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HOMEWORK 2

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Quantum Field Theory II

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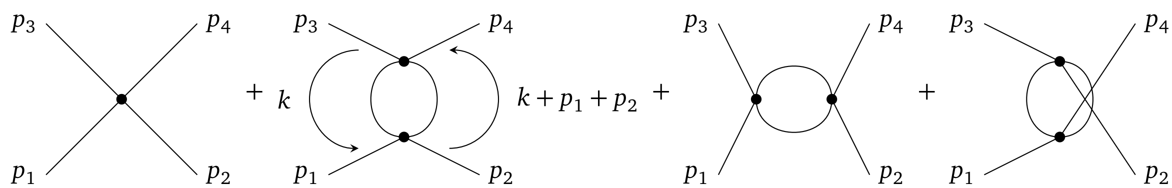
Contents

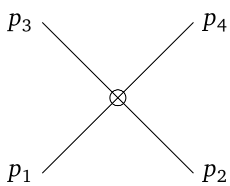
1 Sharp Cutoff Regularizaion

(a) We consider the renomalization of ϕ^4 theory leading to finite physical mass and coupling. At one loop, the renormalized Euclidean action $S_R[\phi] = S[\phi] + \hbar\Delta_1 S[\phi]$ for ϕ^4 theory has two contributions: an action $S[\phi]$ featuring the renormalized parameters (mass m_R and coupling g_R associated to an energy-momentum scale μ) and a counterm action $\hbar\Delta_1 S[\phi]$. Explicitly we have

$$S[\phi] = \int d^4x \left[\frac{1}{2}(\partial\phi)^2 + \frac{m_R^2}{2}\phi^2 + \frac{g_R}{4!}\phi^4\right], \quad \Delta_1 S[\phi] = \int d^4x \left[\frac{B_1}{2}\phi^2 + \frac{C_1}{4!}\phi^4\right]$$

where C_1 and B_1 are UV divergent quantities that are meant to cancel the divergence arising form the one loop corrections to mass and coupling. In what follows we calculate n-point functions using momentum space Feynman rules. We associate different sets of Feynman rules for the S and $\hbar\Delta_1 S[\phi]$. Respectively, their vertices contribute factors g_R ($\circ$) and $\hbar C_1$ ($\otimes$) and their propagators are $\partial^2 - m_R^2$ and $\partial^2 - (B_1\hbar)^2$. Since there is an aditionnal $\hbar$ factor in the counter terms $\hbar\Delta_1 S[\phi]$, their tree level diagrams mix with the S diagrams at one loop order (and the $\hbar\Delta_1 S$ on loop diagram contribute at the truncated two loop order $O(\hbar^2)$). With this mixing in mind, we can approximated the momentum space irreducible 4-point function of momenta p_1, p_2, p_3, p_4 (collectively denoted p_i and all flowing inwards) with the following S diagrams



$+$ 

$$= \Gamma_R^4(p_i) = g_R - \hbar\frac{g_R^2}{2}\big(I(p_1+p_2, m_R) + I(p_1+p_3, m_R) + I(p_1+p_4, m_R)\big) + \hbar C_1 + O(\hbar^2)$$

where I is the loop integral and the factor of $1/2$ is the symmetry factor of the one loop diagrams. The integral I is UV divergent and we regulate it with a sharp cutoff $\pm\Lambda$ on the integration bound to get

$$I(p_1+p_2, m_R) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4}\frac{1}{k^2 + m_R^2}\frac{1}{(k+p_1+p_2)^2 + m_R^2}.$$

The divergence of the integral is more explicit in 4-spherical coordinate with $'z'$ axis along $p_1 + p_2$ ($p = \sqrt{(p_1+p_2)^2}$). In these coordinated, we have the angle measure $\mathrm{d}\Omega$, θ the angle between k and p and $q = \sqrt{k^2}$ leading to the expression

$$I(p_1+p_2, m_R, \Lambda) = \int \mathrm{d}\Omega \int_0^\Lambda \frac{\mathrm{d}q}{(2\pi)^4}\frac{q^3}{q^2 + m_R^2}\frac{1}{q^2 + p^2 + 2\cos(\theta)pq + m_R^2}$$

where we have introduced a sharp momentum cutoff Λ to regulate UV divergence.

(b) Work in progress

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

(k)

(l)

(m)

(n)

(o)

(p)

2 Acknowledgement