



# Special Session & Competitions on Single Objective Bound Constrained Numerical Optimization

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- Introduction
- A review for CEC2021 benchmark problems
- Accepted algorithms
- Analysis of the Results
- Ranking result

# Introduction

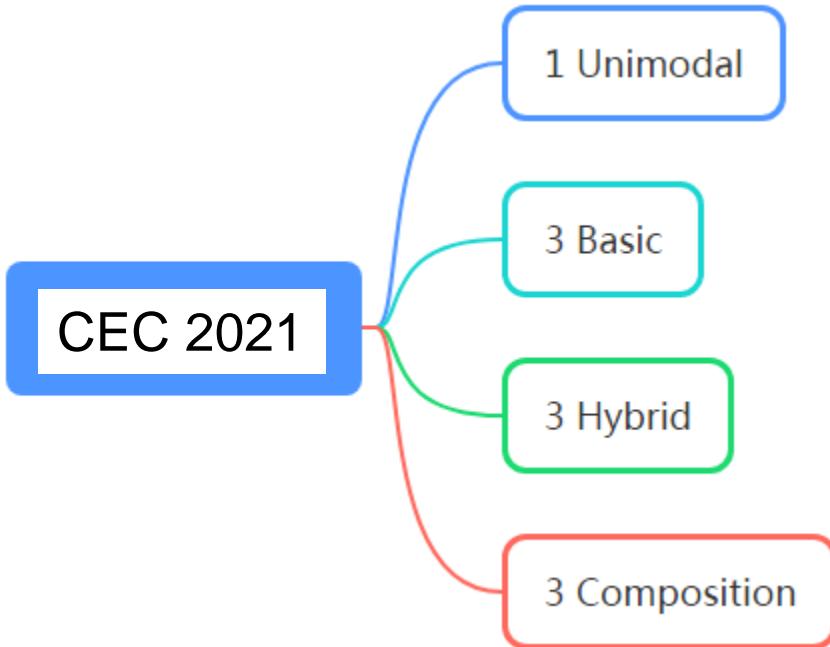
- » Single objective optimization algorithms are the foundation upon which more complex methods, like multi-objective, niching and constrained optimization algorithms, are built.
- » Consequently, improvements to single objective optimization algorithms are important because they can impact other domains as well.
- » This interplay between methods and problems drives progress, so we have developed the CEC'21 Special Session on Real-Parameter Optimization to promote this symbiosis.
- » In this competition, the benchmark objective functions are parameterized by including the operators such as bias, rotation, and translation.
- » The main motive behind the parameterization is to test the effect of all combinations of the operators on all benchmark functions.
- » Parametrized benchmarking is a step towards obtaining multi-faceted insight into algorithmic performance and the optimization problems.
- » For this, 10 scalable benchmark problems are proposed with these binary operators.

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# A review for CEC2021 benchmark problems



**This competition significantly increases the maximum number of allowed function evaluations**

# A review for CEC2021 benchmark problems

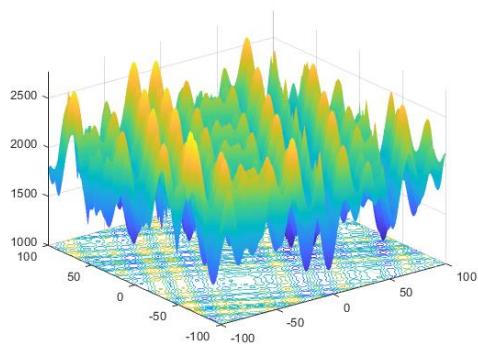
## Information and features of the test problems suite

	No.	Functions	$F_i^* = F_i(\mathbf{x}^*)$
Unimodal Function	1	Shifted and Rotated Bent Cigar Function (CEC 2017 F1)	100
Basic Functions	2	Shifted and Rotated Schwefel's Function (CEC 2014 F11)	1100
	3	Shifted and Rotated Lunacek bi-Rastrigin Function (CEC 2017 F7)	700
	4	Expanded Rosenbrock's plus Griewangk's Function (CEC2017 $f_{19}$ )	1900
Hybrid Functions	5	Hybrid Function 1 ( $N = 3$ ) (CEC 2014 F17)	1700
	6	Hybrid Function 2 ( $N = 4$ ) (CEC 2017 F16)	1600
	7	Hybrid Function 3 ( $N = 5$ ) (CEC 2014 F21)	2100
Composition Functions	8	Composition Function 1 ( $N = 3$ ) (CEC 2017 F22)	2200
	9	Composition Function 2 ( $N = 4$ ) (CEC 2017 F24)	2400
	10	Composition Function 3 ( $N = 5$ ) (CEC 2017 F25)	2500

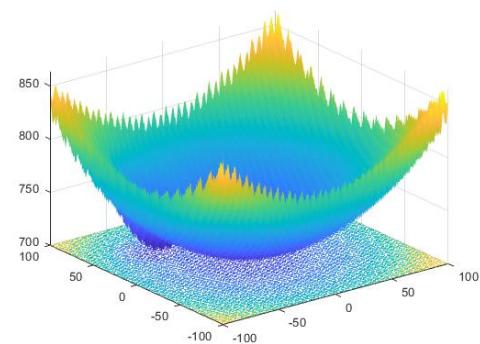
Search range: [-100,100]<sup>D</sup>

# A review for CEC2021 benchmark problems

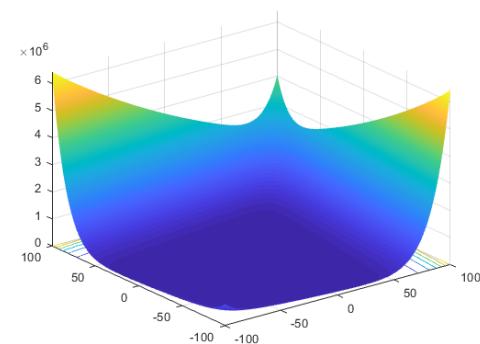
3-D map for some of functions in CEC2021



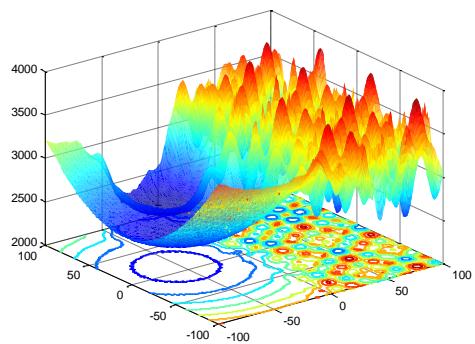
F2



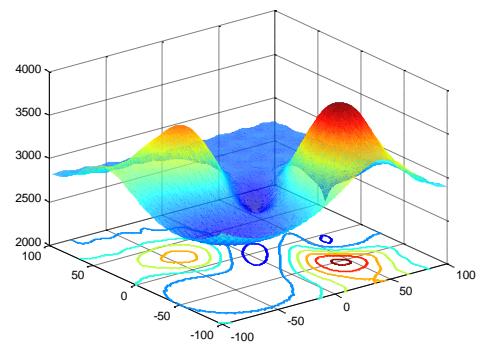
F3



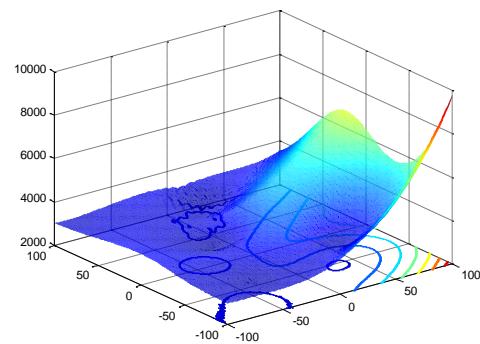
F4



F8



F9



F10

# A review for CEC2021 benchmark problems

- » There are only 8 configurations possible for each function.

Functions	Bias	Shift	Rotation	Parameterized Vector
Basic	0	0	$I$	(000)
Bias	$F_i^*$	0	$I$	(100)
Shift	0	$o_i$	$I$	(010)
Rotation	0	0	$M$	(001)
Bias and Shift	$F_i^*$	$o_i$	$I$	(110)
Bias and Rotation	$F_i^*$	0	$M$	(101)
Shift and Rotation	0	$o_i$	$M$	(011)
Bias, Shift and Rotation	$F_i^*$	$o_i$	$M$	(111)

- » Optimum Value of all test functions for each transformation.

Functions	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10
Basic	0	0	0	0	0	0	0	0	0	0
Bias	100	1100	700	1900	1700	1600	2100	2200	2400	2500
Shift	0	0	0	0	0	0	0	0	0	0
Rotation	0	0	0	0	0	0	0	0	0	0
Bias and Shift	100	1100	700	1900	1700	1600	2100	2200	2400	2500
Bias and Rotation	100	1100	700	1900	1700	1600	2100	2200	2400	2500
Shift and Rotation	0	0	0	0	0	0	0	0	0	0
Bias, Shift and Rotation	100	1100	700	1900	1700	1600	2100	2200	2400	2500

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# Accepted Algorithms

S.N.	Paper ID	Algorithm	Paper Title
1.	E-0453	DEDMNA	Differential Evolution with Distance-based Mutation-selection Applied to CEC 2021 Single Objective Numerical Optimisation
2.	E-0336	APGSK_IMODE	Gaining-Sharing Knowledge Based Algorithm with Adaptive Parameters Hybrid with IMODE Algorithm for Solving CEC 2021 Benchmark Problems
3.	E-0221	MadDE	Improving Differential Evolution through Bayesian Hyperparameter Optimization
4.	E-0204	RB_IPOP_CMAES	A New Step-Size Adaptation Rule for CMA-ES Based on the Population Midpoint Fitness
5.	E-0159	jDE21	Self-adaptive Differential Evolution Algorithm with Population Size Reduction for Single Objective Bound-Constrained Optimization: Algorithm j21
6.	E-0125	NL-SHADE-RSP	NL-SHADE-RSP Algorithm with Adaptive Archive and Selective Pressure for CEC 2021 Numerical Optimization
7.	Com111	SOMA-CLP	SOMA-CLP for Competition on Bound Constrained Single Objective Numerical Optimization Benchmark
8.	Com112	MLS-LSHADE	Technical report: A Multi-start Local Search Algorithm with L-SHADE for Single Objective Bound Constrained Optimization
9.	Com113	LSHADE	An ordered and roulette-wheel-based mutation incorporated L-SHADE algorithm for Solving CEC2021 Single Objective Numerical Optimisation Problems

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# Analysis of the Results

## » Evaluation Criteria:

### 1. $Score_1$ :

$$SNE = 0.5 \left( \sum_{m=1}^n \left( \sum_{i=1}^{10} (ne_{i,m}^{10D}) \right) + \sum_{m=1}^n \left( \sum_{i=1}^{10} (ne_{i,m}^{20D}) \right) \right)$$

where

$$ne = \frac{f(x_{best} - f(x^*))}{f(x_{best})_{max} - f(x^*)}$$

Then,

$$Score_1 = \left( 1 - \frac{SNE - SNE_{min}}{SNE} \right) \times 50$$

### 2. $Score_2$ :

$$SR = 0.5 \left( \sum_{m=1}^n \left( \sum_{i=1}^{10} (rank_{i,m}^{10D}) \right) + \sum_{m=1}^n \left( \sum_{i=1}^{10} (rank_{i,m}^{20D}) \right) \right)$$

where  $rank$  is the algorithm's rank among all algorithms. Then,

$$Score_2 = \left( 1 - \frac{SR - SR_{min}}{SR} \right) \times 50$$

### 3. $Total Score = Score_1 + Score_2$

- » The above evaluation criteria are used to rank the algorithms for analysis in next slides of different scenarios.

# Analysis of the Results

- » Non-shifted (000, 100, 001, and 101) vs. Shifted cases (010, 110, 011, and 111):

S.N.	Paper ID	Algorithm	Non-shift vs. Shift
1.	E-0453	DEDMNA	$6 \Rightarrow 3$
2.	E-0336	APGSK_IMODE	$1 \Rightarrow 4$
3.	E-0221	MadDE	$2 \Rightarrow 6$
4.	E-0204	RB_IPOP_CMAES	$9 \Rightarrow 9$
5.	E-0159	jDE21	$7 \Rightarrow 2$
6.	E-0125	NL-SHADE-RSP	$5 \Rightarrow 1$
7.	Com111	SOMA-CLP	$8 \Rightarrow 7$
8.	Com112	MLS-LSHADE	$4 \Rightarrow 5$
9.	Com113	LSHADE	$3 \Rightarrow 8$

- » Some algorithms, APGSK\_IMODE, MadDE, and LSHADE dominate in Non-shifted cases, where optimum is located at origin.
- » Here, these algorithms use center-biased operator which force the solutions towards the origin.

# Analysis of the Results

- » Non-bias (000, 001, 010, and 011) vs. Bias cases (100, 101, 110, and 111):

S.N.	Paper ID	Algorithm	Non-bias vs. Bias
1.	E-0453	DEDMNA	6 $\Rightarrow$ 6
2.	E-0336	APGSK_IMODE	1 $\Rightarrow$ 1
3.	E-0221	MadDE	2 $\Rightarrow$ 2
4.	E-0204	RB_IPOP_CMAES	9 $\Rightarrow$ 9
5.	E-0159	jDE21	5 $\Rightarrow$ 5
6.	E-0125	NL-SHADE-RSP	3 $\Rightarrow$ 3
7.	Com111	SOMA-CLP	8 $\Rightarrow$ 8
8.	Com112	MLS-LSHADE	4 $\Rightarrow$ 4
9.	Com113	LSHADE	7 $\Rightarrow$ 7

- » Here, Biased cases do not show much effect on the performance of algorithms.

# Analysis of the Results

- » Rotated Non-shifted (001, and 101) vs. Rotated Shifted cases (011, and 111):

S.N.	Paper ID	Algorithm	Non-shift vs. Shift
1.	E-0453	DEDMNA	$6 \Rightarrow 3$
2.	E-0336	APGSK_IMODE	$1 \Rightarrow 4$
3.	E-0221	MadDE	$2 \Rightarrow 6$
4.	E-0204	RB_IPOP_CMAES	$8 \Rightarrow 9$
5.	E-0159	jDE21	$7 \Rightarrow 2$
6.	E-0125	NL-SHADE-RSP	$5 \Rightarrow 1$
7.	Com111	SOMA-CLP	$9 \Rightarrow 7$
8.	Com112	MLS-LSHADE	$4 \Rightarrow 5$
9.	Com113	LSHADE	$3 \Rightarrow 8$

- » Here, performance of algorithms in non-shifted cases dominates in overall ranking due to use of center-biased operator in some algorithms.
- » Centre-biased algorithms solve non-shifted cases with approximately 100% success rate irrespective of the rotation thereby dominating the overall ranking in this case.
- » Moreover, difficulty of a problem is determined predominantly by the presence/absence of the shift operator, not by the rotation operator.
- » Hence, it is meaningful to analyze the effect of rotation using only shifted cases.

# Analysis of the Results

- » Rotated Shifted (011, and 111) vs. Non-Rotated Shifted cases (010, and 110):

S.N.	Paper ID	Algorithn	Non-rotate vs. Rotate	Overall
1.	E-0453	DEDMNA	2 $\Rightarrow$ 3	3
2.	E-0336	APGSK_IMODE	4 $\Rightarrow$ 4	4
3.	E-0221	MadDE	5 $\Rightarrow$ 6	6
4.	E-0204	RB_IPOP_CMAES	9 $\Rightarrow$ 9	9
5.	E-0159	jDE21	1 $\Rightarrow$ 2	2
6.	E-0125	NL-SHADE-RSP	3 $\Rightarrow$ 1	1
7.	Com111	SOMA-CLP	7 $\Rightarrow$ 7	7
8.	Com112	MLS-LSHADE	6 $\Rightarrow$ 5	5
9.	Com113	LSHADE	8 $\Rightarrow$ 8	8

- » Here, better performance of algorithms on rotational cases dominates the overall ranking.

# Analysis of the Results

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- » By the deep analysis of the algorithms' performance, we decide to declare separate ranking for following cases:
  1. Non-shifted Cases (000, 100, 001, and 101).
  2. Shifted Cases (010, 011, 110, and 111).
  3. Non-rotated Shifted Cases (010, and 110).
  4. Rotated Shifted Cases (011, and 111).

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# Ranking Result

**Final Ranking of the accepted algorithms for Non-shifted Cases  
(000, 100, 001, and 101).**

S.N.	Paper ID	Algorithm	$Score_1$	$Score_2$	$Score$	Rank
1.	E-0453	DEDMNA	00.0000	27.3509	27.3509	6
2.	E-0336	APGSK_IMODE	50.0000	50.0000	100.0000	1
3.	E-0221	MadDE	50.0000	49.5842	99.5842	2
4.	E-0204	RB_IPOP_CMAES	00.0000	19.3902	19.3902	9
5.	E-0159	jDE21	00.0000	24.4867	24.4867	7
6.	E-0125	NL-SHADE-RSP	00.0000	29.0854	29.0854	5
7.	Com111	SOMA-CLP	00.0000	20.2462	20.2462	8
8.	Com112	MLS-LSHADE	00.0000	36.1912	36.1912	4
9.	Com113	LSHADE	50.0000	46.8566	96.8566	3

**Superior performance on non-shifted problems and relative performance deterioration on shifted problems flag possible incorporation origin biasing operator(s) in an algorithm. In this situation a closer examination of such algorithm is required.**

# Ranking Result

## Final Ranking of the accepted algorithms for Shifted Cases

(010, 011, 110, and 111).

S.N.	Paper ID	Algorithn	$Score_1$	$Score_2$	$Score$	Rank
1.	E-0453	DEDMNA	41.0315	39.8319	80.8634	3
2.	E-0336	APGSK_IMODE	32.7946	35.2679	68.0625	4
3.	E-0221	MadDE	24.1391	32.0704	56.2095	6
4.	E-0204	RB_IPOP_CMAES	09.5376	19.3312	28.8688	9
5.	E-0159	jDE21	50.0000	35.6391	85.6391	2
6.	E-0125	NL-SHADE-RSP	38.4401	50.0000	88.4401	1
7.	Com111	SOMA-CLP	24.8320	24.6618	49.4938	7
8.	Com112	MLS-LSHADE	26.7024	29.9621	56.6645	5
9.	Com113	LSHADE	15.8431	22.0056	37.8486	8

# Ranking Result

## Final Ranking of the accepted algorithms for Non-rotated Shifted Cases

(010, and 110).

S.N.	Paper ID	Algorithn	$Score_1$	$Score_2$	$Score$	Rank
1.	E-0453	DEDMNA	28.3655	47.7273	76.0928	2
2.	E-0336	APGSK_IMODE	18.6911	34.1716	52.8627	4
3.	E-0221	MadDE	12.6055	31.6438	44.2493	5
4.	E-0204	RB_IPOP_CMAES	04.5240	17.5266	22.0506	9
5.	E-0159	jDE21	50.0000	34.6847	84.6847	1
6.	E-0125	NL-SHADE-RSP	21.8792	50.0000	71.8792	3
7.	Com111	SOMA-CLP	15.8772	25.4967	41.3739	7
8.	Com112	MLS-LSHADE	14.5754	28.6600	43.2355	6
9.	Com113	LSHADE	08.0404	20.0521	28.0925	8

# Ranking Result

## Final Ranking of the accepted algorithms for Rotated Shifted Cases

S.N.	Paper ID	Algorithn	$Score_1$	$Score_2$	$Score$	Rank
1.	E-0453	DEDMNA	47.0225	34.4193	81.4417	3
2.	E-0336	APGSK_IMODE	42.6131	36.3772	78.9903	4
3.	E-0221	MadDE	33.7161	32.4866	66.2027	6
4.	E-0204	RB_IPOP_CMAES	14.6431	21.4286	36.0717	9
5.	E-0159	jDE21	50.0000	34.6847	84.6847	2
6.	E-0125	NL-SHADE-RSP	50.0000	50.0000	100.0000	1
7.	Com111	SOMA-CLP	29.7991	23.9173	53.7165	7
8.	Com112	MLS-LSHADE	35.9052	31.3144	67.2196	5
9.	Com113	LSHADE	22.7171	24.2515	46.9686	8

# Proposal to Edit the Benchmark

- » By the deep analysis of the algorithms' performance, we find that when a problem is not shifted, it is impossible to see the effect of other operators such as rotation, because algorithms incorporating the origin biasing operators can solve these problems irrespective of the other parametrization/transformations.
- » Therefore, it is sufficient to have just a single case without shift.
- » We propose to divide the benchmark into 2 parts:
- » Part I: To be used in the future by other researchers to identify the effects of shift, rotations, etc. on an algorithm: 000 , 010, 011, 110, and 111
- » Part II: Just to be archived to maintain the full record of the competition. Researchers will be asked not to use these cases in the future to evaluate an algorithm: 100, 001, and 101



Thanks for your attention!