

Congugate Gradients

January 15, 2024

```
[7]: using LinearAlgebra
```

```
[8]: function conjugate_gradients(A, b, n)
    x0 = A \ b
    x = zero(b)
    r = b - A*x
    s = r
    for i in 1:n
        println("iter $i    |e|=$(norm(x-x0))    |r|=$(norm(r))")
        beta = (r' * r) / (s' * A * s)
        x1 = x + beta * s
        # r1 = r - beta * A * s
        r1 = r - A * x1
        alpha = (r1' * r1) / (r' * r)
        s1 = r1 + alpha * s
        x = x1
        r = r1
        s = s1
    end
    return x
end
```

```
[8]: conjugate_gradients (generic function with 1 method)
```

```
[9]: A = randn(10, 10)
    A = A + A'
    A = A*A
```

```
[9]: 10×10 Matrix{Float64}:
 11.4945   1.59873  -4.47139   ...  10.2629   2.16137  -3.74698
  1.59873  12.4136   4.72029           -0.597696 -1.4558  -2.90335
 -4.47139  4.72029  15.1851           5.32712  0.30613  2.60648
 -6.5287   2.42204  -2.52085        -11.842   -1.50928 -1.93249
 -5.24847  0.902092  0.0782056       -11.2405  0.186763 4.63587
  2.87422  -3.09525  -7.86021   ...   5.58672  2.9079  7.52427
 -3.68777  -0.311374 -1.70759        -8.31042  2.60199  1.04627
 10.2629  -0.597696  5.32712        30.003   2.02588  5.39021
  2.16137  -1.4558   0.30613         2.02588  13.4744  5.55399
```

-3.74698 -2.90335 2.60648 5.39021 5.55399 16.8701

```
[10]: b = randn(10)
```

```
[10]: 10-element Vector{Float64}:
```

```
 1.8832900793346092
-0.1706865126759039
-0.0639927479417611
 0.09571205062178016
 0.1613766463457089
 0.8735074376302945
-0.07150222841977424
-0.06225564433391346
 1.108533440962643
-0.9976519795305886
```

```
[11]: x0 = A\b
```

```
[11]: 10-element Vector{Float64}:
```

```
 0.9636376002854024
-0.3618249976050089
 0.7939645003648235
 0.025589873946449963
-0.16797804193509983
 0.38335927939251413
-0.11251169241611686
-0.6451483377441326
-0.13127614067329532
 0.10439272727259402
```

```
[12]: x = conjugate_gradients(A, b, 20)
      norm(x - x0), norm(b - A*x)
```

```
iter 1  |e|=1.5240527248013345  |r|=2.571259057047426=2.571259057047426
iter 2  |e|=1.4442834350134488  |r|=2.1481720201832615=2.148172020183261
iter 3  |e|=1.347371404189048  |r|=2.1658594947826195=2.16585949478262
iter 4  |e|=1.1836635039063552  |r|=1.8759364010727508=1.8759364010727515
iter 5  |e|=1.0035238752783366  |r|=1.35071614088406=1.35071614088406
iter 6  |e|=0.9353076959678551  |r|=0.752878345324781=0.752878345324781
iter 7  |e|=0.7391034904415823  |r|=1.1003338521204054=1.100333852120405
iter 8  |e|=0.4965034907660435  |r|=0.632187299372714=0.6321872993727117
iter 9  |e|=0.4876288321528935  |r|=0.04560081757292288=0.0456008175729222
iter 10 |e|=0.4852535525452267  |r|=0.01868198951544235=0.018681989515442445
iter 11 |e|=5.355895016044437e-11
      |r|=2.6971698001819823e-9=2.697163286555595e-9
iter 12 |e|=3.001543200572167e-14
      |r|=1.586992659053566e-13=1.5814319898642047e-13
iter 13 |e|=2.9052029490791163e-14
```

```
|r|=2.36614331702114e-14=2.676266972361379e-14
iter 14   |e|=2.8302601013784463e-14
|r|=2.2333512325441824e-14=2.230002999102391e-14
iter 15   |e|=2.7769982441376475e-14
|r|=1.9367343898069464e-14=1.9657773761207833e-14
iter 16   |e|=2.770617551834015e-14
|r|=6.545209881380098e-15=1.1702524284689016e-14
iter 17   |e|=2.7698788596305558e-14
|r|=3.1255074356178937e-15=9.821751247852607e-15
iter 18   |e|=2.7574261857901847e-14
|r|=5.734257681444875e-15=1.009407791187898e-14
iter 19   |e|=2.76000660992409e-14
|r|=5.608666370461195e-16=5.41532992766315e-15
iter 20   |e|=2.7607718367384693e-14
|r|=2.7187296232362567e-16=6.262516088343962e-15
```

[12]: (3.4449885695026017e-14, 7.48316840865342e-15)

[]: