $D_{1\text{-}4}$. Moreover, the coincidence monitor allows for the filtering out of unwanted random photons coming from the environment, which otherwise would corrupt the data set as noise.

So, what should we expect to see? First, let us take a look at detector D_0 in front of the double slit and scan its optical focusing plane to reconstruct the overall intensity profile considering ***all*** the incoming signal photons.

The answer is straightforward: Despite the fact that D_0 cumulatively measures the photons emerging from two interfering waves from slits A and B, no interference fringes will appear; only the clumpy Gaussian curve is observed. Due to the generation of orthogonal polarizations in front of slit A relative to slit B, this 'marks' the photon and we have the which-path information from which slit we imagine it to have come.

However, with the background knowledge we gained from section 0.2, we can also reach the same conclusion using classical wave optics, without the need to invoke the 'quantum which-way fiction'. We know that two orthogonally interfering waves leave no trace of any double slit interference fringes, as was amply explained in chapter (see Eq. 1, Fig. 7 for $\delta\theta = 90°$ or Fig. 9 or Appendix A I).

The next question is: Which idler photon triggering the other four detectors correlates with a 'joint detection' of the signal photon detected at D_0?

Let us first consider the joint correlation rates for D_3 and D_4. These detectors measure the idler-photon (filtered by the prism and reflected by beamsplitters BS_A or BS_B), which is vertically or horizontally polarized—that is, if it is coming from slit A or B, and therefore 'revealing' the which-way. It is therefore no surprise that, if one were to place a detection screen at D_3 or D_4, one would again observe no interference fringes. After all, how can there be any? The two beams have been separated from each other and we are looking at one or the other slit separately, preventing the two waves from interfering in the first place. (Recall a similar situation in Wheeler's delayed choice experiment in Vol. I with no screen in place and only the two detectors pointing at the slits.) Which polarization is assigned at which photon from which slit is a completely quantum random process that is determined at the instant of the collapse of the idler photon. That is, at each new photon coming from the source, the horizontal and vertical polarization is assigned randomly between path A and path B. The idler photon's polarization will continuously switch sides. Despite the anti-correlation with the signal photons (always keep in mind that the BBO produces type-II entangled photons), this does not determine whether they displace themselves on a fringe or anti-fringe of the interference pattern. Therefore, detectors D_3 and D_4 will reproduce the same intensity curve that the signal photons produce on detector D_0 (with one half of the intensity, because they 'filter out' each 50% of the idler photons on average) and the coincidence joint detection rate, R_{03} and R_{04}, will be that of a bell-shaped curve as well (as shown in Fig. 23).

What about the joint detection rates between detector D_0 and detectors D_1 and D_2? First, note that if one were to place detection screens in front of these detectors, we would again observe the absence of interference fringes. This is because what both detectors D_1 and D_2 'see' is a superposition of a vertically and horizontally polarized idler photon. In fact, follow the paths of the idler photon in Fig. 22. One is coming from path A (B) and is reflected twice at mirror M_A (M_B) and at beamsplitter BS, while the other along path B (A) is reflected at mirror M_B (M_A) and transmitted through the same beamsplitter BS, both reaching detector D_2 (D_1).

The absence of interference here is somewhat surprising. This is because, if the second part of the experimental device comprising the two mirrors M_A, M_b, beamsplitter BS, and detectors D_1 and D_2 are supposed to work as a delayed which-way quantum eraser system, we should expect to see interference fringes at detectors D_1 and D_2.

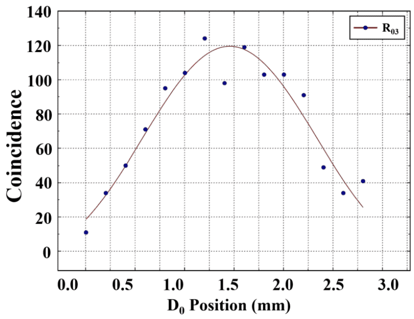


Fig. 23 Joint detection rate R_{03} (Kim et al. [11]*).*

Indeed, a closer inspection reveals that this is not really a quantum eraser, at least not in the conventional sense. Suppose the idler is in a horizontal polarization state on path A and a vertical one on path B. Then, if we were to place a polarizer in front of detectors D_1 and D_2 (say, both with a horizontal polarizer), then if D_1 or D_2 clicks, the idler photon must have traveled along path A. If none click, the idler photon must have been a vertically polarized idler photon traveling along path B. So, in a certain sense, this is a 'fake which-path quantum eraser'. What differentiates detectors D_1 and D_2 from detectors D_3 and D_4 is that the latter 'see' the single slit by pointing at it directly and already determine, in advance, which path they are measuring, independently from 'polarization marks', while the former 'see' both slits at once. However, for anti-correlated photons, this does not prevent us, at least in principle, from determining the path using analyzing polarizers.

The catch is that, as already mentioned, we never know which idler photon has which polarization on which path. The BBO produces anti-correlated entangled photons, but which polarization is associated with which slit remains a quantum random event. The indistinguishability arises due to the random polarization labeling of each idler photon or, if you prefer, the random switching between slits A and B. It is this continuous quantum random side switching that prevents us from knowing the which-path. It is not the beamsplitter that makes the paths indistinguishable.

The question, then, is: Is this a quantum eraser or not? If we regard the existence or the lack of interference fringes as the ultimate test for the which-way information, we must conclude that this is not a quantum eraser. However, the fact is, we are unable to determine the which-way information, not even in principle, because the labeling of the photons by the BBO crystal is a purely quantum process over which we have no control.

So, an ambiguity arises, which should make it clear again how misleading it is to think in terms of single photons traveling on deterministic paths. The supposed interrelation between interference patterns and the which-way information should always be taken with a grain of salt.

In the end, this doesn't look like a particularly interesting experiment: Whatever detector we choose to look at, we see only boring clumps of normal distribution patterns that show no signs of interference fringes.

The interesting part, however, comes from the coincidence counts—that is, the joint detection rates between detector D_0 and detector D_1 or detector D_2. Say we count all the events in which detectors D_1 and D_0 clicked jointly. That is, while the step motor provides for a linear spatial displacement of detector D_0 scanning the interference pattern of the signal photons, one keeps only those events in which detector D_1 also clicks, revealing the idler photon. By doing so, one obtains the joint detection rate between detector D_0 and detector D_1, R_{01}, interference fringes appear, like in Fig. 24 left.

While, looking at the joint detection rates between detector D_0 and detector D_2, R_{02}, interference phenomena are again manifest, but in form of anti-fringes, like that of of Fig. 24 right. Equivalently, one could say that the graph of R_{02} is a π-phase shift of the graph of R_{01}.

Because it has been a source of much confusion, it can't be emphasized enough that (similar to what we saw in the SWE and Walborn experiments), these are not the representations of the interference patterns one would measure at some detector. Instead, what Fig. 23 and Fig. 24 show is the correlation between a coincidence count of detector D_0 with that of detectors D_3, D_1, and D_2, respectively.

They are jointly detected subsets of the signal photons of D_0 filtered out from a superimposed data stream. The interference pattern registered at D_0 can be extracted from the bell-shaped curve only after the idler photons have triggered $D_{1\text{-}4}$. They can be seen only retroactively.

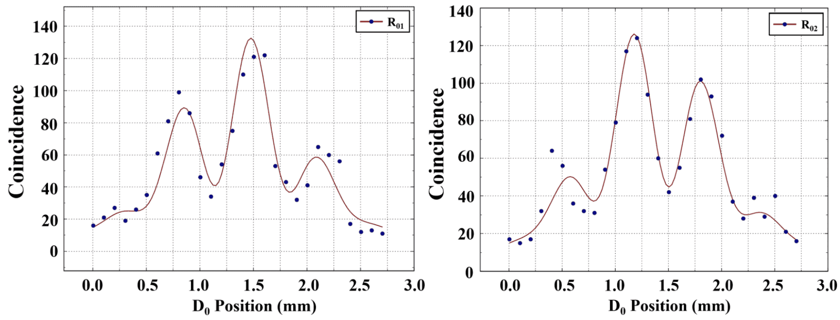


Fig. 24 Joint detection rates R_{01} and R_{02} (Kim et al. [11].*)*

It is not possible to deduce what will happen to the idler-photons by observing the signal photons alone. To put it in other words, one 'selects' and 'picks out' only the data of the clumpy pattern without fringes where two detectors 'clicked' almost simultaneously—that is, those pixels activated by the signal photon at D_0 that correlate with an event triggered by the idler photon at D_1, D_2, or D_3. However, as already pointed out, a detection screen in front of these detectors would not show any interference fringes—not at detector D_0 nor at any of the other ones. They would all display the normal distribution. The sum of the data R_{01}+R_{02} of the left and right graphs of Fig. 24 represents the real pattern measured by detector D_0, which would then appear similar to the curve of Fig. 23.

However, the that fact is, because these correlations are made, interference fringes and anti-fringes appear and every time a signal photon displaces itself on a fringe at detector D_0, then detector D_1 clicks. On the other hand, if it 'runs into' an anti-fringe, detector D_2 clicks. The apparently mysterious and weird thing about all this is that the question arises: How does the idler photon know whether it has to trigger detector D1 or detector D2 considering that the signal photon and detector D_0 are too far away (space-like separated) to convey any information? Once the signal photon collapses onto detector D_0, say, onto a fringe, it has no time to 'inform' the idler photon *"I hit the fringe, please trigger detector D_1"*. However, nevertheless, facts show that the idler photon indeed does 'know' whether the signal photon was a fringe or anti-fringe one and, thus, behaves accordingly.

At first, we might speculate that some form of FTL action from detector D_0 to the idler photon is at work here. Or, as many did, that this implies a temporal quantum retro-causation must be invoked to resolve the paradox. According to this latter idea, the process goes the other way around: The idler photon, once it has randomly triggered either detector D1 or detector D2, eventually also, long after the signal photon triggered D_0, acts back into the past, telling the signal photon what to do. Or, if you prefer, the signal photon receives, from the future, information about whether the idler photon triggered detector D1 or detector D2 and, therefore, behaves accordingly, hitting the fringe or anti-fringe along the scan of detector D_0. The delayed choice seems to change the outcome of an event in the past. Effects seem to precede the cause, changing the order of the causal sequence.

This has caused a plethora of speculations and discussions that persist and continue to be spread all over the Internet. You will find YouTube videos and lots of discussions on forums, blogs, and social media, falsely claiming that through this experiment (and the others we aforementioned), QM has supposedly shown the existence of retro-causal action. While there are, indeed, good reasons to believe that the physics we know of does not necessarily rule out retro-causation (see, later, the chapter on the time-symmetric interpretation of QM), we are going to show that there is no need to resort to 'back-to-the-future' or 'time-machine' narratives to explain this experiment inside the conventional temporal order of the cause-and-effect paradigm.

First, let us not forget that we are dealing with a quantum system of two photons which are in a superposition state (both the signal and idler photons emerge from both slits) and, at the same time, are entangled (with anti-correlated orthogonal polarization states). There is an interplay and simultaneity of quantum entanglement and superposition—something with which we are already familiar due to the experiment of Walborn et al.

Second, the whole phenomenon can become meaningful only when we accept that the state collapse of the signal photon at detector D_0 does ***not*** cause a complete collapse at the idler photon which will remain in a superposition state. The state reduction of the signal photon to a particle state (a dot, a pixel on a screen, or a 'click' of detector D_0) 'removes' the entanglement, but does not cause state reduction to a particle state of the once-entangled idler photon, which remains in a superposition state. (In more technical terms, the measurement at D_0 reduces the system from a 'pure state' to a 'mixed state'). If the signal photon is absorbed, the idler is still 'in-flight' in a superposition state. Eventually, what is going on is quantum steering.

To clarify this formally in detail let us first use the following nomenclature: $|P_\alpha\rangle_S$ is the ket-quantum state in Dirac notation with polarization P= H, V, $\pm45°$ with α = s, i standing for the signal or idler photon and S= A, B for slits A or B. For example, $|H_s\rangle_A$ is the state vector for the signal photon with horizontal polarization and emerging from slit A; $|V_i\rangle_B$ stands for the idler photon with vertical polarization from slit B, and so on.

With this convention let us express how the signal photon must be in an orthogonal polarization superposition state ($|V_s\rangle_A + |H_s\rangle_B$ or $|H_s\rangle_A + |V_s\rangle_B$) and also be entangled with the idler photon which must be in the opposite (anti-correlated) superposition ($|H_i\rangle_A + |V_i\rangle_B$ or $|V_i\rangle_A + |H_i\rangle_B$, respectively). Therefore, the overall state function before detection at D_0 is:

$$|\Psi\rangle = \frac{|H_i\rangle_A|V_s\rangle_A + |V_i\rangle_B|H_s\rangle_B}{\sqrt{2}} + \frac{|V_i\rangle_A|H_s\rangle_A + |H_i\rangle_B|V_s\rangle_B}{\sqrt{2}} \,. \quad \textit{Eq. 9}$$

Before writing down the final process which is physically taking place, let us adopt a pedagogical bottom-up approach which first clarifies the different 'pieces' of the overall picture with which we are dealing. To fix the ideas, and by keeping in mind the anti-correlation of type-II entangled photons, consider, for example, that when the signal photon collapses at detector D_0 to the vertical polarization as coming from slit A (state $|V_s\rangle_A$), then the idler photon, if it is traveling towards the other detectors $D_{1\text{-}4}$ must still be in superposition as coming from the same slit A but with the opposite horizontal polarization state (state $|H_i\rangle_A$), while that from slit B must be in the vertical polarization state (state $|V_i\rangle_B$). The same applies when the signal photon collapses to the horizontal polarization state at slit B (state $|H_s\rangle_B$). That is, the quantum state of the idler photon after detection of the signal photon at D_0, but before being detected at $D_{1\text{-}4}$, is in shorthand:

Measurement at D_0: $|V_s\rangle_A$ or $|H_s\rangle_B$

$\downarrow$

State of idler photon: $|\Psi_i\rangle_1 = \dfrac{|H_i\rangle_A + |V_i\rangle_B}{\sqrt{2}}$

Similarly, for the other two possible outcomes.

Measurement at D_0: $|H_s\rangle_A$ or $|V_s\rangle_B$

$\downarrow$

State of idler photon: $|\Psi_i\rangle_2 = \dfrac{|V_i\rangle_A + |H_i\rangle_B}{\sqrt{2}}$

If you see that, we can refine this picture and note that the signal photons detected at D_0 as $|V_s\rangle_A$ or $|H_s\rangle_B$ must have been the result of the collapse of the signal photon superposition state

$$|\Psi_s\rangle_1 = \frac{|V_s\rangle_A + |H_s\rangle_B}{\sqrt{2}},$$

while $|H_s\rangle_A$ or $|V_s\rangle_B$ that of

$$|\Psi_s\rangle_2 = \frac{|H_s\rangle_A + |V_s\rangle_B}{\sqrt{2}}\,.$$

If you see that, we can refine this picture and note that the signal photons detected at D_0 as $|V_s\rangle_A$ or $|H_s\rangle_B$ must have been the result of the collapse of the signal photon superposition state

$$|\Psi_s\rangle_1 = \frac{|V_s\rangle_A + |H_s\rangle_B}{\sqrt{2}},$$

while $|H_s\rangle_A$ or $|V_s\rangle_B$ that of

$$|\Psi_s\rangle_2 = \frac{|H_s\rangle_A + |V_s\rangle_B}{\sqrt{2}}\,.$$

Because of what we discussed in chapter 0.2 (especially with Eq. 1), the polarizations are orthogonal, $|\Psi_s\rangle_1$ and $|\Psi_s\rangle_2$ as also $|\Psi_i\rangle_1$ and $|\Psi_i\rangle_2$, represent the clumpy refraction pattern without any interference fringes. However, we also saw that, in the diagonal basis given by *Eq. 3* and *Eq. 4*, these split up into symmetric and anti-symmetric components, as described by Eq. *5* and illustrated in Fig. 11.

Therefore, we can summarize the two cases which show the photon's state in the rectilinear and diagonal base with the following diagrams throughout the next two pages. These illustrate the process that leads, at the end of the line, to the observed detector responses.

Case I: detector D_0 measures $|V_s\rangle_A$ or $|H_s\rangle_B$.
Before measurement at D_0:

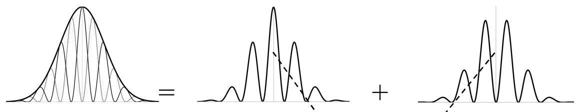
$$|\Psi\rangle = \frac{|H_i\rangle_A|V_s\rangle_A + |V_i\rangle_B|H_s\rangle_B}{\sqrt{2}} + \frac{|V_i\rangle_A|H_s\rangle_A + |H_i\rangle_B|V_s\rangle_B}{\sqrt{2}}$$

Collapse at D_0

$|V_s\rangle_A$ or $|H_s\rangle_B$.

These originated from the signal photon state vectors:

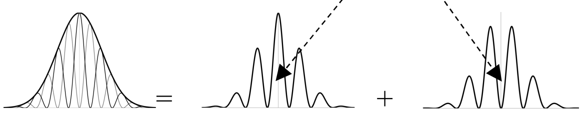
$$|\Psi_s\rangle_1 = \frac{|V_s\rangle_A + |H_s\rangle_B}{\sqrt{2}} = \frac{|45°_s\rangle_A + |45°_s\rangle_B}{\sqrt{2}} (a) + \frac{-|-45°_s\rangle_A + |-45°_s\rangle_B}{\sqrt{2}} (b) \quad \textit{Eq. 10}$$



$|\Psi_s\rangle_1 = |\Psi_s\rangle_1^{\mathrm{Sym}} + |\Psi_s\rangle_1^{\mathrm{Asym}}$

Measurement at D_0 projects also the ('in-flight') anti-correlated idler photon into:

$$|\Psi_i\rangle_1 = \frac{|H_i\rangle_A + |V_i\rangle_B}{\sqrt{2}} = \frac{|45°_i\rangle_A + |45°_i\rangle_B}{\sqrt{2}} (a) + \frac{|-45°_i\rangle_A - |-45°_i\rangle_B}{\sqrt{2}} (b) \quad \textit{Eq. 11}$$



$|\Psi_i\rangle_1 = |\Psi_i\rangle_1^{\mathrm{Sym}} + |\Psi_i\rangle_1^{\mathrm{Asym}}$

Collapse at D_1:	Anti-fringe	Fringe
Collapse at D_2:	Fringe	Anti-fringe

Diagram 1

The difference in the signature of the anti-symmetric component (between Eq. 10b and Eq. 11b or between Eq. 12b and Eq. 13b) is a mere algebraic one which appears by exchanging the order of polarizations or slits (Convince yourself by inserting Eq. 3 and Eq. 4 into $|\Psi_s\rangle_{1/2}$ and $|\Psi_i\rangle_{1/2}$). You may now realize the meaning of the dashed arrows. As an example, consider case I. If detector D_0 detects one signal photon in a fringe position, that is, collapses to symmetric state vector $|\Psi_s\rangle_1^{\mathrm{Sym}}$ (Eq. 10a), it must project the anti-correlated idler photon into the 'anti-fringe state', that is, to $|\Psi_i\rangle_1^{\mathrm{Asym}}$ (Eq. 11b).

Whereas, if detector D_0 detects the signal photon at an anti-fringe position, that is, collapses to anti-symmetric state vector $|\Psi_s\rangle_1^{\mathrm{Asym}}$ (Eq. 10b), it must project the anti-correlated idler photon into the 'fringe-state', $|\Psi_i\rangle_1^{\mathrm{Sym}}$ (Eq. 11a). Exactly the same sort of correlations applies to case II.

Of course, which of the two cases will happen each time is again a completely quantum random process. That's why, after several fringe and anti-fringe signal photons accumulate at D_0 in a normal distribution curve, leaving no trace of interference fringes. The very same process takes place for the idler photons which will display the bell-shaped curve at the other detectors as the sum of the symmetric and anti-symmetric interference patterns as well.

At this point, the decisive insight that becomes clear from all this is that, due to quantum steering, immediately after the measurement of the signal photon at detector D_0, but before the still 'in flight' idler photon is measured at detectors $D_{1\text{-}4}$, it carries this information. At this temporal stage, the idler photon already ‘knows’ whether its signal partner was projected onto a fringe or anti-fringe and will behave accordingly.

Case II: detector D_0 measures $|H_s\rangle_A$ or $|V_s\rangle_B$.
Before measurement at D_0:

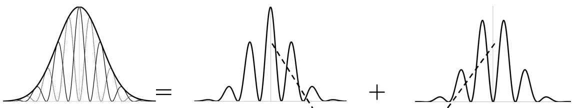
$$|\Psi\rangle = \frac{|H_i\rangle_A|V_s\rangle_A + |V_i\rangle_B|H_s\rangle_B}{\sqrt{2}} + \frac{|V_i\rangle_A|H_s\rangle_A + |H_i\rangle_B|V_s\rangle_B}{\sqrt{2}}$$

Collapse at D_0

$|H_s\rangle_A$ or $|V_s\rangle_B$.

These originated from the signal photon state vectors:

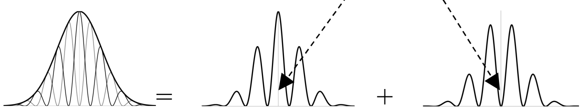
$$|\Psi_s\rangle_2 = \frac{|H_s\rangle_A + |V_s\rangle_B}{\sqrt{2}} = \frac{|45°_s\rangle_A + |45°_s\rangle_B}{\sqrt{2}}(a) + \frac{|-45°_s\rangle_A - |-45°_s\rangle_B}{\sqrt{2}}(b) \quad \textit{Eq. 12}$$



$|\Psi_s\rangle_2 \quad = \quad |\Psi_s\rangle_2^{\mathrm{Sym}} \quad + \quad |\Psi_s\rangle_2^{\mathrm{Asym}}$

Measurement at D_0 projects also the (‘in-flight’) anti-correlated idler photon into:

$$|\Psi_i\rangle_2 = \frac{|V_i\rangle_A + |H_i\rangle_B}{\sqrt{2}} = \frac{|45°_i\rangle_A + |45°_i\rangle_B}{\sqrt{2}}(a) + \frac{-|-45°_i\rangle_A + |-45°_i\rangle_B}{\sqrt{2}}(b) \quad \textit{Eq. 13}$$



$|\Psi_i\rangle_2 \quad = \quad |\Psi_i\rangle_2^{\mathrm{Sym}} \quad + \quad |\Psi_i\rangle_2^{\mathrm{Asym}}$

Collapse at D_1:	Anti-fringe	Fringe
Collapse at D_2:	Fringe	Anti-fringe

Diagram 2

No quantum retro-causal effect must be invoked. The measurement at D_0 of the signal photon already determines, a priori, the probabilities that the idler-photon will hit either $D_{1\text{-}4}$. The signal photon appears to be ‘clairvoyant’ only if we overlook this step of the process and forget to look at things from the perspective of the diagonal eigenbasis.

Still, this does not completely resolve the apparent issue with a supposed retro-causal action of the idler photon when it makes detector D_1 or detector D_2 click. The attentive reader, who has thought this all through, might have realized that the mystery contains another piece requiring an answer. This is because, even if the idler photon already carries in its state vector the information about whether the signal photon hit a fringe or anti-fringe in D_0, how does beamsplitter BS ‘know’ whether it must direct the idler photon towards detector D_1 or detector D_2? After all, a beamsplitter is simply a piece of glass or a crystal that is not supposed to be a ‘receptionist’ that forwards messages according to its informational content. Rather, it is a transparent medium that splits a photonic stream with a prefixed probability (usually 50%) towards two (usually perpendicular) directions. There is no reason to believe that it will treat photons in a symmetric quantum state differently from those in an anti-symmetric state.

Therefore, the next step is to clarify how the idler photon which, after the measurement in D_0 is no longer entangled with the signal photon, but is still ‘in-flight’ in one of the four possible quantum superposition states $|\Psi_i\rangle_1^{\mathrm{Sym}}$, $|\Psi_i\rangle_1^{\mathrm{Asym}}$, $|\Psi_i\rangle_2^{\mathrm{Sym}}$ or $|\Psi_i\rangle_2^{\mathrm{Asym}}$, will reach detector D_1 or D_2. What kind of signal will it trigger?

First of all do not allow your mind to fall into the separation and measurement fallacy and don't forget that there is only ***one*** photon taking ***two*** paths A and B, being reflected at ***both*** mirrors M_A and M_B and being reflected ***and*** transmitted at beamsplitter BS. Second, recognize that, despite a perfect physical symmetry represented by the part of the experimental setup made of the mirrors M_A and M_B, beamsplitter BS, and detectors D_1 and D_2, an optical anti-symmetry nevertheless holds: This part of the system behaves differently according to the photon's diagonal polarization being in a $+45°$ or $-45°$ superposition state. It can be shown (for a detailed discussion, see Appendix A II) that, if a measurement at D_0 selects the symmetric part of the signal photon ($|\Psi_s\rangle_1^{\mathrm{Sym}}$ or $|\Psi_s\rangle_2^{\mathrm{Sym}}$), projecting the idler photon into the anti-symmetric state ($|\Psi_i\rangle_1^{\mathrm{Asym}}$ or $|\Psi_i\rangle_2^{\mathrm{Asym}}$), once reflected at ***both*** mirrors M_A and M_B, and after being reflected ***and*** transmitted at beamsplitter BS, the so transformed idler photon's state function will displace it on a fringe at D_1 and on an anti-fringe at D_2. Vice versa, if a measurement at D_0 selects the anti-symmetric part of the signal photon ($|\Psi_s\rangle_1^{\mathrm{Asym}}$ or $|\Psi_s\rangle_2^{\mathrm{Asym}}$), projecting the idler photon into the symmetric state ($|\Psi_i\rangle_1^{\mathrm{Sym}}$ or $|\Psi_i\rangle_2^{\mathrm{Sym}}$), once reflected at ***both*** mirrors M_A and M_B, and after being reflected ***and*** transmitted at beamsplitter BS, the so transformed idler photon's state function will be shifted by 90° compared to that of the previous case and will displace it on an anti-fringe at D_1 and on a fringe at D_2.

To cut a long story short: Detectors D_1 and D_2 will always react in a complementary fashion and in accordance with the signal photon's symmetry state. The two cases switch permanently and randomly for each photon, as described above, and the overall result is nevertheless the normal distribution. However, if one considers how detector D_0 takes the measurements of the signal photons guided by a step motor along the perpendicular direction of its propagation, this explains why the joint detection rate between detectors D_0 and D_1, R_{01}, displays a standard Young's double slit interference pattern, while the joint detection rate between detectors D_0 and D_2, R_{02}, displays the complementary π-phase shifted interference pattern of Fig. 24.

Finally, also in this case, just as with what we have seen with in the experiment with the polarizers of Fig. 10, the SWE and Walborn et al. experiments, one can again explain everything inside an orthodox cause and effect paradigm without any resorting to retro-causality. Once we become aware of the separation and measurement fallacy, avoid retro-ductive reasonings of counterfactual definiteness, and keep the potential quantum steering effects in mind, then the retro-causal hypothesis appears in all its deceitfulness. On the other hand, these sorts of experiments reinforce our deeper understanding of the foundations of QP and show how 'quantum ubiquity' is at work. It is not just a mathematical figment, rather, must have an 'inherent element of physical reality', as 'someone' used to say, even though he was looking at it from the opposite standpoint.

II. Appendix

A I. Interference of light waves with different polarizations

The interference between two light beams having the same polarization (like in the case of the Young double slit experiment) can be generalized to two waves with different linear polarization states. Recall how waves are described in time and space with their amplitudes and phases with the complex Euler numbers, as described in appendix of Vol. I.

Consider (Fig. 25 left) an EM wave with vertical polarization vector, that is, with an electric field amplitude oscillating along the y-axis, $|E_1| = E_{1y}$, with angular frequency ω and none for the component E_{1x} along the x-axis:

$$E_{1x} = 0 \quad ; \quad E_{1y} = |E_1|e^{i\omega t}$$

Similarly, consider the second vector in Fig. 25 as representing the polarization vector another wave with an electric field amplitude $|E_2|$ having the horizontal and vertical components oscillating along both the x-axis and y-axis, E_{2x} and E_{2y} respectively, forming an angle $\delta\theta$ (with respect to the vertical vector E_1), with angular frequency ω and relative phase $\delta\phi + \delta\varepsilon$ (induced by the two slits path difference plus the eventual retarding optical plate phase shift):

$$E_{2x} = |E_2|\, e^{i(\omega t+\delta\phi+\delta\varepsilon)} sin(\theta) \quad ; \quad E_{2y} = |E_2| e^{i(\omega t+\delta\phi+\delta\varepsilon)} cos(\theta);$$

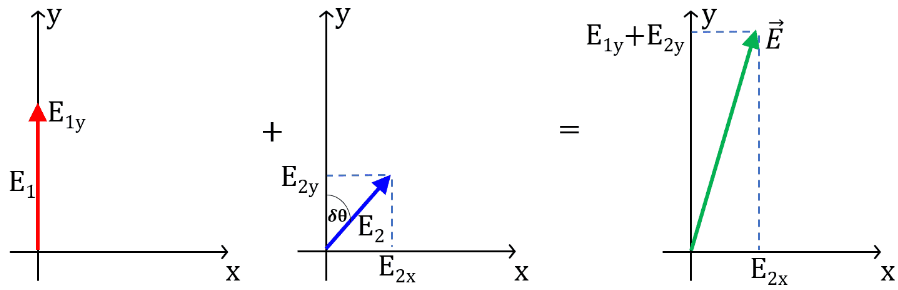


Fig. 25 Sum of two polarization vectors and its result.

To simplify the notation let us write in more compact form $\Delta = \delta\phi + \delta\varepsilon$. Then, the superposition of the two polarization vectors along the x- and y-axis is:

$$E_x = E_{1x} + E_{2x} = |E_2|e^{i(\omega t+\Delta)}sin(\delta\theta);$$

$$E_y = E_{1y} + E_{2y} = |E_1|e^{i\omega t} + |E_2|e^{i(\omega t+\Delta)}cos(\delta\theta) = e^{i(\omega t)}(|E_1| + |E_2|\, e^{i\Delta}cos(\delta\theta)).$$

To obtain the contribution to the intensity of the EM beam from each component we have to modulus square it:

$$|E_x|^2 = E_x E_x^* = |E_2|^2\, sin^2(\delta\theta);$$

$$|E_y|^2 = E_y E_y^* = (|E_1| + |E_2|e^{i\Delta}cos(\delta\theta))(|E_1| + |E_2|e^{-i\Delta}cos(\delta\theta))$$

$$= |E_1|^2 + |E_1||E_2|e^{-i\Delta}cos(\delta\theta) + |E_1||E_2|e^{+i\Delta}cos(\delta\theta) + |E_2|^2cos(\delta\theta)^2$$

$$= |E_1|^2 + |E_1||E_2|cos(\delta\theta)(e^{-i\Delta} + e^{+i\Delta}) + |E_2|^2cos(\delta\theta)^2$$

$$= |E_1|^2 + |E_2|^2cos(\delta\theta)^2 + 2|E_1||E_2|cos(\Delta)cos(\delta\theta),$$

with the last passage because of Euler's identity for the cosine function.

Putting this together, to obtain the resulting vector we can use the good old Pythagorean theorem and, noting that $cos^2\theta + sin^2\theta = 1$, it becomes:

$$|E|^2 = |E_x|^2 + |E_y|^2 = |E_1|^2 + |E_2|^2 + 2|E_1||E_2|cos(\Delta)cos(\delta\theta).$$

Recalling that the modulus squared of an electric field gives the intensity of a beam, we can label the intensity of the first and second beam as $|E_1|^2 = I_1$ and $|E_2|^2 = I_2$, respectively, to finally obtain:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\, cos(\Delta)\, cos(\delta\theta). \quad \textit{Eq. 14}$$

If the two beams have the same intensity $I_1 = I_2 = I_0$ (say because the two slits are equal), then Eq. 14 simplifies to:

$$I = 2I_0 + 2I_0\, cos(\Delta)\, cos(\theta) = 2I_0[1 + cos(\Delta)\, cos(\delta\theta)]. \quad \textit{Eq. 15}$$

A II. Interference at detectors D_1 and D_2 in the delayed choice quantum eraser of Kim et al.

To see what really happens after the signal photon has been measured at detector D_0, we must continue to follow the idler photon along its journey towards detectors D_1 or D_2. We already discussed what is going on at detectors D_3 and D_4. Strictly speaking, the transmission of a photon across a beamsplitter induces a $\frac{\pi}{2}$ phase shift, but since this happens at both beamsplitters BS_A and BS_B, we can ignore this phase shift. This phase shift affects the idler photon state function equally on both paths A and B, leaving the relative phase of the ket-vectors of $|\Psi_i\rangle_1$ or $|\Psi_i\rangle_2$ (Eq. 11 and Eq. 13) unaffected. Therefore, from now on, we will only consider the idler photon after beamsplitters BS_A and BS_B.

Despite the apparently perfect physical path symmetry of the experimental setup of Fig. 22, one cannot say the same thing about the optical symmetry, once the phase shifts induced by the reflection of the idler photon at mirrors M_A, M_B and beamsplitter BS are considered. A phase-shift anti-symmetry holds! It turns out that there is a little but decisive difference if one considers the photon's propagation along the optical path A reflected at mirror M_A and transmitted through beamsplitter BS towards detector

D_1 or the same photon propagating along optical path B, reflected at mirror M_B and transmitted through the other side of the same beamsplitter BS towards detector D_2. So, let us follow the idler's propagation step by step.

First of all, keep in mind that when a photon (or an EM wave) is reflected by a mirror or beamsplitter it undergoes a π-phase shift (180°-phase shift), while, as already mentioned, when it is transmitted through a beamsplitter it undergoes a $\frac{\pi}{2}$-phase shift (90°-phase shift). In mathematical general terms, a phase shift φ applied to a state vector (or wavefunction) $|\Psi\rangle$ is represented by multiplying it by the complex Euler exponential $e^{i\varphi}$, that is: $|\Psi\rangle \rightarrow e^{i\varphi}|\Psi\rangle$.

With this we can proceed by analyzing specifically how the photons of Eq. 11 or Eq. 13 behave. The expression of the photon's state in the rectilinear polarization basis $\mathcal{L} = \{|H\rangle, |V\rangle\}$ is not that interesting because we already know that these, being orthogonal, will not produce any interference pattern and reveal us anything about the fringe and anti-fringe interference components. We can expect more insight by choosing to inspect the idler photon's state in the diagonal basis $\mathcal{D} = \{|45°\rangle, |-45°\rangle\}$.

Begin with the symmetric part of the of Eq. 11 or Eq. 13, the two possible idler photon states that result after collapse at detector D_0, that is:

$$|\Psi_i\rangle_1^{Sym} = |\Psi_i\rangle_2^{Sym} = \frac{|45°_i\rangle_A + |45°_i\rangle_B}{\sqrt{2}} . \quad \textit{Eq. 16}$$

Consider first the photon traveling along path A (after BS_A) reaching detector D_1, first by being reflected at mirror M_A (π-phase shift) and then being transmitted through beamsplitter BS ($\frac{\pi}{2}$-phase shift). Therefore, it undergoes a total $\frac{3}{2}\pi$-phase shift. Since $e^{i\frac{3}{2}\pi} = -i$, then the idler photon's state vector on path A, $|45°_i\rangle_A$, transform as: $|45°_i\rangle_A \rightarrow -i\,|45°_i\rangle_A$.

However, the idler photon is in superposition with itself and is also traveling along path B (after BS_B) reaching the same detector D_1 by being reflected twice, at mirror M_B (π-phase shift) and then at beamsplitter BS (π -phase shift). It undergoes a total 2π-phase shift. Since $e^{i2\pi} = 1$, the idler photon's state vector on path B, $|45°_i\rangle_B$, is invariant under such a transformation, that is: $|45°_i\rangle_B \rightarrow |45°_i\rangle_B$.

Therefore, detector D_1 will measure quantum state:

$$|\Psi_{D_1}\rangle^1 = \frac{-i|45°_i\rangle_A + |45°_i\rangle_B}{\sqrt{2}} . \quad \textit{Eq. 17}$$

Now let's do this the other way around, towards detector D_2. The idler photon traveling along path A towards detector D_2 is reflected twice, at mirror M_A (π-phase shift) and then at beamsplitter BS (π-phase shift). It undergoes a total 2π-phase shift and, since $e^{i2\pi} = 1$, the idler photon's state vector coming from path A, $|45°_i\rangle_A$, is invariant under such a transformation, that is: $|45°_i\rangle_A \rightarrow |45°_i\rangle_A$.

However, being in superposition the idler photon is also traveling along path B towards detector D_2 and is first reflected at mirror M_B (π-phase shift) and then transmitted through beamsplitter BS ($\frac{\pi}{2}$ -phase shift). It undergoes a total $\frac{3}{2}\pi$-phase shift and since $e^{i\frac{3}{2}\pi} = -i$, then the idler photon's state vector on path B, $|45°_i\rangle_B$, transform as: $|45°_i\rangle_B \rightarrow -i\,|45°_i\rangle_B$.

Therefore, detector D_2 will measure quantum state:

$$|\Psi_{D_2}\rangle^1 = \frac{|45°_i\rangle_A - i\,|45°_i\rangle_B}{\sqrt{2}} . \quad \textit{Eq. 18}$$

Eq. 17 and Eq. 18 are not the same. We obtained:

$$|\Psi_{D_2}\rangle^1 = -\,i\,|\Psi_{D_1}\rangle^1 ,$$

that is, they differ by a multiplicative factor, the negative imaginary number $-\,i = e^{i\frac{3}{2}\pi} = e^{-i\frac{\pi}{2}}$ (recall how complex numbers are represented on the unitary complex circle, see also the mathematical appendix of Vol. I). This can be interpreted as a relative phase shift of 90° of the wavefunction between detector D_1 and D_2.

The question at this point is: what kind of signal will these photon quantum states produce at the two detectors? Fringes, anti-fringes or a Gaussian profile? To see this, what remains to do is to modulus square the two wavefunctions of Eq. 17 and Eq. 18. For sake of brevity let us simplify. First omit the probability normalization coefficient which will play no role in the interference. Then, instead of Dirac notation let us use the wavefunction notation and and replace the kets of the diagonal polarization basis $\mathcal{D}$ as follows: $|+45°_i\rangle_A \rightarrow \mathcal{D}^+_{i,A}$, $|-45°_i\rangle_A \rightarrow \mathcal{D}^-_{i,A}$, $|+45°_i\rangle_B \rightarrow \mathcal{D}^+_{i,B}$, $|-45°_i\rangle_B \rightarrow \mathcal{D}^-_{i,B}$ and then also the final state functions at detector D_1 and D_2 with $|\Psi_{D_1}\rangle^1 \rightarrow \Psi^1_{D_1}$, $|\Psi_{D_2}\rangle^1 \rightarrow \Psi^1_{D_2}$.

Then, Eq. 17 can be written as wavefunction:

$$\Psi^1_{D_1} = -i\,\mathcal{D}^+_{i,A} + \mathcal{D}^+_{i,B}.$$

Taking the modulus square:

$$\begin{aligned}|\Psi^1_{D_1}|^2 = \Psi^1_{D_1}\cdot\Psi^{1*}_{D_1} &= (-i\,\mathcal{D}^+_{i,A} + \mathcal{D}^+_{i,B})(-i\mathcal{D}^+_{i,A} + \mathcal{D}^+_{i,B})^* \\ &= (-i\,\mathcal{D}^+_{i,A} + \mathcal{D}^+_{i,B})(i\,\mathcal{D}^{+*}_{i,A} + \mathcal{D}^{+*}_{i,B}) \\ &= |\mathcal{D}^+_{i,A}|^2 - i\,\mathcal{D}^+_{i,A}\mathcal{D}^{+*}_{i,B} + i\,\mathcal{D}^{+*}_{i,A}\mathcal{D}^+_{i,B} + |\mathcal{D}^+_{i,B}|^2 \\ &= |\mathcal{D}^+_{i,A}|^2 + |\mathcal{D}^+_{i,B}|^2 - i\,(\mathcal{D}^+_{i,A}\mathcal{D}^{+*}_{i,B} - \mathcal{D}^{+*}_{i,A}\mathcal{D}^+_{i,B}). \quad \textit{Eq. 19}\end{aligned}$$

This is the celebrated two slits intensity profile (see Eq. 1, or Eq. 14, or Young's double slit experiment in Vol. I) with the third term of the last line the interference term (one can show it to be equivalent to the interference term of Eq. 14 sifted by a $-\frac{\pi}{2}$ phase, a proof we omit here because not essential for the present discussion). The negative signature representing an anti-fringe interference pattern.

If one repeats exactly the same calculation, but for Eq. 18, that is, in wavefunction notation for:

$$\Psi^1_{D_2} = \mathcal{D}^+_{i,A} - i\,\mathcal{D}^+_{i,B},$$

then one obtains:

$$|\Psi^1_{D_2}|^2 = |\mathcal{D}^+_{i,A}|^2 + |\mathcal{D}^+_{i,B}|^2 + i\,(\mathcal{D}^+_{i,A}\mathcal{D}^{+*}_{i,B} - \mathcal{D}^{+*}_{i,A}\mathcal{D}^+_{i,B}). \quad \textit{Eq. 20}$$

The positive signature of the interference term represents a fringe interference pattern. By comparing Eq. 19 and Eq. 20, it turns out that what distinguishes the measurements between the two detectors is only the signature in front of the interference term. This might suggest at first that we should see a symmetric interference pattern on one detector and an anti-symmetric pattern on the other one.

However, we must repeat the same calculations (this time we will furnish only the results, check yourself as exercise) by including also the other two possible idler photon states that result after collapse at detector D_0, namely, from the anti-symmetric states $|\Psi_i\rangle_1^{Asym}$ and $|\Psi_i\rangle_2^{Asym}$ (Eq. 11b and Eq. 13b).

That of Eq. 11 being:

$$|\Psi_i\rangle_1^{Asym} = \frac{|-45°_i\rangle_A - |-45°_i\rangle_B}{\sqrt{2}}. \quad \textit{Eq. 21}$$

Therefore, detector D_1 will measure quantum state:

$$|\Psi_{D_1}\rangle^2 = \frac{-i|-45°_i\rangle_A - |-45°_i\rangle_B}{\sqrt{2}}. \quad \textit{Eq. 22}$$

In wavefunction notation:

$$\Psi^2_{D_1} = -i\,\mathcal{D}^-_{i,A} - \mathcal{D}^-_{i,B}\,,$$

leads to the intensity profile at detector D_1:

$$|\Psi^2_{D_1}|^2 = |\mathcal{D}^-_{i,A}|^2 + |\mathcal{D}^-_{i,B}|^2 + i\,(\mathcal{D}^-_{i,A}\mathcal{D}^{-*}_{i,B} - \mathcal{D}^{-*}_{i,A}\mathcal{D}^-_{i,B}). \quad \textit{Eq. 23}$$

Whereas, detector D_2 will measure quantum state:

$$|\Psi_{D_2}\rangle^1 = \frac{|-45°_i\rangle_A + i\,|-45°_i\rangle_B}{\sqrt{2}}. \quad \textit{Eq. 24}$$

In wavefunction notation:

$$\Psi^2_{D_2} = \mathcal{D}^-_{\mathrm{i},A} + i\,\mathcal{D}^-_{\mathrm{i},B}\ ,$$

leads to the intensity profile at detector D_2:

$$|\Psi^2_{D_2}|^2 = |\mathcal{D}^-_{i,A}|^2 + |\mathcal{D}^-_{i,B}|^2 - i(\mathcal{D}^-_{i,A}\mathcal{D}^{-*}_{i,B} - \mathcal{D}^{-*}_{i,A}\mathcal{D}^-_{i,B}). \quad \textit{Eq. 25}$$

Again, notice the signature difference of the interference term between Eq. 23 and Eq. 25.

Finally, the anti-symmetric part of Eq. 13 being:

$$|\Psi_i\rangle_2^{Asym} = \frac{-|-45°_i\rangle_A + |-45°_i\rangle_B}{\sqrt{2}}, \quad \textit{Eq. 26}$$

then detector D_1 will measure quantum state:

$$|\Psi_{\mathrm{D}_1}\rangle^3 = \frac{i|-45°_{\mathrm{i}}\rangle_A + |-45°_{\mathrm{i}}\rangle_B}{\sqrt{2}}.$$

In wavefunction notation:

$$\Psi^3_{D_1} = i\,\mathcal{D}^-_{\mathrm{i},A} + \mathcal{D}^-_{\mathrm{i},B}\ ,$$

leads to the intensity profile at detector D_1:

$$|\Psi^3_{D_1}|^2 = |\mathcal{D}^-_{i,A}|^2 + |\mathcal{D}^-_{i,B}|^2 + i\,(\mathcal{D}^-_{i,A}\mathcal{D}^{-*}_{i,B} - \mathcal{D}^{-*}_{i,A}\mathcal{D}^-_{i,B}). \quad \textit{Eq. 27}$$

Whereas, detector D_2 will measure quantum state:

$$|\Psi_{\mathrm{D}_2}\rangle^3 = \frac{-|-45°_{\mathrm{i}}\rangle_A - i\,|-45°_{\mathrm{i}}\rangle_B}{\sqrt{2}}.$$

In wavefunction notation:

$$\Psi^3_{D_2} = -\mathcal{D}^-_{\mathrm{i},A} - i\,\mathcal{D}^-_{\mathrm{i},B}\ ,$$

leads to the intensity profile at detector D_2:

$$|\Psi^3_{D_2}|^2 = |\mathcal{D}^-_{i,A}|^2 + |\mathcal{D}^-_{i,B}|^2 - i(\mathcal{D}^-_{i,A}\mathcal{D}^{-*}_{i,B} - \mathcal{D}^{-*}_{i,A}\mathcal{D}^-_{i,B}). \quad \textit{Eq. 28}$$

With the opposite signature difference of the interference term between Eq. 27 and Eq. 28, as expected.

Wrapping it all up first in words, four cases can occur.

When the signal photon collapses onto a anti-symmetric wavefunction of its diagonal basis (anti-fringe - Eq. 10b or Eq. 12b) the idler photon reduces to the symmetric state (Eq. 11a or Eq. 13a, see also Eq. 16), after reflections/transmissions at the mirrors/beamsplitter is transformed (Eq. 17 or Eq. 18), and, in both cases, 'falls' onto an anti-fringe of detector D_1 (Eq. 19) and fringe of detector D_2 (Eq. 20).

When the signal photon collapses onto the symmetric wavefunction of its diagonal basis (fringe - Eq. 10a or Eq. 12a) the idler photon reduces to one of two possible anti-symmetric states (Eq. 11b or Eq. 13b, see also Eq. 21 or Eq. 26), after reflections/transmissions at the mirrors/beamsplitter is transformed (Eq. 22 or Eq. 24) and, in both cases, 'falls' onto a fringe of detector D_1 (Eq. 23 or Eq. 27) and anti-fringe of detector D_2 (Eq. 25 or Eq. 28).

Restating the above in symbols:

$$\left.\begin{array}{l} |\Psi_{\mathrm{s}}\rangle_1^{\mathrm{Asym}} \to |\Psi_{\mathrm{i}}\rangle_1^{\mathrm{Sym}} \to \mathrm{D_1\text{: anti-fringes ; } D_2\text{: fringes}} \\ |\Psi_{\mathrm{s}}\rangle_2^{\mathrm{ASym}} \to |\Psi_{\mathrm{i}}\rangle_2^{\mathrm{Sym}} \to \mathrm{D_1\text{: anti-fringes ; } D_2\text{: fringes}} \end{array}\right\} \quad R_{02}$$

$$\left.\begin{array}{l} |\Psi_{\mathrm{s}}\rangle_1^{\mathrm{Sym}} \to |\Psi_{\mathrm{i}}\rangle_1^{\mathrm{Asym}} \to \mathrm{D_1\text{: fringes ; } D_2\text{: anti-fringes}} \\ |\Psi_{\mathrm{s}}\rangle_2^{\mathrm{Sym}} \to |\Psi_{\mathrm{i}}\rangle_2^{\mathrm{Asym}} \to \mathrm{D_1\text{: fringes ; } D_2\text{: anti-fringes}} \end{array}\right\} \quad R_{01}$$

where R_{01} and R_{02} indicate the joint detection rates explained in the text.

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