

Near Optimal Adaptive Robust Beamforming

The performance degradation in traditional adaptive beamformers can be attributed to the imprecise knowledge of the array steering vector and inaccurate estimation of the covariance matrix. The inaccurate estimation of the covariance matrix is due to the limited data samples and presence of desired signal components in the training data, especially when the desired signal is the dominant signal in the training data. The mismatch between the actual and the presumed steering vectors can be due to the error in the position (geometry) and/or in the look direction estimate. Due to mismatch in the actual and the presumed steering vectors and the inaccurate estimation of covariance matrix, the output SINR of the adaptive beamformers cannot adapt to the increase in the input SNR. In other words the output SINR of the adaptive beamformers saturates as the input SNR increases. From the following graphs, it can be observed that the proposed algorithms in the following references were able to overcome the effect of saturation. The results of the proposed algorithms were compared with some of the state-of-the-art algorithms such as RCB [1], IRCB [2], SQP [3-5]. We present the performance graphs of different algorithms presented in our papers in terms of output SINR for different input SNR.

Mallipeddi, R., J. P. Lie, P. N. Suganthan, S. G. Razul, and C. M. S. See, “Near optimal robust adaptive beamforming approach based on evolutionary algorithm”, *Progress In Electromagnetics Research B*, Vol. 29, 157-174, 2011.

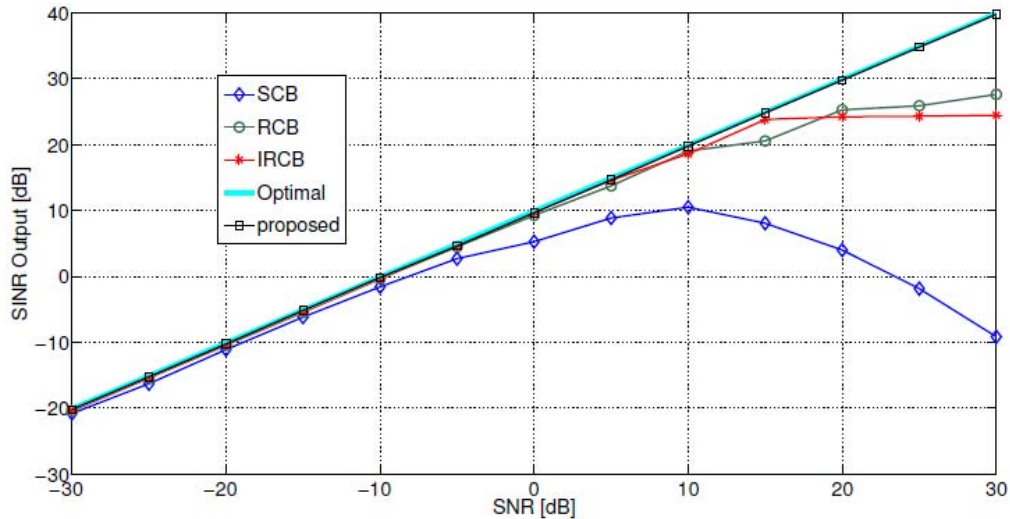


Figure 2.2: Best output SINR versus SNR for no geometry error case. Our proposed approach yields SINR which is approximately equal to the theoretical maximum SINR

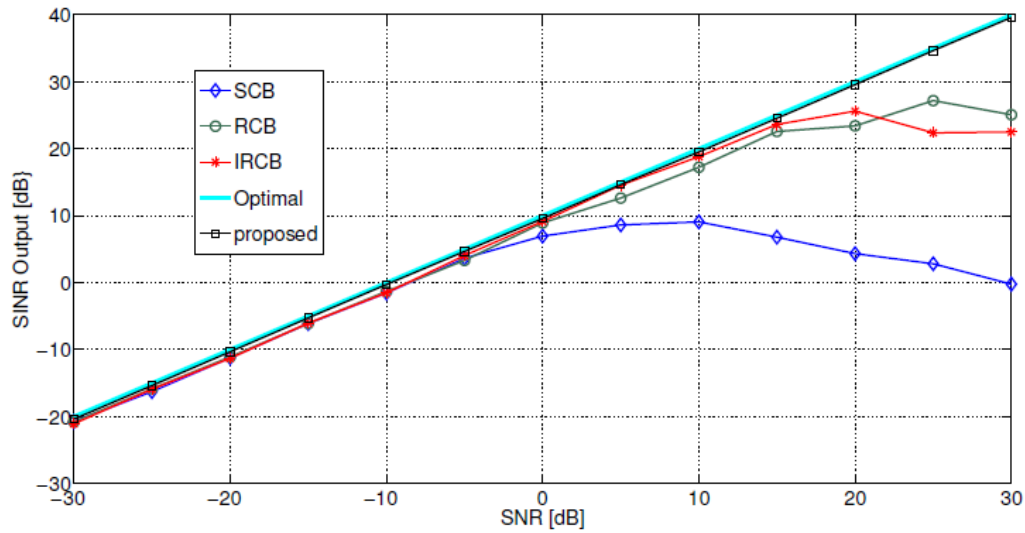


Figure 2.3: Best output SINR versus SNR for 15% geometry error case. Our proposed approach yields SINR which is approximately equal to the theoretical maximum SINR

Mallipeddi, R., J. P. Lie, P. N. Suganthan, S. G. Razul, and C. M. S. See, “Robust adaptive beamforming based on covariance matrix reconstruction for look direction mismatch”, *Progress In Electromagnetics Research - Letters*, Vol. 25, pp 37-46, 2011.

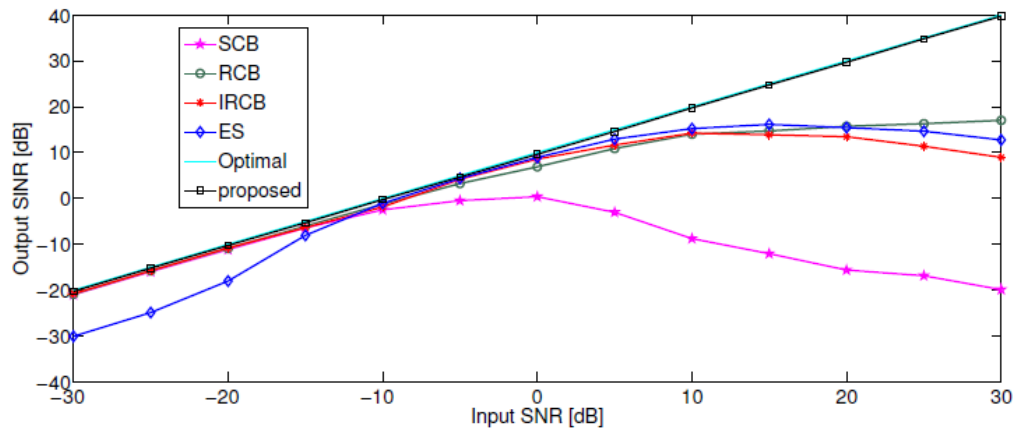


Figure 3.1: SINR(Median) Vs SNR for Look Direction Error Only. Our proposed approach yields SINR which is approximately equal to the theoretical maximum SINR

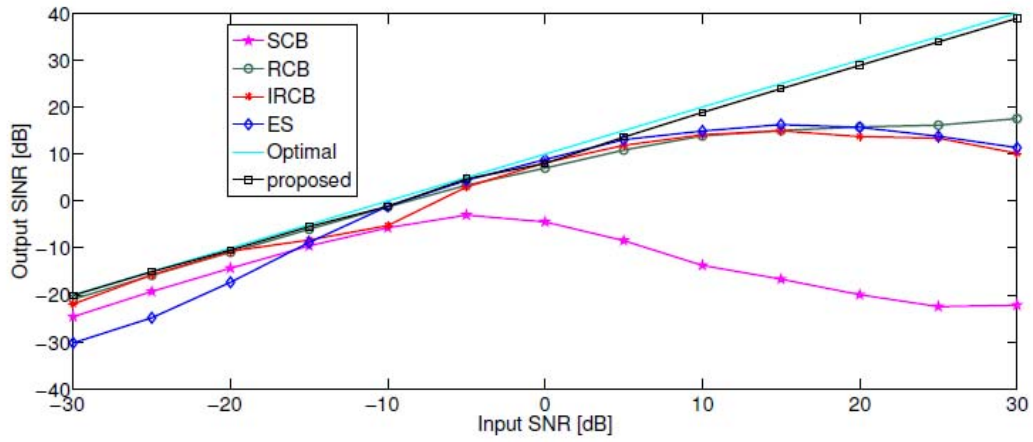


Figure 3.2: SINR(Median) Vs SNR for Look Direction & 15% Geometry Error. Our proposed approach yields SINR which is approximately equal to the theoretical maximum SINR

R. Mallipeddi, J. P. Lie, P. N. Suganthan, S. G. Razul, and C. M. S. See, "A differential evolution approach for robust adaptive beamforming based on joint estimation of look direction and array geometry," *Progress In Electromagnetics Research*, Vol. 119, 381-394, 2011.

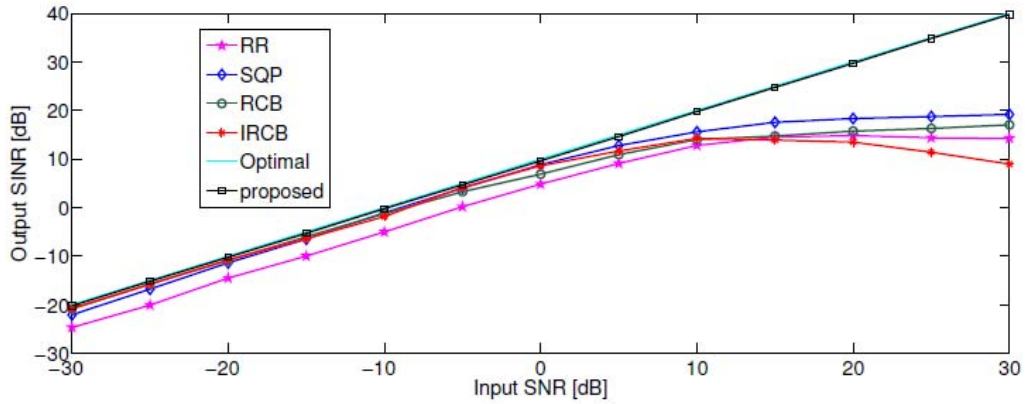


Figure 4.2: Median output SINR versus SNR for 0% geometry error. Our proposed approach yields SINR which is approximately equal to the theoretical maximum SINR

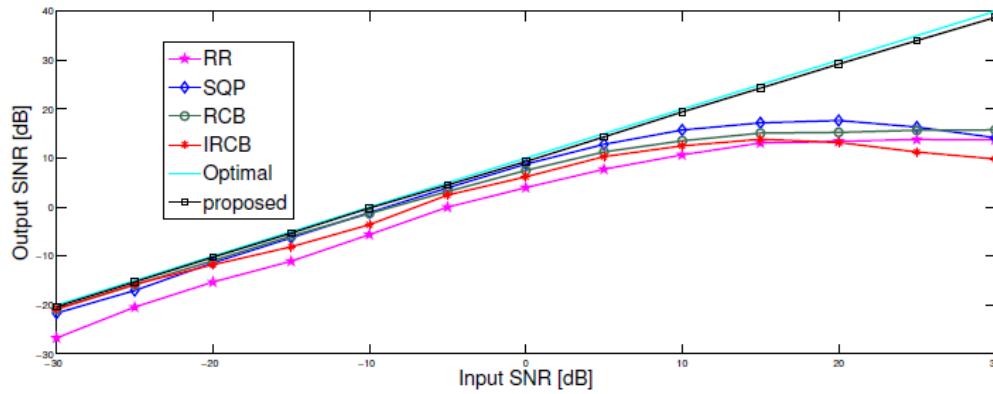


Figure 4.3: Median output SINR versus SNR for 50% geometry error. Our proposed approach yields SINR which is approximately equal to the theoretical maximum SINR

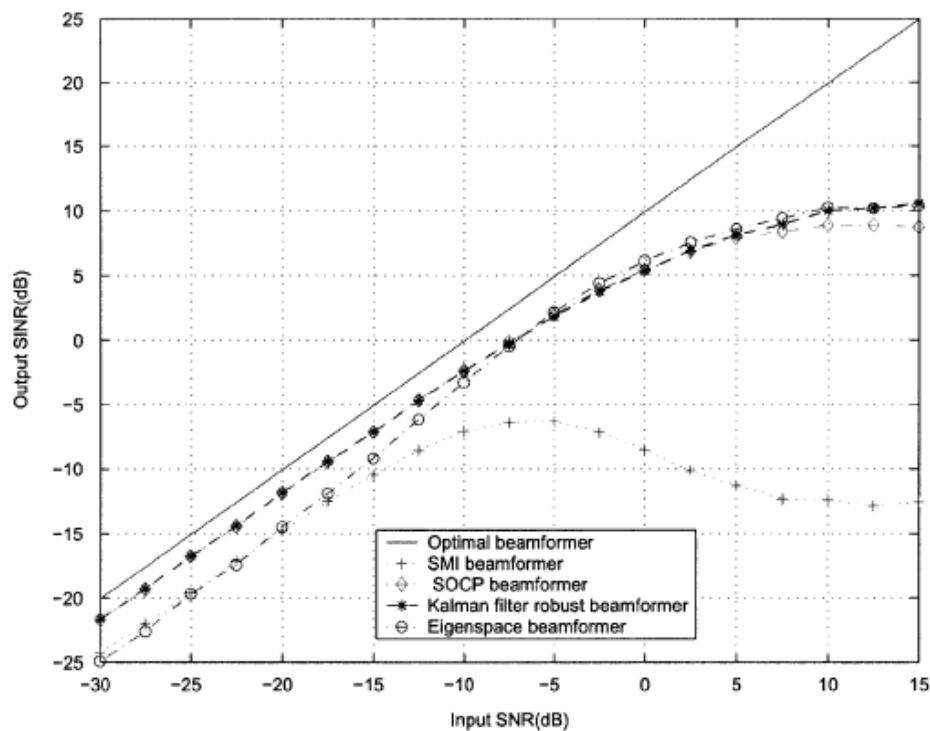
References

- [1] J. Li, P. Stoica, and Z. Wang, "On robust capon beamforming and diagonal loading," *IEEE Transactions on Signal Processing*, vol. 51, no. 7, pp. 1702–1715, 2003.
- [2] J. P. Lie, X. Li, W. Ser, C. M. S. See, and L. Lei, "Adaptive uncertainty based iterative robust capon beamformer," *Proc. IEEE Int Acoustics Speech and Signal Processing (ICASSP)*, pp. 2526-2529, 2010.
- [3] Z. L. Yu, Z. Gu, J. Zhou, Y. Li, S. Wee, M. H. Er, "A Robust Adaptive Beamformer Based on Worst-Case Semi-Definite Programming," *IEEE Transactions on Signal Processing*, vol. 58, no. 11, pp. 5914-5919, 2010.
- [4] Won, S. Y. and Chong K. U., "Robust adaptive beamforming based on the eigenstructure method," *IEEE Transactions on Signal Processing*, Vol. 42, No.6, 1543-1547, June 1994.
- [5] Y. Sel'en, R. Abrahamsson, and P. Stoica, "Automatic robust adaptive beamforming via ridge regression," *Signal Processing*, vol. 88, no. 1, pp. 33–49, 2008.

For better viewing, we also present the abstract and the results presented in some of the papers. From the graphs below, it can be observed that all the algorithms suffer from the effect of saturation.

El-Keyi, A. Kirubarajan, T. Gershman, A.B. Robust Adaptive Beamforming Based on the Kalman Filter, *IEEE Transactions on Signal Processing*, Vol. 53, Issue 8, pp. 3032-3041, August, 2005.

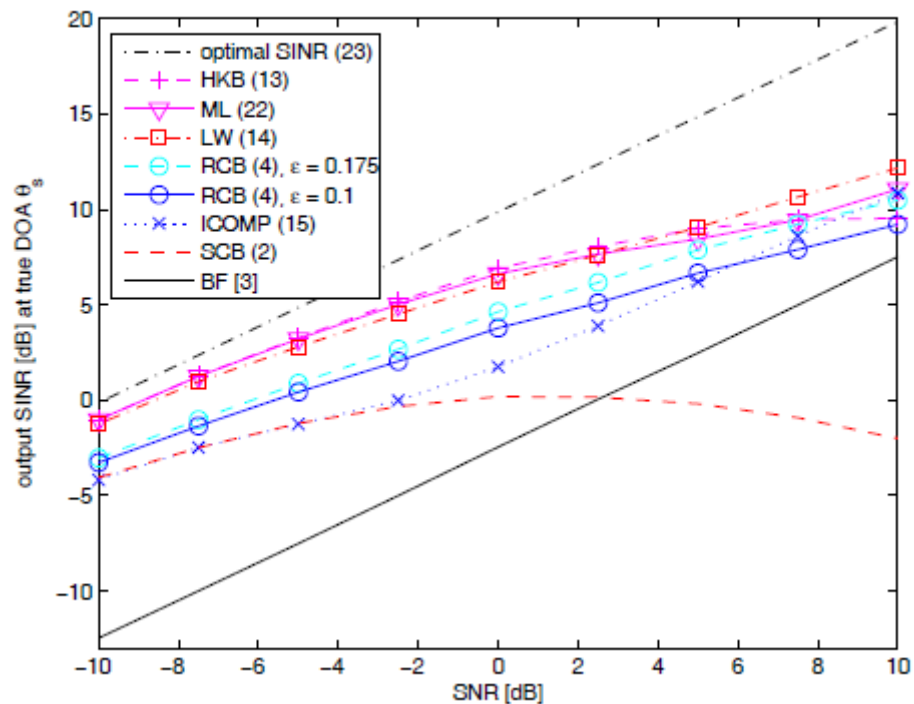
In this paper, we present a novel approach to implement the robust minimum variance distortionless response (MVDR) beamformer. This beamformer is based on worst-case performance optimization and has been shown to provide an excellent robustness against arbitrary but norm-bounded mismatches in the desired signal steering vector. However, the existing algorithms to solve this problem do not have direct computationally efficient online implementations. In this paper, we develop a new algorithm for the robust MVDR beamformer, which is based on the constrained Kalman filter and can be implemented online with a low computational cost. Our algorithm is shown to have a similar performance to that of the original second-order cone programming (SOCP)-based implementation of the robust MVDR beamformer. We also present two improved modifications of the proposed algorithm to additionally account for nonstationary environments. These modifications are based on model switching and hypothesis merging techniques that further improve the robustness of the beamformer against rapid (abrupt) environmental changes.



Output SINR vs SNR (Fig. 7 in the reference above)

Selen, Y. Abrahamsson, R. Stoica, P. Automatic Robust Adaptive Beamforming via Ridge Regression Acoustics, *IEEE international conference on Speech and Signal Processing, ICASSP* , pp. II-965 - II-968, April, 2007.

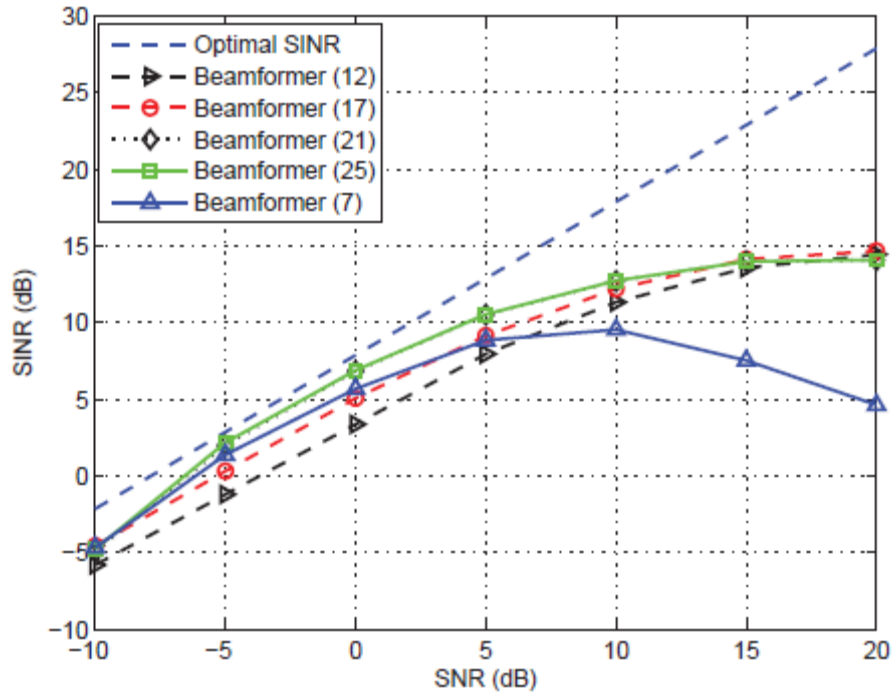
In this paper we derive a class of new parameter free robust adaptive beamformers using the generalized sidelobe canceler reparameterization of the Capon beamformer. In this parameterization the minimum variance beamformer is obtained as the solution of a linear least squares problem. In the case of an inaccurate steering vector and/or few data snapshots this marginally overdetermined system gives an ill fit causing signal cancellation in the standard minimum variance solution. By regularizing the problem using ridge regression techniques we get a whole class of robust adaptive beamformers, none of which requires the choice of a user parameter. We also propose a novel empirical Bayes-based ridge regression technique. The performance is compared to other robust adaptive beamformers.



Mean output SINRs vs varying input SNR (Fig. 1 in the reference above)

Lei Lei Lie, J.P. Gershman, A.B. See, C.M.S. Robust adaptive beamforming and steering vector estimation in partly calibrated sensor arrays: A structured uncertainty approach, *Acoustics, IEEE international conference on Speech and Signal Processing, ICASSP*, pp. 2534 – 2537, March, 2010.

Two new approaches to adaptive beamforming in sparse subarray based sensor arrays are proposed. Each subarray is assumed to be well calibrated but the intersubarray gain and/or phase mismatches are assumed to remain unknown or imperfectly known. Our first approach is based on a worst-case beamformer design that, unlike the existing worst-case designs, exploits a *structured* ellipsoidal uncertainty model for the signal steering vector. Our second approach exploits the idea of estimating the signal steering vector by maximizing the output power of the minimum variance beamformer. Several modifications of our second approach are developed for the cases of gain-and-phase and phase-only intersubarray distortions.



Output beamformer SINRs vs SNR (Fig. 2 in the reference above)

To overcome the signal-to-interference-and-noise ratio (SINR) performance degradation in the presence of large steering vector mismatches, we propose an iterative robust Capon beamformer (IRCB) with adaptive uncertainty level. The approach iteratively estimates the actual steering vector (SV) based on conventional robust Capon beamformer (RCB) formulation that uses an uncertainty sphere to model the mismatch between the actual and presumed SV. At each iteration, the adaptive uncertainty algorithm self-adjusts the uncertainty sphere according to the estimated mismatch SV. This estimation is derived based on the geometrical interpretation of the mismatch and can be expressed as a simple closed-form expression as a function of the presumed SV and the signal-subspace projection. The other variant of the proposed algorithm that uses a flat ellipsoid to model the mismatch is also proposed. Simulation results show that the proposed approaches offer better interference suppression capability and achieve higher output SINR, as compared to other diagonal-loading-based approaches.

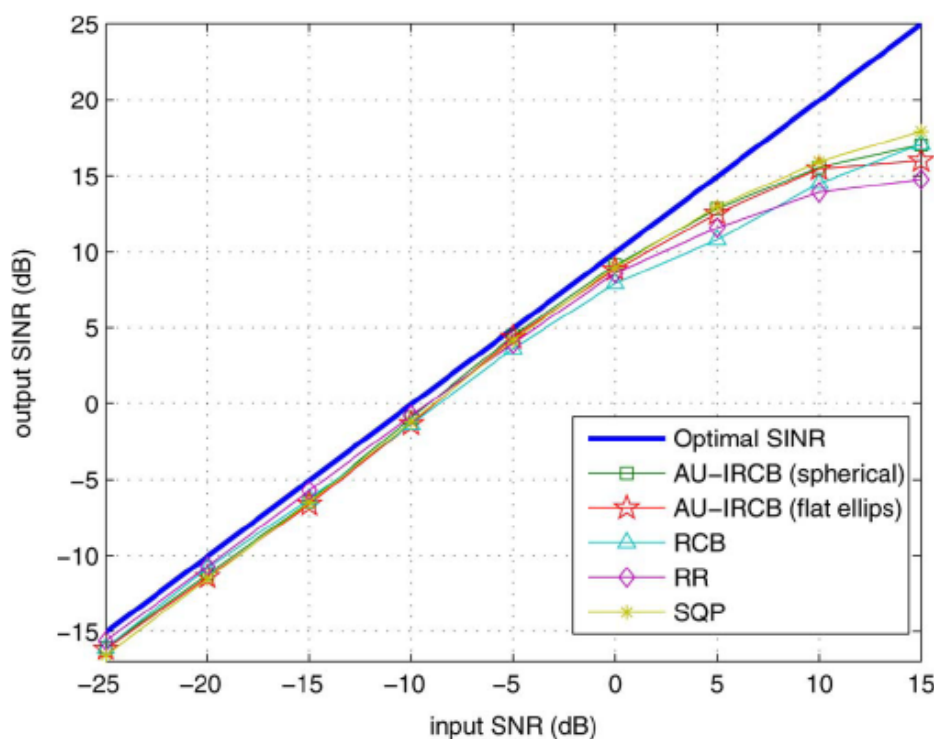


Fig. 5. Output SINR versus SNR. The dominant interferences are at $\theta_i = \{-50^\circ, -20^\circ\}$.

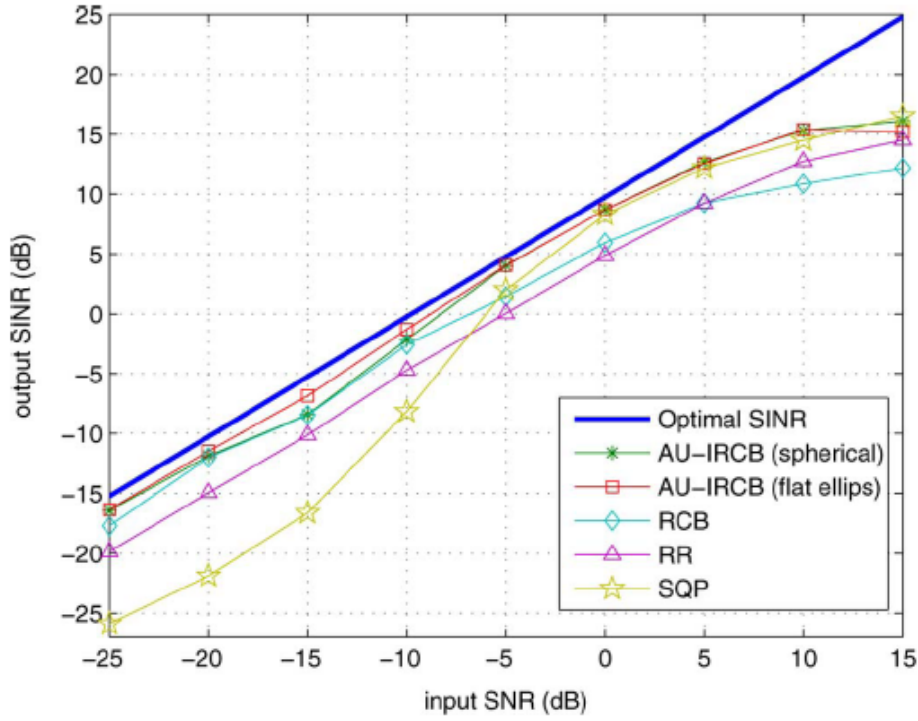
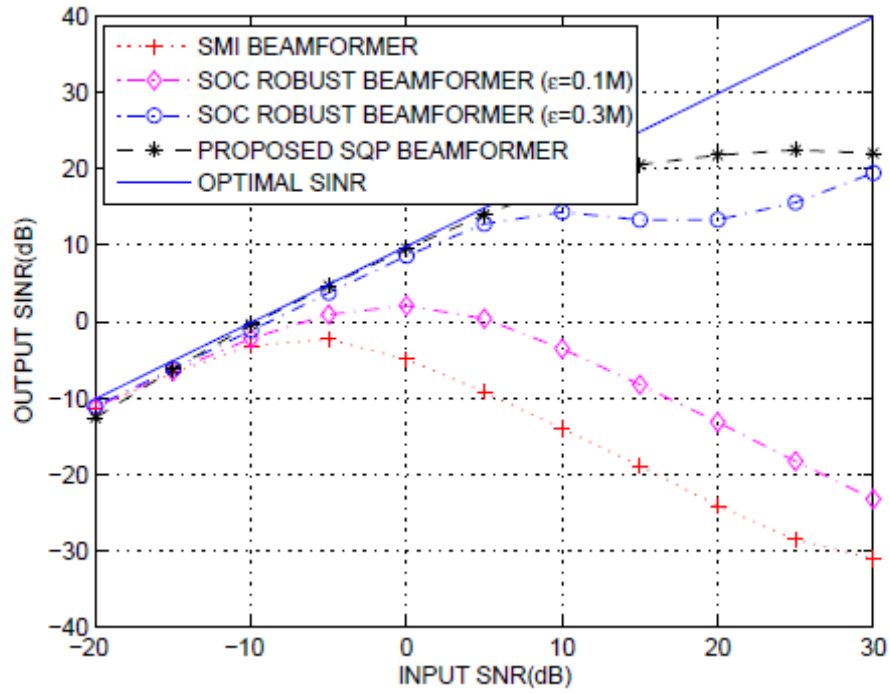


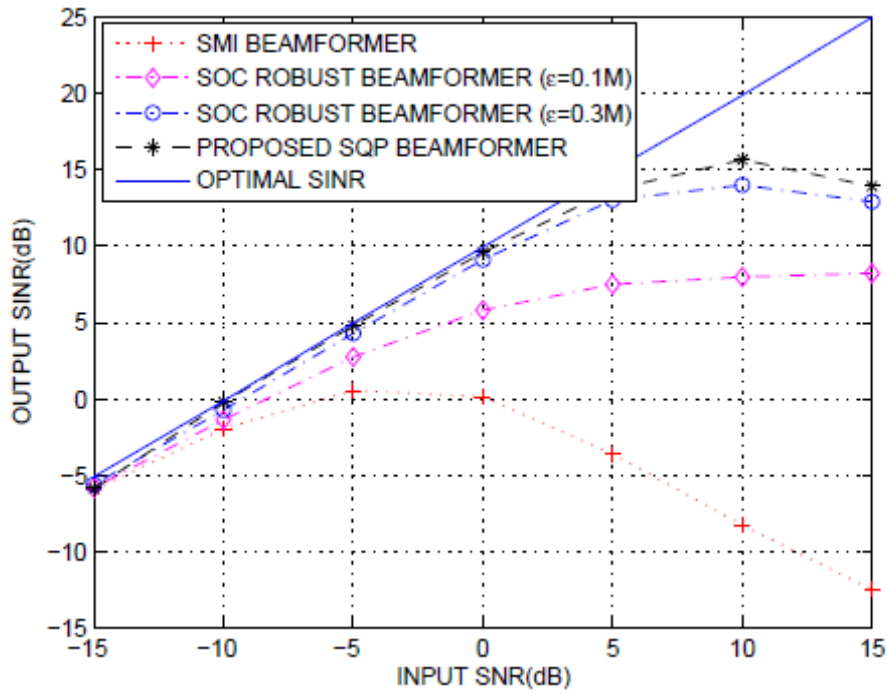
Fig. 4. Output SINR versus SNR. The dominant interferences are at $\theta_i = \{20^\circ, 30^\circ\}$.

Hassanien, A.; Vorobyov, S.A.; Wong, K.M.; Robust Adaptive Beamforming using Sequential Quadratic Programming, *IEEE international conference on Speech and Signal Processing, ICASSP*, pp. 2345-2348, 2008

In this paper, a new algorithm for robust adaptive beamforming is developed. The basic idea of the proposed algorithm is to estimate the difference between the actual and presumed steering vectors and to use this difference to correct the erroneous presumed steering vector. The estimation process is performed iteratively where a quadratic convex optimization problem is solved at each iteration. Unlike other robust beamforming techniques, our algorithm does not assume that the norm of the mismatch vector is upper bounded, and hence it does not suffer from the negative effects of over/under estimation of the upper bound. Simulation results show the effectiveness of the proposed algorithm.



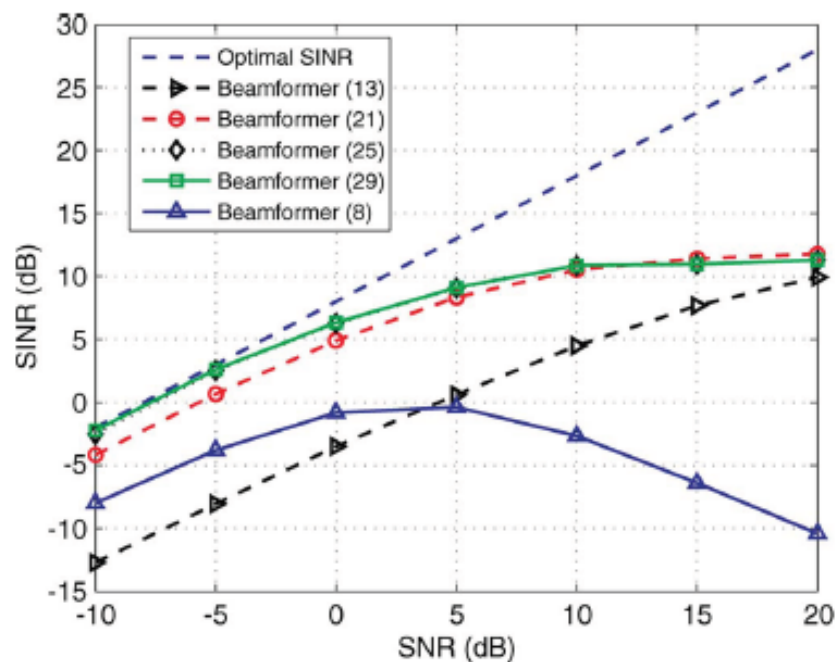
Output SINR vs SNR. (Look Direction Mismatch of 3°) (Fig. 2 in reference above)



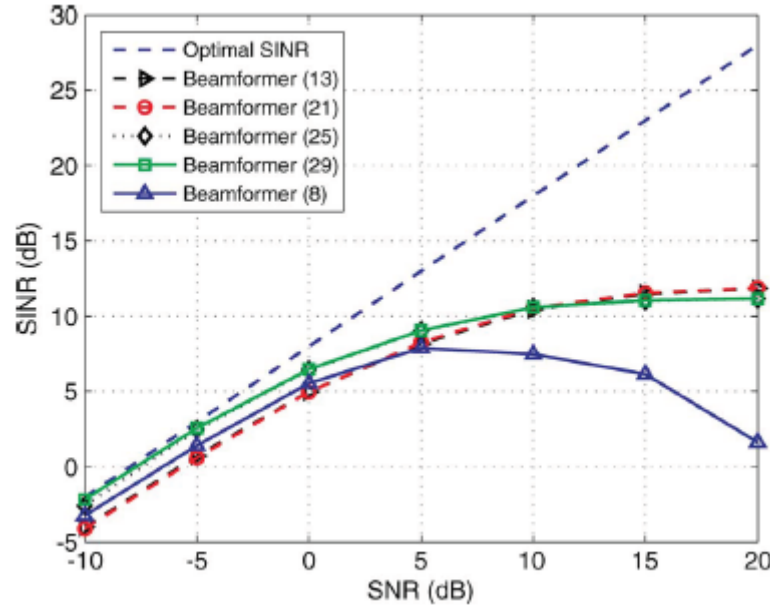
Output SINR vs SNR. (Look Direction Mismatch and errors in sensor positions) (Fig. 3 in reference above)

Lei Lei; Lie, J.P.; Gershman, A.B.; See, C.M.S.; Robust Adaptive Beamforming in Partly Calibrated Sparse Sensor Arrays, *IEEE Transactions on Signal Processing*, Vol. 58, Issue 3, March 2010.

Two new approaches to adaptive beamforming in sparse subarray-based partly calibrated sensor arrays are developed. Each subarray is assumed to be well calibrated, so that the steering vectors of all subarrays are exactly known. However, the intersubarray gain and/or phase mismatches are known imperfectly or remain completely unknown. Our first approach is based on a worst-case beamformer design which, in contrast to the existing worst-case designs, exploits a specific *structured* ellipsoidal uncertainty model for the signal steering vector rather than the commonly used unstructured uncertainty models. Our second approach is based on estimating the unknown intersubarray parameters by maximizing the output power of the minimum variance beamformer subject to a proper constraint that helps to avoid trivial solution of the resulting optimization problem. Different modifications of the second approach are developed for the cases of gain-and-phase and phase-only intersubarray distortions.



Output SINRs vs SNR (Completely Unknown intersubarray displacements) (Fig. 2 of Reference above)



Output SINRs vs SNR (Intersubarray displacements assumed to be known precisely. Fig. 4 of Reference above)

Siew Eng Nai; Wee Ser; Zhu Liang Yu; Huawei Chen; Iterative Robust Minimum Variance Beamforming, *IEEE Transactions on Signal Processing*, VOL. 59, NO. 4, APRIL 2011

Based on worst-case performance optimization, the recently developed adaptive beamformers utilize the uncertainty set of the desired array steering vector to achieve robustness against steering vector mismatches. In the presence of large steering vector mismatches, the uncertainty set has to expand to accommodate the increased error. This degrades the output signal-to-interference-plus-noise ratios (SINRs) of these beamformers since their interference-plus-noise suppression abilities are weakened. In this paper, an iterative robust minimum variance beamformer (IRMVB) is proposed which uses a small uncertainty sphere (and a small flat ellipsoid) to search for the desired array steering vector iteratively. This preserves the interference-plus-noise suppression ability of the proposed beamformer and results in a higher output SINR. Theoretical analysis and simulation results are presented to show the effectiveness of the proposed beamformer.

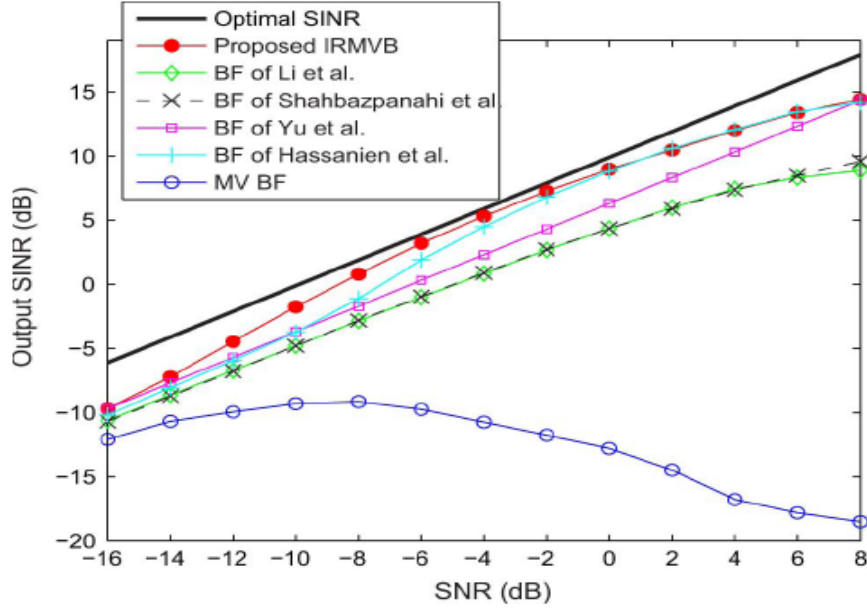


Fig. 4. Optimal SINR and output SINRs of the proposed IRMVB, the beamformer of Li *et al.* [4], the beamformer of Shahbazpanahi *et al.* [3], the beamformer of Yu *et al.* [11], the beamformer of Hassanien *et al.* [18], and the MV beamformer. There is a steering direction error of 6° .

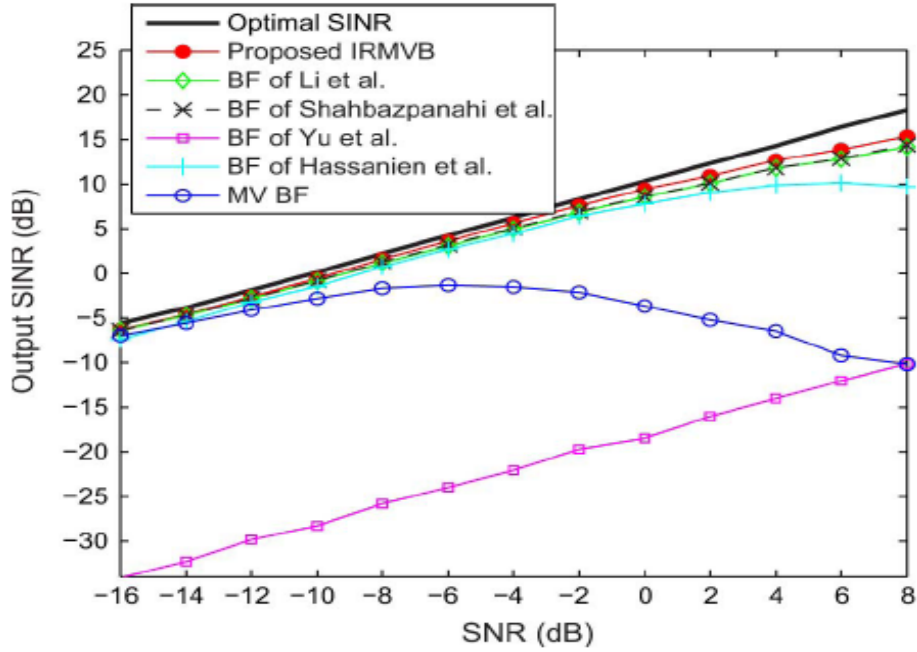


Fig. 7. Optimal SINR and output SINRs of the proposed IRMVB, the beamformer of Li *et al.* [4], the beamformer of Shahbazpanahi *et al.* [3], the beamformer of Yu *et al.* [11], the beamformer of Hassanien *et al.* [18], and the MV beamformer. There are array calibration errors.

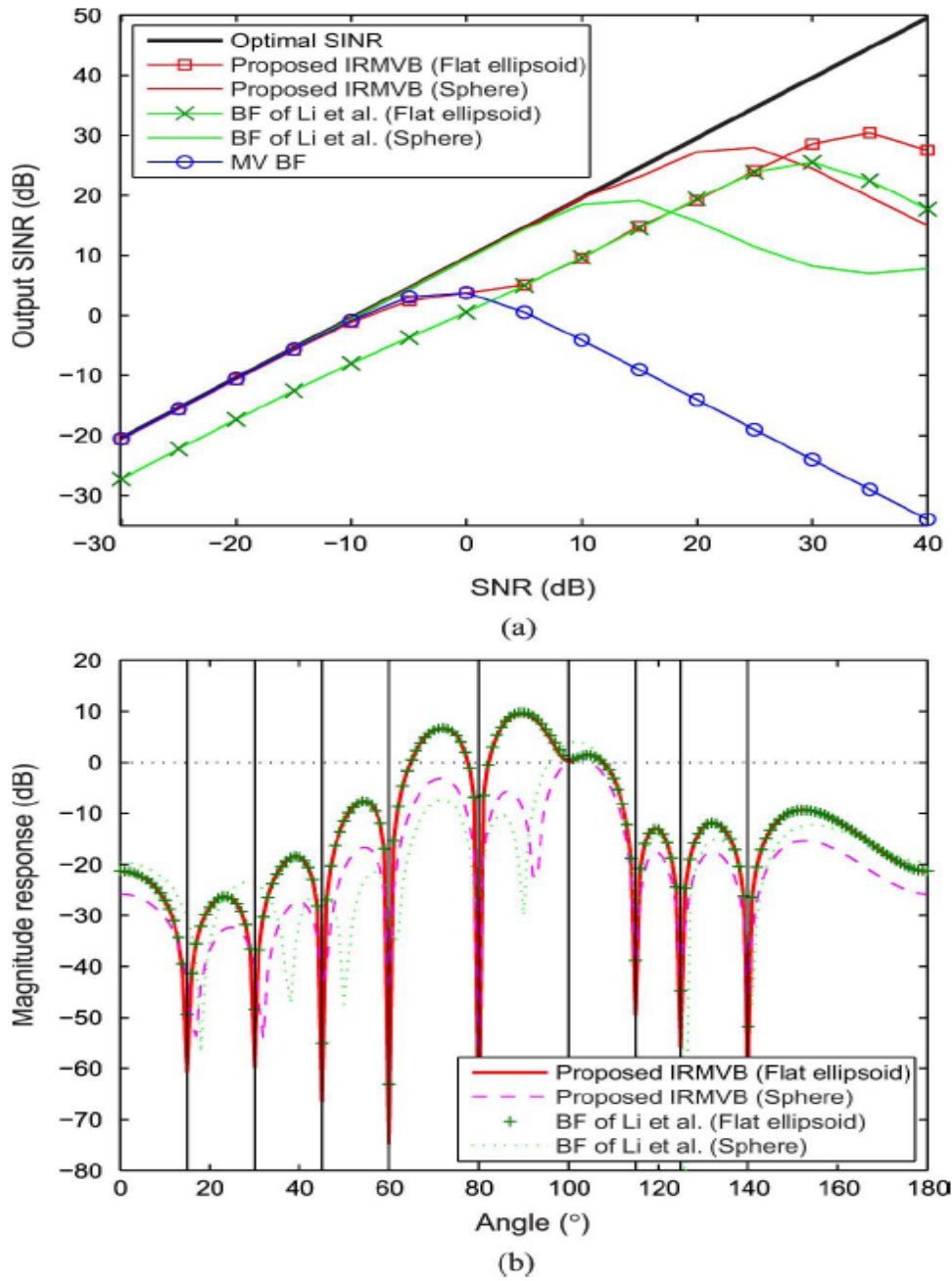


Fig. 8. Top: Optimal SINR and output SINRs of the proposed IRMVBs using flat ellipsoidal and spherical constraints, respectively, the beamformers of Li *et al.* [4] using flat ellipsoidal and spherical constraints, respectively, and the MV beamformer. There is a steering direction error of 2° . Bottom: Beampatterns of the tested beamformers at SNR = 35 dB. Solid vertical lines indicate the impinging signals' DOAs. Dotted horizontal line indicates the 0 dB gain level (a) Output SINR versus SNR (b) Beampatterns.