# Cartan in a FLRW universe

Main deadline: Friday 19 Jan @ 9PM https://www.dropbox.com/request/7AvcLt7K4gBjvVyz4PJC
Late deadline: Friday 26 Jan @ 9PM https://www.dropbox.com/request/Smpsx5vtAWEdJX3UVwQN
Pass-or-fail deadline: Friday 9 Feb @ 9PM – `contact Aldo & Dan`

A spatially homogeneous and isotropic universe is described by the Friedmann-Lemaitre-Robinson-Walker (FLRW) metric:

$$ds^2_{FLRW} = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\vartheta d\phi^2\right) \equiv g_{\mu\nu}\underline{d}x^\mu \otimes \underline{d}x^\nu, \tag{1}$$

which depends on the *scale factor* $a(t) > 0$ and on the number $k = 0, \pm 1$ which determines whether the spatial slices are flat or positively/negatively curved. Positively curved spatial slices are closed; they are open and of infinite volume otherwise. Einstein equations determine the time evolution of the scale factor $a(t)$ (provided the matter content is also assumed spatially homogeneous and isotropic!).

The FLRW metric is starting point of our (simplest) cosmological models.

a) Write down an orthonormal basis of 1-forms, $\{\underline{\omega}^a\}_{a=0}^3$, such that this metric takes the form

$$ds^2 = \eta_{ab}\underline{\omega}^a \otimes \underline{\omega}^b. \tag{2}$$

b) Using Cartan's first structure equation for the torsion, i.e.

$$\underline{T}^a = \underline{d\omega}^a + \underline{\theta}^a{}_b \wedge \underline{\omega}^b \tag{3}$$

compute the Levi-Civita connection 1-forms $\underline{\theta}^a{}_b$. Recall: the Levi-Civita connection is uniquely determined by the demands that the connection is metric, $\underline{\theta}_{ab} + \underline{\theta}_{ba} = 0$, and torsion free, $\underline{T}^a = 0$.

[HINT – Don't speed up the computation more than warranted: Cartan's structure equation involves a sum over the index $b$, as well as an (antisymmetric) wedge product. In particular pay attention that expressions like $\underline{d}\phi \wedge \underline{d}\phi \equiv 0$.]

c) Using Cartan's second structure equation for the curvature, i.e.

$$\underline{R}^a{}_b = \underline{d\theta}^a{}_b + \underline{\theta}^a{}_c \wedge \underline{\theta}^c{}_b, \tag{4}$$

compute the curvature 2-forms $\underline{R}^a{}_b$.

d) Write down the Riemann and Ricci tensor components in the FRLW *coordniate basis*.