

$$\begin{aligned}
 1) \max_{x_{ij}} \quad & \sum_{i=1}^n \sum_{j=1}^m R_{ij} x_{ij} \\
 \text{s.t.:} \quad & \sum_{j=1}^m R_{ij} x_{ij} \leq b_i \\
 & \sum_{i=1}^n x_{ij} \leq q_j \\
 & x_{ij} \geq 0 \quad \forall i, j
 \end{aligned}$$

$$\begin{aligned}
 2) \max \quad & Z = R^T x \\
 \text{s.t.:} \quad & Ax = b \\
 & x \geq 0
 \end{aligned}
 \quad R = (1 \ 0.75 \ 5 \ 0.5 \ 0.5 \ 2 \ 0.5 \ 3 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)^T$$

$$x = (x_{A1} \ x_{A2} \ x_{A3} \ x_{B1} \ x_{B2} \ x_{B3} \ x_{C1} \ x_{C2} \ x_{C3} \ z_1 \ z_2 \ \dots \ z_6)^T$$

$$x_{A1} + 0.75x_{A2} + 5x_{A3} + z_1 = 200$$

$$0.5x_{B1} + 0.5x_{B2} + 2x_{B3} + z_2 = 150$$

$$0.5x_{C1} + 3x_{C2} + x_{C3} + z_3 = 180$$

$$x_{A1} + x_{B1} + x_{C1} + z_4 = 150$$

$$x_{A2} + x_{B2} + x_{C2} + z_5 = 90$$

$$x_{A3} + x_{B3} + x_{C3} + z_6 = 80$$

$$b = \begin{bmatrix} 200 \\ 150 \\ 180 \\ 150 \\ 90 \\ 80 \end{bmatrix}$$

$$A = \begin{bmatrix}
 1 & 0.75 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & & & \\
 0 & 0 & 0 & 0.5 & 0.5 & 2 & 0 & 0 & 0 & 0 & & 1 & & \\
 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 3 & 1 & 0 & & & 1 & \\
 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & & & & \\
 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & & & & \\
 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & & & &
 \end{bmatrix}$$

$$4) \begin{array}{l} \max_{x_{ij}} Z = R^T x \\ Ax \leq b \\ x \geq 0 \end{array} \quad \xrightarrow{\text{dual}} \quad \begin{array}{l} \min_{x_{ij}, z_i} Z_p = (-R^T)x \\ Ax = b \\ x \geq 0 \end{array}$$

$$\xrightarrow{\text{dual}} \begin{array}{l} \max_{y_i} Z_D = b^T y \\ A^T y \leq (-R) \end{array}$$

$$\Rightarrow \max_{y_i} Z_D = [200 \ 150 \ 180 \ 150 \ 90 \ 80] \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix}$$

$$\max_{y_i} Z = 200y_1 + 150y_2 + 180y_3 + 150y_4 + 90y_5 + 80y_6$$

$$\Rightarrow A^T y \leq (-R)$$

$$\rightarrow A^T = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0.75 & 0 & 0 & 0 & 1 & 0 \\ 5 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0.5 & 0 & 1 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0.5 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow y_1 + y_4 \leq -1$$

$$0.75y_1 + y_5 \leq -0.75$$

$$5y_1 + y_6 \leq -5$$

$$0.5y_2 + y_4 \leq -0.5$$

$$0.5y_2 + y_5 \leq -0.5$$

$$2y_2 + y_6 \leq -2$$

$$0.5y_3 + y_4 \leq -0.5$$

$$3y_3 + y_5 \leq -3$$

$$y_3 + y_6 \leq -1$$

$$\cancel{y_i \geq 0}$$

$$y_1, y_2, y_3, y_4, y_5, y_6 \leq 0$$