Supplementary File of “Differential Evolution with Neighborhood Mutation for Multimodal Optimization”

Table XIV. Success Rate for test function set 1

fnc	NCDE	NShDE	NSDE	CDE [6]	ShDE	SDE [13]	FER-PSO [28]	SPSO [28]	r2pso [28]	r3pso [28]	r2pso-lhc [28]	r3pso-lhc [28]	SCMA-ES [35]
E1-F1	100 (1)	100 (1)	100 (1)	100 (1)	100 (1)	100 (1)	72 (10)	48 (13)	76 (9)	84 (8)	56 (12)	60 (11)	100 (1)
E1-F2	100 (1)	100 (1)	100 (1)	100 (1)	100 (1)	100 (1)	100 (1)	44 (12.5)	88 (10)	96 (9)	44 (12.5)	56 (11)	100 (1)
E1-F3	100 (1)	100 (1)	100 (1)	100 (1)	100 (1)	96 (7)	20 (8)	4 (12.5)	8 (10)	8 (10)	4 (12.5)	8 (10)	100 (1)
E1-F4	100 (1)	100 (1)	100 (1)	28 (11)	4 (12)	72 (10)	84 (9)	88 (7.5)	92 (5.5)	88 (7.5)	100 (1)	92 (5.5)	0 (13)
E1-F5	100 (1)	100 (1)	100 (1)	72 (12)	44 (13)	100 (1)	100 (1)	100 (1)	100 (1)	100 (1)	100 (1)	100 (1)	100 (1)
E1-F6	100 (1)	100 (1)	100 (1)	28 (11)	8 (12)	60 (10)	100 (1)	92 (6)	88 (8)	72 (9)	92 (6)	92 (6)	0 (13)
E1-F7	100 (1)	100 (1)	100 (1)	60 (12)	40 (13)	100 (1)	100 (1)	100 (1)	100 (1)	100 (1)	100 (1)	100 (1)	96 (11)
E1-F8	100 (1)	92 (3)	100 (1)	0 (12)	0 (12)	72 (4.5)	72 (4.5)	0 (12)	28 (7.5)	24 (9.5)	28 (7.5)	24 (9.5)	44 (6)
E1-F9	100 (1)	100 (1)	100 (1)	0 (12)	0 (12)	100 (1)	96 (6)	0 (12)	56 (8.5)	60 (7)	56 (8.5)	52 (10)	100 (1)
E1-F10	100 (1)	96 (4.5)	100 (1)	52 (11)	96 (4.5)	32 (12)	100 (1)	56 (10)	88 (6)	76 (7)	72 (8)	60 (9)	4 (13)
E1-F11	100 (1)	100 (1)	100 (1)	72 (4)	28 (6)	0 (12)	52 (5)	0 (12)	4 (9)	4 (9)	4 (9)	20 (7)	0 (12)
E1-F12	84 (2.5)	88 (1)	84 (2.5)	56 (8.5)	68 (5.5)	48 (11.5)	60 (7)	72 (4)	68 (5.5)	56 (8.5)	52 (10)	48 (11.5)	0 (13)
E1-F13	88 (3)	96 (1)	24 (4)	8 (5)	92 (2)	0 (9.5)	0 (9.5)	0 (9.5)	0 (9.5)	0 (9.5)	0 (9.5)	0 (9.5)	0 (9.5)
E1-F14	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)
Total Rank	**17.5**	**19.5**	**18.5**	105.5	83	82.5	65	114	91.5	97	99.5	103	96.5

Table XV. Average number of peaks found for test function set 1

fnc	NCDE	NShDE	NSDE	CDE [6]	ShDE	SDE [13]	FER-PSO [28]	SPSO [28]	r2pso [28]	r3pso [28]	r2pso-lhc [28]	r3pso-lhc [28]	SCMA-ES [35]
E1-F1	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	0.72 (10)	0.48 (13)	0.76 (9)	0.84 (8)	0.56 (12)	0.6 (11)	1 (1)
E1-F2	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	0.44 (12.5)	0.88 (10)	0.96 (9)	0.44 (12.5)	0.56 (11)	1 (1)
E1-F3	2 (1)	2 (1)	2 (1)	2 (1)	2 (1)	1.96 (7)	0.8 (8)	0.24 (13)	0.48 (11.5)	0.6 (9.5)	0.48 (11.5)	0.6 (9.5)	2 (1)
E1-F4	5 (1)	5 (1)	5 (1)	3.84 (11)	3.28 (12)	4.72 (10)	4.84 (9)	4.88 (7.5)	4.92 (5.5)	4.88 (7.5)	5 (1)	4.92 (5.5)	0.04 (13)
E1-F5	1 (1)	1 (1)	1 (1)	0.72 (12)	0.44 (13)	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)
E1-F6	5 (1)	5 (1)	5 (1)	3.96 (11)	3.28 (12)	4.6 (10)	5 (1)	4.92 (5.5)	4.88 (7.5)	4.72 (9)	4.92 (5.5)	4.88 (7.5)	0 (13)
E1-F7	1 (1)	1 (1)	1 (1)	0.6 (12)	0.4 (13)	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	0.96 (11)
E1-F8	4 (1)	3.92 (3)	4 (1)	0.32 (12)	0.16 (13)	3.72 (4)	3.68 (5)	0.84 (11)	2.92 (9)	2.76 (10)	3 (8)	3.12 (7)	3.44 (6)
E1-F9	2 (1)	2 (1)	2 (1)	0.04 (12.5)	0.04 (12.5)	2 (1)	1.96 (6)	0.08 (11)	1.44 (10)	1.56 (7.5)	1.56 (7.5)	1.48 (9)	2 (1)
E1-F10	1 (1)	0.96 (4.5)	1 (1)	0.52 (11)	0.96 (4.5)	0.32 (12)	1 (1)	0.56 (10)	0.88 (6)	0.76 (7)	0.72 (8)	0.6 (9)	0.04 (13)
E1-F11	18 (1)	18 (1)	18 (1)	17.7 (4)	16.56 (6)	12.4 (11)	17.4 (5)	8.52 (12)	15.2 (9)	15.6 (8)	15.1 (10)	16.2 (7)	2.16 (13)
E1-F12	5.8 (2)	5.88 (1)	5.84 (3)	5.56 (6)	5.6 (4.5)	4.88 (12)	5.36 (8.5)	5.6 (4.5)	5.52 (7)	5.16 (11)	5.36 (8.5)	5.28 (10)	1.52 (13)
E1-F13	35.9 (3)	35.96 (1)	30.6 (5)	33.8 (4)	35.92 (2)	22.8 (9)	23.6 (7)	25.7 (6)	21.8 (12)	22.2 (11)	22.5 (10)	23.1 (8)	1.4 (13)
E1-F14	179 (3)	198.96 (1)	84.28 (5)	152 (4)	197.88 (2)	50.6 (8)	68.6 (7)	70.1 (6)	40.6 (12)	45.4 (9)	42.2 (11)	43.3 (10)	0.04 (13)
Total Rank	**19**	**19.5**	**24**	102.5	97.5	88	70.5	114	110.5	108.5	107.5	106.5	113

Table XVI. *t*-test and Wilcoxon test on the average number of peaks found for test function set 1 presented as "t-test / Wilcoxon"

fnc	NCDE	NShDE	NSDE	CDE [6]	ShDE	SDE [13]	FER-PSO [28]	SPSO [28]	r2pso [28]	r3pso [28]	r2pso-lhc [28]	r3pso-lhc [28]	SCMA-ES [35]
E1-F1	N.A.	0/0	0/0	0/0	0/0	0/0	1/1	1/1	1/1	1/1	1/1	1/1	0/0
E1-F2	N.A	0/0	0/0	0/0	0/0	0/0	0/0	1/1	1/1	1/0	1/1	1/1	0/0
E1-F3	N.A.	0/0	0/0	0/0	0/0	0/0	1/1	1/1	1/1	1/1	1/1	1/1	0/0
E1-F4	N.A.	0/0	0/0	1/1	1/1	1/1	1/1	0/0	0/0	0/0	0/0	0/0	1/1
E1-F5	N.A.	0/0	0/0	0/0	1/1	1/1	0/0	0/0	0/0	0/0	0/0	0/0	0/0
E1-F6	N.A.	0/0	0/0	1/1	1/1	1/1	0/0	0/0	0/0	1/1	0/0	0/0	1/1
E1-F7	N.A	0/0	0/0	0/0	1/1	1/1	0/0	0/0	0/0	0/0	0/0	0/0	0/0
E1-F8	N.A.	1/0	0/0	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1
E1-F9	N.A.	0/0	0/0	1/1	1/1	0/0	0/0	1/1	1/1	1/1	1/1	1/1	0/0
E1-F10	N.A.	0/0	0/0	1/1	0/0	1/1	0/0	1/1	0/0	1/1	1/1	1/1	1/1
E1-F11	N.A.	0/0	0/0	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1
E1-F12	0/0	N.A.	0/0	0/0	0/0	1/1	1/1	0/0	0/0	1/1	1/1	1/1	1/1
E1-F13	0/0	N.A.	1/1	1/1	0/0	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1
E1-F14	1/1	N.A.	1/1	1/1	0/0	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1

Table XVII. Average number of peaks found for test function set 2

fnc	NCDE	NSHDE	NSDE	CDE [6]	SHDE	SDE [13]	FER-PSO [28]	SPSO [28]	r2pso [28]	r3pso [28]	r2pso-lhc [28]	r3pso-lhc [28]	SCMA-ES [35]
E1-CF1	5.18 (2)	3.7 (3)	6.7 (1)	0 (10)	0 (10)	1.79 (5)	1.08 (6)	0 (10)	0 (10)	0 (10)	0 (10)	0 (10)	2 (4)
E1-CF2	3.6 (2)	2.8 (3)	4 (1)	1.2 (6.5)	1.1 (8)	1.2 (6.5)	2 (4)	0 (11)	0 (11)	0 (11)	0 (11)	0 (11)	1.9 (5)
E1-CF3	5.8 (2)	4 (3)	6 (1)	0.7 (8)	1.11 (7)	1.5 (6)	2.5 (5)	0 (11)	0 (11)	0 (11)	0 (11)	0 (11)	2.7 (4)
E1-CF4	4.8 (2)	4.5 (3)	5.4 (1)	0 (9)	0 (9)	0 (9)	0 (9)	0 (9)	0 (9)	0 (9)	0 (9)	0 (9)	0.2 (4)

E1-CF5	5.2 (2)	3.6 (3)	5.9 (1)	1.1 (8)	1.3 (7)	1.3 (6)	2 (4)	0 (11)	0 (11)	0 (11)	0 (11)	0 (11)	1.9 (5)
E1-CF6	3 (1)	3 (1)	3 (1)	0 (10)	0 (10)	1.4 (5)	1.2 (6)	0 (10)	0 (10)	0 (10)	0 (10)	0 (10)	2.6 (4)
E1-CF7	1.8 (2)	1 (4)	1.9 (1)	0 (10)	0 (10)	1 (4)	0.5 (6)	0 (10)	0 (10)	0 (10)	0 (10)	0 (10)	1 (4)
E1-CF8	3 (1)	3 (1)	3 (1)	0 (10)	0 (10)	1.4 (6)	1.5 (5)	0 (10)	0 (10)	0 (10)	0 (10)	0 (10)	2.3 (4)
E1-CF9	3 (1)	3 (1)	3 (1)	0 (10)	0 (10)	1.8 (4)	1.5 (6)	0 (10)	0 (10)	0 (10)	0 (10)	0 (10)	1.7 (5)
E1-CF10	1.3 (2)	1 (6)	2 (1)	0 (10)	0 (10)	1.1 (4.5)	1.1 (4.5)	0 (10)	0 (10)	0 (10)	0 (10)	0 (10)	1.2 (3)
E1-CF11	2.8 (2)	2.2 (3)	4 (1)	0 (9.5)	0 (9.5)	1.3 (4)	0 (9.5)	0 (9.5)	0 (9.5)	0 (9.5)	0 (9.5)	0 (9.5)	0.7 (5)
E1-CF12	2.5 (2)	2 (3)	2.9 (1)	0 (10)	0 (10)	1.6 (5.5)	1.6 (5.5)	0 (10)	0 (10)	0 (10)	0 (10)	0 (10)	1.7 (4)
E1-CF13	2.3 (2)	1 (4)	3.8 (1)	0 (10)	0 (10)	0.9 (5)	0.3 (6)	0 (10)	0 (10)	0 (10)	0 (10)	0 (10)	1.4 (3)
E1-CF14	1 (1)	1 (1)	1 (1)	0 (10)	0 (10)	1 (1)	1 (1)	0 (10)	0 (10)	0 (10)	0 (10)	0 (10)	1 (1)
CF15	3.8 (2)	2.4 (3)	4 (1)	0 (10)	0 (10)	1.6 (5)	1.2 (6)	0 (10)	0 (10)	0 (10)	0 (10)	0 (10)	2 (4)
Total Rank	**26**	**42**	**15**	141	140.5	76.5	82	151.5	151.5	151.5	151.5	151.5	59

Table XVIII. *t*-test and Wilcoxon test on the average number of peaks found for test function set 2 presented as “t-test / Wilcoxon”

fnc	NCDE	NSHDE	NSDE	CDE [6]	SHDE	SDE [13]	FER-PSO [28]	SPSO [28]	r2pso [28]	r3pso [28]	r2pso-lhc [28]	r3pso-lhc [28]	SCMA-ES [35]
E1-CF1	1/1	1/1	N.A.	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1
E1-CF2	1/0	1/1	N.A.	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1
E1-CF3	0/0	1/1	N.A.	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1
E1-CF4	0/0	1/1	N.A.	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1
E1-CF5	0/0	1/1	N.A.	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1

E1-CF6	0/0	0/0	N.A.	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/0
E1-CF7	0/0	1/1	N.A.	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1
E1-CF8	0/0	0/0	N.A.	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1
E1-CF9	0/0	0/0	N.A.	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1
E1-CF10	1/1	1/1	N.A.	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1
E1-CF11	1/1	1/1	N.A.	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1
E1-CF12	0/0	1/1	N.A.	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1
E1-CF13	1/1	1/1	N.A.	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1
E1-CF14	0/0	0/0	N.A.	1/1	1/1	0/0	0/0	1/1	1/1	1/1	1/1	1/1	0/0
E1-CF15	0/0	1/1	N.A.	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1

Table XIX. Success rate in locating both global and local peaks

fnc	NCDE	NSHDE	NSDE	CDE [6]	SHDE	SDE [13]	FER-PSO [28]	SPSO [28]	r2pso [28]	r3pso [28]	r2pso-lhc [28]	r3pso-lhc [28]	SCMA-ES [35]
E1-F1	100 (1)	100 (1)	100 (1)	100 (1)	100 (1)	84 (7)	64 (9)	44 (13)	72 (8)	56 (10)	48 (12)	52 (11)	100 (1)
E1-F2	100 (1)	100 (1)	100 (1)	100 (1)	84 (7)	68 (10)	88 (6)	72 (9)	56 (11)	32 (13)	52 (12)	76 (8)	100 (1)
E1-F3	100 (1)	100 (1)	40 (4)	44 (3)	12 (5)	4 (7)	0 (10.5)	0 (10.5)	0 (10.5)	0 (10.5)	8 (6)	0 (10.5)	0 (10.5)
E1-F5	100 (1)	100 (1)	76 (4)	48 (6)	4 (7.5)	0 (11)	0 (11)	100 (1)	0 (11)	0 (11)	64 (5)	4 (7.5)	0 (11)
E1-F10	100 (1)	100 (1)	100 (1)	0 (11.5)	96 (4)	0 (11.5)	0 (11.5)	92 (5)	60 (8)	52 (9)	84 (6)	76 (7)	0 (11.5)
Total Rank	5	5	11	22.5	24.5	46.5	48	38.5	48.5	53.5	41	44	35

Table XX. Average of optima found in locating both global and local peaks

fnc	NCDE	NSHDE	NSDE	CDE [6]	SHDE	SDE [13]	FER-PSO [28]	SPSO [28]	r2pso [28]	r3pso [28]	r2pso-lhc [28]	r3pso-lhc [28]	SCMA-ES [35]
E1-F1	2 (1)	2 (1)	2 (1)	2 (1)	2 (1)	1.84 (7)	1.48 (11)	1.44 (13)	1.72 (8)	1.48 (11)	1.48 (11)	1.52 (9)	2 (1)
E1-F2	2 (1)	2 (1)	2 (1)	2 (1)	1.84 (7)	1.68 (10)	1.88 (6)	1.72 (9)	1.36 (12)	1.24 (13)	1.52 (11)	1.76 (8)	2 (1)
E1-F3	5 (1)	5 (1)	3.76 (4)	4.44 (3)	3.6 (5)	3.04 (7)	0.64 (12)	3.08 (6)	0.8 (11)	0.4 (13)	3 (8)	2.16 (9)	2 (10)
E1-F5	5 (1)	5 (1)	4.76 (4)	4.28 (6)	3.12 (7)	1.52 (9)	1 (11)	5 (1)	1 (11)	1 (11)	4.52 (5)	2.8 (8)	0.48 (13)
E1-F10	25 (1)	25 (1)	25 (1)	12.5 (10)	24.96 (4)	1.32 (12)	5.16 (11)	24.9 (5)	24.4 (8)	24.3 (9)	24.8 (6)	24.6 (7)	0.88 (13)
Total Rank	5	5	11	21	24	45	51	34	50	57	41	41	38

Table XXI. The effect of varying neighborhood size parameter *m*

fnc	ε	*m*	SR	Average no. of optima found
E1-F1 (two-peak trap)	0.05	5	1	1
E1-F1 (two-peak trap)	0.05	10	1	1
E1-F4 (Equal Maxima)	0.000001	5	1	5
E1-F4 (Equal Maxima)	0.000001	10	1	5
E1-F11 (2D inverted Shubert function)	0.05	20	0.96	17.96
E1-F11 (2D inverted Shubert function)	0.05	30	1	18
E1-F11 (2D inverted Shubert function)	0.05	40	1	18
E1-F11 (2D inverted Shubert function)	0.05	50	1	18

Table XXII. The results of peak accuracy

	NCDE	NShDE	NSDE	TSC2	CDE	TSC	NCMA-ES	SCGA	DFS
E2-F1	0.10 (1)	0.14 (2)	6.72 (5)	1.84 (4)	1.59 (3)	7.7 (6)	8.89 (7)	18.59 (8)	20.93 (9)
E2-F2	2.75E-04 (1)	0.14 (2)	5.14 (6)	2.91 (3)	3.3 (4)	6.18 (7)	3.9 (5)	6.37 (8)	7.27 (9)
E2-F3	3.01E-33 (2)	6.85E-30 (3)	6.56E-47 (1)	1.81E-07 (4)	4.48E-04 (9)	4.90E-06 (8)	3.92E-06 (6)	2.86E-07 (5)	4.17E-06 (7)
E2-F4	0 (1)	0.0028 (3)	0 (1)	1.63 (8)	0.11 (6)	1.73 (9)	0.19 (7)	0.01 (4)	0.02 (5)
E2-F5	51.10 (2)	179.56 (5)	10.92 (1)	369.93 (7)	134.64 (3)	934.45 (8)	1840 (9)	317.24 (6)	164.85 (4)
E2-F6	1.95E-04 (1)	0.21 (2)	5.8 (7)	2.77 (4)	1.78 (3)	2.99 (5)	5.12 (6)	7.06 (9)	6.89 (8)
E2-F7	1.70E-04 (2)	3.51E-04 (3)	1.58E-05 (1)	0.02 (5)	0.1 (6)	1.79 (8)	1.96 (9)	0.73 (7)	3.42E-04 (4)
E2-F8	4.99 (3)	6.09 (4)	2.96 (2)	727.9 (6)	115.4 (7)	1628.46 (9)	52.6 (5)	1381.05 (8)	0.11 (1)
E2-F9	9.73E-04 (2)	1.10E-03 (3)	7.60E-14 (1)	0.85 (9)	0.24 (8)	0.23 (7)	0.01 (6)	0.003 (4.5)	0.003 (4.5)
E2-F10	4.39E-05 (1)	4.39E-05 (1)	0.02 (6)	0.009 (4)	0.006 (3)	0.01 (5)	0.07 (7)	0.48 (9)	0.37 (8)
E2-F11	3.18E-05 (1)	3.18E-05 (1)	3.18E-05 (1)	0.1 (6)	0.002 (4)	0.005 (5)	0.64 (7)	0.76 (8)	0.92 (9)
E2-F12	1.69E-04 (1)	4.26E-04 (2)	0.40 (5)	0.32 (4)	0.1 (3)	1.68 (6)	1.89 (7)	4.28 (8)	4.3 (9)
E2-F13	1.37E-06 (1)	9.20E-04 (3)	1.37E-06 (1)	0.28 (5)	0.12 (4)	0.89 (6)	1.02 (7)	2.53 (9)	2.11 (8)
Total Rank	**19**	**34**	**38**	69	63	89	88	93.5	85.5

Table XXIII. The results of distance accuracy

	NCDE	NShDE	NSDE	TSC2	CDE	TSC	NCMA-ES	SCGA	DFS
E2-F1	0.02 (1)	0.037 (2)	1.55 (5)	0.79 (4)	0.41 (3)	3.26 (6)	3.87 (7)	11.56 (9)	11.52 (8)
E2-F2	4.80E-03 (1)	0.10 (2)	2.23 (5)	2.09 (4)	1.99 (3)	6.18 (7)	3.19 (6)	7.02 (9)	6.22 (8)
E2-F3	2.63E-17 (2)	6.36E-016 (3)	1.87E-24 (1)	9.32E-05 (5)	5.25E-03 (9)	9.08E-05 (4)	5.84E-04 (7)	1.65E-04 (6)	8.12E-04 (8)
E2-F4	8.00E-05 (2)	7.72E-04 (3)	0 (1)	0.05 (7)	0.03 (6)	0.07 (8)	0.14 (9)	0.01 (4)	0.02 (5)
E2-F5	0.02 (2)	0.0304 (3)	6.30E-03 (1)	0.49 (5)	0.07 (4)	1.07 (7)	0.71 (6)	2.51 (8)	3.05 (9)
E2-F6	3.30E-03 (1)	0.03 (2)	0.28 (4)	0.54 (5)	0.22 (3)	3.94 (7)	1.48 (6)	5.31 (9)	4.55 (8)
E2-F7	0.01 (2)	0.02 (3)	1.80E-03 (1)	0.45 (5)	0.21 (4)	5.48 (8)	4.56 (7)	6.04 (9)	3.63 (6)
E2-F8	0.16 (2)	0.18 (3)	0.03 (1)	33.2 (7)	3.12 (4)	59.2 (8)	31.0 (6)	22.1 (5)	88.2 (9)
E2-F9	3.42E-04 (2)	3.76E-04 (3)	2.70E-14 (1)	0.21 (9)	0.05 (7)	0.06 (8)	2.96E-03 (6)	9.78E-04 (5)	8.18E-04 (4)
E2-F10	1.00E-05 (1)	3.20E-05 (2)	0.05 (6)	0.02 (4)	0.01 (3)	0.03 (5)	0.15 (7)	0.92 (9)	0.72 (8)
E2-F11	1.32E-06 (2)	6.80E-04 (3)	1.28E-09 (1)	0.21 (6)	9.00E-03 (4)	0.01 (5)	1.42 (7)	1.55 (8)	1.87 (9)
E2-F12	6.81E-04 (1)	1.00E-03 (2)	0.43 (5)	0.34 (4)	0.12 (3)	1.77 (6)	1.99 (7)	4.32 (9)	4.29 (8)
E2-F13	4.93E-10 (2)	6.93E-04 (3)	1.05E-16 (1)	0.95 (5)	036 (4)	7 (7)	5.42 (6)	7.68 (9)	7.36 (8)
Total Rank	**21**	**34**	**33**	65	57	86	87	99	98

Description of Test Function

Test Function Set 1

F1: Two-Peak Trap

$$f_1(x) = \begin{cases} \frac{160}{15}(15 - x), & \text{for } 0 \le x \le 15 \\ \frac{200}{5}(x - 15), & \text{for } 15 \le x \le 20 \end{cases}$$

Range: $0 \le x \le 20$

F2: Central Two-Peak Trap

$$f_2(x) = \begin{cases} \frac{160}{10}x, & \text{for } 0 \le x \le 10 \\ \frac{160}{5}(15 - x) & \text{for } 10 \le x \le 15 \\ \frac{200}{5}(x - 15), & \text{for } 15 \le x \le 20 \end{cases}$$

Range: $0 \le x \le 20$

F3: Five-Uneven-Peak Trap

$$f_3(x) = \begin{cases} 80(2.5 - x) & \text{for } 0 \le x < 2.5 \\ 64(x - 2.5) & \text{for } 2.5 \le x < 5 \\ 64(7.5 - x) & \text{for } 5 \le x < 7.5 \\ 28(x - 7.5) & \text{for } 7.5 \le x < 12.5 \\ 28(17.5 - x) & \text{for } 12.5 \le x < 17.5 \\ 32(x - 17.5) & \text{for } 17.5 \le x < 22.5 \\ 32(27.5 - x) & \text{for } 22.5 \le x < 27.5 \\ 80(x - 27.5) & \text{for } 27.5 \le x \le 30 \end{cases}$$

Range: $0 \le x \le 20$

F4: Equal Maxima

$$f_4(x) = \sin^6(5\pi x)$$

Range: $0 \le x \le 1$

F5: Decreasing Maxima

$$f_5(x) = \exp[-2\log(2)\cdot(\frac{x-0.1}{0.8})^2]\cdot\sin^6(5\pi x)$$

Range: $0 \le x \le 1$

F6: Uneven Maxima

$$f_6(x) = \sin^6(5\pi(x^{3/4} - 0.05))$$

Range: $0 \le x \le 1$

F7: Uneven Decreasing Maxima

$$f_7(x) = \exp[-2\log(2)\cdot(\frac{x-0.08}{0.854})^2]\cdot\sin^6(5\pi(x^{3/4} - 0.05))$$

Range: $0 \le x \le 1$

F8: Himmelblau's function

$$f_8(x, y) = 200 - (x^2 + y - 11)^2 - (x + y^2 - 7)^2$$

Range: $-6 \le x, y \le 6$

F9: Six-Hump Camel Back

$$f_9(x, y) = -4[(4 - 2.1x^2 + \frac{x^4}{3})x^2 + xy + (-4 + 4y^2)y^2]$$

Range: $-1.9 \le x \le 1.9;$
$-1.1 \le y \le 1.1$

F10: Shekel's foxholes

$$f_{10}(x, y) = 500 - \frac{1}{0.002 + \sum_{i=0}^{24} \frac{1}{1 + i + (x - a(i))^6 + (y - b(i))^6}}$$

where $a(i) = 16(i \bmod 5) - 2)$, and $b(i) = 16(\lfloor (i/5) \rfloor - 2)$

Range: $-65.536 \le x, y \le 65.535$

F11: 2D Inverted Shubert function

$$f_{11}(\vec{x}) = -\prod_{i=1}^{2}\sum_{j=1}^{5} j\cos[(j+1)x_i + j]$$

Range: $-10 \le x_1, x_2 \le 10$

F12-14: Inverted Vincent function

$$f\ (\vec{x}) = \frac{1}{n}\sum_{i=1}^{n}\sin(10.\log(x_i))$$

where n is the dimesnion of the problem

Range: $0.25 \le x_i \le 10$

Test Function Set 2

The set 2 composition function are defined as follow:

$F(x)$: new composition function

$f_i(x)$: i[th] basic function used to construct the composition function.

n: number of basic functions (number of optima)

D: dimensions (can be chosen from 1-100)

M_i : linear transformation matrix for each $f_i(x)$

o_i : new shifted optima position for each $f_i(x)$

$$F(x) = \sum_{i=1}^{n}\left\{w_i * [f_i'((x-o_i)/\lambda_i * M_i)]\right\}$$

w_i : weight value for each $f_i(x)$, calculated as follow:

$$w_i = \exp(-\frac{\sum_{k=1}^{D}(x_k - o_{ik})}{2D\sigma_i^2})$$

$$w_i = \begin{cases} w_i & w_i == \max(w_i) \\ w_i * (1-\max(w_i).\wedge 10) & w_i \neq \max(w_i) \end{cases}$$

Then normalize the weight $w_i = w_i / \sum_{i=1}^{n} w_i$

σ_i : used to control each $f_i(x)$'s coverage range.

λ_i : used to stretch compress the function.

$f_i'(x) = C * f_i(x) / \left| f_{\max i} \right|$, C is a predefined constant.

$\left| f_{\max i} \right|$ is estimated using: $\left| f_{\max i} \right| = f_i((x' / \lambda_i) * M_i), x' = [5, 5, ..., 5]$

Composition Function 1 (F15, *n*=8)

$f_{1-2}(x)$: Rastrigin's Function

$$f_i(x) = \sum_{i=1}^{D} (x_i^2 - 10\cos(2\pi x_i) + 10)$$

$f_{3-4}(x)$: Weierstrass Function

$$f_i(x) = \sum_{i=1}^{D} (\sum_{k=0}^{k\max} [a^k \cos(2\pi b^k (x_i + 0.5))]) - D \sum_{k=0}^{k\max} [a^k \cos(2\pi b^k \bullet 0.5)])$$

$a = 0.5, b = 3, k_{\max} = 20$

$f_{5-6}(x)$: Griewank's Function

$$f_i(x) = \sum_{i=1}^{D} \frac{x_i^2}{4000} - \prod_{i=1}^{D} \cos(\frac{x_i}{\sqrt{i}}) + 1$$

$f_{7-8}(x)$: Sphere Function

$$f_i(x) = \sum_{i=1}^{D} x_i^2$$

$\sigma_i = 1$ for all i

λ = [1, 1, 10, 10, 5/60, 5/60, 5/32, 5/32]

M_i : are all identity matrices

These formulas are basic functions; shift and rotation should be added to these functions. Take f_1 as an example, the following function should be evaluated:

$$f_i(z) = \sum_{i=1}^{D} (z_i^2 - 10\cos(2\pi z_i) + 10)$$

where $z = ((x - o_i) / \lambda_1) * M_1$.

Composition Function 2 (F16 *n*=6)

$f_{1-2}(x)$: Griewank's Function

$f_{3-4}(x)$: Weierstrass Function

$f_{5-6}(x)$: Sphere Function

$\sigma_i = 1$ for all *i*

λ = [1, 1, 10, 10, 5/60, 5/60,]

M_i : are all identity matrices

Composition Function 3 (F17 *n*=6)

$f_{1-2}(x)$: Rastrigin's Function

$f_{3-4}(x)$: Griewank's Function

$f_{5-6}(x)$: Sphere Function

$\sigma_i = 1$ for all *i*

λ = [1, 1, 10, 10, 5/60, 5/60,]

M_i : are all identity matrices

Composition Function 4 (F18 *n*=6)

$f_{1-2}(x)$: Rastrigin's Function

$f_{3-4}(x)$: Weierstrass Function

$f_{5-6}(x)$: Griewank's Function

$\sigma_i = 1$ for all *i*

λ = [1, 1, 10, 10, 5/60, 5/60,]

M_i : are all identity matrices

Composition Function 5 (F19 *n*=6)

$f_{1-2}(x)$: Rastrigin's Function

$f_{3-4}(x)$: Weierstrass Function

$f_{5-6}(x)$: Sphere Function

$\sigma_i = 1$ for all *i*

λ = [1, 1, 10, 10, 5/60, 5/60,]

M_i : are all identity matrices

Composition Function 6 (F20 *n*=6)

$f_{1-2}(x)$: F8F2 Function

$$F8(x) = \sum_{i=1}^{D} \frac{x_i^2}{4000} - \prod_{i=1}^{D} \cos(\frac{x_i}{\sqrt{i}}) + 1$$

$$F2(x) = \sum_{i=1}^{D-1} (100(x_i^2 - x_{i+1})^2 + (x_i - 1)^2)$$

$$f_i(x) = F8(F2(x_1, x_2)) + F8(F2(x_2, x_3)) + ... + F8(F2(x_{D-1}, x_D) + F8(F2(x_{D,} x_1))$$

$f_{3-4}(x)$: Weierstrass Function

$f_{5-6}(x)$: Griewank's Function

$\sigma = [1,1,1,1,1,2]$,

$\lambda = [5*5/100; 5/100; 5*1; 1; 5*1; 1]$

M_i : are all orthogonal matrix

Composition Function 7 (F21 *n*=6)

$f_{1-2}(x)$: Rotated Expanded Scaffer's F6 Function $F(x,y) = 0.5 + \frac{(\sin^2(\sqrt{x^2+y^2}) - 0.5)}{(1+0.001(x^2+y^2))^2}$

$f_i(x) = F(x_1, x_2) + F(x_2, x_3) + ... + F(x_{D-1}, x_D) + F(x_D, x_1)$

$f_{3-4}(x)$: F8F2 Function

$f_{5-6}(x)$: Weierstrass Function

$\sigma = [1,1,1,1,1,2]$,

$\lambda = [5;10;5;1;5*5/100;5/100]$

M_i : are all orthogonal matrix

Composition Function 8 (F22 *n*=6)

$f_{1-2}(x)$: Rotated Expanded Scaffer's F6 Function

$f_{3-4}(x)$: F8F2 Function

$f_{5-6}(x)$: Griewank's Function

$\sigma = [1,1,1,1,1,2]$,

$\lambda = [5*5/100;5/100;5*1;1;5*1;1]$

M_i : are all orthogonal matrix

Composition Function 9 (F23 *n*=6)

$f_{1-2}(x)$: Rotated Expanded Scaffer's F6 Function

$f_{3-4}(x)$: Weierstrass Function

$f_{5-6}(x)$: Griewank's Function

$\sigma = [1,1,1,1,1,2]$,

$\lambda = [5;10;5*5/100;5/100;5;1]$

M_i : are all orthogonal matrix

Composition Function 10 (F24 *n*=6)

$f_{1-2}(x)$ ： Rastrigin’s Function

$f_{3-4}(x)$ ： F8F2 Function

$f_{5-6}(x)$: Weierstrass Function

$\sigma = [1,1,1,1,1,2]$,

$\lambda = [5;10;5*5/100;5/100;5;1]$

M_i : are all orthogonal matrix

Composition Function 11 (F25 *n*=8)

$f_{1-2}(x)$ ： Rastrigin’s Function

$f_{3-4}(x)$ ： F8F2 Function

$f_{5-6}(x)$: Weierstrass Function

$f_{7-8}(x)$: Griewank’s Function

$\sigma = [1,1,1,1,1,2,2,2]$,

$\lambda = [5;1;5;1;50;10;5*5/200;5/200]$

M_i : are all orthogonal matrix

Composition Function 12 (F26 *n*=8)

$f_{1-2}(x)$ ： Rotated Expanded Scaffer’s F6 Function

$f_{3-4}(x)$ ： F8F2 Function

$f_{5-6}(x)$: Weierstrass Function

$f_{7-8}(x)$: Griewank’s Function

$\sigma = [1,1,1,1,1,2,2,2]$,

$\lambda = [5*5/100; 5/100; 5; 1; 5; 1; 50; 10]$

M_i : are all orthogonal matrix

Composition Function 13 (F27 *n*=10)

$f_{1-2}(x)$: Rotated Expanded Scaffer's F6 Function

$f_{3-4}(x)$: Rastrigin's Function

$f_{5-6}(x)$: F8F2 Function

$f_{7-8}(x)$: Weierstrass Function

$f_{9-10}(x)$: Griewank's Function

$\sigma = [1,1,1,1,1,2,2,2,2,2]$,

$\lambda = [5*5/100; 5/100; 5; 1; 5; 1; 50; 10; 5*5/200; 5/200]$

M_i : are all orthogonal matrix

Composition Function 14 (F28 *n*=10)

All settings are the same as F13, except M_i's condition numbers are [10 20 50 100 200 1000 2000 3000 4000 5000]

Composition Function 15 (F29 *n*=10)

$f_1(x)$: Weierstrass Function

$f_2(x)$: Rotated Expanded Scaffer's F6 Function

$f_3(x)$: F8F2 Function

$f_4(x)$: Ackley's Function

$f_i(x) = -20\exp(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}) - \exp(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i)) + 20 + e$ $f_5(x)$: Rastrigin's Function

$f_6(x)$: Griewank's Function

$f_7(x)$: Non-Continuous Expanded Scaffer's F6 Function

$$F(x,y)=0.5+\frac{(\sin^2(\sqrt{x^2+y^2})-0.5)}{(1+0.001(x^2+y^2))^2}$$

$$f_i(x)=F(y_1,y_2)+F(y_2,y_3)+...+F(y_{D-1},y_D)+F(y_D,y_1)$$

$$y_i=\begin{cases}x_j & |x_j|<1/2\\ round(2x_j)/2 & |x_j|>1/2\end{cases} \text{ for } j=1,2,...,D$$

$$round(x)=\begin{cases}a-1 \text{ if } x\le 0 \,\&\, b\ge 0.5\\ a \quad \text{if } b<0.5\\ a+1 \text{ if } x>0 \,\&\, b\ge 0.5\end{cases}$$

$f_8(x)$: Non-Continuous Rastrigin's Function

$$f_i(x)=\sum_{i=1}^{D}(y_i^2-10\cos(2\pi y_i)+10)$$

$$y_i=\begin{cases}x_j & |x_j|<1/2\\ round(2x_j)/2 & |x_j|>1/2\end{cases} \text{ for } j=1,2,...,D$$

$f_9(x)$: High Conditioned Elliptic Function

$$f(x)=\sum_{i=1}^{D}(10^6)^{\frac{i-1}{D-1}}x_i^2$$

$f_{10}(x)$: Sphere Function with Noise in Fitness

$$f_i(x)=(\sum_{i=1}^{D}x_i^2)(1+0.1|N(0,1)|)$$

n=10

$\sigma_i=2$ for all i

$\lambda=[10;5/20;1;5/32;1;5/100;5/50;1;5/100;5/100]$

M_i are all rotation matrices, condition number are [100 50 30 10 5 5 4 3 2 2];