

$$\frac{n_k}{n_{k-1}} = \frac{(k-1)(1-p)}{1+(1-p)k}$$

$$\Gamma(k) = (k-1)!$$

$$n_k = \left[\frac{(k-1)(1-p)}{1+(1-p)k} \right] \left[\frac{(k-2)(1-p)}{1+(1-p)(k-1)} \right] n_{k-2}$$

\uparrow n_{k-1}
 \downarrow
 $\left[\frac{(k-3)(1-p)}{1+(1-p)(k-2)} \right] n_{k-3}$
 \uparrow

$$\dots \left[\frac{(2)(1-p)}{1+(1-p)3} \right] n_1$$

$$\Gamma(x+1) = x \Gamma(x)$$

$$x = n+1 \quad \Gamma(n+1) = n \Gamma(n) = \dots = n! \quad \Gamma(1) = 1$$

example $0 < z < 1$

$$\begin{aligned} & (1+zk)(1+z(k-1)) \dots (1+z1) \\ &= z^k \left(\frac{1}{z} + k \right) \left(\frac{1}{z} + k-1 \right) \dots \left(\frac{1}{z} + 1 \right) = z^k \frac{\left(\frac{1}{z} + k \right) \left(\frac{1}{z} + k-1 \right) \dots}{\frac{1}{z} \cdot \left(\frac{1}{z} - 1 \right) \left(\frac{1}{z} - 2 \right) \dots} \\ & \quad \text{differ by 1} \\ &= z^k \frac{\Gamma\left(\frac{1}{z} + k + 1\right)}{\Gamma\left(\frac{1}{z} + 1\right)}. \end{aligned}$$