A Benchmark-Suite of Real-World Constrained Multi-Objective Optimization Problems and some Baseline Results

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Abstract

Generally, Synthetic Benchmark Problems (SBPs) are utilized to assess the performance of metaheuristics. However, these SBPs may include various unrealistic properties. As a consequence, performance assessment may lead to underestimation or overestimation. To address this issue, few benchmark suits containing real-world problems have been proposed for all kinds of metaheuristics except for Constrained Multi-objective Optimization Evolutionary Algorithms (CMOEAs). To fill this gap, we develop a benchmark suite of Real-world Constrained Multi-objective Optimization Problems (RWCMOPs) for performance assessment of CMOEAs. This benchmark suite includes 50 problems collected from various streams of research. We also present the baseline results of this benchmark suite by using state-of-the-art algorithms. Besides, for comparative analysis, a ranking scheme is also proposed.

Keywords: Metaheuristics, Performance Assessment, Real-World Problems, Multi-Objective Constrained Optimization, Benchmark-Suite, Ranking Scheme

1. Introduction

During the past decades, Constrained Multi-objective Optimization Problems (CMOPs) has gained a lot of attention since the majority of optimization problems of real-world applications contain constraints. Generally, a CMOP has multiple conflicting objectives with one or more constraints that demand to optimize these objectives while satisfying the constraints simultaneously. In CMOPs, Evolutionary Algorithms (EAs) have to provide proper tradeoffs among the conflicting objectives while satisfying all constraints, which is a great challenge to them [1, 2].

Without losing generality, a CMOP can be defined mathematically:

$$\text{Minimize } f_1(\bar{x}), f_2(\bar{x}), ..., f_M(\bar{x}), \tag{1}$$

$$\text{Subject to } g_i(\bar{x}) \leq 0, i \in \{1, 2, ..., ng\}$$

$$h_j(x) = 0, j \in \{ng+1, ng+2, ..., ng+nh\}$$

$$L_k \leq x_k \leq U_k, k \in \{1, ..., D\}$$

where f_i represents the i-th objective function, M is the total number of the conflicting objective functions, $\bar{x}$ $(= (x_1, x_2, ...x_D)^T)$ is a solution vector of length D, L_k and U_k are the lower and upper bound of the search-space at k-th dimension. Here, solution $\bar{x}$ can be of two types: feasible and infeasible solution. The feasible solutions satisfy

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all $(ng + nh)$ constraints of the given problem and a set of all possible feasible solutions within the bound of the search-space creates a subspace in the search-space, called a feasible region. While a solution that does not lie in the feasible region is called an infeasible solution. Similarly, a set of all possible infeasible solutions formed an infeasible subspace in the search-space.

The constraint violation of a solution, $\bar{x}_i$, over a j-th constraint can be calculated by the following equation:

$$\nu_j = \begin{cases} max\left(0, g_j(\bar{x}_i)\right), & j \leq ng \\ max\left(0, |h_j(\bar{x}_i)| - \epsilon\right), & ng < j \leq (ng + nh) \end{cases}, \tag{2}$$

where ν_j is the value of constraint violation for $\bar{x}_i$ on j-th constraint and ϵ is a very small value for relaxing the equality constraints. On the basis of this definition, a solution can be called as a feasible solution if that solution has zero constraint violation at each constraint or the sum of total constraint violations of that solution is zero, i.e.

$$CV(\bar{x}_i) = \sum_{i=1}^{ng+nh} \nu_i = 0, \tag{3}$$

where $CV(\bar{x}_i)$ is the total constraint violation at solution $\bar{x}_i$. In the case of a nonzero total constraint violation, the solution is termed as an infeasible solution.

For given two solutions $\bar{a}$ and $\bar{b}$ in Constrained Multi-objective Optimization (CMOO), $\bar{a}$ constrained Pareto dominates $\bar{b}$ (can be denoted as $\bar{a} \prec_c \bar{b}$), if and only if

1. $f_i(\bar{b}) \geq f_i(\bar{a}) \; \forall \; i \in \{1, 2, ..., M\}$,
2. $f_j(\bar{b}) > f_j(\bar{a}) \; \exists \; i \in \{1, 2, ..., M\}$, and
3. $CV(\bar{b}) \geq CV(\bar{a})$.

Here, a feasible solution $\bar{x}^*$ can be said constrained Pareto optimal solution if all possible feasible solutions do not Pareto dominates $\bar{x}^*$. The set of all possible constrained Pareto solutions is termed as Pareto set, and the image formed by this Pareto set on objective space is called Pareto front.

In the majority of CMOPs, some solutions of bound-constrained Pareto front become infeasible and looses its optimility due to some constraints. Therefore, CMOPs cannot be solved by using Multi-objective Optimization Evolutionary Algorithms (MOEAs). We need to incorporate a Constraint Handling Technique (CHT) in the framework of the MOEAs to handle the constraints. Several CHTs have been utilized with MOEAs in the literature, such as constrained dominance principle [3], self-adaptive penalty function [4], and stochastic ranking [5].

As compared to bound-constrained Pareto front, CMOPs can be divided into four types [6].

1. ***Type I:*** In this case, the constrained Pareto front is the same as bound-constrained Pareto front, i.e., both Pareto fronts have the same Pareto set.
2. ***Type II:*** In this case, the constrained Pareto set is the subset of the bound-constrained Pareto set.
3. ***Type III:*** In this case, some portions of the constrained Pareto front are the same as bound-constrained Pareto front, i.e., the interSection of both Pareto sets is not a null set.
4. ***Type IV:*** In this case, the interSection of both Pareto set is a null set, i.e., there is no common region in both Pareto fronts.

While solving the CMOPs, there is a need for proper balance between minimizing the objective functions and minimizing the constraint violations. Consequently, we can characterize the above-mentioned types of CMOPs according to their required level of balance between minimizing objective functions and minimizing constraint violations. From ***Type I*** to ***Type IV***, the required level is gradually increased. Therefore, in case of ***Type I*** CMOPs, there is no need of minimizing constraint violation to calculate the constrained Pareto front. While in case of ***Type IV*** CMOPs, more focus is required on minimizing the constraint violations as compared to objective functions.

Generally, theoretical evaluation of the performance of algorithms is difficult due to their stochastic behavior. This is the major reason behind the use of benchmark problems to assess the performance of algorithms empirically. SBPs have been usually used in the performance assessment of the algorithms. The main reasons are that performance evaluation on a real-world application requires domain knowledge of that real-world application and assessment on one problem cannot effectively demonstrate the generality of an algorithm.

To cope with this issue, several test-suites having artificial test problems have been designed for CMOPs, see, for example, MFs [6], CFs [7], C-DTLZs [8], SRN [9], TNK [10], OSY [11], and CTPs [12]. There are several advantages to these artificial test suites. They can be easily represented by simple mathematical equations and calculations of objective functions and constraints are computationally cheap and usually fast. Pareto front of these problems is known. Thus, different indicators can be used to represent the experimental results. Most of these problems are scalable to a different number of objectives, the number of decision variables, and the number of constraints. Despite these all advantages, these test problems have been suffered from serious drawbacks. Usually, they have synthetic properties that maybe never appeared in real-world applications [13, 14]. Consequently, the performance of CMOEAs can become overrated on some problems and underrated on other problems. For example, most of the problems of these test-suites are ***Type-I*** or ***Type-II*** having regular Pareto front, which can be easily calculated by some of the decomposition-based algorithms [13] (MOEA-D [15] and NSGAIII [8]). Since artificial test problems may contain undesirable characteristics, there is a requirement of a test-suite of problems of real-world applications to assess the performance of newly developed algorithms more reliably and effectively. In literature, several benchmark suites have been proposed for assessing the performance of the different class of optimization algorithms, see, for example, [16, 17, 18]. However, a benchmark suite of RWCMOPs has not been existed, where problems have advantages similar to artificial test problems such as easy to implement, computationally cheap, etc.

To overcome the above-mentioned issues, an easy-to-use test-suite having RWCMOPs is proposed for assessing the performance of CMOPs in this paper. This test-suite contains 50 RWCMOPs collected from several areas from mechanical design problems to power system problems. The proposed test-suite provides a diverse set of computationally cheap problems where all problems are implemented by simple mathematical equations. In contrast, the difficulty level of these problems has been maintained at different difficulty levels from moderate to high levels. Additionally, these problems have not unrealistic features as compared to SBPs. However, we do not claim that the proposed test problems must always have better properties than existing synthetic or artificial problems in terms of the performance assessment of CMOEAs. We develop this test suite to provide a better tool for conducting the performance assessment of CMOEAs over problems of real-world applications in a more realistic way.

The main contributions of this work can be summarized as follows:

1. A test-suite of 50 RWCMOPs is proposed where problems are collected from different scientific and engineering fields.
2. In this paper, we have described all RWCMOPs mathematically. Therefore, there is no need to refer to each original article to implement these problems as this paper is self-contained.
3. Moreover, we have implemented this test-suite on MATLAB and other languages and uploaded it on the official GITHUB page. Researchers can easily download this test-suite for examining their CMOEAs on RWCMOPs with minimum assistance.
4. The performance of seven state-of-the-art algorithms is assessed on these problems and some baseline results are included in this study.
5. A ranking scheme is also proposed to compare the performance of CMOEAs on this test suite.

The remaining parts of this paper are organized as follows. In Section 2, we describe the 50 RWCMOPs mathematically. In Section 3, experimental settings and a ranking scheme are presented for conducting the experiment for the performance assessment of CMOEAs on the proposed test-suite. In Section 4, the baseline results of this test-suite calculated by seven state-of-the-art algorithms are reported. Finally, Section 5 concludes the works of this paper.

2. Real-World Constrained Multi-objective Optimization Test-suite

In this section, the RWCMOPs are described. These problems are classified into five parts according to their domain: mechanical design problems; chemical engineering problems; process design and synthesis problems; power electronics problems; and power system problems.

2.1. Mechanical Design Problems

From mechanical design applications, we have collected 21 RWCMOPs where M, D, and ng vary from 2 to 5, 2 to 10, and 1 to 11, respectively.

2.1.1. Pressure Vessel Design (RCM01) [19]

Minimize :

$$f_1 = 1.7781 z_2 x_3^2 + 0.6224 z_1 x_3 x_4 + 3.1661 z_1^2 x_4 + 19.84 z_1^2 x_3 \tag{4}$$

$$f_2 = -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 \tag{5}$$

subject to :

$$g_1(\bar{x}) = 0.00954 x_3 \le z_2,$$

$$g_2(\bar{x}) = 0.0193 x_3 \le z_1,$$

where :

$$z_1 = 0.0625 x_1,$$
$$z_2 = 0.0625 x_2.$$

with bounds :

$$10 \le x_4, x_3 \le 200$$
$$1 \le x_2, x_1 \le 99 \text{ (integer variables).}$$

2.1.2. Vibrating Platform Design (RCM02) [20]

Minimize:

$$f_1 = -\frac{\pi}{2L^2}\sqrt{\frac{EI}{\mu}} \tag{6}$$

$$f_2 = 2bL\left(c_1 d_1 + c_2(d_2 - d_1) + c_3(d_3 - d_2)\right) \tag{7}$$

subject to:

$$g_1 = \mu L - 2800 \le 0,$$

$$g_2 = d_1 - d_2 \le 0,$$

$$g_3 = d_2 - d_1 - 0.15 \le 0,$$

$$g_4 = d_2 - d_3 \le 0,$$

$$g_5 = d_3 - d_2 - 0.01 \le 0$$

where,

$$EI = \frac{2b}{3}\left(E_1 d_1^3 + E_2(d_2^3 - d_1^3) + E_3(d_3^3 - d_2^3)\right),$$
$$\mu = 2b\left(\rho_1 d_1 + \rho_2(d_2 - d_1) + \rho_3(d_3 - d_2)\right)$$
$$\rho_1 = 100, \quad \rho_2 = 2770, \rho_3 = 7780,$$
$$E_1 = 1.6, \qquad E_2 = 70, E_3 = 200,$$
$$c_1 = 500, \quad c_2 = 1500, c_3 = 800$$

with bounds:

$$0.05 \le d_1 \le 0.5$$
$$0.2 \le d_2 \le 0.5$$
$$0.2 \le d_3 \le 0.6$$
$$0.35 \le b \le 0.5$$
$$3 \le L \le 6$$

2.1.3. Two Bar Truss Design (RCM03) [21]

Minimize:

$$f_1(x) = x_1\sqrt{16 + x_3^2} + x_2\sqrt{1 + x_3^2}, \tag{8}$$

$$f_2(x) = \frac{20\sqrt{16 + x_3^2}}{x_3 x_1} \tag{9}$$

subject to:

$$g_1(x) = f_1(x) - 0.1 \leq 0,$$

$$g_2(x) = f_2(x) - 10^5 \leq 0,$$

$$g_3(x) = \frac{80\sqrt{1 + x_3^2}}{x_3 x_2} - 10^5 \leq 0$$

with bounds:

$$10^{-5} \leq x_1 \leq 100,$$
$$10^{-5} \leq x_2 \leq 100,$$
$$1 \leq x_3 \leq 3$$

2.1.4. Welded Beam Design (RCM04) [22]

Minimize:

$$f_1(x) = 1.10471x_1^2 x_2 + 0.04811x_3 x_4(14 + x_2), \tag{10}$$

$$f_2(x) = \frac{4PL^3}{Ex_4 x_3^3} \tag{11}$$

subject to:

$$g_1(x) = \tau(x) - \tau_{max} \leq 0,$$

$$g_2(x) = \sigma(x) - \sigma_{max} \leq 0,$$

$$g_3(x) = x_1 - x_4 \leq 0,$$

$$g_4(x) = P - P_c(x) \leq 0,$$

where,

$$\tau(x) = \sqrt{(\tau')^2 + \frac{2\tau'\tau'' x_2}{2R} + (\tau'')^2},$$

$$\tau' = \frac{P}{\sqrt{2}x_1 x_2},$$

$$M = P\left(L + \frac{x_2}{2}\right),$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}$$

$$J = 2\left(\sqrt{2}x_1 x_2\left(\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right)\right),$$

$$\sigma(x) = \frac{6PL}{x_4 x_3^2}$$

$$P_c(x) = \frac{4.013E\sqrt{\frac{x_3^2 x_6^4}{36}}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right),$$

$$P = 6000,$$

$$L = 14,$$

$$E = 30 \times 10^6,$$

$$\tau_{max} = 13600.$$

$$\sigma_{max} = 30,000.$$

with bounds:

$$0.125 \leq x_1 \leq 5,$$
$$0.1 \leq x_2 \leq 10,$$
$$0.1 \leq x_3 \leq 10,$$
$$0.125 \leq x_4 \leq 5.$$

2.1.5. Disc Brake Design (RCM05) [23]

Minimize:

$$f_1(x) = 4.9 \times 10^{-5}\left(x_2^2 - x_1^2\right)(x_4 - 1), \tag{12}$$

$$f_2(x) = 9.82 \times 10^6\left(\frac{x_2^2 - x_1^2}{x_3 x_4\left(x_2^3 - x_1^3\right)}\right), \tag{13}$$

subject to:

$$g_1(x) = 20 - (x_2 - x_1) \leq 0,$$

$$g_2(x) = \frac{x_3}{3.14\left(x_2^2 - x_1^2\right)} - 0.4 \leq 0,$$

$$g_3(x) = \frac{2.22 \times 10^{-3} x_3\left(x_2^3 - x_1^3\right)}{\left(x_2^2 - x_1^2\right)^2} - 1 \leq 0,$$

$$g_4(x) = 900 - 2.66 \times 10^{-2}\frac{x_3 x_4\left(x_2^3 - x_1^3\right)}{(x_2^2 - x_1^2)} \leq 0$$

with bounds:

$$55 \leq x_1 \leq 80$$
$$75 \leq x_2 \leq 110$$
$$1000 \leq x_3 \leq 3000$$
$$11 \leq x_4 \leq 20.$$

2.1.6. *Speed Reducer Design (RCM06) [24]*

Minimize:

$$
\begin{aligned}
f_1(x) =& 0.7854 x_1 x_2^2 \left(\frac{10x_3^2}{3} + 14.933x_3 - 43.0934 \right) \\
& - 1.508x_1(x_6^2 + x_7^2) + 7.477(x_6^3 + x_7^3) \\
& + 0.7854(x_4 x_6^2 + x_5 x_7^2)
\end{aligned}
\tag{14}
$$

$$
f_2(x) = \frac{\sqrt{\left(\frac{745x_4}{x_2x_3}\right)^2 + 1.69 \times 10^7}}{0.1x_6^3}
\tag{15}
$$

subject to:

$$
\begin{aligned}
g_1(x) &= \frac{1}{x_1x_2^2x_3} - \frac{1}{27} \leq 0, \\
g_2(x) &= \frac{1}{x_1x_2^2x_3^2} - \frac{1}{397.5} \leq 0, \\
g_3(x) &= \frac{x_4^3}{x_2x_3x_6^4} - \frac{1}{1.93} \leq 0, \\
g_4(x) &= \frac{x_5^3}{x_2x_3x_7^4} - \frac{1}{1.93} \leq 0, \\
g_5(x) &= x_2x_3 - 40 \leq 0, \\
g_6(x) &= \frac{x_1}{x_2} - 12 \leq 0, \\
g_7(x) &= -\frac{x_1}{x_2} + 5 \leq 0, \\
g_8(x) &= 1.9 - x_4 + 1.5x_6 \leq 0, \\
g_9(x) &= 1.9 - x_5 + 1.1x_7 \leq 0, \\
g_{10}(x) &= f_2(x) - 1300 \leq 0, \\
g_{11}(x) &= \frac{\sqrt{\left(\frac{745x_5}{x_2x_3}\right)^2 + 1.575 \times 10^8}}{0.1x_7^3} - 110 \leq 0.
\end{aligned}
$$

with bounds:

$$
\begin{aligned}
& 2.6 \leq x_1 \leq 3.6 \\
& 0.7 \leq x_2 \leq 0.8 \\
& x_3 \in \{17, ..., 28\}(integer) \\
& 7.3 \leq x_4 \leq 8.3 \\
& 7.3 \leq x_5 \leq 8.3 \\
& 2.9 \leq x_6 \leq 3.9 \\
& 5 \leq x_7 \leq 5.5
\end{aligned}
$$

2.1.7. Gear Train Design (RCM07) [25]

Minimize:

$$f_1(x) = \left|6.931 - \frac{x_3 x_4}{x_1 x_2}\right|, \tag{16}$$

$$f_2(x) = max\{x_1, x_2.x_3, x_4\} \tag{17}$$

subject to:

$$g_1(x) = \frac{f_1(x)}{6.931} - 0.5 \leq 0$$

with bounds:

$$x_1.x_2, x_3, x_4 \in \{12, ..., 60\}(interger)$$

2.1.8. Car Side Impact Design (RCM08) [8]

Minimize:

$$f_1(x) = 1.98 + 4.9x_1 6.67x_2 + 6.98x_3 + 4.01x_4 + 1.78x_5 + 10^{-5}x_6 + 2.73x_7, \tag{18}$$

$$f_2(x) = 4.72 - 0.5x_4 - 0.19x_2x_3, \tag{19}$$

$$f_3(x) = 0.5\,(V_{MBP}(x) + V_{FD}(x)) \tag{20}$$

subject to:

$$g_1(x) = -1 + 1.16 - 0.3717x_2x_4 - 0.0092928x_3 \leq 0,$$

$$g_2(x) = -0.32 + 0.261 - 0.0159x_1x_2 - 0.06486x_1 - 0.019x_2x_7 + 0, 0144x_3x_5 + 0.0154464x_6 \leq 0,$$

$$g_3(x) = -0.32 + 0.74 - 0.61x_2 - 0.031296x_3 - 0.031872x_7 + 0.227x_2^2 \leq 0,$$

$$\begin{aligned} g_4(x) = &- 0.32 + 0.214 + 0.00817x_5 - 0.045195x_1 - 0.0135168x_1 + 0.03099x_2x_6 - 0.018x_2x_7 \\ &+ 0.007176x_3 + 0.023232x_3 - 0.00364x_5x_6 - 0.018x_2^2 \leq 0, \end{aligned}$$

$$g_5(x) = -32 + 33.86 + 2.95x_3 - 5.057x_1x_2 - 3.795x_2 - 3.4431x_7 + 1.45728 \leq 0,$$

$$g_6(x) = -32 + 28.98 + 3.818x_3 - 4.2x_1x_2 + 1.27296x_6 - 2.68065x_7 \leq 0,$$

$$g_7(x) = -32 + 46.36 - 9.9x_2 - 4.4505x_1 \leq 0,$$

$$g_8(x) = f_1(x) - 4 \leq 0,$$

$$g_9(x) = V_{MBP} - 9.9 \leq 0,$$

$$g_{10}(x) = V_{FD}(x) - 15.7 \leq 0$$

where,

$$V_{MBP}(x) = 10.58 - 0.674x_1x_2 - 0.67275x_2,$$

$$V_{FD}(x) = 16.45 - 0.489x_3x_7 - 0.843x_5x_6$$

with bounds:

$$0.5 \leq x_1 \leq 1.5$$
$$0.45 \leq x_2 \leq 1.35$$
$$0.5 \leq x_3 \leq 1.5$$
$$0.5 \leq x_4 \leq 1.5$$
$$0.875 \leq x_5 \leq 2.625$$
$$0.4 \leq x_6 \leq 1.2$$
$$0.4 \leq x_7 \leq 1.2$$

2.1.9. *Four Bar Plane Truss (RCM09) [26]*

The minimization of the mass and the compliance of a four bar plane truss respectively described by the two objective functions:

$$f_1(x) = L\left(2x_1 + \sqrt{(2)}x_2 + \sqrt{(2)}x_3 + x_4\right), \tag{21}$$

$$f_2(x) = \frac{FL}{E}\left(\frac{2}{x_1} + \frac{2\sqrt{(2)}}{x_2} - \frac{2\sqrt{(2)}}{x_3} + \frac{2}{x_4}\right), \tag{22}$$

with bounds:

$$\begin{aligned}
&\frac{F}{\sigma} \leq x_1 \leq 3\frac{F}{\sigma}\\
&\sqrt{2}\frac{F}{\sigma} \leq x_2 \leq 3\frac{F}{\sigma}\\
&\sqrt{2}\frac{F}{\sigma} \leq x_3 \leq 3\frac{F}{\sigma}\\
&\frac{F}{\sigma} \leq x_4 \leq 3\frac{F}{\sigma}
\end{aligned}$$

where,

$$F = 10kN,\ \ E = 2\times 10^5 kN/cm^2,\ \ L = 200cm,\ \ \sigma = 10kN/cm^2.$$

2.1.10. *Two Bar Plane Truss (RCM10)*

Minimize,

$$f_1(x) = 2\rho h x_2\sqrt{1 + x_1^2}, \tag{23}$$

$$f_2(x) = \frac{\rho h(1 + x_1^2)^{1.5}(1 + x_1^4)^{0.5}}{2\sqrt{2}Ex_1^2 x_2}, \tag{24}$$

subject to:

$$\begin{aligned}
g_1 &= \frac{P(1 + x_1)(1 + x_1^2)^{0.5}}{2\sqrt{2}x_1 x_2} - \sigma_0 \leq 0,\\
g_2 &= \frac{P(-x_1 + 1)(1 + x_1^2)^{0.5}}{2\sqrt{2}x_1 x_2} - \sigma_0 \leq 0
\end{aligned}$$

with bounds:

$$\begin{aligned}
&0.1 \leq x_1 \leq 2,\\
&0.5 \leq x_2 \leq 2.5
\end{aligned}$$

where,

$$\rho = 0.283lb/in^3,\ \ h = 100in,\ \ P = 104lb,\ \ E = 3\times 10^7 lb/in^2,$$

$$\sigma_0 = 2\times 10^4 lb/in^2,\ \ A_{min} = 1in^2.$$

2.1.11. Water Resources Management (RCM11)

Minimize :

$$f_1 = 106780.37\,(x_2 + x_3) + 61704.67, \tag{25}$$

$$f_2 = 3000x_1, \tag{26}$$

$$f_3 = 2.62314586 \times 10^3 x_2, \tag{27}$$

$$f_4 = 572250e^{-3.975x_2+9.9x_3+2.74}, \tag{28}$$

$$f_5 = 25\left(\frac{1.39}{x_1x_2} + 4940x_3 - 80\right). \tag{29}$$

subject to :

$$
\begin{aligned}
g_1 &= -1 + \left(\frac{0.00139}{x_1x_2} + 4.94x_3 - 0.08\right),\\
g_2 &= -1 + \left(\frac{0.000306}{x_1x_2} + 1.082x_3 - 0.0986\right),\\
g_3 &= -50000 + \left(\frac{12.307}{x_1x_2} + 49408.24x_3 + 4051.02\right),\\
g_4 &= -16000 + \left(\frac{12.098}{x_1x_2} + 8046.33x_3 - 696.71\right),\\
g_5 &= -10000 + \left(\frac{2.138}{x_1x_2} + 7883.39x_3 - 705.04\right),\\
g_6 &= -2000 + \left(0.417x_1x_2 + 1721.26x_3 - 136.54\right),\\
g_7 &= -550 + \left(\frac{0.164}{x_1x_2} + 631.13x_3 - 54.48\right).
\end{aligned}
$$

with bounds :

$0.01 \le x_1 \le 0.45$

$0.01 \le x_2 \le 0.1$

$0.01 \le x_3 \le 0.1.$

2.1.12. Simply Supported I-beam Design (RCM12) [27]

Minimize :

$$f_1 = 2x_2x_4 + x_3\,(x_1 - 2x_4), \tag{30}$$

$$f_2 = \frac{PL^3}{4E\left(x_3\,(x_1 - 2x_4)^3 + 2x_2x_4\left(4x_4^2 + 3x_1\,(x_1 - 2x_4)\right)\right)}. \tag{31}$$

where,

$P = 600,\ \ L = 200,\ \ E = 20000,$

subject to :

$$g_1 = -16 + \frac{180000x_1}{x_3\,(x_1 - 2x_4)^3 + 2x_2x_4\left(4x_4^2 + 3x_1\,(x_1 - 2x_4)\right)} + \frac{15000x_2}{\left((x_1 - 2x_4)\,x_3^3 + 2x_4x_2^3\right)},$$

with bounds :

$10 \le x_1 \le 80,$

$10 \le x_2 \le 50,$

$0.9 \le x_3 \le 5,$

$0.9 \le x_4 \le 5.$

2.1.13. Gear Box Design (RCM13)

Minimize :

$$
\begin{aligned}
f_1 &= 0.7854x_2^2x_1\left(\frac{14.9334}{x_3} - 43.0934 + 3.3333x_3^2\right) + 0.7854(x_5x_7^2 + x_4x_6^2) - 1.508x_1(x_7^2 + x_6^2) + 7.477(x_7^3 + x_6^3) \\
f_2 &= 10x_6^{-3}\sqrt{16.91 \times 10^6 + (745x_4x_2^{-1}x_3^{-1})^2} \\
f_3 &= 10x_7^{-3}\sqrt{157.5 \times 10^6 + (745x_5x_2^{-1}x_3^{-1})^2}
\end{aligned}
\tag{32}
$$

subject to :

$$
\begin{aligned}
g_1(\bar{x}) &= \frac{1}{x_1x_2^2x_3} - \frac{1}{27} \le 0, \\
g_2(\bar{x}) &= \frac{1}{x_1x_2^2x_3^2} - \frac{1}{397.5} \le 0, \\
g_3(\bar{x}) &= \frac{1}{x_2x_6^4x_3x_4^{-3}} - \frac{1}{1.93} \le 0, \\
g_4(\bar{x}) &= \frac{1}{x_2x_7^4x_3x_5^{-3}} - \frac{1}{1.93} \le 0, \\
g_5(\bar{x}) &= 10x_6^{-3}\sqrt{16.91 \times 10^6 + (745x_4x_2^{-1}x_3^{-1})^2} - 1100 \le 0, \\
g_6(\bar{x}) &= 10x_7^{-3}\sqrt{157.5 \times 10^6 + (745x_5x_2^{-1}x_3^{-1})^2} - 850 \le 0, \\
g_7(\bar{x}) &= x_2x_3 - 40 \le 0, \\
g_8(\bar{x}) &= -x_1x_2^{-1} + 5 \le 0, \\
g_9(\bar{x}) &= x_1x_2^{-1} - 12 \le 0, \\
g_{10}(\bar{x}) &= 1.5x_6 - x_4 + 1.9 \le 0, \\
g_{11}(\bar{x}) &= 1.1x_7 - x_5 + 1.9 \le 0,
\end{aligned}
$$

with bounds :

$$
\begin{aligned}
&0.7 \le x_2 \le 0.8, x_3 \in \{17, 28\}, 2.6 \le x_1 \le 3.6, \\
&5 \le x_7 \le 5.5, 7.3 \le x_5, x_4 \le 8.3, 2.9 \le x_6 \le 3.9.
\end{aligned}
$$

2.1.14. Multiple Disk Clutch Brake Design (RCM14) [28]

Minimize :

$$
f_1 = \pi(x_2^2 - x_1^2)x_3(x_5 + 1)\rho, \tag{33}
$$

$$
f_2 = T. \tag{34}
$$

subject to :

$$
\begin{aligned}
g_1(\bar{x}) &= -p_{max} + p_{rz} \le 0, \\
g_2(\bar{x}) &= p_{rz}V_{sr} - V_{sr,max}p_{max} \le 0, \\
g_3(\bar{x}) &= \triangle R + x_1 - x_2 \le 0,
\end{aligned}
$$

$$g_4(\bar{x}) = -L_{max} + (x_5 + 1)(x_3 + \delta) \leq 0,$$
$$g_5(\bar{x}) = sM_s - M_h \leq 0,$$
$$g_6(\bar{x}) = T \geq 0,$$
$$g_7(\bar{x}) = -V_{sr,max} + V_{sr} \leq 0,$$
$$g_7(\bar{x}) = T - T_{max} \leq 0,$$

where,

$$M_h = \frac{2}{3}\mu x_4 x_5 \frac{x_2^3 - x_1^3}{x_2^2 - x_1^2} \text{ N.mm},$$
$$\omega = \frac{\pi n}{30} \text{ rad/s},$$
$$A = \pi(x_2^2 - x_1^2) \text{ mm}^2,$$
$$p_{rz} = \frac{x_4}{A} \text{ N/mm}^2,$$
$$V_{sr} = \frac{\pi R_{sr} n}{30} \text{ mm/s},$$
$$R_{sr} = \frac{2}{3}\frac{x_2^3 - x_1^3}{x_2^2 x_1^2} \text{ mm},$$
$$T = \frac{I_z \omega}{M_h + M_f},$$
$$\triangle R = 20 \text{ mm},\ L_{max} = 30 \text{ mm},\ \mu = 0.6,$$
$$V_{sr,max} = 10 \text{ m/s},\ \delta = 0.5 \text{ mm},\ s = 1.5,$$
$$T_{max} = 15 \text{ s},\ n = 250 \text{ rpm},\ I_z = 55 \text{ Kg.m}^2,$$
$$M_s = 40 \text{ Nm},\ M_f = 3 \text{ Nm},\ \text{and } p_{max} = 1.$$

with bounds :

$$60 \leq x_1 \leq 80,\ 90 \leq x_2 \leq 110,\ 1 \leq x_3 \leq 3,$$
$$0 \leq x_4 \leq 1000,\ 2 \leq x_5 \leq 9.$$

2.1.15. Spring Design (RCM15) [19]

Minimize :

$$f_1 = \frac{\pi^2 x_2 x_3^2 (x_1 + 2)}{4}, \tag{35}$$

$$f_2 = \frac{8000 C_f x_2}{\pi x_3^3}. \tag{36}$$

subject to :

$$g_1(\bar{x}) = \frac{8000 C_f x_2}{\pi x_3^3} - 189000 \leq 0,$$
$$g_2(\bar{x}) = l_f - 14 \leq 0,$$
$$g_3(\bar{x}) = 0.2 - x_3 \leq 0,$$

$$g_4(\bar{x}) = x_2 - 3 \leq 0,$$
$$g_5(\bar{x}) = 3 - \frac{x_2}{x_3} \leq 0,$$
$$g_6(\bar{x}) = \sigma_p - 6 \leq 0,$$
$$g_7(\bar{x}) = \sigma_p + \frac{700}{K} + 1.05(x_1 + 2)x_3 - l_f \leq 0,$$
$$g_8(\bar{x}) = 1.25 - \frac{700}{K} \leq 0,$$

where,

$$C_f = \frac{4\frac{x_2}{x_3} - 1}{4\frac{x_2}{x_3} - 4} + \frac{0.615x_3}{x_2},\ K = \frac{11.5 \times 10^6 x_3^4}{8x_1 x_2^3},\ \sigma_p = \frac{300}{K},\ l_f = \frac{1000}{K} + 1.05(x_1 + 2)x_3.$$

with bounds :

$$1 \leq x_1 \text{ (integer) } \leq 70,$$
$$x_3 \text{ (discreate) } \in \{0.009, 0.0095, 0.0104, 0.0118, 0.0128, 0.0132, 0.014, 0.015, 0.0162, 0.0173, 0.018, 0.020, 0.023, 0.025, 0.028, 0.032, 0.035, 0.041, 0.047, 0.054, 0.063, 0.072, 0.080, 0.092, 0.0105, 0.120, 0.135, 0.148, 0.162, 0.177, 0.192, 0.207, 0.225, 0.244, 0.263, 0.283, 0.307, 0.0331, 0.362, 0.394, 0.4375, 0.500\}$$
$$0.6 \leq x_2 \text{ (continuous) } \leq 3.$$

2.1.16. Cantilever Beam Design (RCM16) [29]

Minimize :

$$f_1 = 0.25\rho\pi x_2 x_1^2, \tag{37}$$
$$f_2 = \frac{64Px_2^3}{3E\pi x_1^4} \tag{38}$$

where,

$$P = 1,\ E = 307 \times 10^8, \rho = 7800.$$

subject to:

$$g_1 = -Sy + \frac{32Px_2}{\pi x_1^3},$$
$$g_2 = -\delta_{max} + \frac{64Px_2^3}{3E\pi x_1^4}$$

where,

$$Sy = 3 \times 10^5,\ \delta_{max} = 0.05.$$

with bounds:

$$0.01 \leq x_1 \leq 0.05,\ 0.20 \leq x_2 \leq 1.$$

2.1.17. Bulk Carrier Design (RCM17) [30]

Minimize :

$$f_1 = \frac{(C_c + C_r + C_v)}{ac}, \tag{39}$$

$$f_2 = ls, \tag{40}$$

$$f_3 = -ac \tag{41}$$

where,

$$\begin{aligned}
a &= 4977.06C_B^2 - 8105.61C_B + 4456.51,\\
b &= -10847.2C_B^2 + 12817C_B - 6960.32,\\
F_n &= \frac{0.5144}{(9.8065L)^{0.5}},\\
P &= \frac{(1.025LBTC_B)^{0.67}V_k^3}{a + bF_n},\\
W_s &= 0.034L^{1.7}B^{0.6}D^{0.4}C_B^{0.5},\\
W_o &= L^{0.8}B^{0.6}D^{0.3}C_B^{0.1},\\
W_m &= 0.17P^{0.9},\\
ls &= W_s + W_o + W_m,\\
D_{wt} &= 1.025LBTC_B - ls,\\
F_c &= 4.56 \times 10^{-5}P + 0.2,\\
D_{cwt} &= D_{wt} - F_c\left(\frac{5000V_k}{24} + 5\right) - 2D_{wt}^{0.5},\\
R_{trp} &= \frac{350}{\frac{5000.*V_k}{24} + 2\left(\frac{D_{cwt}}{8000} + 0.5\right)},\\
ac &= D_{cwt}R_{trp},\\
S_d &= \frac{5000V_k}{24},\\
C_c &= 0.26\left(2000W_s^{0.85} + 3500W_o + 2400P^{0.8}\right),\\
C_r &= 40000D_{wt}^{0.3},\\
C_v &= \left(105F_cS_d + 6.3D_{wt}^{0.8}\right)R_{trp}
\end{aligned} \tag{42}$$

subject to :

$$\begin{aligned}
g_1 &= -\frac{L}{B} + 6,\\
g_1 &= -15 + \frac{L}{D},\\
g_3 &= -19 + \frac{L}{T},\\
g_4 &= -0.45D_{wt}^{0.31} + T,\\
g_5 &= -0.7D - 0.7 + T,\\
g_6 &= -0.32 + F_n,
\end{aligned}$$

$$g_7 = -0.53T - \frac{(0.085. * C_B - 0.002). * B.^2}{(TC_B)} + (1 + 0.52D) + 0.07B,$$

$$g_8 = -D_{wt} + 3000,$$

$$g_9 = -500000 + D_{wt}.$$

with bounds :

$$150 \leq L \leq 274.32$$
$$20 \leq B \leq 32.31$$
$$13 \leq D \leq 25$$
$$10 \leq T \leq 11.71$$
$$14 \leq V_k \leq 18$$
$$0.63 \leq C_B \leq 0.75$$

2.1.18. Front Rail Design (RCM18) [31]

Minimize :

$$f_1 = \frac{Ea}{E}, \tag{43}$$

$$f_2 = \frac{F}{Fa}. \tag{44}$$

where,

$$Ea = 14496.5, \quad Fa = 234.9, E = -70973.4 + 958.656w + 614.173hh - 3.827whh + 57.023wt + 63.274hht - 3.582w^2 - 1.4842hh$$

subject to :

$$g_1 = -(hh - 136)(146 - hh),$$

$$g_2 = -(w - 58)(66 - w),$$

$$g_3 = -(t - 1.4)(2.2 - t).$$

with bounds :

$$136 \leq hh \leq 146,$$
$$56 \leq w \leq 68,$$
$$1.4 \leq t \leq 2.2$$

2.1.19. Multi-product Batch Plant (RCM19) [32]

Minimize :

$$f_1 = \sum_{j=1}^{M} \alpha_j N_j V_j^{\beta_j}, \tag{45}$$

$$f_2 = 65\left(\frac{Q_1}{B_1} + \frac{Q_2}{B_2}\right) + 0.08Q_1 + 0.1Q_2, \tag{46}$$

$$f_3 = Q_1 \frac{T_{L1}}{B_1} + Q_2 \frac{T_{L2}}{B_2}. \tag{47}$$

subject to :

$$g_1(\bar{x}) = S_{ij}B_i - V_j \leq 0,$$

$$g_2(\bar{x}) = -H + \sum_{i=1}^{N} \frac{Q_i T_{Li}}{B_i} \leq 0,$$

$$g_3(\bar{x}) = t_{ij} - N_j T_{Li} \leq 0,$$

with bounds :

$$1 \leq N_i \leq N_j^u,$$
$$V_j^l \leq V_j \leq V_j^u,$$
$$T_{Li}^l \leq T_{Li} \leq T_{Li}^u,$$
$$B_j^l \leq B_j \leq B_j^u.$$

where, $N = 2$, $M = 3$, $\alpha_j = 250$, $H = 6000$, $\beta_j = 0.6$, $N_j^u = 3$, $V_j^l = 250$, and $V_j^u = 2500$. The value of other parameters are calculated by

$$T_{Li}^l = \max\left(\frac{t_{ij}}{N_j^u}\right), \tag{48}$$

$$T_{Li}^u = \max\left(t_{ij}\right), \tag{49}$$

$$B_j^l = \frac{Q_i^* T_{Li}}{H}, \tag{50}$$

$$B_j^u = \min\left(Q_i, \min_j\left(\frac{V_j^u}{S_{ij}}\right)\right) \tag{51}$$

Parameters S_{ij} and t_{ij} are given in Table 1.

Table 1: Values of S_{ij} and t_{ij}.

S_{ij}				t_{ij}		
2	3	4		8	20	8
4	6	3		16	4	4

2.1.20. Hydro-static Thrust Bearing Design (RCM20) [33]

Minimize :

$$f_1 = \left(\frac{QP_0}{0.7} + E_f\right)\frac{1}{12}, \tag{52}$$

$$f_2 = \frac{0.0307}{386.4P_0}\frac{Q}{2\pi Rh}. \tag{53}$$

subject to :

$$g_1(\bar{x}) = 1000 - P_0 \leq 0,$$

$$g_2(\bar{x}) = W - 101000 \leq 0,$$

$$g_3(\bar{x}) = 5000 - \frac{W}{\pi(R^2 - R_0^2)} \leq 0,$$

$$g_4(\bar{x}) = 50 - P_0 \leq 0,$$

$$g_5(\bar{x}) = 0.001 - \frac{0.0307}{386.4P_0}\left(\frac{Q}{2\pi Rh}\right) \leq 0,$$

$$g_6(\bar{x}) = R - R_0 \leq 0,$$

$$g_7(\bar{x}) = h - 0.001 \leq 0,$$

where,

$$W = \frac{\pi P_0}{2} \frac{R^2 - R_0^2}{ln\left(\frac{R}{R_0}\right)},\ P_0 = \frac{6\mu Q}{\pi h^3} ln\left(\frac{R}{R_0}\right),$$

$$E_f = 9336Q \times 0.0307 \times 0.5\triangle T,\ \triangle T = 2(10^P - 559.7),$$

$$P = \frac{log_{10}log_{10}\left(8.122 \times 10^6\mu + 0.8\right) + 3.55}{10.04},$$

$$h = \left(\frac{2\pi \times 750}{60}\right)^2 \frac{2\pi\mu}{E_f}\left(\frac{R^4}{4} - \frac{R_0^4}{4}\right)$$

with bounds :

$$1 \leq R \leq 16,\ 1 \leq R_0 \leq 16,$$

$$1 \times 10^{-6} \leq \mu \leq 16 \times 10^{-6},\ 1 \leq Q \leq 16.$$

2.1.21. Crash Energy Management for High-speed Train (RCM21) [34]

Minimize :

$$\begin{aligned} f_1 =& 1.3667145844797 - 0.00904459793976106x_1 - 0.0016193573938033x_2 - 0.00758531275221425x_3 \\ & - 0.00440727360327102x_4 - 0.00572216860791644x_5 - 0.00936039926190721x_6 + 2.62510221107328 \\ & \times 10^{-6}(x_1^2) + 4.92982681358861 \times 10^{-7}(x_2^2) + 2.25524989067108 \times 10^{-6} \\ & (x_3^2) + 1.84605439400301 \times 10^{-6}(x_4^2) + 2.17175358243416 \times 10^{-6}(x_5^2) \\ & + 3.90158043948054 \times 10^{-6}(x_6^2) + 4.55276994245781 \times 10^{-7}x_1x_2 - 6.37013576290982 \\ & \times 10^{-7}x_1x_3 + 8.26736480446359 \times 10^{-7}x_1x_4 + 5.66352809442276 \times 10^{-8}x_1x_5 \\ & - 3.20213897443278 \times 10^{-7}x_1x_6 + 1.18015467772812 \times 10^{-8}x_2x_3 + 9.25820391546515 \\ & \times 10^{-8}x_2x_4 - 1.05705364119837 \times 10^{-7}x_2x_5 - 4.74797783014687 \times 10^{-7}x_2x_6 \\ & - 5.02319867013788 \times 10^{-7}x_3x_4 + 9.54284258085225 \times 10^{-7}x_3x_5 + 1.80533309229454 \\ & \times 10^{-7}x_3x_6 - 1.07938022118477 \times 10^{-6}x_4x_5 - 1.81370642220182 \times 10^{-7}x_4x_6 \\ & - 2.24238851688047 \times 10^{-7}x_5x_6, \end{aligned} \tag{54}$$

$$
\begin{aligned}
f_2 =& - 1.19896668942683 + 3.04107017009774x_1 + 1.23535701600191x_2 + 2.13882039381528x_3 \\
& + 2.33495178382303x_4 + 2.68632494801975x_5 + 3.43918953617606x_6 - 7.89144544980703 \\
& \times 10^{-4}(x_1^2) - 2.06085185698215 \times 10^{-4}(x_2^2) - 7.15269900037858 \\
& \times 10^{-4}(x_3^2) - 7.8449237573837 \times 10^{-4}(x_4^2) - 9.31396896237177 \\
& \times 10^{-4}(x_5^2) - 1.40826531972195 \times 10^{-3}(x_6^2) - 1.60434988248392 \\
& \times 10^{-4}x_1x_2 + 2.0824655419411 \times 10^{-4}x_1x_3 - 3.0530659653553 \times 10^{-4} \\
& x_1x_4 - 8.10145973591615 \times 10^{-5}x_1x_5 + 6.94728759651311 \times 10^{-5}x_1x_6 \\
& + 1.18015467772812 \times 10^{-8}x_2x_3 + 9.25820391546515 \times 10^{-8}x_2x_4 \\
& - 1.05705364119837 \times 10^{-7}x_2x_5 + 1.69935290196781 \times 10^{-4}x_2x_6 \\
& + 2.32421829190088 \times 10^{-5}x_3x_4 - 2.0808624041163476 \times 10^{-4}x_3x_5 \\
& + 1.75576341867273 \times 10^{-5}x_3x_6 + 2.68422081654044 \times 10^{-4}x_4x_5 \\
& + 4.39852066801981 \times 10^{-5}x_4x_6 + 2.96785446021357 \times 10^{-5}x_5x_6,
\end{aligned}
\tag{55}
$$

subject to :

$$
\begin{aligned}
g_1 &= f_1 - 5, \\
g_2 &= -f_1, \\
g_3 &= f_2 - 28, \\
g_4 &= -f_2
\end{aligned}
$$

with bounds :

$$
\begin{aligned}
1.3 &\le x_1 \le 1.7 \\
2.5 &\le x_2 \le 3.5 \\
1.3 &\le x_3 \le 1.7 \\
1.3 &\le x_4 \le 1.7 \\
1.3 &\le x_5 \le 1.7 \\
1.3 &\le x_6 \le 1.7
\end{aligned}
$$

2.2. *Chemical Engineering Problems*

From chemical engineering applications, we have collected 3 RWCMOPs where M, D, ng, and nh vary from 2 to 3, 6 to 9, 0 to 2 and 4 to 6, respectively.

2.2.1. *Haverly's Pooling Problem (RCM22) [35]*

Minimize :

$$f_1 = -9x_1 - 15x_2 + 6x_3 + 16x_4, \tag{56}$$

$$f_2 = 10(x_5 + x_6). \tag{57}$$

subject to :

$$
\begin{aligned}
h_1(\bar{x}) &= x_7 + x_8 - x_4 - x_3 = 0, \\
h_2(\bar{x}) &= x_1 - x_5 - x_7 = 0, \\
h_3(\bar{x}) &= x_2 - x_6 - x_8 = 0, \\
h_4(\bar{x}) &= x_9x_7 + x_9x_8 - 3x_3 - x_4 = 0,
\end{aligned}
$$

$$g_1(\bar{x}) = x_9 x_7 + 2x_5 - 2.5x_1 \leq 0,$$
$$g_2(\bar{x}) = x_9 x_8 + 2x_6 - 1.5x_2 \leq 0,$$

with bounds :

$$0 \leq x_1, x_3, x_4, x_5, x_6, x_8 \leq 100, 0 \leq x_2, x_7, x_9 \leq 200.$$

2.2.2. Reactor Network Design (RCM23) [36]

Minimize :

$$f_1 = -x_4, \tag{58}$$
$$f_2 = x_5^{0.5} + x_6^{0.5}. \tag{59}$$

subject to:

$$h_1(\bar{x}) = k_1 x_5 x_2 + x_1 - 1 = 0,$$
$$h_2(\bar{x}) = k_3 x_5 x_3 + x_3 + x_1 - 1 = 0,$$
$$h_3(\bar{x}) = k_2 x_6 x_2 - x_1 + x_2 = 0,$$
$$h_4(\bar{x}) = k_4 x_6 x_4 + x_2 - x_1 + x_4 - x_3 = 0,$$
$$g_1(\bar{x}) = x_5^{0.5} + x_6^{0.5} \leq 4$$

with bounds :

$$0 \leq x_4, x_3, x_2, x_1 \leq 1,$$
$$0.00001 \leq x_6, x_5 \leq 16.$$

where, $k_3 = 0.0391908$, $k_4 = 0.9k_3$, $k_1 = 0.09755988$, and $k_2 = 0.99k_1$.

2.2.3. Heat Exchanger Nwteork Design (RCM24) [37]

Minimize :

$$f_1 = 35x_1^{0.6} + 35x_2^{0.6}, \tag{60}$$
$$f_2 = 200x_1 x_4 - x3, \tag{61}$$
$$f_3 = 200x_1 x_6 - x_5. \tag{62}$$

subject to :

$$h_1(\bar{x}) = 200x_1 x_4 - x_3 = 0,$$
$$h_2(\bar{x}) = 200x_2 x_6 - x_5 = 0,$$
$$h_3(\bar{x}) = x_3 - 10000(x_7 - 100) = 0,$$
$$h_4(\bar{x}) = x_5 - 10000(300 - x_7) = 0,$$
$$h_5(\bar{x}) = x_3 - 10000(600 - x_8) = 0,$$
$$h_6(\bar{x}) = x_5 - 10000(900 - x_9) = 0,$$
$$h_7(\bar{x}) = x_4 \ln(x_8 - 100) - x_4 \ln(600 - x_7) - x_8 + x_7 + 500 = 0,$$
$$h_8(\bar{x}) = x_6 \ln(x_9 - x_7) - x_6 \ln(600) - x_9 + x_7 + 600 = 0$$

with bounds :

$$0 \leq x_1 \leq 10, 0 \leq x_2 \leq 200, 0 \leq x_3 \leq 100, 0 \leq x_4 \leq 200,$$
$$1000 \leq x_5 \leq 2000000, 0 \leq x_6 \leq 600, 100 \leq x_7 \leq 600, 100 \leq x_8 \leq 600,$$
$$100 \leq x_9 \leq 900.$$

2.3. Process, Design and Synthesis Problems

From this domain, we have collected bi-objective 5 RWCMOPs where *D*, *ng*, and *nh* vary from 2 to 8, 1 to 9, and 0 to 5, respectively.

2.3.1. Process Synthesis Problem (RCM25) [38]

Minimize :

$$f_1 = x_2 + 2x_1, \tag{63}$$

$$f_2 = -x_1^2 - x_2 \tag{64}$$

subject to :

$$g_1(\bar{x}) = -x_1^2 - x_2 + 1.25 \leq 0,$$

$$g_2(\bar{x}) = x_1 + x_2 \leq 1.6.$$

with bounds :

$$0 \leq x_1 \leq 1.6$$
$$x_2 \in \{0, 1\}$$

2.3.2. Process Synthesis and Design Problem (RCM26) [39]

Minimize :

$$f_1 = -x_3 + x_2 + 2x_1, \tag{65}$$

$$f_2 = -x_1^2 - x_2 + x_1 x_3. \tag{66}$$

subject to :

$$h_1(\bar{x}) = -2\exp(-x_2) + x_1 = 0,$$

$$g_1(\bar{x}) = x_2 - x_1 + x_3 \leq 0.$$

with bounds :

$$0.5 \leq x_1, x_2 \leq 1.4,$$
$$x_3 \in \{0, 1\}.$$

2.3.3. Process Flow Sheeting Problem (RCM27) [40]

Minimize :

$$f_1 = -0.7x_3 + 0.8 + 5(0.5 - x_1)^2, \tag{67}$$

$$f_2 = x_1 - x_3. \tag{68}$$

subject to :

$$g_1(\bar{x}) = -exp(x_1 - 0.2) - x_2 \leq 0,$$

$$g_2(\bar{x}) = x_2 + 1.1x_3 \leq -1.0,$$

$$g_3(\bar{x}) = x_1 - x_3 \leq 0.2.$$

with bounds :

$$-2.22554 \leq x_2 \leq -1,\ 0.2 \leq x_1 \leq 1,$$
$$x_3 \in \{0, 1\}.$$

2.3.4. Two Reactor Problem (RCM28) [38]

Minimize :

$$f_1 = 7.5x_7 + 5.5x_8 + 7x_5 + 6x_6 + 5(x_1 + x_2), \tag{69}$$

$$f_2 = x_1 + x_2. \tag{70}$$

subject to :

$$h_1(\bar{x}) = x_7 + x_8 - 1 = 0,$$

$$\begin{aligned}
h_2(\bar{x}) &= x_3 - 0.9(1 - \exp(0.5x_5))x_1 = 0,\\
h_3(\bar{x}) &= x_4 - 0.8(1 - \exp(0.4x_6))x_2 = 0,\\
h_4(\bar{x}) &= x_3 + x_4 - 10 = 0,\\
h_5(\bar{x}) &= x_3x_7 + x_4x_8 - 10 = 0,\\
g_1(\bar{x}) &= x_5 - 10x_7 \leq 0,\\
g_2(\bar{x}) &= x_6 - 10x_8 \leq 0,\\
g_3(\bar{x}) &= x_1 - 20x_7 \leq 0,\\
g_4(\bar{x}) &= x_2 - 20x_8 \leq 0
\end{aligned}$$

with bounds :

$$0 \leq x_6, x_5, x_4, x_3, x_2, x_1 \leq 100$$
$$x_8, x_7 \in \{0, 1\}.$$

2.3.5. Process Synthesis Problem (RCM29) [38]

Minimize :

$$f_1 = (1 - x_4)^2 + (1 - x_5)^2 + (1 - x_6)^2 - \ln(1 + x_7) + (1 - x_1)^2 + (2 - x_2)^2 + (3 - x_3)^2, \tag{71}$$

$$f_2 = (1 - x_1)^2 + (2 - x_2)^2 + (3 - x_3)^2. \tag{72}$$

subject to :

$$g_1(\bar{x}) = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 - 5 \leq 0,$$

$$\begin{aligned}
g_2(\bar{x}) &= x_6^3 + x_1^2 + x_2^2 + x_3^2 = 5.5 \leq 0,\\
g_3(\bar{x}) &= x_1 + x_4 - 1.2 \leq 0,\\
g_4(\bar{x}) &= x_2 + x_5 - 1.8 \leq 0,\\
g_5(\bar{x}) &= x_3 + x_6 - 2.5 \leq 0,\\
g_6(\bar{x}) &= x_1 + x_7 - 1.2 \leq 0,\\
g_7(\bar{x}) &= x_5^2 + x_2^2 - 1.64 \leq 0,\\
g_8(\bar{x}) &= x_6^2 + x_3^2 - 4.25 \leq 0,\\
g_9(\bar{x}) &= x_5^2 + x_3^2 - 4.64 \leq 0,
\end{aligned}$$

with bounds :

$$0 \leq x_2, x_3, x_1 \leq 100,$$
$$x_7, x_6, x_5, x_4 \in \{0, 1\}.$$

2.4. Power Electronics Problems

From this domain, we have collected bi-objective 6 RWCMOPs where D, ng, and nh vary from 2 to 8, 1 to 9, and 0 to 5, respectively.

2.4.1. Synchronous Optimal Pulse-width Modulation of 3-level Inverters (RCM30) [41]

Minimize:

$$f_1 = \frac{\sqrt{\sum_k (k^{-4}) (\sum_{i=1}^{N} s(i) cos(k x_i))^2}}{\sqrt{\sum_k k^{-4}}} \tag{73}$$

$$f_2 = \left(m - \sum_{i=1}^{N} s(i) cos(x_i) \right)^2 \tag{74}$$

where, $k = 5, 7, 11, 13.....97$, $N = \lfloor \frac{f_{s,max}}{f.m} \rfloor$, and $s(i) = (-1)^{i+1}$
subject to:

$$x_{i+1} - x_i - 10^{-5} > 0, \ i = 1, 2, ...N - 1,$$
$$0 < x_i < \frac{\pi}{2}, \ , i = 1, 2, ...N.$$

2.4.2. Synchronous Optimal Pulse-width Modulation of 5-level Inverters (RCM31) [42]

Minimize:

$$f_1 = \frac{\sqrt{\sum_k (k^{-4}) (\sum_{i=1}^{N} s(i) cos(k x_i))^2}}{2 \sqrt{\sum_k k^{-4}}} \tag{75}$$

$$f_2 = \left(2m - \sum_{i=1}^{N} s(i) cos(x_i) \right)^2 \tag{76}$$

where, $k = 5, 7, 11, 13.....97$, $N = \lfloor \frac{2.f_{s,max}}{f.m} \rfloor$, and $s = [1, -1, 1, 1, -1, 1, -1, 1, -1, -1]$
subject to:

$$x_{i+1} - x_i - 10^{-5} > 0, \ i = 1, 2, ...N - 1,$$
$$0 < x_i < \frac{\pi}{2}, \ , i = 1, 2, ...N.$$

2.4.3. Synchronous Optimal Pulse-width Modulation of 7-level Inverters (RCM32) [43]

Minimize:

$$f_1 = \frac{\sqrt{\sum_k (k^{-4}) (\sum_{i=1}^{N} s(i) cos(k x_i))^2}}{3 \sqrt{\sum_k k^{-4}}} \tag{77}$$

$$f_2 = \left(3m - \sum_{i=1}^{N} s(i) cos(x_i) \right)^2 \tag{78}$$

where, $k = 5, 7, 11, 13.....97$, $N = \lfloor \frac{3.f_{s,max}}{f.m} \rfloor$, and $s = [1, -1, 1, 1, 1, -1, -1, -1, 1, 1, -1]$
subject to:

$$x_{i+1} - x_i - 10^{-5} > 0, \ i = 1, 2, ...N - 1,$$
$$0 < x_i < \frac{\pi}{2}, \ , i = 1, 2, ...N.$$

2.4.4. Synchronous Optimal Pulse-width Modulation of 9-level Inverters (RCM33) [44]

Minimize:

$$f_1 = \frac{\sqrt{\sum_k (k^{-4})(\sum_{i=1}^{N} s(i)cos(kx_i))^2}}{4\sqrt{\sum_k k^{-4}}} \tag{79}$$

$$f_2 = \left(4m - \sum_{i=1}^{N} s(i)cos(x_i)\right)^2 \tag{80}$$

where, $k = 5, 7, 11, 13.....97$, $N = \lfloor \frac{4.f_{s,max}}{f.m} \rfloor$, and $s = [1, 1, 1, 1, -1, 1, -1 - 1, -1, 1, -1, -1]$
subject to:

$$x_{i+1} - x_i - 10^{-5} > 0,\ i = 1, 2, ...N - 1,$$

$$0 < x_i < \frac{\pi}{2},\ , i = 1, 2, ...N.$$

2.4.5. Synchronous Optimal Pulse-width Modulation of 11-level Inverters (RCM34) [45]

Minimize:

$$f_2 = \frac{\sqrt{\sum_k (k^{-4})(\sum_{i=1}^{N} s(i)cos(kx_i))^2}}{5\sqrt{\sum_k k^{-4}}} \tag{81}$$

$$f_2 = \left(5m - \sum_{i=1}^{N} s(i)cos(x_i)\right)^2 \tag{82}$$

where, $k = 5, 7, 11, 13.....97$, $N = \lfloor \frac{5.f_{s,max}}{f.m} \rfloor$, and $s = [1, -1, 1, 1, 1, -1, -1, -1, 1, 1, 1, 1]$
subject to:

$$x_{i+1} - x_i - 10^{-5} > 0,\ i = 1, 2, ...N - 1,$$

$$0 < x_i < \frac{\pi}{2},\ , i = 1, 2, ...N.$$

2.4.6. Synchronous Optimal Pulse-width Modulation of 13-level Inverters (RCM35) [45]

Minimize:

$$f_1 = \frac{\sqrt{\sum_k (k^{-4})(\sum_{i=1}^{N} s(i)cos(k\alpha_i))^2}}{6\sqrt{\sum_k k^{-4}}} \tag{83}$$

$$f_2 = \left(6m - \sum_{i=1}^{N} s(i)cos(\alpha_i)\right)^2 \tag{84}$$

where, $k = 5, 7, 11, 13.....97$, $N = \lfloor \frac{6.f_{s,max}}{f.m} \rfloor$, and $s = [1, 1, 1, -1, 1, -1, 1, -1, 1, 1, 1, 1]$
subject to:

$$\alpha_{i+1} - \alpha_i - 10^{-5} > 0,\ i = 1, 2, ...N - 1,$$

$$0 < \alpha_i < \frac{\pi}{2},\ , i = 1, 2, ...N.$$

2.5. Power System Optimization Problems

From this domain, we have collected 15 RWCMOPs where *M*, *D*, and *nh* vary from 2 to 4, 6 to 34, and 1 to 26, respectively.

2.5.1. Optimal Sizing of Single Phase Distribution Generation with Reactive Power support for phase balancing at Main Transformer/Grid and Minimizing Active Power Loss (RCM36) [46]

This problem can be formulated as a constrained multiobjective optimization problem, which is as follows.
Minimize:

$$\begin{aligned} f_1 =& \left(I^a_{r,1} + I^b_{r,1} + I^c_{r,1}\right)^2 + \left(I^a_{m,1} + I^b_{m,1} + I^c_{m,1}\right)^2 \\ &+ \left(I^a_{r,1} - 0.5\left(I^b_{r,1} + I^c_{r,1}\right) - 0.5\sqrt{3}\left(I^b_{m,1} - I^c_{m,1}\right)\right)^2 \\ &+ \left(I^a_{m,1} - 0.5\left(I^b_{m,1} + I^c_{m,1}\right) + 0.5\sqrt{3}\left(I^b_{r,1} - I^c_{r,1}\right)\right)^2, \end{aligned} \tag{85}$$

$$f_2 = \sum_{i=1}^{N} \sum_{j\in\{a,b,c\}} P^j_i$$

where,

$$I^s_{r,1} = \sum_{k\in\{a,b,c\}} \sum_{i=1}^{N} \left(G^{sk}_{1,i} V^k_{r,i} - B^{sk}_{1i} V^k_{m,i}\right)$$

$$I^s_{m,1} = \sum_{k\in\{a,b,c\}} \sum_{i=1}^{N} \left(B^{sk}_{1,i} V^k_{r,i} + G^{sk}_{1i} V^k_{m,i}\right)$$

subject to:

$$\begin{aligned} &\sum_{s\in\{a,b,c\}} \sum_{i=1}^{N} \left(G^{js}_{k,i} V^s_{r,i} - B^{js}_{ki} V^s_{m,i}\right) - \frac{P^j_k V^j_{r,k} + Q^j_k V^j_{m,k}}{(V^j_{r,k})^2 + (V^j_{m,k})^2} = 0,\ k = 1, ...N \text{ and } j = \{a,b,c\}, \\ &\sum_{s\in\{a,b,c\}} \sum_{i=1}^{N} (B^{js}_{ki} V^s_{r,i} + G^{js}_{ki} V^s_{m,i}) - \frac{P^j_k V^j_{m,k} - Q^j_k V^j_{r,k}}{(V^j_{r,k})^2 + (V^j_{m,k})^2} = 0,\ k = 1, ...N \text{ and } j = \{a,b,c\}, \\ &P^j_k - P^j_{dg,k} + P^j_{l,k} = 0,\ k = 1, ...N \text{ and } j = \{a,b,c\}, \\ &Q^j_k - Q^j_{dg,k} + Q^j_{l,k} = 0,\ k = 1, ...N \text{ and } j = \{a,b,c\}, \\ &V_{min} \leq V^j_{r,k}, V^j_{m,k} ...k = 1, 2, ...N \text{ and } j = \{a,b,c\} \leq V_{max} \\ &P_{min} \leq P^j_k ....k = 1, 2, ...N \text{ and } j = \{a,b,c\} \leq P_{max} \\ &Q_{min} \leq Q^j_k ....k = 1, 2, ...N \text{ and } j = \{a,b,c\} \leq Q_{max} \\ &P_{dg,min} \leq P^j_{dg,k} ....k = 1, 2, ...N \text{ and } j = \{a,b,c\} \leq P_{dg,max} \\ &Q_{dg,min} \leq Q^j_{dg,k} ....k = 1, 2, ...N \text{ and } j = \{a,b,c\} \leq Q_{dg,max} \end{aligned}$$

where P^j_i and Q^j_i represent the active and reactive injected power, respectively, at i-th bus in j-th phase, $Ybus^{st}_{ij}(= G^{st}_{ij} + 1jB^{st}_{ij})$ is ij-th element of st-th block of admittance matrix, $V^j_i(= V^j_{r,i} + 1jV^j_{m,i})$ is bus voltage at i-th bus in j-th phase, $P^j_{dg,k}$ and $Q^j_{dg,k}$ represent the active and reactive power generation, respectively, at k-th DG in j-th phase and N represents the total number of buses in system.

2.5.2. Optimal Sizing of Single Phase Distribution Generation with Reactive Power support for phase balancing at Main Transformer/Grid and Minimizing Reactive Power Loss (RCM37) [46]

Minimize:

$$\begin{aligned} f_1 =& \left(I^a_{r,1} + I^b_{r,1} + I^c_{r,1}\right)^2 + \left(I^a_{m,1} + I^b_{m,1} + I^c_{m,1}\right)^2 \\ &+ \left(I^a_{r,1} - 0.5\left(I^b_{r,1} + I^c_{r,1}\right) - 0.5\sqrt{3}\left(I^b_{m,1} - I^c_{m,1}\right)\right)^2 \\ &+ \left(I^a_{m,1} - 0.5\left(I^b_{m,1} + I^c_{m,1}\right) + 0.5\sqrt{3}\left(I^b_{r,1} - I^c_{r,1}\right)\right)^2, \end{aligned} \tag{86}$$

$$f_2 = \sum_{i=1}^{N} \sum_{j\in\{a,b,c\}} Q^j_i$$

where,

$$I^s_{r,1} = \sum_{k\in\{a,b,c\}} \sum_{i=1}^{N} \left(G^{sk}_{1,i} V^k_{r,i} - B^{sk}_{1i} V^k_{m,i}\right)$$

$$I^s_{m,1} = \sum_{k\in\{a,b,c\}} \sum_{i=1}^{N} \left(B^{sk}_{1,i} V^k_{r,i} + G^{sk}_{1i} V^k_{m,i}\right)$$

subject to:

$$\begin{aligned} &\sum_{s\in\{a,b,c\}} \sum_{i=1}^{N} \left(G^{js}_{k,i} V^s_{r,i} - B^{js}_{ki} V^s_{m,i}\right) - \frac{P^j_k V^j_{r,k} + Q^j_k V^j_{m,k}}{(V^j_{r,k})^2 + (V^j_{m,k})^2} = 0,\ k = 1,...N \text{ and } j = \{a,b,c\}, \\ &\sum_{s\in\{a,b,c\}} \sum_{i=1}^{N} (B^{js}_{ki} V^s_{r,i} + G^{js}_{ki} V^s_{m,i}) - \frac{P^j_k V^j_{m,k} - Q^j_k V^j_{r,k}}{(V^j_{r,k})^2 + (V^j_{m,k})^2} = 0,\ k = 1,...N \text{ and } j = \{a,b,c\}, \\ &P^j_k - P^j_{dg,k} + P^j_{l,k} = 0,\ k = 1,...N \text{ and } j = \{a,b,c\}, \\ &Q^j_k - Q^j_{dg,k} + Q^j_{l,k} = 0,\ k = 1,...N \text{ and } j = \{a,b,c\}, \end{aligned}$$

$$V_{min} \leq V^j_{r,k}, V^j_{m,k} ...k = 1,2,...N \text{ and } j = \{a,b,c\} \leq V_{max}$$

$$P_{min} \leq P^j_k ....k = 1,2,...N \text{ and } j = \{a,b,c\} \leq P_{max}$$

$$Q_{min} \leq Q^j_k ....k = 1,2,...N \text{ and } j = \{a,b,c\} \leq Q_{max}$$

$$P_{dg,min} \leq P^j_{dg,k} ....k = 1,2,...N \text{ and } j = \{a,b,c\} \leq P_{dg,max}$$

$$Q_{dg,min} \leq Q^j_{dg,k} ....k = 1,2,...N \text{ and } j = \{a,b,c\} \leq Q_{dg,max}$$

where P^j_i and Q^j_i represent the active and reactive injected power, respectively, at i-th bus in j-th phase, $Ybus^{st}_{ij}(= G^{st}_{ij} + 1jB^{st}_{ij})$ is ij-th element of st-th block of admittance matrix, $V^j_i(= V^j_{r,i} + 1jV^j_{m,i})$ is bus voltage at i-th bus in j-th phase, $P^j_{dg,k}$ and $Q^j_{dg,k}$ represent the active and reactive power generation, respectively, at k-th DG in j-th phase and N represents the total number of buses in system.

2.5.3. Optimal Sizing of Single Phase Distribution Generation with Reactive Power support for Minimizing Active and Reactive Power Loss (RCM38) [46]

Minimize:

$$
\begin{aligned}
f_1 =& \left(I^a_{r,1} + I^b_{r,1} + I^c_{r,1}\right)^2 + \left(I^a_{m,1} + I^b_{m,1} + I^c_{m,1}\right)^2 \\
&+ \left(I^a_{r,1} - 0.5\left(I^b_{r,1} + I^c_{r,1}\right) - 0.5\sqrt{3}\left(I^b_{m,1} - I^c_{m,1}\right)\right)^2 \\
&+ \left(I^a_{m,1} - 0.5\left(I^b_{m,1} + I^c_{m,1}\right) + 0.5\sqrt{3}\left(I^b_{r,1} - I^c_{r,1}\right)\right)^2,
\end{aligned}
\tag{87}
$$

$$f_2 = \sum_{i=1}^{N} \sum_{j\in\{a,b,c\}} P^j_i$$

$$f_3 = \sum_{i=1}^{N} \sum_{j\in\{a,b,c\}} Q^j_i$$

where,

$$I^s_{r,1} = \sum_{k\in\{a,b,c\}} \sum_{i=1}^{N} \left(G^{sk}_{1,i} V^k_{r,i} - B^{sk}_{1i} V^k_{m,i}\right)$$

$$I^s_{m,1} = \sum_{k\in\{a,b,c\}} \sum_{i=1}^{N} \left(B^{sk}_{1,i} V^k_{r,i} + G^{sk}_{1i} V^k_{m,i}\right)$$

subject to:

$$
\begin{aligned}
&\sum_{s\in\{a,b,c\}} \sum_{i=1}^{N} \left(G^{js}_{k,i} V^s_{r,i} - B^{js}_{ki} V^s_{m,i}\right) - \frac{P^j_k V^j_{r,k} + Q^j_k V^j_{m,k}}{(V^j_{r,k})^2 + (V^j_{m,k})^2} = 0,\ k = 1, ...N \text{ and } j = \{a,b,c\}, \\
&\sum_{s\in\{a,b,c\}} \sum_{i=1}^{N} (B^{js}_{ki} V^s_{r,i} + G^{js}_{ki} V^s_{m,i}) - \frac{P^j_k V^j_{m,k} - Q^j_k V^j_{r,k}}{(V^j_{r,k})^2 + (V^j_{m,k})^2} = 0,\ k = 1, ...N \text{ and } j = \{a,b,c\}, \\
&P^j_k - P^j_{dg,k} + P^j_{l,k} = 0,\ k = 1, ...N \text{ and } j = \{a,b,c\}, \\
&Q^j_k - Q^j_{dg,k} + Q^j_{l,k} = 0,\ k = 1, ...N \text{ and } j = \{a,b,c\}, \\
&V_{min} \le V^j_{r,k}, V^j_{m,k} ...k = 1,2,...N \text{ and } j = \{a,b,c\} \le V_{max} \\
&P_{min} \le P^j_k ....k = 1,2,...N \text{ and } j = \{a,b,c\} \le P_{max} \\
&Q_{min} \le Q^j_k ....k = 1,2,...N \text{ and } j = \{a,b,c\} \le Q_{max} \\
&P_{dg,min} \le P^j_{dg,k} ....k = 1,2,...N \text{ and } j = \{a,b,c\} \le P_{dg,max} \\
&Q_{dg,min} \le Q^j_{dg,k} ....k = 1,2,...N \text{ and } j = \{a,b,c\} \le Q_{dg,max}
\end{aligned}
$$

where P^j_i and Q^j_i represent the active and reactive injected power, respectively, at i-th bus in j-th phase, $Ybus^{st}_{ij}(= G^{st}_{ij} + 1jB^{st}_{ij})$ is ij-th element of st-th block of admittance matrix, $V^j_i(= V^j_{r,i} + 1jV^j_{m,i})$ is bus voltage at i-th bus in j-th phase, $P^j_{dg,k}$ and $Q^j_{dg,k}$ represent the active and reactive power generation, respectively, at k-th DG in j-th phase and N represents the total number of buses in system.

2.5.4. Optimal Sizing of Single Phase Distribution Generation with Reactive Power support for phase balancing at Main Transformer/Grid and Minimizing Active and Reactive Power Loss (RCM39) [46]

Minimize:

$$f_1 = \sum_{i=1}^{N} \sum_{j \in \{a,b,c\}} P_i^j$$

$$f_2 = \sum_{i=1}^{N} \sum_{j \in \{a,b,c\}} Q_i^j$$

where,

$$I_{r,1}^s = \sum_{k \in \{a,b,c\}} \sum_{i=1}^{N} \left(G_{1,i}^{sk} V_{r,i}^k - B_{1i}^{sk} V_{m,i}^k \right)$$

$$I_{m,1}^s = \sum_{k \in \{a,b,c\}} \sum_{i=1}^{N} \left(B_{1,i}^{sk} V_{r,i}^k + G_{1i}^{sk} V_{m,i}^k \right)$$

subject to:

$$\sum_{s \in \{a,b,c\}} \sum_{i=1}^{N} \left(G_{k,i}^{js} V_{r,i}^s - B_{ki}^{js} V_{m,i}^s \right) - \frac{P_k^j V_{r,k}^j + Q_k^j V_{m,k}^j}{(V_{r,k}^j)^2 + (V_{m,k}^j)^2} = 0,\ k = 1, ...N \text{ and } j = \{a, b, c\},$$

$$\sum_{s \in \{a,b,c\}} \sum_{i=1}^{N} (B_{ki}^{js} V_{r,i}^s + G_{ki}^{js} V_{m,i}^s) - \frac{P_k^j V_{m,k}^j - Q_k^j V_{r,k}^j}{(V_{r,k}^j)^2 + (V_{m,k}^j)^2} = 0,\ k = 1, ...N \text{ and } j = \{a, b, c\},$$

$$P_k^j - P_{dg,k}^j + P_{l,k}^j = 0,\ k = 1, ...N \text{ and } j = \{a, b, c\},$$

$$Q_k^j - Q_{dg,k}^j + Q_{l,k}^j = 0,\ k = 1, ...N \text{ and } j = \{a, b, c\},$$

$$V_{min} \leq V_{r,k}^j, V_{m,k}^j ...k = 1, 2, ...N \text{ and } j = \{a, b, c\} \leq V_{max}$$

$$P_{min} \leq P_k^j ....k = 1, 2, ...N \text{ and } j = \{a, b, c\} \leq P_{max}$$

$$Q_{min} \leq Q_k^j ....k = 1, 2, ...N \text{ and } j = \{a, b, c\} \leq Q_{max}$$

$$P_{dg,min} \leq P_{dg,k}^j ....k = 1, 2, ...N \text{ and } j = \{a, b, c\} \leq P_{dg,max}$$

$$Q_{dg,min} \leq Q_{dg,k}^j ....k = 1, 2, ...N \text{ and } j = \{a, b, c\} \leq Q_{dg,max}$$

where P_i^j and Q_i^j represent the active and reactive injected power, respectively, at i-th bus in j-th phase, $Ybus_{ij}^{st}(= G_{ij}^{st} + 1jB_{ij}^{st})$ is ij-th element of st-th block of admittance matrix, $V_i^j(= V_{r,i}^j + 1jV_{m,i}^j)$ is bus voltage at i-th bus in j-th phase, $P_{dg,k}^j$ and $Q_{dg,k}^j$ represent the active and reactive power generation, respectively, at k-th DG in j-th phase and N represents the total number of buses in system.

2.5.5. Optimal Power Flow for Minimizing Active and Reactive Power Loss (RCM40) [47]

Minimize:

$$f_1 = \sum_{i=1}^{N} P_i \tag{88}$$

$$f_2 = \sum_{i=1}^{N} Q_i \tag{89}$$

subject to:

$$\sum_{i=1}^{N}(G_{k,i}V_{r,i} - B_{ki}V_{m,i}) - \frac{P_kV_{r,k} + Q_kV_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0, \; k = 1, ...N,$$
$$\sum_{i=1}^{N}(B_{ki}V_{r,i} + G_{ki}V_{m,i}) - \frac{P_kV_{m,k} - Q_kV_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0, \; k = 1, ...N,$$
$$P_k - P_{dg,k} + P_{l,k} = 0, \; k = 1, ...N,$$
$$Q_k + Q_{l,k} = 0, \; k = 1, ...N,$$
$$V_{min} \leq V_{r,k}, V_{m,k}...k = 1, 2, ...N \leq V_{max}$$
$$P_{min} \leq P_k.......k = 1, 2, ...N \leq P_{max}$$
$$Q_{min} \leq Q_k.......k = 1, 2, ...N \leq Q_{max}$$
$$P_{min,dg} \leq P_{dg,k}.......k = 1, 2, ...N \leq P_{max,dg}$$

where P_i and Q_i represent the active and reactive injected power, respectively, at i-th bus, $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix, $V_i(= V_{r,i} + 1jV_{m,i})$ is bus voltage at i-th bus, $P_{dg,k}$ represents the active power generation of DG at $k-$th bus and N represents the total number of buses in system.

2.5.6. Optimal Power Flow for Minimizing Voltage deviation, Active and Reactive Power Loss (RCM41) [47]

Minimize:

$$f_1 = \sum_{i=1}^{N} P_i \tag{90}$$

$$f_2 = \sum_{i=1}^{N} Q_i \tag{91}$$

$$f_3 = \sum_{i=1}^{N}(1 - |V_i|) \tag{92}$$

subject to:

$$\sum_{i=1}^{N}(G_{k,i}V_{r,i} - B_{ki}V_{m,i}) - \frac{P_kV_{r,k} + Q_kV_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0, \; k = 1, ...N,$$
$$\sum_{i=1}^{N}(B_{ki}V_{r,i} + G_{ki}V_{m,i}) - \frac{P_kV_{m,k} - Q_kV_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0, \; k = 1, ...N,$$
$$P_k - P_{dg,k} + P_{l,k} = 0, \; k = 1, ...N,$$
$$Q_k + Q_{l,k} = 0, \; k = 1, ...N,$$
$$V_{min} \leq V_{r,k}, V_{m,k}...k = 1, 2, ...N \leq V_{max}$$
$$P_{min} \leq P_k.......k = 1, 2, ...N \leq P_{max}$$
$$Q_{min} \leq Q_k.......k = 1, 2, ...N \leq Q_{max}$$
$$P_{min,dg} \leq P_{dg,k}.......k = 1, 2, ...N \leq P_{max,dg}$$

where P_i and Q_i represent the active and reactive injected power, respectively, at i-th bus, $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix, $V_i(= V_{r,i} + 1jV_{m,i})$ is bus voltage at i-th bus, $P_{dg,k}$ represents the active power generation of DG at $k-$th bus and N represents the total number of buses in system.

2.5.7. Optimal Power Flow for Minimizing Voltage deviation, and Active Power Loss (RCM42) [47]

Minimize:

$$f_1 = \sum_{i=1}^{N} P_i \tag{93}$$

$$f_2 = \sum_{i=1}^{N} (1 - |V_i|) \tag{94}$$

subject to:

$$\begin{aligned}
&\sum_{i=1}^{N} (G_{k,i} V_{r,i} - B_{ki} V_{m,i}) - \frac{P_k V_{r,k} + Q_k V_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\ k = 1, ...N,\\
&\sum_{i=1}^{N} (B_{ki} V_{r,i} + G_{ki} V_{m,i}) - \frac{P_k V_{m,k} - Q_k V_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\ k = 1, ...N,\\
&P_k - P_{dg,k} + P_{l,k} = 0,\ k = 1, ...N,\\
&Q_k + Q_{l,k} = 0,\ k = 1, ...N,\\
&V_{min} \leq V_{r,k}, V_{m,k} ...k = 1, 2, ...N \leq V_{max}\\
&P_{min} \leq P_k ....k = 1, 2, ...N \leq P_{max}\\
&Q_{min} \leq Q_k ....k = 1, 2, ...N \leq Q_{max}\\
&P_{min,dg} \leq P_{dg,k} ....k = 1, 2, ...N \leq P_{max,dg}
\end{aligned}$$

where P_i and Q_i represent the active and reactive injected power, respectively, at i-th bus, $Ybus_{ij} (= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix, $V_i (= V_{r,i} + 1jV_{m,i})$ is bus voltage at i-th bus, $P_{dg,k}$ represents the active power generation of DG at $k-$th bus and N represents the total number of buses in system.

2.5.8. Optimal Power Flow for Minimizing Fuel Cost, and Active Power Loss (RCM43) [47]

Minimize:

$$f_1 = \sum_{i=1}^{N} \left(P_{g,i} - P_{l,i}\right) \tag{95}$$

$$f_2 = \sum_{i=1}^{N} \left(a_i + b_i P_{g,i} + c_i P_{g,i}^2\right) \tag{96}$$

where a_i, b_i, and c_i are the cost coefficient of i-th bus generator,
subject to:

$$\begin{aligned}
&P_{g,i} - P_{l,i} - V_i \sum_{j=1}^{N} V_j \left(G_{ij} cos(\delta_i - \delta_j) + B_{ij} sin(\delta_i - \delta_j)\right) = 0,\ i = 1, 2, ...N,\\
&Q_{g,i} - Q_{l,i} - V_i \sum_{j=1}^{N} V_j \left(G_{ij} sin(\delta_i - \delta_j) - B_{ij} cos(\delta_i - \delta_j)\right) = 0,\ i = 1, 2, ...N,\\
&V_{min} \leq V_k ...k = 1, 2, ..N \leq V_{max},\\
&\delta_{min} \leq \delta_k ...k = 1, 2, ..N \leq \delta_{max},\\
&P_{min} \leq P_{g,k} ....k = 1, 2, ...N \leq P_{max}\\
&Q_{min} \leq Q_{g,k} ....k = 1, 2, ...N \leq Q_{max}
\end{aligned}$$

where $P_i (= P_{g,i} - P_{l,i})$ and $Q_i (= Q_{g,i} - Q_{l,i})$ represent the active and reactive injected power, respectively, at i-th bus, $Ybus_{ij} (= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix, $V_i (= V_i \angle \delta_i)$ is bus voltage at i-th bus and N represents the total number of buses in system.

2.5.9. Optimal Power Flow for Minimizing Fuel Cost, Active and Reactive Power Loss (RCM44) [47]

Minimize:

$$f_1 = \sum_{i=1}^{N} \left(P_{g,i} - P_{l,i}\right) \tag{97}$$

$$f_2 = \sum_{i=1}^{N} \left(Q_{g,i} - Q_{l,i}\right) \tag{98}$$

$$f_3 = \sum_{i=1}^{N} \left(a_i + b_i P_{g,i} + c_i P_{g,i}^2\right) \tag{99}$$

where a_i, b_i, and c_i are the cost coefficient of i-th bus generator,
subject to:

$$P_{g,i} - P_{l,i} - V_i \sum_{j=1}^{N} V_j \left(G_{ij} cos(\delta_i - \delta_j) + B_{ij} sin(\delta_i - \delta_j)\right) = 0,\ i = 1, 2, ...N,$$

$$Q_{g,i} - Q_{l,i} - V_i \sum_{j=1}^{N} V_j \left(G_{ij} sin(\delta_i - \delta_j) - B_{ij} cos(\delta_i - \delta_j)\right) = 0,\ i = 1, 2, ...N,$$

$$V_{min} \leq V_k ... k = 1, 2, ..N \leq V_{max},$$

$$\delta_{min} \leq \delta_k ... k = 1, 2, ..N \leq \delta_{max},$$

$$P_{min} \leq P_{g,k} .... k = 1, 2, ...N \leq P_{max}$$

$$Q_{min} \leq Q_{g,k} .... k = 1, 2, ...N \leq Q_{max}$$

where $P_i (= P_{g,i} - P_{l,i})$ and $Q_i (= Q_{g,i} - Q_{l,i})$ represent the active and reactive injected power, respectively, at i-th bus, $Ybus_{ij} (= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix, $V_i (= V_i \angle \delta_i)$ is bus voltage at i-th bus and N represents the total number of buses in system.

2.5.10. Optimal Power Flow for Minimizing Fuel Cost, Voltage deviation, and Active Power Loss (RCM45) [47]

Minimize:

$$f_1 = \sum_{i=1}^{N} \left(P_{g,i} - P_{l,i}\right) \tag{100}$$

$$f_2 = \sum_{i=1}^{N} (1 - |V_i|) \tag{101}$$

$$f_3 = \sum_{i=1}^{N} \left(a_i + b_i P_{g,i} + c_i P_{g,i}^2\right) \tag{102}$$

where a_i, b_i, and c_i are the cost coefficient of i-th bus generator,
subject to:

$$P_{g,i} - P_{l,i} - V_i \sum_{j=1}^{N} V_j \left(G_{ij} cos(\delta_i - \delta_j) + B_{ij} sin(\delta_i - \delta_j)\right) = 0,\ i = 1, 2, ...N,$$

$$Q_{g,i} - Q_{l,i} - V_i \sum_{j=1}^{N} V_j \left(G_{ij} sin(\delta_i - \delta_j) - B_{ij} cos(\delta_i - \delta_j)\right) = 0,\ i = 1, 2, ...N,$$

$V_{min} \leq V_k ...k = 1, 2, ..N \leq V_{max}$,

$\delta_{min} \leq \delta_k ...k = 1, 2, ..N \leq \delta_{max}$,

$P_{min} \leq P_{g,k} ....k = 1, 2, ...N \leq P_{max}$

$Q_{min} \leq Q_{g,k} ....k = 1, 2, ...N \leq Q_{max}$

where $P_i (= P_{g,i} - P_{l,i})$ and $Q_i (= Q_{g,i} - Q_{l,i})$ represent the active and reactive injected power, respectively, at i-th bus, $Ybus_{ij} (= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix, $V_i (= V_i \angle \delta_i)$ is bus voltage at i-th bus and N represents the total number of buses in system.

2.5.11. Optimal Power Flow for Minimizing Fuel Cost, Voltage deviation, Active and Reactive Power Loss (RCM46) [47]

Minimize:

$$f_1 = \sum_{i=1}^{N} \left(P_{g,i} - P_{l,i} \right) \tag{103}$$

$$f_2 = \sum_{i=1}^{N} \left(Q_{g,i} - Q_{l,i} \right) \tag{104}$$

$$f_3 = \sum_{i=1}^{N} \left(a_i + b_i P_{g,i} + c_i P_{g,i}^2 \right) \tag{105}$$

where a_i, b_i, and c_i are the cost coefficient of i-th bus generator,

$$f_4 = \sum_{i=1}^{N} (1 - |V_i|) \tag{106}$$

subject to:

$$P_{g,i} - P_{l,i} - V_i \sum_{j=1}^{N} V_j \left(G_{ij} cos(\delta_i - \delta_j) + B_{ij} sin(\delta_i - \delta_j) \right) = 0, \; i = 1, 2, ...N,$$

$$Q_{g,i} - Q_{l,i} - V_i \sum_{j=1}^{N} V_j \left(G_{ij} sin(\delta_i - \delta_j) - B_{ij} cos(\delta_i - \delta_j) \right) = 0, \; i = 1, 2, ...N,$$

$V_{min} \leq V_k ...k = 1, 2, ..N \leq V_{max}$,

$\delta_{min} \leq \delta_k ...k = 1, 2, ..N \leq \delta_{max}$,

$P_{min} \leq P_{g,k} ....k = 1, 2, ...N \leq P_{max}$

$Q_{min} \leq Q_{g,k} ....k = 1, 2, ...N \leq Q_{max}$

where $P_i (= P_{g,i} - P_{l,i})$ and $Q_i (= Q_{g,i} - Q_{l,i})$ represent the active and reactive injected power, respectively, at i-th bus, $Ybus_{ij} (= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix, $V_i (= V_i \angle \delta_i)$ is bus voltage at i-th bus and N represents the total number of buses in system.

2.5.12. Optimal Droop Setting for Minimizing Active and Reactive Power Loss (RCM47) [48]

Minimize:

$$f_1 = \sum_{i=1}^{N} P_i \tag{107}$$

$$f_2 = \sum_{i=1}^{N} Q_i \tag{108}$$

subject to:

$$\begin{aligned}
&\sum_{i=1}^{N}(G_{k,i}V_{r,i} - B_{ki}V_{m,i}) - \frac{P_kV_{r,k} + Q_kV_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\ k = 1, ...N,\\
&\sum_{i=1}^{N}(B_{ki}V_{r,i} + G_{ki}V_{m,i}) - \frac{P_kV_{m,k} - Q_kV_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\ k = 1, ...N,\\
&P_k - Cp_k(w_k^* - w) + P_{l,k} = 0,\ k = 1, ...N,\\
&Q_k - Cq_k\left(V_k^* - \sqrt{(V_{r,k})^2 + (V_{m,k})^2}\right) + Q_{l,k} = \qquad 0,\ k = 1, ...N,\\
&V_{min} \le V_{r,k}, V_{m,k}...k = 1, 2, ...N \le V_{max}\\
&P_{min} \le P_k.......k = 1, 2, ...N \le P_{max}\\
&Q_{min} \le Q_k.......k = 1, 2, ...N \le Q_{max}\\
&Cp_{min,k} \le Cp_k.......k = 1, 2, ...N \le Cp_{max,k}\\
&Cq_{min,k} \le Cq_k.......k = 1, 2, ...N \le Cq_{max,k}\\
&w_{min} \le w \le w_{max}
\end{aligned}$$

where P_i and Q_i represent the active and reactive injected power, respectively, at i-th bus, $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix, $V_i(= V_{r,i} + 1jV_{m,i})$ is bus voltage at i-th bus, Cp_k and Cq_k represent the active and reactive power droop parameters of controllers, respectively, w is operating frequency and N represents the total number of buses in system.

2.5.13. Optimal Droop Setting for Minimizing Voltage Deviation and Active Power Loss (RCM48) [49]

Minimize:

$$f_1 = \sum_{i=1}^{N} P_i \tag{109}$$

$$f_2 = \sum_{i=1}^{N} \left(1 - |V_i|)^2\right) \tag{110}$$

subject to:

$$\begin{aligned}
&\sum_{i=1}^{N}(G_{k,i}V_{r,i} - B_{ki}V_{m,i}) - \frac{P_kV_{r,k} + Q_kV_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\ k = 1, ...N,\\
&\sum_{i=1}^{N}(B_{ki}V_{r,i} + G_{ki}V_{m,i}) - \frac{P_kV_{m,k} - Q_kV_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\ k = 1, ...N,\\
&P_k - Cp_k(w_k^* - w) + P_{l,k} = 0,\ k = 1, ...N,\\
&Q_k - Cq_k\left(V_k^* - \sqrt{(V_{r,k})^2 + (V_{m,k})^2}\right) + Q_{l,k} = \qquad 0,\ k = 1, ...N,
\end{aligned}$$

$$V_{min} \leq V_{r,k}, V_{m,k}...k = 1, 2, ...N \leq V_{max}$$
$$P_{min} \leq P_k.......k = 1, 2, ...N \leq P_{max}$$
$$Q_{min} \leq Q_k.......k = 1, 2, ...N \leq Q_{max}$$
$$Cp_{min,k} \leq Cp_k.......k = 1, 2, ...N \leq Cp_{max,k}$$
$$Cq_{min,k} \leq Cq_k.......k = 1, 2, ...N \leq Cq_{max,k}$$
$$w_{min} \leq w \leq w_{max}$$

where P_i and Q_i represent the active and reactive injected power, respectively, at i-th bus, $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix, $V_i(= V_{r,i} + 1jV_{m,i})$ is bus voltage at i-th bus, Cp_k and Cq_k represent the active and reactive power droop parameters of controllers, respectively, w is operating frequency and N represents the total number of buses in system.

2.5.14. Optimal Droop Setting for Minimizing Voltage Deviation, Active, and Reactive Power Loss (RCM49) [50]

Minimize:

$$f_1 = \sum_{i=1}^{N} P_i \tag{111}$$

$$f_1 = \sum_{i=1}^{N} Q_i \tag{112}$$

$$f_1 = \sum_{i=1}^{N} \left(1 - |V_i|)^2\right) \tag{113}$$

subject to:

$$\sum_{i=1}^{N}(G_{k,i}V_{r,i} - B_{ki}V_{m,i}) - \frac{P_kV_{r,k} + Q_kV_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\ k = 1, ...N,$$
$$\sum_{i=1}^{N}(B_{ki}V_{r,i} + G_{ki}V_{m,i}) - \frac{P_kV_{m,k} - Q_kV_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\ k = 1, ...N,$$
$$P_k - Cp_k(w_k^* - w) + P_{l,k} = 0,\ k = 1, ...N,$$
$$Q_k - Cq_k\left(V_k^* - \sqrt{(V_{r,k})^2 + (V_{m,k})^2}\right) + Q_{l,k} = \qquad 0,\ k = 1, ...N,$$
$$V_{min} \leq V_{r,k}, V_{m,k}...k = 1, 2, ...N \leq V_{max}$$
$$P_{min} \leq P_k.......k = 1, 2, ...N \leq P_{max}$$
$$Q_{min} \leq Q_k.......k = 1, 2, ...N \leq Q_{max}$$
$$Cp_{min,k} \leq Cp_k.......k = 1, 2, ...N \leq Cp_{max,k}$$
$$Cq_{min,k} \leq Cq_k.......k = 1, 2, ...N \leq Cq_{max,k}$$
$$w_{min} \leq w \leq w_{max}$$

where P_i and Q_i represent the active and reactive injected power, respectively, at i-th bus, $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix, $V_i(= V_{r,i} + 1jV_{m,i})$ is bus voltage at i-th bus, Cp_k and Cq_k represent the active and reactive power droop parameters of controllers, respectively, w is operating frequency and N represents the total number of buses in system.

2.5.15. Power Distribution System Planning (RCM50) [51]

Minimize :

$$f_1 = \sum_{i=1}^{6} \left(a_i + b_i x_i + c_i x_i^2\right), \tag{114}$$

$$f_2 = \sum_{i=1}^{6} \left(\alpha_i + \beta_i x_i + \gamma_i x_i^2\right) \tag{115}$$

where,

i	a_i	b_i	c_i	α_i	β_i	γ_i
1	756.7988	38.5390	0.15247	13.8593	0.32767	0.00419
2	451.3251	46.1591	0.10587	13.8593	0.32767	0.00419
3	1243.5311	38.3055	0.03546	40.2669	-0.54551	0.00683
4	1049.9977	40.3965	0.02803	40.2669	-0.54551	0.00683
5	1356.6592	38.2704	0.01799	42.8955	-0.51116	0.00461
6	1658.5696	36.3278	0.02111	42.8955	-0.51116	0.00461

Subject to :

$$h_1 = \sum_{i=1}^{6} (x_i - PD - PL)$$

where,

$$PD = 12000$$

$$PL = \sum_{i=1}^{6} \sum_{j=1}^{6} \left(x_i x_j B_{ij} \times 10^{-6}\right)$$

B_{ij}	1	2	3	4	5	6
1	140	17	15	19	26	22
2	17	60	13	16	15	20
3	15	13	65	17	24	19
4	19	16	17	71	30	25
5	26	15	24	30	69	32
6	22	20	19	25	32	85

2.6. Proposed Test-suite of RWCMOPs

The above-mentioned 50 problems are combined to create a test-suite for evaluating the performance of CMOEAs. The basic details of these problems such as the number of objective functions, number of decision variables, number of equality constraints and inequality constraints are reported in Table 2. As shown in Table 2. the number of objective functions varies from 2 to 5, the number of decision variable varies from 2 to 34, the number of inequality constraints varies from 0 to 29, and the number of equality constraints vary from 0 to 26.

3. Evaluation of the Proposed Test-suite

In this section, we evaluate the performance of six state-of-the-art CMOEAs on the problems of the proposed test-suite. These six algorithms are ToP [52], TiGE_2 [53], cNSGAIII [8], cMOEA/D [8], CCMO [54], cARMOEA [55], AnD [56]. The source codes of these algorithms are taken from PLATEMO [57], a MATLAB platform for multi- or many-objective optimization.

Table 2: Details of the 50 RWCMOPs. M is the total number of objectives, D is the total number of decision variables of the problem, ng is the number of inequality constraints and nh is the number of equality constraints

Prob	Name	M	D	ng	nh
	Mechanical Design Problems				
RCM01	Pressure Vessel Design	2	2	2	2
RCM02	Vibrating Platform Design	2	5	5	0
RCM03	Two Bar Truss Design	2	3	3	0
RCM04	Welded Beam Design	2	4	4	0
RCM05	Disc Brake Design	2	4	4	0
RCM06	Speed Reducer Design	2	7	11	0
RCM07	Gear Train Design	2	4	1	0
RCM08	Car Side Impact Design	3	7	9	0
RCM09	Four Bar Plane Truss	2	4	1	0
RCM10	Two Bar Plane Truss	2	2	2	0
RCM11	Water Resources Management	5	3	7	0
RCM12	Simply Supported I-beam Design	2	4	1	0
RCM13	Gear Box Design	3	7	11	0
RCM14	Multiple Disk Clutch Brake Design	2	5	8	0
RCM15	Spring Design	2	3	8	0
RCM16	Cantilever Beam Design	2	2	2	0
RCM17	Bulk Carrier Design	3	6	9	0
RCM18	Front Rail Design	2	3	3	0
RCM19	Multi-product Batch Plant	3	10	10	0
RCM20	Hydro-static Thrust Bearing Design	2	4	7	0
RCM21	Crash Energy Management for High-speed Train	2	6	4	0
	Chemical Engineering Problems				
RCM22	Haverly's Pooling Problem	2	9	2	4
RCM23	Reactor Network Design	2	6	1	4
RCM24	Heat Exchanger Nwteork Design	3	9	0	6
	Process, Design and Synthesis Problems				
RCM25	Process Synthesis Problem	2	2	2	0
RCM26	Process Synthesis and Design Problem	2	3	1	1
RCM27	Process Flow Sheeting Problem	2	3	3	0
RCM28	Two Reactor Problem	2	7	4	4
RCM29	Process Synthesis Problem	2	7	9	0
	Power Electronics Problems				
RCM30	Synchronous Optimal Pulse-width Modulation of 3-level Inverters	2	25	24	0
RCM31	Synchronous Optimal Pulse-width Modulation of 5-level Inverters	2	25	24	0
RCM32	Synchronous Optimal Pulse-width Modulation of 7-level Inverters	2	25	24	0
RCM33	Synchronous Optimal Pulse-width Modulation of 9-level Inverters	2	30	29	0
RCM34	Synchronous Optimal Pulse-width Modulation of 11-level Inverters	2	30	29	0
RCM35	Synchronous Optimal Pulse-width Modulation of 13-level Inverters	2	30	29	0
	Power System Optimization Problems				
RCM36	Optimal Sizing of Single Phase Distribution Generation with Reactive Power support for phase balancing at Main Transformer/Grid and Minimizing Active Power Loss	2	28	0	24
RCM37	Optimal Sizing of Single Phase Distribution Generation with Reactive Power support for phase balancing at Main Transformer/Grid and Minimizing Reactive Power Loss	2	28	0	24
RCM38	Optimal Sizing of Single Phase Distribution Generation with Reactive Power support for Minimizing Active and Reactive Power Loss	2	28	0	24
RCM39	Optimal Sizing of Single Phase Distribution Generation with Reactive Power support for phase balancing at Main Transformer/Grid and Minimizing Active and Reactive Power Loss	3	28	0	24
RCM40	Optimal Power Flow for Minimizing Active and Reactive Power Loss	2	34	0	26
RCM41	Optimal Power Flow for Minimizing Voltage deviation, Active and Reactive Power Loss	3	34	0	26
RCM42	Optimal Power Flow for Minimizing Voltage deviation, and Active Power Loss	2	34	0	26
RCM43	Optimal Power Flow for Minimizing Fuel Cost, and Active Power Loss	2	34	0	26
RCM44	Optimal Power Flow for Minimizing Fuel Cost, Active and Reactive Power Loss	3	34	0	26
RCM45	Optimal Power Flow for Minimizing Fuel Cost, Voltage deviation, and Active Power Loss	3	34	0	26
RCM46	Optimal Power Flow for Minimizing Fuel Cost, Voltage deviation, Active and Reactive Power Loss	4	34	0	26
RCM47	Optimal Droop Setting for Minimizing Active and Reactive Power Loss	2	18	0	12
RCM48	Optimal Droop Setting for Minimizing Voltage Deviation and Active Power Loss	2	18	0	12
RCM49	Optimal Droop Setting for Minimizing Voltage Deviation, Active, and Reactive Power Loss	2	18	0	12
RCM50	Power Distribution System Planning	2	6	0	1

3.1. Performance Indicator

In general, performance indicators are used to assess the quality of the obtained Pareto fronts in case of CMOPs. Here, we utilize the Hypervolume Indicator (HV) for giving a score to the Pareto fronts obtained by all algorithms as HV has been the only Pareto-compliant indicator available currently in the literature [58]. A larger value of HV of a given Pareto front indicates the better approximation of the original Pareto front of the given problem. Usually, Pareto front of the real-world problem is not known. This is the main reason for not utilizing the other performance indicator which requires a set of reference vectors. As suggested in [59, 13], we set the reference vector of length M to

$[1.1, 1.1, ..., 1.1]^T$ for the calculation of HV in the normalized objective-space. For normalization of objective-space, we use approximated ideal and nadir points of actual objective-space and the normalized i-th objective function value, $f_i(\bar{x})$, for a solution $\bar{x}$ can be obtained by the following equation.

$$\hat{f}_i(\bar{x}) = \frac{f_i(\bar{x}) - f_i^{ideal}}{f_i^{nadir} - f_i^{ideal}} \tag{116}$$

where $\hat{f}_i(\bar{x})$ is the normalized i-th objective function value at solution $barx$; f_i^{ideal} and f_i^{nadir} are the ideal and nadir points of i-th dimension of the original objective-space. respectively. Here, we use two algorithms, SASS [60] and sCMAgES [61] to calculate the ideal and nadir points of all objectives of all problems of the proposed test-suite as these algorithms are the top-ranked algorithms of *Special Session & Competition on Real-world Constrained Optimization* organised at WCCI 2020 and GECCO 2020 [17].

3.2. Experimental Settings

All algorithms are implemented on MATLAB r2017b in a PC with Windows 10 operating system, INTEL Core i7 CPU, and 16 GB RSM. The parameters of all algorithms are set on values suggested in their respective papers. For stopping the optimization process, we apply the same stopping criterion on each algorithm, which is based on the number of objective functions and decision variables. In this stopping criterion, we allot a fixed budget of function evaluations for each problem separately, and we stop the optimization process when the number of consumed function evaluations exceeds this budget. The budget of function evaluation, Max_{FEs}, for each problem is set as follows.

$$Max_{FEs} = \begin{cases} 2 \times 10^4, & \text{if } (M == 2) \ \& \ (D \leq 10) \\ 8 \times 10^4, & \text{elseif } (M == 2) \ \& \ (D > 10) \\ 2.6250 \times 10^4, & \text{elseif } (M == 3) \ \& \ (D \leq 10) \\ 1.05 \times 10^5, & \text{elseif } (M == 3) \ \& \ (D > 10) \\ 3.575 \times 10^4, & \text{elseif } (M == 4) \ \& \ (D \leq 10) \\ 1.43 \times 10^5, & \text{elseif } (M == 4) \ \& \ (D > 10) \\ 5.3 \times 10^4, & \text{elseif } (M == 5) \ \& \ (D \leq 10) \\ 2.12 \times 10^5, & \text{elseif } (M == 5) \ \& \ (D > 10) \end{cases} \tag{117}$$

3.3. Difficulty Level Evaluation of Problems of Proposed Test-suite

The difficulty level of each problem of the proposed test-suite is different from each other. To assess the relative difficulty level of these problems, we adopt the following procedure.

1. All algorithms are implemented 25 times independently on each problem to calculate the statistical data for the assessment of performance.
2. This statistical data contains best, mean, worst, and standard deviation of HV values and degree of CV obtained from 25 times independent implementation. In addition, we also calculate the Feasibility Rate (FR) of algorithms on each problem.
   - *Degree of CV:* The degree of CV, CV^d, is the average of the CV of all solutions of the final output population obtained by the algorithm.
   - *FR:* FR is the ratio of total runs in which CV^d is zero versus total independent runs.
3. Finally, we evaluate the difficulty level of problems by using the following criteria.
   - Evaluation is done on the basis of the FR values of all algorithms.
   - Then, evaluation is done on the basis of CV^d values of all algorithms.

3.3.1. Mechanical Design Problems

The baseline results of mechanical design problems are shown in Tables 3-5. By analyzing these tables, we get that the FR of all algorithms is 100 for all mechanical design problems. Therefore, we can conclude that the difficulty level of these problems is relatively low, as state-of-the-art algorithms except cMOEA/D easily locate the feasible solutions of the constrained Pareto front of these problems.

Table 3: Baseline results of mechanical design problems (RCM01-RCM08).

Problem		ToP [52]	TiGE_2 [53]	cNSGAIII [8]	cMOEA/D [8]	CCMO [54]	cARMOEA [55]	AnD [56]
RCM01	HV_best	0.606717	0.538106	0.60758	0.108992	0.605144	0.607879	0.603085
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.605832	0.510526	0.606391	0.108888	0.603743	0.606639	0.599144
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.604649	0.455252	0.603023	0.108025	0.601598	0.605276	0.593941
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000511	0.021003	0.00096	0.000173	0.000855	0.000742	0.001756
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM02	HV_best	0.16646	0.134328	0.166457	0.176394	0.166382	0.16642	0.16519
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.149764	0.021593	0.053488	0.052203	0.063709	0.035963	0.038431
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0	0	0	0	0	0	0
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.049921	0.037817	0.067281	0.069297	0.072199	0.059449	0.061729
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM03	HV_best	0.901391	0.851123	0.896894	0.298947	0.898968	0.898881	0.89865
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.781179	0.687886	0.891818	0.120981	0.89715	0.897511	0.897299
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.236575	0.285117	0.889927	0.089525	0.895395	0.895233	0.894421
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.194751	0.146899	0.00162	0.044027	0.000984	0.001035	0.000992
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM04	HV_best	0.861896	0.729125	0.861236	0.088416	0.859299	0.860785	0.858811
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.861354	0.472491	0.853826	0.014423	0.853488	0.852793	0.85333
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.860162	0.254392	0.840075	0	0.826305	0.831737	0.835492
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000332	0.140715	0.006158	0.028662	0.007333	0.007443	0.005027
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM05	HV_best	0.434245	0.412378	0.434245	0.427847	0.434311	0.434516	0.432729
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.433843	0.396852	0.432672	0.420501	0.432985	0.433091	0.430543
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.433401	0.369139	0.428064	0.400399	0.427832	0.428895	0.427071
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000194	0.011186	0.001367	0.006833	0.001467	0.001447	0.00132
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM06	HV_best	0.277233	0.274287	0.277145	0.276696	0.27738	0.277163	0.276948
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.274139	0.271504	0.276964	0.276548	0.276606	0.277025	0.276551
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.231964	0.26809	0.276253	0.273695	0.269111	0.276878	0.275255
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.008451	0.001677	0.00018	0.000531	0.001431	9.42E-05	0.000281
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM07	HV_best	0.226953	0.215711	0.226861	0.222935	0.226971	0.227019	0.225198
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.22671	0.199665	0.225854	0.220981	0.226712	0.226378	0.224103
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.226375	0.106722	0.222861	0.215165	0.226341	0.225192	0.222241
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000117	0.023012	0.000815	0.001425	0.000148	0.00045	0.000637
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM08	HV_best	0.025865	0.021195	0.025616	0.013168	0.025976	0.026053	0.026062
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.025617	0.020437	0.025358	0.009369	0.025828	0.02591	0.02584
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.025388	0.020236	0.024966	0.008119	0.02547	0.025616	0.025259
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000109	0.00024	0.000164	0.001043	9.65E-05	0.00012	0.000155
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100

Table 4: Baseline results of mechanical design problems RC09-RC16

Problem		ToP [52]	TiGE_2 [53]	cNSGAIII [8]	cMOEA/D [8]	CCMO [54]	cARMOEA [55]	AnD [56]
RCM09	HV_best	0.408889	0.358098	0.409689	0.05316	0.408948	0.40966	0.408287
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.40851	0.323399	0.409477	0.053057	0.40864	0.409568	0.407496
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.408112	0.295263	0.409166	0.052973	0.408196	0.409307	0.4066
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000169	0.01341	0.00012	3.86E-05	0.000177	6.48E-05	0.000418
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM10	HV_best	0.847364	0.84456	0.837576	0.080044	0.842495	0.843914	0.845939
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.847146	0.840968	0.833455	0.079487	0.839439	0.841241	0.844971
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.846424	0.834233	0.832868	0.078753	0.832774	0.837581	0.843441
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000182	0.00205	0.001023	0.000419	0.002253	0.002155	0.000598
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM11	HV_best	0.09929	0.098926	0.100442	0.061329	0.099962	0.099998	0.099662
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.097266	0.097945	0.099746	0.060353	0.099165	0.097146	0.098868
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.094654	0.094981	0.099195	0.059309	0.098126	0.092464	0.098031
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.001183	0.000851	0.000348	0.000505	0.000456	0.001526	0.000394
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM12	HV_best	0.723422	0.711611	0.722742	0.101861	0.722206	0.722755	0.720785
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.723073	0.698009	0.721768	0.064495	0.719671	0.722192	0.718469
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.722601	0.668655	0.720212	0.012029	0.714833	0.719849	0.713855
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000244	0.00993	0.000578	0.02106	0.002199	0.000579	0.001523
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM13	HV_best	0.089673	0.088687	0.090348	0.090343	0.08925	0.090421	0.090365
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.089243	0.086669	0.090125	0.090201	0.088845	0.090296	0.090291
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.088436	0.085158	0.089524	0.089093	0.088388	0.089942	0.090166
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000267	0.000699	0.000205	0.000226	0.000184	0.000108	5.68E-05
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM14	HV_best	0.617478	0.495242	0.617891	0.172877	0.61637	0.618066	0.61418
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.616706	0.330275	0.61628	0.120625	0.614199	0.616625	0.606465
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.616112	0.097988	0.610967	0.076989	0.61144	0.612748	0.587042
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000328	0.10877	0.001446	0.024785	0.001396	0.0013	0.004093
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM15	HV_best	0.543199	0.521655	0.542396	0.24499	0.540387	0.542542	0.541475
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.542927	0.509063	0.540606	0.071978	0.535172	0.54117	0.53927
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.542363	0.480435	0.536591	0.065994	0.516745	0.539697	0.537395
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000178	0.009041	0.001018	0.032127	0.006226	0.00072	0.000915
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM16	HV_best	0.763379	0.752025	0.762657	0.079343	0.762161	0.762473	0.761334
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.762977	0.742237	0.762404	0.079087	0.761688	0.762449	0.758998
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.761876	0.708289	0.762299	0.079055	0.760799	0.76243	0.754586
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000306	0.008109	7.92E-05	5.67E-05	0.000303	1.29E-05	0.001495
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100

Table 5: Baseline results of mechanical design problems (RC17-RC21).

Problem		ToP [52]	TiGE_2 [53]	cNSGAIII [8]	cMOEA/D [8]	CCMO [54]	cARMOEA [55]	AnD [56]
RCM17	HV_best	0.343355	0.329025	0.272668	0.300459	0.34287	0.27689	0.275577
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.265468	0.20413	0.247039	0.196528	0.271101	0.253003	0.209068
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.04228	0.086825	0.190024	0.100199	0.227364	0.239668	0.058186
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.059101	0.059935	0.017716	0.055155	0.031926	0.007278	0.042127
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM18	HV_best	0.040475	0.03988	0.040508	0.040316	0.0405	0.040509	0.040469
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.040463	0.039305	0.040504	0.040259	0.040494	0.040507	0.040435
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.04045	0.038049	0.040492	0.04019	0.040487	0.040499	0.04024
	CV_worst	0	0	0	0	0	0	0
	HV_sd	5.86E-06	0.000445	4.25E-06	2.44E-05	3.61E-06	2.01E-06	4.10E-05
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM19	HV_best	0.332801	0.301435	0.30792	0.244087	0.306362	0.303361	0.304245
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.284636	0.277989	0.284733	0.171237	0.281467	0.280489	0.284262
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.15376	0.257503	0.254717	0.087701	0.218761	0.254298	0.270568
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.05626	0.011308	0.009991	0.039532	0.016711	0.011653	0.00897
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM20	HV_best	0.207864	0.129792	0.179272	0	0.163509	0.055214	0.179222
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.108881	0.024426	0.138998	0	0.114169	0.003069	0.115754
	CV_mean	0.000401	63.24543	0.000186	0	0	0	0
	HV_worst	0	0	0	0	0	0	0
	CV_worst	0.012044	1897.363	0.00557	0	0	0	0
	HV_sd	0.083168	0.042439	0.037054	0	0.049259	0.011727	0.043542
	CV_sd	0.002162	340.5871	0.001	0	0	0	0
	FR	96.66667	96.66667	96.66667	100	100	100	100
RCM21	HV_best	0.031753	0.028646	0.031757	0.02933	0.031753	0.031758	0.031749
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.03175	0.021239	0.031711	0.029322	0.0317	0.03167	0.031706
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.031748	0.019965	0.031405	0.029317	0.030878	0.030542	0.030975
	CV_worst	0	0	0	0	0	0	0
	HV_sd	1.48E-06	0.002286	7.14E-05	3.16E-06	0.000177	0.000237	0.000141
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100

Table 6: Baseline results of chemical engineering problems (RC22-RC24).

Problem		ToP [52]	TiGE_2 [53]	cNSGAIII [8]	cMOEA/D [8]	CCMO [54]	cARMOEA [55]	AnD [56]
RCM22	HV_best	0	0	0	0	0	0	0
	CV_best	1.280592	0.010923	0.024605	0.001647	0.014998	0.011363	0.021321
	HV_mean	0	0	0	0	0	0	0
	CV_mean	29.05799	4.663478	3.022009	2.202614	8.295304	7.225637	6.29337
	HV_worst	0	0	0	0	0	0	0
	CV_worst	81.36874	25.05874	16.5864	14.88096	30.08038	49.2384	28.30793
	HV_sd	0	0	0	0	0	0	0
	CV_sd	21.68135	6.241971	4.173321	3.3434	9.066057	10.30937	7.585384
	FR	0	0	0	0	0	0	0
RCM23	HV_best	0.998563	0.990669	0.467709	0.689092	0.447149	0.577967	0.487608
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.033285	0.456674	0.14435	0.175798	0.0611	0.138216	0.108034
	CV_mean	0.039073	1.73E-05	3.36E-05	1.42E-05	0.00019	3.65E-05	5.11E-05
	HV_worst	0	0	0	0	0	0	0
	CV_worst	0.347136	0.000377	0.000253	0.000194	0.000827	0.000186	0.000265
	HV_sd	0.179247	0.279928	0.158177	0.163518	0.115609	0.172869	0.139781
	CV_sd	0.082742	6.91E-05	5.95E-05	3.99E-05	0.000252	5.78E-05	7.39E-05
	FR	3.333333	90	56.66667	73.33333	26.66667	50	43.33333
RCM24	HV_best	0	2.86E+08	0	0	0	0	0
	CV_best	165.3936	0	0.003802	0.166729	6.838311	0.675435	0.518473
	HV_mean	0	10259231	0	0	0	0	0
	CV_mean	251582.2	1.317266	137.2594	81.94926	631.407	257.567	144.8416
	HV_worst	0	0	0	0	0	0	0
	CV_worst	878333.6	11.63664	861.3564	413.3518	5123.33	1715.384	630.6579
	HV_sd	0	51387539	0	0	0	0	0
	CV_sd	303702.5	2.781827	198.909	99.4577	1050.21	406.839	148.7518
	FR	0	10	0	0	0	0	0

3.3.2. Chemical Engineering Problems

The baseline results of chemical engineering problems are shown in Table (6). It can be seen from this table, FR of two problems out of three is zero. Therefore, it can be concluded that the difficulty level of these problems is relatively high, as state-of-the-art algorithms cannot locate a single feasible solution of two out of three problems. In the case of RCM23, the algorithms locate the feasible solutions in some of the runs, but these feasible solutions are not located on its constrained Pareto front.

3.3.3. Process Design and Synthesis Problems

In Table (7), the baseline results of process design and synthesis problems. From Table (7), FR of all problems is 100 on all problems except RCM28 as RCM28 contains four equality constraints. Therefore, it can be concluded that the difficulty level of these problems is relatively low.

3.3.4. Power Electronic Problems

In Table (8), the baseline results of power electronics problems are shown. It can be seen from the Table that the FR of these problems is relatively low for most of the algorithms. All algorithms cannot locate the feasible solutions in each run. Therefore, the difficulty level of power electronics problems is relatively high.

3.3.5. Power System Problems

The baseline results of power system problems are depicted in Tables 9 and 10. As shown in these tables, FR of these problems is zero for all algorithms as these problems contain a higher number of equality constraints. Therefore, these problems are relatively more difficult for the problems of other streams.

From the above analysis, it can be concluded that the proposed test suite contains all a variety of problems having different difficulty levels, and it can be utilized to determine the robustness and efficacy of newly proposed algorithms. Due to the higher difficulty level, state-of-the-art algorithms cannot find a single feasible solution in case of majority problems. This phenomenon inspires others to develop more robust CMOEAs and CHTs than currently available in literature.

3.4. Evaluation of Performance of Algorithms

For ranking the CMOEAs based on the performance over the proposed benchmark suite, we propose a ranking scheme inspired from [62]. Supposing N algorithms CMOEA_1, CMOEA_2.....CMOEA_N participates in the compara-

Table 7: Baseline results of process design and synthesis problems (RCM25-RCM29).

Problem		ToP [52]	TiGE_2 [53]	cNSGAIII [8]	cMOEA/D [8]	CCMO [54]	cARMOEA [55]	AnD [56]
RCM25	HV_best	0.240906	0.217111	0.241086	0.237308	0.241187	0.241118	0.240922
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.24077	0.198929	0.24106	0.236795	0.241164	0.240776	0.240808
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.240643	0.164597	0.241033	0.236569	0.241122	0.23134	0.240572
	CV_worst	0	0	0	0	0	0	0
	HV_sd	7.29E-05	0.01378	1.30E-05	0.000215	1.19E-05	0.001752	8.04E-05
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM26	HV_best	0.188765	0.166356	0.188171	0.194468	0.20437	0.200758	0.200818
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.155657	0.124086	0.152925	0.144699	0.154502	0.159318	0.144786
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.110303	0.090911	0.095389	0.116597	0.091719	0.117439	0.093144
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.019194	0.019445	0.028217	0.018733	0.029585	0.023262	0.030851
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM27	HV_best	0.718792	0.795056	0.719229	0.721782	0.719622	0.719229	0.721013
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.718359	0.747882	0.719224	0.721672	0.71956	0.719221	0.717436
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.717772	0.690259	0.719218	0.721631	0.719482	0.719213	0.714571
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.000262	0.0291	2.35E-06	2.41E-05	3.67E-05	4.54E-06	0.001573
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100
RCM28	HV_best	0	0.07228	0	0.070562	0	0	0
	CV_best	0.014281	0	0.9999	0	0.000635	0.00145	0.000234
	HV_mean	0	0.03953	0	0.004974	0	0	0
	CV_mean	0.936119	8.03E-09	1.000205	0.899936	0.933887	0.933675	0.967071
	HV_worst	0	0	0	0	0	0	0
	CV_worst	1.019421	2.41E-07	1.002147	1.000507	1.002939	1.00318	1.003175
	HV_sd	0	0.018531	0	0.015634	0	0	0
	CV_sd	0.244706	4.32E-08	0.000465	0.299979	0.249196	0.2491	0.179539
	FR	0	96.66667	0	10	0	0	0
RCM29	HV_best	0.471774	0.442004	0.51602	0.518846	0.504352	0.520551	0.517712
	CV_best	0	0	0	0	0	0	0
	HV_mean	0.335559	0.390516	0.447617	0.452675	0.404413	0.432106	0.417833
	CV_mean	0	0	0	0	0	0	0
	HV_worst	0.200477	0.20332	0.351614	0.384726	0.285779	0.321582	0.285964
	CV_worst	0	0	0	0	0	0	0
	HV_sd	0.071275	0.046371	0.043386	0.033962	0.04798	0.053401	0.060436
	CV_sd	0	0	0	0	0	0	0
	FR	100	100	100	100	100	100	100

Table 8: Baseline results of power electronics problems (RCM30-RCM35).

Problem		ToP [52]	TiGE_2 [53]	cNSGAIII [8]	cMOEA/D [8]	CCMO [54]	cARMOEA [55]	AnD [56]
RCM30	HV_best	0	0.662965	0.701013	0.839462	0.666132	0.697997	0.725822
	CV_best	0	0	0	0	0	0	0
	HV_mean	0	0.034649	0.289495	0.391785	0.144395	0.217875	0.34759
	CV_mean	11.56486	21.04043	1.205781	0.90871	1.914152	1.628609	1.888535
	HV_worst	0	0	0	0	0	0	0
	CV_worst	28.29356	40.98365	9.124346	6.781095	15.41303	12.44645	12.42719
	HV_sd	0	0.134816	0.300872	0.326206	0.247237	0.290774	0.311574
	CV_sd	9.229781	12.84853	2.211664	1.802905	3.428305	2.838586	3.302945
	FR	10	6.666667	50	60	26.66667	40	56.66667
RCM31	HV_best	0	0.393877	0.827003	0.812276	0.806101	0.857183	0.80836
	CV_best	0.042148	0	0	0	0	0	0
	HV_mean	0	0.022851	0.306911	0.240442	0.227716	0.229353	0.279842
	CV_mean	18.27013	20.82913	3.073342	1.35446	2.117136	3.009694	1.363218
	HV_worst	0	0	0	0	0	0	0
	CV_worst	56.90556	55.72129	21.15783	19.16384	14.1716	18.19538	15.92129
	HV_sd	0	0.086513	0.341371	0.295125	0.305414	0.317861	0.298428
	CV_sd	12.51585	14.13454	5.573602	3.558056	3.41207	4.371542	3.179246
	FR	0	6.666667	46.66667	46.66667	40	36.66667	53.33333
RCM32	HV_best	0	0.793533	0.825149	0.835984	0.822397	0.792387	0.878748
	CV_best	2.20E-05	0	0	0	0	0	0
	HV_mean	0	0.07489	0.445862	0.4641	0.229616	0.333093	0.398617
	CV_mean	18.99021	18.54321	0.99685	1.459053	4.089679	1.675006	1.521516
	HV_worst	0	0	0	0	0	0	0
	CV_worst	51.56746	59.46963	12.24312	12.10024	19.75116	21.96807	9.837396
	HV_sd	0	0.2251	0.366448	0.367472	0.335455	0.364356	0.380229
	CV_sd	13.01789	15.25797	2.310026	2.837696	6.196577	4.433198	2.497286
	FR	0	10	60	63.33333	33.33333	46.66667	53.33333
RCM33	HV_best	0	0	0.23966	0.052622	0.025531	0.065701	0.288832
	CV_best	10.40772	0	0	0	0	0	0
	HV_mean	0	0	0.007989	0.003471	0.000851	0.00553	0.009716
	CV_mean	39.10995	28.67943	6.270309	3.470084	8.276413	6.438681	4.812328
	HV_worst	0	0	0	0	0	0	0
	CV_worst	66.10341	68.29036	32.1277	19.78779	28.90257	20.99447	19.5327
	HV_sd	0	0	0.04302	0.012987	0.004583	0.016825	0.051833
	CV_sd	13.50099	17.38156	7.893064	4.853179	7.719334	6.105385	5.112044
	FR	0	3.333333	20	30	13.33333	13.33333	16.66667
RCM34	HV_best	0	0.113921	0.479272	0.320512	0.437299	0.487213	0.431892
	CV_best	10.38784	0	0	0	0	0	0
	HV_mean	0	0.006823	0.088819	0.055524	0.063454	0.049491	0.066807
	CV_mean	34.29755	28.19329	3.049706	4.922927	7.650207	3.139325	4.122926
	HV_worst	0	0	0	0	0	0	0
	CV_worst	65.58168	75.36825	23.41344	21.54223	25.95273	14.5725	26.77143
	HV_sd	0	0.025705	0.16467	0.095984	0.127719	0.119012	0.123938
	CV_sd	12.98598	17.48042	5.038477	6.423627	7.846577	3.837989	6.573328
	FR	0	6.666667	23.33333	36.66667	26.66667	23.33333	26.66667
RCM35	HV_best	0	0	0.686211	0.651583	0.590883	0.752999	0.676141
	CV_best	15.4508	1.546835	0	0	0	0	0
	HV_mean	0	0	0.167352	0.215711	0.019696	0.10397	0.134673
	CV_mean	37.43432	26.005	4.989614	4.099303	6.324352	6.595854	3.217712
	HV_worst	0	0	0	0	0	0	0
	CV_worst	64.91172	62.33122	20.83738	16.38286	23.85962	32.50146	18.87406
	HV_sd	0	0	0.26105	0.267693	0.106067	0.236295	0.246232
	CV_sd	12.63062	14.89643	6.323394	5.494804	6.28649	8.393579	4.408667
	FR	0	0	30	40	3.333333	16.66667	23.33333

Table 9: Baseline results of power system Optimization problems (RCM36-RCM44)

Problem		ToP [52]	TiGE_2 [53]	cNSGAIII [8]	cMOEA/D [8]	CCMO [54]	cARMOEA [55]	AnD [56]
RCM36	HV_best	0	0	0	0	0	0	0
	CV_best	82.3121	6.643412	4.25498	3.651223	7.154865	4.415033	4.435388
	HV_mean	0	0	0	0	0	0	0
	CV_mean	274.4762	35.2284	53.70688	15.06921	94.37566	52.79134	54.93299
	HV_worst	0	0	0	0	0	0	0
	CV_worst	678.7059	182.8126	161.8934	51.0971	335.3863	258.121	331.5996
	HV_sd	0	0	0	0	0	0	0
	CV_sd	142.171	45.21685	43.41143	13.72349	75.30395	63.66908	68.93145
	FR	0	0	0	0	0	0	0
RCM37	HV_best	0	0	0	0	0	0	0
	CV_best	127.732	4.963391	4.404994	3.553089	4.683178	4.311428	4.417435
	HV_mean	0	0	0	0	0	0	0
	CV_mean	293.7674	71.53191	71.71703	26.24301	102.6528	54.66656	59.19636
	HV_worst	0	0	0	0	0	0	0
	CV_worst	530.8988	309.3974	404.9752	162.1423	325.4873	251.9391	229.573
	HV_sd	0	0	0	0	0	0	0
	CV_sd	122.1939	78.49083	77.53464	39.93584	83.6963	55.73482	57.36992
	FR	0	0	0	0	0	0	0
RCM38	HV_best	0	0	0	0	0	0	0
	CV_best	106.7765	4.581639	4.263193	3.341614	26.41366	4.293962	4.262062
	HV_mean	0	0	0	0	0	0	0
	CV_mean	316.6513	121.5665	48.41404	13.9309	61.61325	57.32413	28.06424
	HV_worst	0	0	0	0	0	0	0
	CV_worst	592.1684	385.4081	225.0023	89.6667	95.77121	321.0996	111.3932
	HV_sd	0	0	0	0	0	0	0
	CV_sd	130.8002	107.8309	56.57967	19.90946	15.04164	81.33751	29.04501
	FR	0	0	0	0	0	0	0
RCM39	HV_best	0	0	0	0	0	0	0
	CV_best	65.16863	4.972818	4.338738	3.348161	4.455859	4.371101	4.178467
	HV_mean	0	0	0	0	0	0	0
	CV_mean	199.7498	20.87616	38.46924	53.59397	51.13681	47.85619	78.27483
	HV_worst	0	0	0	0	0	0	0
	CV_worst	555.3932	92.60474	194.8944	412.8086	189.879	173.4359	496.7103
	HV_sd	0	0	0	0	0	0	0
	CV_sd	112.1574	22.54907	49.33667	78.29306	46.79739	50.81561	109.7538
	FR	0	0	0	0	0	0	0
RCM40	HV_best	0	0	0	0	0	0	0
	CV_best	3.150392	0.660076	0.999967	0.848469	1.066144	0.91699	0.952824
	HV_mean	0	0	0	0	0	0	0
	CV_mean	5.972555	2.512599	1.623799	1.329999	1.536825	1.689827	1.493907
	HV_worst	0	0	0	0	0	0	0
	CV_worst	9.37611	5.529642	3.957437	1.942884	2.858858	2.994664	3.674098
	HV_sd	0	0	0	0	0	0	0
	CV_sd	1.636978	1.375104	0.586713	0.222756	0.328697	0.511366	0.456711
	FR	0	0	0	0	0	0	0
RCM41	HV_best	0	0	0	0	0	0	0
	CV_best	3.301155	0.806728	0.965293	0.914012	1.081988	1.093108	0.927847
	HV_mean	0	0	0	0	0	0	0
	CV_mean	5.662143	2.628734	1.365783	1.561768	1.596323	1.541367	1.44663
	HV_worst	0	0	0	0	0	0	0
	CV_worst	9.892571	7.870352	1.962484	5.534691	4.006354	4.262531	2.180599
	HV_sd	0	0	0	0	0	0	0
	CV_sd	2.068608	1.632579	0.245068	0.767151	0.534491	0.572747	0.255961
	FR	0	0	0	0	0	0	0
RCM42	HV_best	0	0	0	0	0	0	0
	CV_best	3.519447	1.31777	0.894709	1.061619	0.843062	0.901273	1.069173
	HV_mean	0	0	0	0	0	0	0
	CV_mean	6.049385	3.469379	1.527958	1.388699	1.758078	1.420168	1.54281
	HV_worst	0	0	0	0	0	0	0
	CV_worst	12.73405	7.037393	2.851103	1.903395	4.349191	2.266922	2.615807
	HV_sd	0	0	0	0	0	0	0
	CV_sd	2.067198	1.446002	0.37419	0.180178	0.647684	0.260657	0.307961
	FR	0	0	0	0	0	0	0
RCM43	HV_best	0	0	0	0	0	0	0
	CV_best	3.511517	1.040588	0.904851	0.905841	1.070411	1.012481	1.126768
	HV_mean	0	0	0	0	0	0	0
	CV_mean	6.14353	2.330865	1.461826	1.331803	1.74988	1.783643	1.665836
	HV_worst	0	0	0	0	0	0	0
	CV_worst	12.7558	4.918305	2.640186	1.79928	4.462654	6.152961	4.349584
	HV_sd	0	0	0	0	0	0	0
	CV_sd	1.996193	1.041009	0.287781	0.1635	0.768109	0.905813	0.638118
	FR	0	0	0	0	0	0	0

Table 10: Baseline results of power system optimization problems RCM44-RCM50

Problem		ToP [52]	TiGE_2 [53]	cNSGAIII [8]	cMOEA/D [8]	CCMO [54]	cARMOEA [55]	AnD [56]
RCM44	HV_best	0	0	0	0	0	0	0
	CV_best	3.121608	0.36281	1.104857	1.024321	1.047254	0.905816	1.089238
	HV_mean	0	0	0	0	0	0	0
	CV_mean	5.157192	1.207907	1.504204	1.574824	1.627674	1.479146	1.448399
	HV_worst	0	0	0	0	0	0	0
	CV_worst	8.110355	2.523043	2.263849	2.26727	3.727173	2.508425	2.442546
	HV_sd	0	0	0	0	0	0	0
	CV_sd	1.30777	0.491465	0.272569	0.307687	0.713274	0.281505	0.253467
	FR	0	0	0	0	0	0	0
RCM45	HV_best	0	0	0	0	0	0	0
	CV_best	3.145494	1.190196	1.201944	1.105595	1.117485	1.088376	1.142505
	HV_mean	0	0	0	0	0	0	0
	CV_mean	5.624684	2.578918	1.523252	1.396802	1.572634	1.502721	1.476502
	HV_worst	0	0	0	0	0	0	0
	CV_worst	11.34453	5.824603	1.886354	1.857372	3.14689	2.177195	1.723471
	HV_sd	0	0	0	0	0	0	0
	CV_sd	1.648326	1.13158	0.167319	0.195778	0.359486	0.24672	0.116904
	FR	0	0	0	0	0	0	0
RCM46	HV_best	0	0	0	0	0	0	0
	CV_best	2.030229	0.681656	0.81054	1.016072	0.922549	1.070911	1.043286
	HV_mean	0	0	0	0	0	0	0
	CV_mean	4.360037	1.538903	1.427434	1.600378	1.46879	1.44803	1.375263
	HV_worst	0	0	0	0	0	0	0
	CV_worst	7.516484	3.203139	2.292921	2.685567	1.875628	2.099201	1.688632
	HV_sd	0	0	0	0	0	0	0
	CV_sd	1.314467	0.640264	0.253772	0.38678	0.220315	0.214225	0.183594
	FR	0	0	0	0	0	0	0
RCM47	HV_best	0	0	0	0	0	0	0
	CV_best	0.468588	0.074105	0.182643	0.065264	0.144811	0.06228	0.407403
	HV_mean	0	0	0	0	0	0	0
	CV_mean	4.348969	3.9884	2.985086	1.465222	3.68641	2.600211	3.314637
	HV_worst	0	0	0	0	0	0	0
	CV_worst	15.00573	14.62155	9.434895	6.233328	9.565402	12.24001	10.97261
	HV_sd	0	0	0	0	0	0	0
	CV_sd	3.196686	2.97268	2.473667	1.520414	2.431509	2.880452	2.925691
	FR	0	0	0	0	0	0	0
RCM48	HV_best	0	0	0	0	0	0	0
	CV_best	0.392895	0.057091	0.037706	0.007331	0.161581	0.138793	0.21525
	HV_mean	0	0	0	0	0	0	0
	CV_mean	4.851161	3.39968	3.055395	1.603796	3.451364	3.367045	2.337885
	HV_worst	0	0	0	0	0	0	0
	CV_worst	13.50848	11.2626	17.45677	5.529828	12.0752	12.85875	8.437814
	HV_sd	0	0	0	0	0	0	0
	CV_sd	3.864569	2.713269	3.760928	1.461697	3.045447	3.498318	2.106418
	FR	0	0	0	0	0	0	0
RCM49	HV_best	0	0	0	0	0	0	0
	CV_best	0.538135	0.103872	0.064077	0.030818	0.212473	0.194484	0.114581
	HV_mean	0	0	0	0	0	0	0
	CV_mean	4.71745	5.415072	2.405423	2.595872	2.374781	1.751124	2.978641
	HV_worst	0	0	0	0	0	0	0
	CV_worst	14.30472	40.78255	8.001206	7.585709	7.680618	5.466936	8.212466
	HV_sd	0	0	0	0	0	0	0
	CV_sd	3.722273	7.241503	2.248118	1.849062	1.98031	1.471382	2.413002
	FR	0	0	0	0	0	0	0
RCM50	HV_best	0	0	0	0	0	0	0
	CV_best	0	0	0	0	0	0	0
	HV_mean	0	0	0	0	0	0	0
	CV_mean	0.002172	0.001959	0.000193	0	0.000785	0.000183	0.000489
	HV_worst	0	0	0	0	0	0	0
	CV_worst	0.008623	0.007746	0.001493	0	0.005396	0.000968	0.001869
	HV_sd	0	0	0	0	0	0	0
	CV_sd	0.001843	0.002066	0.000319	0	0.001178	0.000299	0.000566
	FR	6.666667	6.666667	40	100	20	40	10

tive analysis done on P problems. The performance score, S, can be defined as follows:

$$S(CMOEA_i) = \frac{1}{P}\left(\sum_{j=1}^{P} \frac{1}{N-1}\left(\sum_{k=1}^{N} \delta^i_{j,k}\right)\right), \tag{118}$$

where,

$$\delta^i_{j,k} = \begin{cases} 1, & \text{if } CMOEA_j \text{ significantly outperforms } CMOEA_i \text{ on a problem } k, \\ 0, & \text{otherwise} \end{cases} \tag{119}$$

Here, we use the Wilcoxon rank-sum test at a 0.05 significance level to determine the significant difference between the performance of two algorithms on a problem. The lower value of S of an algorithm suggests that the algorithm performs better on the proposed test-suite.

The performance score of all algorithms on the proposed benchmark suite is shown in Table (11). As shown in Table (11), cNSGAIII and cARMOEA provide the lowest performance score, i.e., performs better than other algorithms.

Table 11: Ranking of all algorithms on the proposed benchmark suite.

Algorithm	Performance Score	Rank
ToP [52]	0.6567	6
TiGE_2 [53]	0.7300	7
cNSGAIII [8]	0.3433	1.5
cMOEA/D [8]	0.4900	4
CCMO [54]	0.5033	5
cARMOEA [55]	0.3433	1.5
AnD [56]	0.4333	3

4. Conclusion

While evaluation on RWCMOPs is an important aspect of performance assessment of newly developed CMOEAs, it is a difficult task to establish due to domain knowledge requirements and other obstacles. To resolve this issue, we develop a test-suite containing RWCMOPs selected from various engineering streams. This test-suite contains 50 RWCMOPs of different difficulty levels from low to high level. To evaluate the difficulty level of these problems, we select seven state-of-the-art algorithms for calculating the baseline results of these problems. The baseline results obtained from the experiment suggest that some of the problems are not solved by all algorithms, i.e., hard to solve by currently available algorithms. We also present the performance comparison of the selected algorithms on the proposed benchmark suite.

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