P o C S What's The Story?

Principles of Complex Systems, Vols. 1, 2, & 3D
CSYS/MATH 300, 303, & 394
University of Vermont, Fall 2022
Solutions to Assignment 15
Dangerous Beans

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- Please use Overleaf for writing up your project.
- Build your paper using:
  https://github.com/petersheridandodds/universal-paper-template
- Please use Github and Gitlab to share the code and data things you make.
- For this first assignment, just getting the paper template up is enough.

Solution:

The template for this paper can be found at:
https://github.com/P-Harvey/papers/tree/main/2023-05supply-optimization

□

1. Come up with some rich, text-based stories for analysis.

   For example: One (longish) book, or a book series, or a TV series.

   Data would be the original text (books), subtitles, screenplay, or scripts (TV series).

   - You must be able to obtain the full text.
   - You will want something with at least around 10^5 words. More than 10^6 would be great.
   - Transcripts of shows may be good for extracting temporal character interaction networks.

   Please talk about possibilities with others in the class.

   For this assignment, simply list at least one possibility, noting the approximate text size in number of words.

Solution:

For this problem I have selected the book "Blood Meridian" by Cormac McCarthy[1], which has greater than 10^4 unique words after pre-processing the text for punctuation and white-space.

The initial examination of this text can be found at https://github.com/PHarvey/csys303_assignments/blob/642aae80fb5d5627a5acd203fe1a0374310c8fc/plharvey_15/plharvey_15.ipynb.

□

2. Tokunaga's law is statistical but we can consider a rigid version. Take $T_1 = 2$ and $R_T = 2$ and draw an example network of order $\Omega = 4$ with these parameters.

   Please take some effort to make your network look somewhat like a river network.

   Solution: □

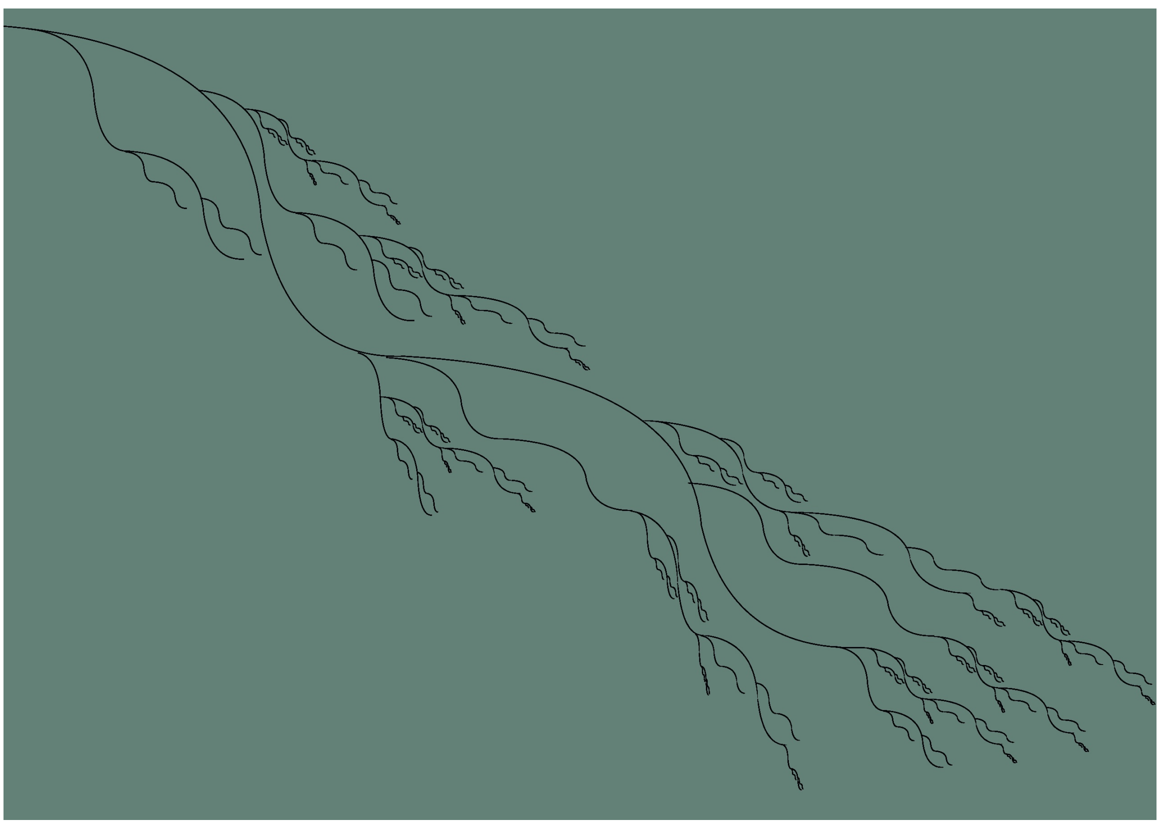

Figure 1: Branching network with $T_1 = 2$ and $R_T = 2$ of order $\Omega = 4$.

3. Show $R_s = R_\ell$. In other words show that Horton's law of stream segments matches that of main stream lengths, and do this by showing they imply each other.

Solution:

Let $R_s = \frac{\bar{s}_{\omega+1}}{\bar{s}_\omega}$

then:

$$
\begin{aligned}
\bar{s}_{\omega+1} &= \bar{s}_\omega R_s && (1)\\
&= \bar{s}_{\omega-1}(R_s)^2 && (2)\\
&= \bar{s}_{\omega-2}(R_s)^3 && (3)\\
&\cdots && (4)\\
&= \bar{s}_1(R_s)^\omega && (5)\\
& && (6)
\end{aligned}
$$

In other words: if we take this as a bifurcation ratio ($R_b = \frac{1}{\alpha}$ also expressed as $R_b = \frac{n_i}{n_i+1}$), and raise it to the $N-1^{\text{th}}$ power, then we have a function that lineally maps x to n ($R_b^{N-1} \therefore n(x) = ax^b$).

If we take the average length of order ω:

$$
\begin{aligned}
\bar{\ell}_\omega &= \sum_{i=1}^{\omega} \bar{s}_i && (7)\\
&= \bar{s}_1 + \cdots + \bar{s}_\omega && (8)\\
&= \sum_{i=1}^{\omega} R_s^{i-1}\bar{s}_1 && (9)\\
\therefore \bar{\ell}_\omega &= \bar{s}_1 \frac{1-R_s^\omega}{1-R_s} \text{for} R_s \neq 1 && (10)\\
& && (11)
\end{aligned}
$$

If we assume that $R_\ell = \frac{\bar{\ell}_{\omega+1}}{\bar{\ell}_\omega}$, then we have $\bar{s}_\omega = \bar{\ell}_\omega - \bar{\ell}_{\omega-1}$

$$
\begin{align}
\frac{\bar{s}_{\omega+1}}{\bar{s}_{\omega}} &= \frac{\bar{\ell}_{\omega+1} - \bar{\ell}_{\omega}}{\bar{\ell}_{\omega} - \bar{\ell}_{\omega-1}} \tag{12}\\
&= \frac{\bar{\ell}_1 R_\ell^{\omega} - \bar{\ell}_1 R_\ell^{\omega-1}}{\bar{\ell}_1 R_\ell^{\omega-1} - \bar{\ell}_1 R_\ell^{\omega-2}} \tag{13}\\
&= \frac{R_\ell^{\omega} - R_\ell^{\omega-1}}{R_\ell^{\omega-1} - R_\ell^{\omega-2}} \tag{14}\\
&= \frac{R_\ell^{\omega-1}(R_\ell - 1)}{R_\ell^{\omega-2}(R_\ell - 1)} \tag{15}\\
&= R_\ell \tag{16}\\
\therefore R_s &= R_\ell \tag{17}\\
& \tag{18}
\end{align}
$$

$\square$

References

[1] C. McCarthy, Blood meridian. Picador, 1985.