FEE321 – E.C.T IIA – Oct 2020

Lecture 7: Laplace Transform (1 hr)

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Overview

Today's class looks at Laplace transform and its calculation

- Sample LT properties
- Sample LT calculation

Content

- **Sample LT properties**

Laplace transform[5]

LT properties

1. Linearity

$$\mathcal{L}[a_1 f_1(t) + a_2 f_2(t)] = a_1 \mathcal{L}[f_1(t)] + a_2 \mathcal{L}[f_2(t)] = a_1 F_1(s) + a_2 F_2(s)$$

2. Derivative

$$\mathcal{L}\left[\frac{df(t)}{dt}\right] = sF(s) - f(0^+)$$

3. nth order derivative

$$\mathcal{L}\left[\frac{d^n f(t)}{dt^n}\right] = s^n F(s) - s^{n-1} f(0^+) - \cdots - s f^{n-2}(0^+) - f^{n-1}(0^+)$$

4. Integral

$$\mathcal{L}\left[\int_0^t f(\tau)\, d\tau\right] = \frac{F(s)}{s}$$

Laplace transform[6]

LT properties

5. Real shifting

$$\mathcal{L}[f(t-t_o)u(t-t_o)] = e^{-t_o s}F(s)$$

6. Complex shifting

$$\mathcal{L}[e^{-at}f(t)] = F(s+a)$$

7. Initial value

$$\lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s)$$

8. Final value

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

Laplace transform[7]

LT properties

9. Multiplication by t

$$\mathcal{L}[tf(t)] = -\frac{dF(s)}{ds}$$

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$$

10. Time transformation ($a > 0$, $b \geq 0$)

$$\mathcal{L}[f(at-b)u(at-b)] = \frac{e^{-s\frac{b}{a}}}{a} F\left(\frac{s}{a}\right)$$

11. Convolution, $f_1(t) * f_2\,(t)$

$$\mathcal{L}^{-1}[F_1(s)F_2(s)] = \int_0^t f_1(t-\tau)f_2(\tau)\,d\tau = \int_0^t f_1(\tau)f_2(t-\tau)\,d\tau$$

12. Time periodicity $f(t) = f(t+T), t \geq 0$

$$\mathcal{L}[f(t)] = \frac{1}{1-e^{-sT}} F_1(s)$$

$$F_1(s) = \int_0^T f(t)\,e^{-st}dt$$

Content

- Sample LT properties
- **Sample LT calculation**

Laplace transform[8]

Examples

1. Unit step, $f(t) = u(t)$

$$\mathcal{L}[f(t)] = \int_0^{\infty} f(t)e^{-st}dt = \int_0^{\infty} u(t)e^{-st}dt = \int_0^{\infty} 1 \cdot e^{-st}dt$$

$$= -\frac{1}{s}\{e^{-st}\}_0^{\infty} = -\frac{1}{s}\left\{e^{-\sigma t}e^{-j\omega t}\right\}_0^{\infty}$$

$$= \frac{1}{s}\left\{1 - e^{-\sigma\infty}e^{-j\omega\infty}\right\}$$

Exponential terms only vanish under some condition, i.e. $\sigma > 0$

$$= \frac{1}{s} \qquad \text{ROC: } (Re[s] = \sigma) > 0$$

LT of the function only exists in the region of convergence (ROC). c in the ILT chosen so that the integration path lies in the ROC

Thus $u(t) \rightleftharpoons \frac{1}{s}$

Summary

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QUESTIONS?