Supplementary File of Oblique Decision Tree Ensemble via Multisurface Proximal Support Vector Machine

I. Detail of Rotation Forest algorithm

Table A- I: Rotation Forest

Rotation Forest algorithm:

· Training phase:
Given:
$X := N \times n$ is the training dataset, where N is the number of the training data, n is the dimension of each data.
F is the feature set.
$Y : N \times 1$ is the labels of the training set.
L is the ensemble size, which means the number of trees in the forests.
T_i refers to each random tree in the Rotation Forest, $i = 1...L$.
m is the number of subsets. $\{w_1, ..., w_c\}$ the set of class labels
For $i = 1...L$:

1) prepare the Rotation Matrix R_i
   - Split F into m subsets:$F_{i,j}$ (for $j = 1...m$)
   - For $j = 1...m$
     - Let $X_{i,j}$ be the data set X for the features in $F_{i,j}$
     - Eliminate from $X_{i,j}$ a random subset of classes
     - Select a bootstrap sample from $X_{i,j}$ of size 75% of the number of objects in $X_{i,j}$. Denote the new set by $X'_{i,j}$
     - Apply PCA on $X'_{i,j}$ to obtain the coefficients in a matrix $C_{i,j}$
   - Arrange the $C_{i,j}$ for $j = 1...m$ in a rotation matrix as in equation (1)
   - Construct ${R_i}^a$ by rearranging the columns of R_i so as to match the order of features in F.
2) Building classifier using $(X{R_i}^a, Y)$as the training set

· Classification phase:

1) For a given x, let $d_{i,j}(x{R_i}^a)$ be the probability assigned by classifier D_i to the hypothesis that x comes from class w_j. Calculate the confidence for each class, w_j, by the average combination method:

$$\mu_j(x) = \frac{1}{L}\sum_{i=1}^{n} d_{i,j}(x{R_i}^a),\ j = 1, ...c \tag{1}$$

2) Assign x to the class with the largest confidence.

II. SPECIFICATIONS OF THE 44 CLASSIFICATION PROBLEMS

Table A- II: Specifications of the 44 Classification Problems

Datasets	Classes	Samples	Features
Adult	2	32561	123
AFP-Pred	2	9974	119
Australian	2	690	14
Balance scale	3	625	4
Banknote	2	1372	4
Biodeg	2	1055	41
Blood transfusion	2	748	4
BLProt	2	18943	545
Breast cancer	2	699	9
Breast tissue	6	106	9
Climate model	2	540	18
DNA	3	3186	180
Ecoli	8	336	7
Fertility	2	100	9
Glass	7	214	9
Harberman	2	306	3
Heart	2	270	13
Hepatitis	2	155	19
segment	7	2310	19
Ionosphere	2	351	34
Iris	3	150	4
Lymphography	4	296	18
Mam-masses	2	961	5
Orl	40	400	1024
Ozone	2	2536	72
Page block	5	5473	10
Parkinsons	2	195	22
Pima-diabetes	2	768	8
Planning Relax	2	182	12
Seeds	3	210	7
Sonar	2	208	60
Spambase	2	4601	57
Statlog-segment	7	2310	19
Teaching assistant	3	151	5
Twonorm	2	7400	20
User Knowledge	4	258	5
Vehicle	4	846	18
Vertebral-2C	2	310	6
Vertebral-3C	3	310	6
Waveform1	3	5000	21
Waveform2	3	5000	40
Wine	3	178	13
Wine quality(red)	6	1599	11
Yale	15	165	1024

III. Details about "Bias-Variance" decomposition

Let X and Y be the input and output spaces, respectively with cardinalities $|X|$ and $|Y|$ and elements x and y, respectively. The target f is a conditional probability distribution $P(Y_F = y_F|x)$ where Y_F is a Y-valued random variable. Then for a single test sample:

$$E(C) = 1 - \sum_{y \in Y} P(Y_F = Y_H = y) \tag{2}$$

$$\begin{aligned} E(C) = & - \sum_{y \in Y} P(Y_F = Y_H = y) \\ & + \sum_{y \in Y} P(Y_F = y) P(Y_H = y) \\ & + \sum_{y \in Y} [-P(Y_H = y) P(Y_F = y) \\ & + \frac{1}{2} P(Y_H = y)^2 + \frac{1}{2} P(Y_F = y)^2] \\ & + [\frac{1}{2} - \frac{1}{2} \sum_{y \in Y} P(Y_H = y)^2] \\ & + [\frac{1}{2} - \frac{1}{2} \sum_{y \in Y} P(Y_F = y)^2] \end{aligned} \tag{3}$$

Rearranging the terms, we have

$$\begin{aligned} E(C) = & \sum_{y \in Y} [P(Y_F = y) P(Y_H = y) \\ & - P(Y_F = Y_H = y)] \\ & + \frac{1}{2} \sum_{y \in Y} [P(Y_F = y) - P(Y_H = y)]^2 \\ & + \frac{1}{2} [1 - \sum_{y \in Y} P(Y_H = y)^2] \\ & + \frac{1}{2} [1 - \sum_{y \in Y} P(Y_F = y)^2] \end{aligned} \tag{4}$$

As Y_F and Y_H are conditionally independent given f and a test sample x, the "covariance" term vanishes. Hence,

$$E(C) = \sum_{x} P(X) [(bias_x)^2 + \sigma_x^2 + variance_x]; \tag{5}$$

where

$$\begin{aligned} (bias_x)^2 &= \frac{1}{2} \sum_{y \in Y} [P(Y_F = y) - P(Y_H = y)]^2 \\ variance_x &= \frac{1}{2} [1 - \sum_{y \in Y} P(Y_H = y)^2] \\ \sigma_x^2 &= \frac{1}{2} [1 - \sum_{y \in Y} P(Y_H = y)^2] \end{aligned} \tag{6}$$

The $(bias_x)^2$ term measures the squared difference between the target's average output and the algorithm's average output. It is a real valued non-negative quantity and equals zero only if $P(Y_F = y|x) = P(Y_H = y|x)$ for all x and y. The variance term measures the variability (over Y_H) of $P(Y_H = y|x)$. It is a real-valued non-negative quantity and equals zero for an algorithm that always makes the same guess regardless of the training set (e.g. the Bayes optimal classifier). As the algorithm becomes more sensitive to changes in the training set, the variance increases. Moreover, given a distribution over the training set, the variance only measures the sensitivity of the learning algorithm to changes in the training set and is independent of the underlying target. The noise measures the $variance$ of the target in that the definitions of variance and noise are identical except for the interchange of Y_F and Y_H. In addition, the noise is independent of the learning algorithm.

IV. "BIAS-VARIANCE" FOR EACH CASE

Table A- III: "$(Bias, variance)$" of Random Forest and its MPSVM based variants.

Datasets	RaF	MPRaF-T	MPRaF-P	MPRaF-N	Datasets	RaF	MPRaF-T	MPRaF-P	MPRaF-N
Adult	(**13.93**,1.69)	(15.59,**1.01**)	(13.98,1.68)	(16.16,1.78)	Mam-masses	(15.14,2.76)	(15.25,2.51)	(**15.11**,2.96)	(15.47,**2.15**)
AFP-Pred	(21.87,**4.13**)	(21.48,5.09)	(22.39,4.83)	(**16.38**,4.71)	Orl	(7.91,14.95)	(4.23,**1.68**)	(4.08,7.73)	(**3.23**,3.33)
Australian	(**10.39**,2.59)	(11.15,2.41)	(10.59,2.52)	(11.51,**2.25**)	Ozone	(2.88,0.05)	(2.87,**0.01**)	(**2.86**,0.04)	(**2.86**,0.02)
Balance scale	(9.64,5.60)	(**7.42**,3.56)	(7.80,4.01)	(7.92,**2.98**)	Page block	(2.12,0.55)	(2.15,0.57)	(**2.10**,**0.54**)	(4.03,0.60)
Banknote	(0.63,0.20)	(**0.06**,0.03)	(0.08,**0.02**)	(**0.06**,0.05)	Parkinsons	(7.13,2.66)	(**6.14**,2.63)	(6.56,2.60)	(12.66,**0.98**)
Biodeg	(10.91,2.78)	(11.21,**2.11**)	(**10.83**,2.65)	(11.12,3.10)	Pima-diabetes	(19.67,4.53)	(19.76,4.33)	(19.69,**4.09**)	(**19.14**,5.77)
Blood transfusion	(19.46,3.45)	(**19.40**,2.55)	(19.43,3.09)	(20.08,**2.42**)	Planning Relax	(27.04,3.35)	(27.70,**1.81**)	(27.00,2.34)	(**26.87**,1.92)
BLProt	(13.84,**4.95**)	(13.69,5.04)	(**10.65**,5.49)	(22.88,6.72)	Seeds	(5.56,2.30)	(**4.31**,1.50)	(5.41,1.45)	(6.54,**1.03**)
Breast cancer	(2.97,0.58)	(**2.69**,0.59)	(2.80,0.48)	(2.71,**0.36**)	Sonar	(14.96,**5.90**)	(**11.07**,6.82)	(13.93,7.08)	(11.70,6.86)
Breast tissue	(22.78,9.20)	(**22.04**,**9.09**)	(22.52,9.93)	(24.38,14.69)	Spambase	(4.49,**0.90**)	(5.32,1.00)	(**4.35**,0.96)	(4.70,1.54)
Climate mode	(**7.16**,0.56)	(7.37,0.58)	(7.74,0.40)	(8.28,**0.13**)	Statlog-segment	(1.78,0.85)	(2.03,0.85)	(**1.75**,**0.77**)	(3.17,1.94)
DNA	(4.16,**1.60**)	(4.84,3.86)	(**3.99**,2.09)	(13.27,6.21)	Teaching assistant	(31.58,14.05)	(32.34,**12.56**)	(**30.88**,13.56)	(33.09,13.53)
Ecoli	(11.96,3.49)	(**11.31**,**3.24**)	(11.94,3.29)	(12.17,3.42)	Twonorm	(2.17,1.32)	(**1.92**,0.61)	(**1.92**,**0.57**)	(1.97,0.58)
Fertility	(12.05,0.25)	(12.01,0.09)	(**11.46**,0.34)	(12.00,**0.00**)	User Knowledge	(**5.91**,2.57)	(6.91,**1.89**)	(6.25,1.93)	(7.74,2.11)
Glass	(**1.71**,**1.38**)	(2.96,2.83)	(1.85,1.51)	(25.40,14.93)	Vehicle	(17.98,**7.17**)	(**16.15**,7.55)	(16.44,7.22)	(20.21,9.03)
Harberman	(24.18,5.42)	(**23.69**,**3.92**)	(24.77,4.94)	(24.28,4.38)	Vertebral-2C	(13.25,4.17)	(**11.41**,3.98)	(11.57,3.85)	(12.19,**3.71**)
Heart	(14.36,3.72)	(**13.42**,**2.84**)	(14.00,3.04)	(14.05,2.88)	Vertebral-3C	(12.52,4.13)	(12.19,4.20)	(**12.13**,**4.10**)	(13.02,4.49)
Hepatitis	(13.59,3.45)	(13.07,3.32)	(**12.65**,3.41)	(20.32,**0.19**)	Waveform1	(11.86,3.80)	(**10.91**,3.41)	(11.12,3.38)	(11.39,**2.91**)
Segment	(3.74,0.93)	(4.24,1.33)	(**3.61**,**0.88**)	(6.55,3.92)	Waveform2	(11.29,4.25)	(**10.36**,4.20)	(10.55,4.12)	(10.77,**3.88**)
Ionosphere	(5.45,0.93)	(6.20,1.13)	(**5.31**,**0.73**)	(8.61,1.65)	Wine	(1.82,0.88)	(**1.21**,1.15)	(1.30,**0.44**)	(2.75,1.02)
Iris	(4.34,1.06)	(**1.51**,0.89)	(3.31,1.02)	(1.83,**0.77**)	Wine quality(red)	(25.17,8.22)	(25.00,7.79)	(**24.66**,8.24)	(35.08,**5.51**)
Lymphography	(2.81,4.08)	(**1.51**,**3.12**)	(2.74,4.25)	(4.89,5.85)	Yale	(19.43,24.12)	(13.64,**6.36**)	(15.34,20.10)	(**13.49**,8.29)

"MPRaF-T", "MPRaF-P", "MPRaF-N" means MPSVM based Random Forest with Tikhonov, axis-parallel split, and NULL space regularization, respectively.

Table A- IV: "$(Bias, variance)$" of Random Rotation Forest and its MPSVM based variants.

Datasets	RRoF	MPRRoF-T	MPRRoF-P	MPRRoF-N	Datasets	RRoF	MPRRoF-T	MPRRoF-P	MPRRoF-N
Adult	(**13.88**,2.82)	(14.93,**1.13**)	(**13.88**,2.74)	(16.33,1.76)	Mam-masses	(16.62,4.70)	(15.44,**3.08**)	(16.95,4.45)	(**15.20**,3.42)
AFP-Pred	(19.91,**3.67**)	(21.99,4.85)	(21.36,4.54)	(**18.31**,4.14)	Orl	(4.15,6.45)	(**3.43**,**2.02**)	(3.52,4.53)	(7.38,6.77)
Australian	(10.92,2.66)	(11.04,2.55)	(**10.83**,2.56)	(11.30,**2.21**)	Ozone	(**2.82**,0.08)	(2.83,**0.05**)	(2.84,0.06)	(2.84,0.06)
Balance scale	(9.12,4.22)	(**7.74**,2.97)	(8.12,3.18)	(7.81,**2.38**)	Page block	(2.21,0.58)	(2.26,**0.53**)	(**2.20**,0.56)	(2.67,0.55)
Banknote	(0.03,0.09)	(**0.00**,**0.00**)	(**0.00**,**0.00**)	(**0.00**,**0.00**)	Parkinsons	(6.42,2.25)	(**5.06**,2.78)	(5.59,2.25)	(10.15,**1.44**)
Biodeg	(10.52,2.56)	(10.71,**2.11**)	(10.94,2.40)	(**10.27**,2.18)	Pima-diabetes	(**19.79**,4.71)	(20.88,4.26)	(20.66,**4.08**)	(19.96,4.22)
Blood transfusion	(20.31,3.92)	(19.78,**2.63**)	(20.39,3.47)	(**19.49**,2.88)	Planning Relax	(**26.39**,3.55)	(26.93,1.81)	(26.89,2.01)	(27.12,**1.73**)
BLProt	(13.32,4.50)	(14.08,**4.00**)	(**12.72**,4.71)	(28.29,4.76)	Seeds	(5.10,1.71)	(**3.86**,1.42)	(4.00,**1.00**)	(5.29,1.10)
Breast cancer	(**2.64**,0.63)	(2.96,0.65)	(2.75,0.59)	(2.65,**0.55**)	Sonar	(11.99,6.47)	(11.51,**5.03**)	(13.29,5.94)	(**10.92**,5.90)
Breast tissue	(21.08,9.40)	(21.92,**9.22**)	(**19.94**,9.87)	(22.87,11.28)	Spambase	(3.97,**0.95**)	(4.43,1.06)	(**3.92**,1.04)	(4.39,1.19)
Climate mode	(7.29,0.57)	(**6.90**,0.80)	(7.15,0.59)	(7.88,**0.36**)	Statlog-segment	(1.37,0.70)	(1.51,**0.61**)	(**1.34**,**0.61**)	(1.93,0.83)
DNA	(4.34,**1.28**)	(5.05,3.66)	(**3.99**,2.33)	(5.28,3.84)	Teaching assistan	(**29.26**,14.31)	(32.05,13.25)	(30.15,13.56)	(30.87,**12.11**)
Ecoli	(11.36,3.13)	(11.85,**2.91**)	(**11.08**,3.12)	(11.97,3.65)	Twonorm	(**1.90**,0.76)	(1.94,0.54)	(1.92,**0.53**)	(1.98,0.55)
Fertility	(**11.37**,1.73)	(11.82,0.18)	(11.57,0.73)	(11.81,**0.09**)	User Knowledge	(7.95,2.87)	(8.02,3.06)	(**7.65**,2.93)	(8.08,**2.27**)
Glass	(2.17,**1.15**)	(2.93,2.12)	(**1.39**,1.18)	(16.74,13.41)	Vehicle	(15.94,**6.82**)	(15.87,7.10)	(**15.27**,6.91)	(18.27,8.88)
Harberman	(**24.47**,6.67)	(24.89,**3.90**)	(25.54,4.33)	(24.73,5.60)	Vertebral-2C	(11.32,3.49)	(11.43,**3.31**)	(**11.01**,3.54)	(11.35,3.68)
Heart	(13.96,4.19)	(13.95,2.94)	(14.34,3.29)	(**13.71**,**2.69**)	Vertebral-3C	(11.41,3.69)	(11.48,**3.42**)	(**11.25**,3.69)	(12.05,3.69)
Hepatitis	(13.73,3.82)	(**12.52**,**2.77**)	(12.94,2.93)	(13.37,2.89)	Waveform1	(11.26,3.08)	(**11.04**,3.28)	(11.17,3.20)	(11.17,**2.84**)
Segment	(3.46,0.79)	(3.51,0.87)	(**3.24**,**0.75**)	(4.13,1.50)	Waveform2	(10.87,**3.60**)	(**10.38**,4.11)	(10.45,3.81)	(10.54,3.75)
Ionosphere	(4.67,0.97)	(5.73,0.87)	(**4.52**,**0.76**)	(7.77,1.46)	Wine	(1.97,1.01)	(**1.12**,0.90)	(1.17,**0.74**)	(1.62,1.02)
Iris	(4.39,1.21)	(**2.39**,**1.14**)	(3.29,1.31)	(2.82,1.85)	Wine quality(red)	(24.85,8.07)	(25.78,7.31)	(**24.63**,7.88)	(34.04,**5.50**)
Lymphography	(1.40,3.56)	(**1.26**,**3.34**)	(**1.26**,3.54)	(1.66,4.35)	Yale	(14.16,13.73)	(**12.89**,**4.78**)	(13.44,11.44)	(14.51,6.60)

"MPRRoF-T", "MPRRoF-P", "MPRRoF-N" means MPSVM based Random Rotation Forest with Tikhonov, axis-parallel split, and NULL space regularization, respectively.

V. Average number of Nodes and Computational time for each case

Table A- V: Average number of nodes in a base classifier for each ensemble

Datasets	RaF	MPRaF-T	MPRaF-P	MPRaF-N	RRoF	MPRRoF-T	MPRRoF-P	MPRRoF-N
Adult	995.8	248.5	993.5	589.1	2048.5	849.8	2064.3	1648.1
AFP-Pred	94.32	152.20	97.52	116.65	150.28	241.16	172.32	193.72
Australian	83.40	104.24	87.20	97.28	115.32	165.52	136.24	155.52
Balance scale	97.00	95.88	93.80	54.84	114.04	126.40	115.84	89.68
Banknote	39.72	46.88	38.96	44.00	40.40	41.44	41.08	39.80
Biodeg	112.16	135.77	117.84	85.28	156.89	250.28	189.48	189.44
Blood transfusion	113.64	98.16	120.00	71.84	181.48	127.44	185.57	157.28
BLProt	87.48	151.44	91.52	103.03	145.08	228.28	152.64	161.08
Breast cancer	34.64	40.80	33.00	29.56	41.36	61.32	49.52	56.44
Breast tissue	21.20	31.68	22.68	15.92	31.04	43.16	35.56	28.64
Climate mode	29.96	42.12	34.72	42.04	48.84	70.40	58.88	65.72
DNA	399.44	329.71	415.64	452.12	451.92	676.16	571.68	816.76
Ecoli	46.60	55.04	48.36	51.00	65.88	79.52	75.00	62.24
Fertility	14.96	16.76	15.80	16.64	20.32	26.88	23.36	27.92
Glass	26.96	42.04	28.68	11.08	31.60	58.24	43.36	22.28
Harberman	56.44	56.44	58.72	57.28	81.72	88.52	91.84	95.60
Heart	41.40	45.56	41.60	36.40	56.08	74.72	65.40	67.80
Hepatitis	21.40	25.68	22.88	7.32	29.68	40.52	34.56	24.44
Segment	35.56	59.08	38.96	52.60	44.92	80.44	54.64	70.76
Ionosphere	27.28	41.12	30.20	47.40	38.44	62.52	47.64	72.44
Iris	8.84	10.68	9.92	10.04	13.40	18.04	18.08	15.28
Lymphography	28.24	31.36	30.16	24.36	36.48	47.56	39.52	38.16
Mam-masses	112.84	108.20	117.36	92.88	210.92	205.40	239.72	196.64
Orl	72.84	82.56	76.04	54.28	79.00	100.88	91.16	76.76
Ozone	8.35	4.33	4.79	4.57	9.18	4.98	5.83	4.72
Page block	136.92	205.76	150.64	94.64	206.16	319.84	237.20	191.88
Parkinsons	18.32	30.72	21.08	17.68	27.12	43.68	33.24	27.76
Pima-diabetes	104.36	156.84	132.28	133.32	166.96	241.48	216.96	223.52
Planning Relax	28.68	41.12	35.68	46.20	50.40	65.88	62.96	69.96
Seeds	16.32	24.20	18.24	12.84	24.04	34.08	27.28	20.08
Sonar	22.52	34.28	26.20	30.52	38.32	56.40	46.16	54.48
Spambase	316.44	323.30	335.32	397.60	427.36	707.56	516.56	765.36
Statlog-segment	110.12	212.08	130.48	148.36	146.80	293.28	184.40	244.12
Teaching assistant	36.20	39.28	38.12	31.12	58.20	52.16	59.24	40.60
Twonorm	474.24	455.60	359.44	503.28	633.92	701.32	587.48	757.04
User Knowledge	37.04	47.60	38.96	45.32	60.92	73.44	65.08	65.72
Vehicle	134.16	185.12	145.96	62.00	195.44	269.72	220.92	115.40
Vertebral-2C	35.04	47.96	40.92	42.24	52.44	64.68	64.84	66.28
Vertebral-3C	38.16	56.68	44.76	42.00	60.60	79.12	69.08	63.80
Waveform1	486	688	534	695	776	1098	913	1103
Waveform2	501	761	575	807	817	1207	964	1132
Wine	12.64	16.84	12.52	15.92	19.36	25.88	21.00	23.60
Wine quality(red)	315.16	446.68	343.04	159.44	472.04	615.32	522.08	200.72
Yale	26.08	27.02	25.84	26.68	29.00	29.00	29.00	29.00

"MPRRoF-T", "MPRRoF-P", "MPRRoF-N" means MPSVM based Random Rotation Forest with Tikhonov, axis-parallel split, and NULL space regularization, respectively.

Table A- VI: Average "(Training Time, Testing Time)" for each ensemble method

Datasets	RaF	MPRaF-T	MPRaF-P	MPRaF-N	RRoF	MPRRoF-T	MPRRoF-P	MPRRoF-N
Adult	(237.70,411.11)	(25.80,248.90)	(210.61,389.91)	(40.61,228.42)	(316.53,418.64)	(126.42,350.33)	(377.71,424.01)	(118.21,289.61)
AFP-Pred	(62.01,79.64)	(6.25,73.76)	(54.93,72.09)	(5.31,54.65)	(139.53,79.94)	(15.87,77.82)	(85.01,79.68)	(14.99,66.35)
Australian	(8.16,1.32)	(3.27,1.29)	(7.10,1.28)	(3.24,1.23)	(25.46,1.59)	(5.77,1.57)	(14.15,1.66)	(5.53,1.47)
Balance scale	(3.73,1.17)	(3.94,1.06)	(5.32,1.09)	(2.89,1.91)	(8.59,1.22)	(5.46,1.12)	(8.06,1.19)	(4.60,1.03)
Banknote	(14.58,2.83)	(1.81,2.09)	(2.64,2.04)	(,1.76,2.04)	(21.58,2.38)	(2.10,2.06)	(2.98,1.98)	(2.11,2.09)
Biodeg	(21.60,2.48)	(5.30,2.37)	(20.91,2.43)	(3.61,1.34)	(65.28,2.71)	(10.49,2.66)	(31.81,2.71)	(9.98,2.64)
Blood transfusion	(6.48,1.79)	(3.45,1.33)	(5.74,1.50)	(2.62,1.10)	(13.89,1.87)	(4.46,1.49)	(9.63,1.72)	(5.60,1.64)
BLProt	(127.22,157.01)	(9.16,173.78)	(146.20,149.75)	(9.02,169.84)	(449.67,209.06)	(100.09,182.80)	(219.37,163.27)	(76.46,142.19)
Breast cancer	(4.08,1.08)	(1.88,1.05)	(2.63,1.10)	(1.43,0.97)	(8.37,1.06)	(2.29,1.14)	(4.21,1.24)	(2.40,1.24)
Breast tissue	(1.80,0.14)	(1.83,0.21)	(2.48,0.14)	(0.90,0.10)	(4.15,0.22)	(2.32,0.18)	(3.68,0.17)	(1.97,0.15)
Climate mode	(10.96,0.95)	(1.59,0.73)	(4.69,0.70)	(1.53,0.73)	(23.91,1.13)	(2.86,0.90)	(8.80,0.87)	(3.02,0.93)
DNA	(91.63,10.86)	(21.63,10.80)	(85.54,10.98)	(29.17,10.99)	(208.85,12.94)	(100.69,12.33)	(203.08,12.63)	(114.58,13.09)
Ecoli	(4.04,0.62)	(2.79,0.70)	(5.22,0.62)	(2.80,0.65)	(10.01,0.69)	(4.76,0.75)	(9.70,0.69)	(4.17,0.67)
Fertility	(0.67,0.11)	(0.63,0.11)	(0.58,0.10)	(0.51,0.10)	(1.83,0.13)	2(1.05,0.15)	(1.83,0.15)	(0.90,0.15)
Glass	(2.94,0.30)	(2.30,0.38)	(2.28,0.30)	((0.87,0.17)	(5.16,0.30)	(3.04,0.46)	(5.52,0.39)	(1.69,0.23)
Harberman	(2.89,0.59)	(1.81,0.51)	(2.46,0.53)	(1.73,0.51)	(8.91,0.70)	(2.93,0.61)	(4.38,0.70)	(2.78,0.59)
Heart	(3.39,0.43)	(1.49,0.41)	4(3.09,0.44)	(1.26,0.39)	(9.83,0.57)	(2.64,0.52)	(5.24,0.48)	(2.73,0.52)
Hepatitis	(1.83,0.22)	(0.96,0.20)	(1.79,0.20)	(0.38,0.09)	(4.75,0.27)	(1.77,0.20)	(3.44,0.20)	(1.40,0.19)
Segment	(5.81,10.43)	(2.86,11.33)	(6.10,11.40)	(4.34,12.09)	(13.72,12.19)	(4.90,12.72)	(11.95,12.57)	(5.50,13.27)
Ionosphere	(11.59,0.68)	(1.50,0.69)	(6.83,0.61)	(1.78,0.61)	(28.06,0.97)	(3.26,0.90)	(12.52,0.79)	(3.71,0.86)
Iris	(0.76,0.13)	(0.44,0.13)	(0.52,0.14)	(0.42,0.13)	(1.69,0.16)	(0.79,0.16)	(0.98,0.17)	(0.71,0.16)
Lymphography	(1.32,0.21)	(1.11,0.21)	(2.02,0.22)	(1.00,0.17)	(3.14,0.25)	(1.97,0.25)	(3.81,0.24)	(1.97,0.21)
Mam-masses	(6.72,246)	(4.12,2.14)	(6.52,2.31)	(3.77,2.00)	(16.29,2.77)	(7.34,2.56)	(11.18,2.67)	(6.89,2.48)
Orl	(32.02,2.67)	(7.75,1.19)	(22.70,1.48)	(6.44,0.87)	(59.37,2.46)	(14.76,1.25)	(28.37,1.31)	(13.62,1.07)
Ozone	(67.56,8.35)	(3.73,4.33)	(26.80,4.79)	(5.42,4.57)	(262.71,9.18)	(6.29,4.98)	(101.93,5.83)	(7.50,4.72)
Page block	(100.95,50.44)	(14.11,26.03)	(62.86,34.54)	(9.80,20.11)	(383.42,51.70)	(31.28,27.24)	(200.02,37.61)	(27.12,23.84)
Parkinsons	(4.55,0.25)	3(1.17,0.27)	(3.02,0.27)	(0.72,0.19)	(8.58,0.29)	(1.81,0.31)	(4.80,0.30)	(1.54,0.26)
Pima-diabet	(13.76,1.78)	(4.66,1.92)	(8.02,1.66)	(4.21,1.62)	(36.36,2.24)	(8.30,2.08)	(12.97,1.99)	(7.45,1.97)
Planning Relax	(3.51,0.29)	(1.15,0.26)	(2.24,0.35)	(1.47,0.29)	(7.42,0.40)	(2.06,0.31)	(4.25,0.40)	(2.25,0.33)
Seeds	(2.76,0.25)	(1.03,0.30)	(1.87,0.30)	(0.76,0.28)	(5.27,0.29)	(1.59,0.32)	(2.51,0.26)	(1.01,0.22)
Sonar	(8.27,0.323)	(1.26,0.30)	(5.21,0.33)	(1.19,0.26)	(15.66,0.33)	(2.68,0.36)	(8.67,0.34)	(2.84,0.36)
Spambase	(123.32,26.71)	3(18.43,18.58)	(108.53,23.88)	(21.69,17.08)	4(348.25,27.20)	(39.64,21.32)	(196.67,22.32)	7(41.29,19.67)
Statlog-segment	(43.93,6.48)	(10.99,6.61)	1(35.78,6.18)	(9.57,5.75)	(101.74,6.73)	(18.65,7.19)	1(69.78,6.80)	(19.89,7.10)
Teaching assistant	(1.54,0.25)	(1.72,0.22)	(2.36,0.28)	(1.49,0.20)	(3.22,0.31)	(2.52,0.24)	(4.00,0.30)	(2.18,0.23)
Twonorm	(355.90,43.81)	(20.02,30.78)	(59.33,30.34)	(21.45,31.13)	(627.00,44.78)	(38.22,31.53)	(109.12,32.22)	(40.00,31.64)
User Knowledge	(3.75,0.43)	(2.21,0.47)	(2.66,0.41)	(1.91,0.41)	(6.47,0.52)	(2.79,0.43)	(3.63,0.43)	(2.70,0.43)
Vehicle	(18.19,2.18)	(7.62,2.10)	(17.53,4.44)	(3.76,1.26)	(60.18,2.70)	(15.08,2.57)	(37.58,2.52)	(9.43,1.86)
Vertebral-2C	(3.77,0.46)	(1.78,0.68)	(1.96,0.48)	(1.50,0.47)	(6.73,0.54)	(2.36,0.54)	(3.39,0.53)	(2.79,0.63)
Vertebral-3C	(3.75,0.45)	(1.99,0.48)	(2.71,0.43)	(1.63,0.42)	(6.69,0.53)	(3.01,0.56)	(4.56,0.52)	(2.62,0.49)
Waveform1	(254.20,27.99)	(32.99,23.62)	(113.95,22.31)	(31.88,25.16)	(577.73,28.35)	(48.51,22.03)	(174.16,23.36)	(54.11,23.56)
Waveform2	(303.88,25.44)	(31.23,21.72)	(140.09,21.83)	(35.40,22.52)	(638.94,25.96)	(60.39,22.23)	(233.54,22.76)	(67.30,22.82)
Wine	(2.41,0.19)	(0.83,0.22)	(1.62,0.27)	(0.87,0.22)	(5.01,0.22)	(1.34,0.25)	(3.16,0.24)	(1.15,0.21)
Wine quality(red)	(38.58,6.12)	(20.52,5.51)	(42.17,5.83)	(10.88,3.42)	(94.41,6.10)	(30.25,5.67)	(78.10,5.65)	(13.99,3.48)
Yale	(3.15,0.54)	(1.79,0.37)	(2.60,0.48)	(1.75,0.36)	(5.49,0.51)	(2.60,0.39)	2(6.07,0.44)	(2.68,0.39)

"MPRRoF-T", "MPRRoF-P", "MPRRoF-N" means MPSVM based Random Rotation Forest with Tikhonov, axis-parallel split, and NULL space regularization, respectively. Each number in the bracket stands for the average computational time.

VI. Graphic illustration of "Node Ratio" and "SpeedUp Factor"

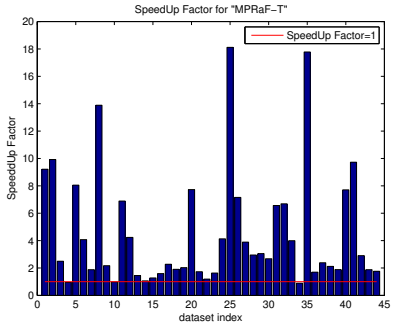
Here we define the "SpeedUp Factor", i.e., the "SpeedUp Factor" for MPRaF-T is:

$$\text{SpeedUp Factor for MPRaF-T} = \frac{\text{Training time of RaF}}{\text{Training time of MPRaF-T}} \tag{7}$$

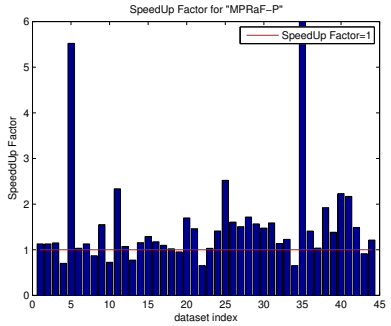
Similarly, we define the "Node Ratio", i.e., the "Node Ratio" for MPRaF-T is:

$$\text{Node Ratio for MPRaF-T} = \frac{N_{av}(\text{MPRaF-T})}{N_{av}(\text{RaF})} \tag{8}$$

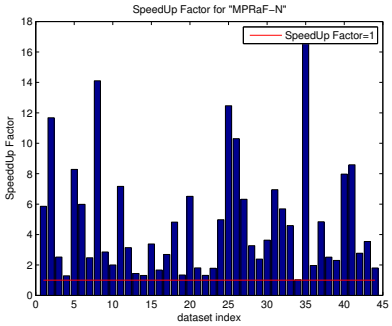
where "$N_{av}(E)$" stands for the average number of nodes in the base classifier of "E" (for example, "E" can be RaF).



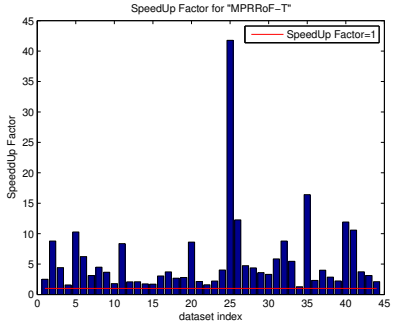
(a) “SpeedUp Fator” for “MPRaF-T”



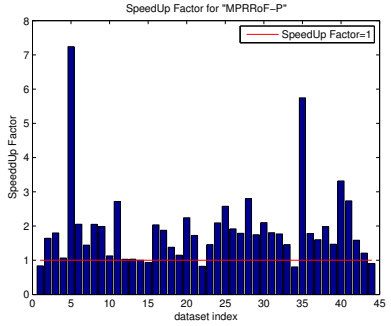
(b) “SpeedUp Fator” for “MPRaF-P”



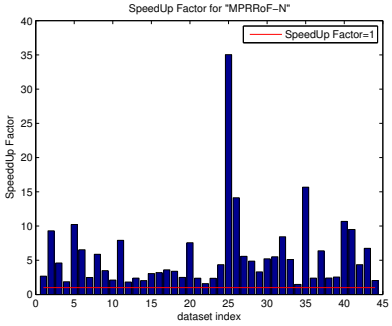
(c) “SpeedUp Fator” for “MPRaF-N”



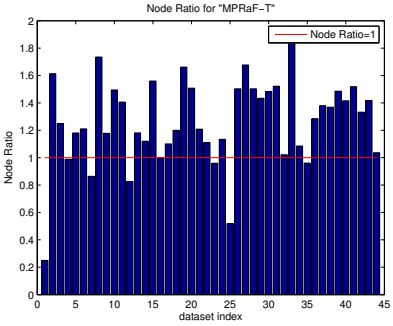
(d) “SpeedUp Fator” for “MPRRoF-T”



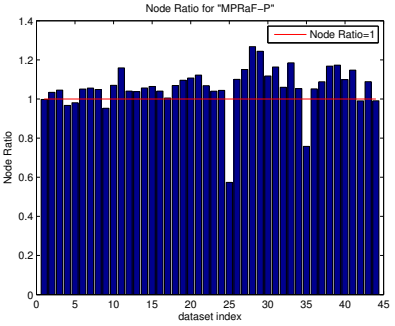
(e) “SpeedUp Fator” for “MPRRoF-P”



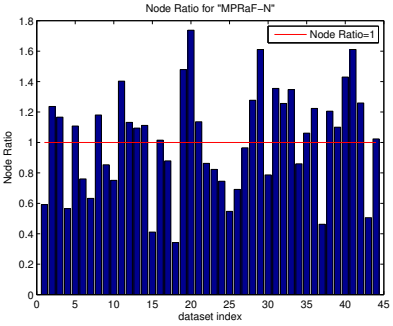
(f) “SpeedUp Fator” for “MPRRoF-N”



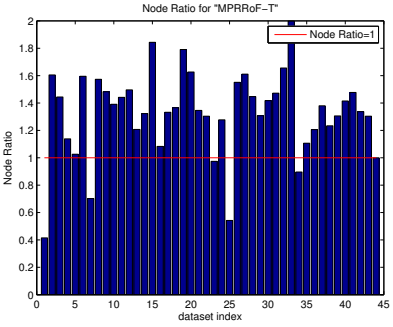
(g) “Node Ratio” for “MPRaF-T”



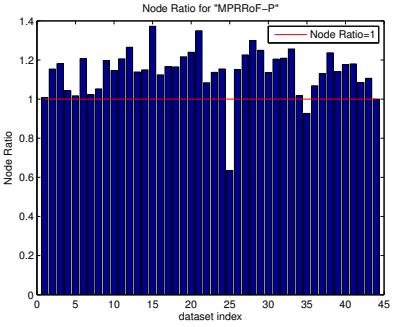
(h) “Node Ratio” for “MPRaF-P”



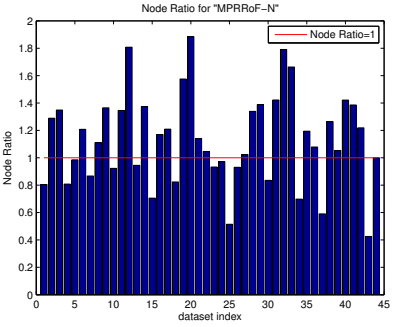
(i) “Node Ratio” for “MPRaF-N”



(j) “Node Ratio” for “MPRRoF-T”



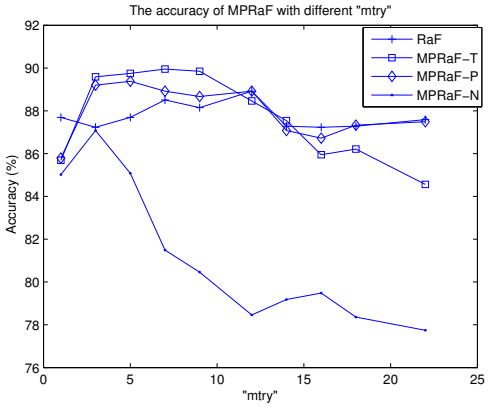
(k) “Node Ratio”for “MPRRoF-P”



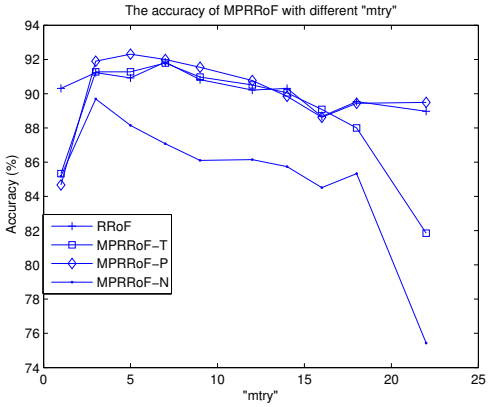
(l) “Node Ratio” for “MPRRoF-N”

Figure A-I :“SpeedUp Factor ” and “Node Ratio” for each ensemble methods, see text (page 10,section H) for detailed explanation

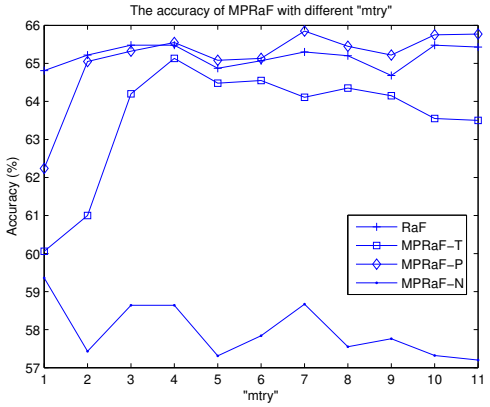
VII. EFFECT OF "MTRY"



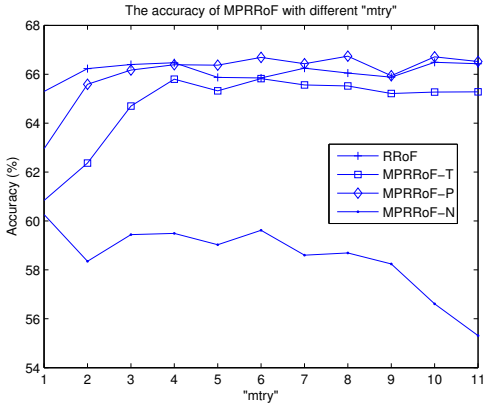
(m) RaF-parkinsons



(n) RRoF-parkinsons



(o) RaF-wine quality(red)



(p) RRoF-wine quality(red)

Figure A-II :Accuracy of different MPRaF and MPRRoF with different "mtry" parameter