Pierre-Antoine Graham

Homework 2

Eduardo Martín-Martínez, Bindiya Arora
Quantum Information

Perimeter Institute for Theoretical Physics
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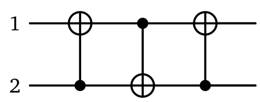
1 Back to basics: quantum circuits

In what follows, we evaluate the matrix expressions representing a quantum circuit unitary acting on a sequence of qubit input. We work in the computational basis $\{|0\rangle, |1\rangle\}$ and use the notation X, Y, Z for the Pauli gates in this basis.

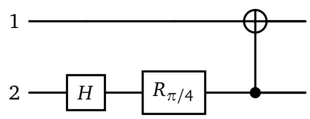
(a) First we consider the conjugation of a `CNOT` by two `CNOT` with control and target qubit reversed:



$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

which exchanges the qubits ($|00\rangle \to |00\rangle, |01\rangle \to |10\rangle, |10\rangle \to |01\rangle, |11\rangle \to |11\rangle$) and constitutes a `SWAP` gate. The matrix expression for the reversed `CNOT` was obtained by writing its action on the computational basis which reads $|00\rangle \to |00\rangle, |01\rangle \to |11\rangle, |10\rangle \to |10\rangle, |11\rangle \to |01\rangle$.

(b) Then we calculate the matrix expression of the entanglement-generating circuit



$$= (1_1 \otimes |0\rangle\langle 0|_2 + X_1 \otimes |1\rangle\langle 1|_2) R_{\pi/4,2} \left(1_1 \otimes \frac{1}{\sqrt{2}}(X_2 + Z_2)\right)$$

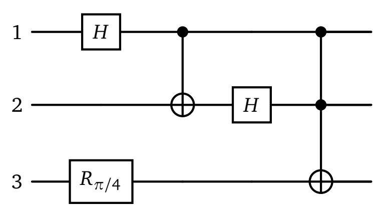
$$= (1_1 \otimes |0\rangle\langle 0|_2 + X_1 \otimes e^{i\pi/4} |1\rangle\langle 1|_2) \left(1_1 \otimes \frac{1}{\sqrt{2}}(X_2 + Z_2)\right)$$

$$= \frac{1}{\sqrt{2}} \left(1_1 \otimes |0\rangle_2 (\langle 0|_2 + \langle 1|_2) + X_1 \otimes e^{i\pi/4} |1\rangle_2 (\langle 0|_2 - \langle 1|_2)\right) = \frac{1}{\sqrt{2}} \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 \\ e^{i\pi/4} & -e^{i\pi/4} \end{pmatrix} \right)$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & e^{i\pi/4} & -e^{i\pi/4} \\ 0 & 0 & 0 & 0 \\ e^{i\pi/4} & -e^{i\pi/4} & 0 & 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & e^{i\pi/4} & -e^{i\pi/4} \\ 0 & 0 & 1 & 1 \\ e^{i\pi/4} & -e^{i\pi/4} & 0 & 0 \end{pmatrix}.$$

If we set the phases to 1, we recover the Bell state mapping $|00\rangle \to (|00\rangle + |11\rangle)/\sqrt{2}$, $|01\rangle \to (|00\rangle - |11\rangle)/\sqrt{2}$, $|10\rangle \to (|01\rangle + |10\rangle)/\sqrt{2}$ and $|11\rangle \to (-|01\rangle + |10\rangle)/\sqrt{2}$.

(c) Finally, we calculate the matrix expression associated with a three-qubit circuit as follows:



$$= \texttt{TOFFOLI}\ R_{\pi/4,3} H_2 (|0\rangle\langle 0|_1 \otimes 1_2 + |1\rangle\langle 1|_1 \otimes X_2) H_1$$

$$= \texttt{TOFFOLI}\ R_{\pi/4,3} \frac{1}{\sqrt{2}} (|0\rangle_1 (\langle 0|_1 + \langle 1|_1) \otimes H_2 + |1\rangle_1 (\langle 0|_1 - \langle 1|_1) \otimes H_2 X_2)$$

$$= \texttt{TOFFOLI}\ R_{\pi/4,3} \frac{1}{2} \left(\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & -1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \right)$$

$$= \texttt{TOFFOLI}\ R_{\pi/4,3} \frac{1}{2} \left(\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 \end{pmatrix} \right)$$

$$= \texttt{TOFFOLI}\ R_{\pi/4,3} \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1_3 & 0 & 0 & 0 \\ 0 & 1_3 & 0 & 0 \\ 0 & 0 & 1_3 & 0 \\ 0 & 0 & 0 & X_3 \end{pmatrix} R_{\pi/4,3} \begin{pmatrix} 1_3 & 1_3 & 1_3 & 1_3 \\ 1_3 & -1_3 & 1_3 & -1_3 \\ 1_3 & 1_3 & -1_3 & -1_3 \\ -1_3 & 1_3 & 1_3 & -1_3 \end{pmatrix}.$$

2 Quantum Adder

(a) Work in progress

(b)

(c)

3 Grover's algorithm on IBM composer

4 Acknowledgement