Trading Strategies for self-financing portfolio

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Introduction

There are many strategies available for investors to gain profits on assets available in the financial market. The development of strategy enables financial institutions and individual investors to automate their trading decisions without human emotion. Each method has its desirable properties depending on the type of asset, the market, and the investors' acceptable risk level. This report will examine two fundamental trading strategies, including trend following (TF) and mean reversion strategies by simulation on synthetic financial time series. The mean reversion is extended to involve the signal generated based on price prediction by the Autoregressive integrated moving average (ARIMA) model. Additionally, the buy and hold strategy is going to be utilised as a performance benchmark. There is no external signal, such as economics signal or news, involved in this study. The strategies are designed for a self-financing portfolio without short-selling opportunities. The top three strategies will then be selected based on the return and performance indicators. Their Sharpe ratio (SR) will be used for further statistical testing with control of Family Wise Error Rate (FWER) to minimise false positives probability.

Data and Methodology

The trading strategy consists of rational rules to implement buying and selling decisions on the assets (Barucca & Firoozye, 2021). Before implementing these rules, data analysis of asset information is required. Usually, the choice of assets is picked; for example, the equity will take a closed price on daily trading days into account. For this study, the synthetic financial time series was generated following the formula 1. The daily asset price was formed for 2,000 days (i.e., around 8 years by 252 business days) with parameters of $d = 0.025$, $\phi = 0.6$, $\theta = -0.4$, and $y_0 = y_1 = 100$ where ϵ is sequence of i.i.d Gaussian random variable. The random number was selected based on a normal distribution with a seed value of 20031488. As a result, the time series (figure 1) shows an increasing price over time with a mean (μ) value of 241.86 and a standard deviation (σ) of 66.70. The standard deviation showed that the market volatility exists as the prices spread from the mean value. This time series reflects the reality that the market price does not constantly follow a straight line (Fong, Tai, & Si, 2011). The data can be split into training, and testing sets the help to define the best hyperparameters for strategy. The training set is historical data for the model to learn, and the testing set contains unseen data to evaluate the model's performance. The time series was split by index with a 7:3 ratio for training and testing samples, respectively. This approach to enforcing continuity of time series across time horizon rather than randomly selected. Once the data is available for analysis, the next step is forming the trading strategies.

$$\Delta y_t - d = \phi(\Delta y_{t-1} - d) + \epsilon_t + \theta\epsilon_{t-1} \tag{1}$$

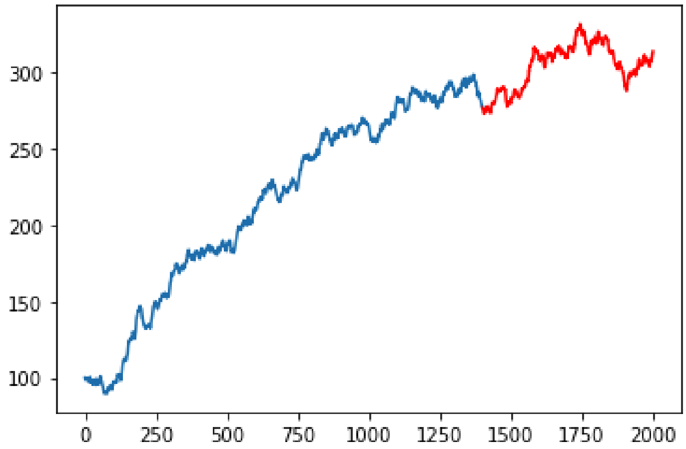


Figure 1: Synthetic price time series of 2,000 days generated from formula 1. The red color represents testing samples of the last 600 days.

As previously mentioned, the two fundamental trading strategies are trend following (TF) and mean reversion. This study also includes buy and hold strategy for performance benchmark purpose. All three strategies are implemented in Python and refactoring the source code provided by Barucca and Firoozye (2021) to accept time window of moving average, have initial cash capital of 10,000, and self-financing conditions. Thus, in each time step, the assets volume ($V(t)$) and capital ($C(t)$) are non-negative. The total value is also updated in each iteration using the formula $TV(t) = C(t) + p(t)V(t) = C(t+1) + p(t) + V(t+1)$ where $p(t)$ represents the asset price at time t. Simple return of each strategy can be calculated using the formula $r_a(t) = log(\frac{TV_a(t)}{TV_a(t-1)})$. Notably, each strategy has it own implementation rules.

The trend following (TF) takes the strategy weight or signal by observing the market change trend for trading decisions. The signal is based on market trends from historical price rather than forecasting or external information, such as news (Fong et al., 2011). If the market trend is positive, the investors can long position by buying all stocks with available capital. Vice versa, the investors can short all assets to obtain capital while the market trend is declining. This decision-making is similar to enter and exit market rules used in Fong et al. (2011)'s studies. These rules help the investors determine when to stay in the market to obtain profit and exit the market. There are various indicators can be used, the most common weight is the simple moving average (SMA) which takes the average of return by time window following this formula $S_{t-1} = \frac{1}{T}\sum_1^T r_{t-k}$ (Barucca & Firoozye, 2021). Then it takes the difference between the current price and moving average to determine whether it is a positive or negative trend. Various studies showed that a SMA could gain an excess return, for example, in the currency market (James, 2003). Zakamulin and Giner (2020) also found that following MA is more profitable than the momentum rule, which relies on the price is above or below the previous value by the time window. However, the return may be reduced due to transaction costs of frequent trading. The strategy will take the common time windows of moving average (MA) [1] on the trend following with training sample of 1,400 points to define the best time window. Then, the out-sample returns

[1]This study uses the common time window for moving average (MA) of 3, 5, 7, 10, 20, 50, 100 and 200 days (Mitchell, 2021)

will be taken with the best time window derived from the training process. The implementation follows the TF rules as previously explained with an additional condition that there is no action required if the asset price and MA are equal.

Next is the mean reversion, which takes the assumption that the change asset price is temporary and eventually reaches a long-term mean, i.e., follows the mean reversion phenomenon (Li, Hoi, Sahoo, & Liu, 2015). It is a contrarian strategy from the TF strategy, which utilises opposite rules to enter and exit the market. The process takes a long position once the price drops and short assets when the trend rises. There are two types of mean reversions, single-period, and multiple-period mean reversion. Li et al. (2015) explained that singe-period mean reversion strategy underperforms given the actual market prices (Borodin, El-Yaniv, & Gogan, 2004). Thus, a multiple-period mean reversion or moving average reversion aims to overcome the problem. The source code takes mean reversion rules using the same time window range from the TF strategy. If the MA is lower than the current price, the model buys assets with available capital. The function also executes a short position once the current price is higher than MA.

The mean reversion can be extended using the estimated market prices for the signal. The autoregressive integrated moving average (ARIMA) model is one way to predict the future price from the stationary time series. The ARIMA is constructed based on the assumption that the historical time series can generate future values without other predictors. The model consists of three components: autoregression (AR), integration of observation (I), and moving average (MA) (Box & Jenkins, 1976). This study adds both AR and MA terms by looking on the first-order autoregressive model, assuming that the series is autocorrelated and apply one lagged forest errors (Nau, n.d.). The number of nonseasonal differences is not to be taken into the model, i.e., I = 0. This ARIMA(1,0,1) follows $Y_t = rY_{t-1} + e_t + ae_tt - 1$ where r represents AR parameter, e_t is the error term and a for MA parameter (Chernick, 2016). The model is trained to minimise the mean squared error (MSE) between the predicted and actual prices, thus, return optimal hyperparameters. The trading rules are based on the difference of predicted price by the ARIMA model and its MA and subtract with the actual cumulative prices. Then the strategy executes long and short decisions using contrarian strategy following the traditional mean reversion method.

Unlike the above two strategies, the buy and hold acts as a long-term strategy with minimum frequency (Barucca & Firoozye, 2021). It acts as a passive investment regardless of the fluctuation in the financial market (Ling, Ng, & Muhamad, 2014). When entering the market, the strategy takes all capitals to purchase assets given the initial price, i.e., p_0. Therefore, the total value is the same across time step by using formula $cash_0 * S/S_0$ and no short decisions during the trading. This approach gives the benefit of low commission fees and transaction costs. It is a suitable strategy for investors who believe that the assets' value will grow over time and work well on well-performing companies (Ling et al., 2014). In this study, the buy and hold acts as a benchmark to evaluate whether the above two strategies, mean-reversion and trend following, will outperform the simple buy and hold approach. The return of strategy can give essential information regarding portfolio performance. Nevertheless, other factors should be considered, such as the risk of loss and volatility of the market price.

There are three performance indicators: Sharpe ratio (SR), Value-at-Risk (VaR), and Calmar ratio. Notably, there is no risk-free rate of return considered in this study, thus zero value. The Sharpe ratio (SR) is formed on Markowitz's mean-variance paradigm, which takes the mean and standard deviation of strategy returns computed based on historical time series (Sharpe, 1994). It assumes that these two moments are sufficient for performance evaluation. The SR is the measurement of volatility-adjusted performance, which defines excess return above the risk-free rate (Israelsen, 2005). This ratio can support the comparison of return among a choice of strategies. The better strategy should have a higher SR. If the SR is negative, there are two possible reasons: the negative expected value of returns or the strategy's performance is below the defined risk-free rate (Fernando, 2021). The strategy with negative SR is not a preferable choice to be deployed. The SR can be computed on an annual basis by multiply with a square root of 252 business days.

The potential loss or the market risk with an unexpected price change should also taking into consideration. Value-at-risk (VaR) represents relative loss of strategy given the confidence level (p) over the time horizon (Duffie & Pan, 1997). The loss in market value can exceed with probability $1 - p$. The return that corresponds to the confidence level can be simply computed by the quantile of overall return over time. This report takes the confidence level (p) as 0.05. The output interprets the 95% chance the strategy will lose less than the output value. If the output is negative, this implies a high probability of making a profit rather than a loss.

Lastly, the Calmar ratio is a drawndown-based performance indicator that takes the mean of strategy return divided by the maximum drawdown (MDD). The MDD is the largest drop from the peak of the portfolio over a time horizon of investment using formula $\frac{TroughValue - PeakValue}{PeakValue}$ (Young, 1991). The ratio measures return per unit of risk to the maximum drawdown (Steinki & Mohammad, 2015). The annualised Calmar ratio can be computed by taking the mean of annualised strategy return. This ratio does not take the volatility into account as Sharpe ratio, to exclude the sensitivity of volatility in short-term. However, the drawback is that only a single event by the maximum drawdown is considered to introduce the bias for performance assessment due to the presence of outliners (Steinki & Mohammad, 2015). All three performance indicators will be used along with the return of each strategy. The strategies which yield negative returns and performance indicators will be discarded for further steps. Next is taking the hypotheses that selected strategies have a non-zero Sharpe ratio.

The statistical test aims to ensure that all selected strategies have a non-zero Sharpe ratio. The p-value can be obtained from the T-statistics distribution by taking the probability where the return is greater than the SR multiply with the root of time horizon, i.e., $p = Pr(r > SR\sqrt{T})$ with the degree of freedom $T - 1$ where T is the size of underlying data. A higher SR will yield higher T-statistics which implies a significance level for investment strategy (Harvey & Liu, 2015). Each method represents a single hypothesis. Testing multiple hypotheses can be done by applying the Family Wise Error Rate (FWER) to reduce false-negatives which is not desirable for this problem. There are two available methods for adjusted p-value: Bonferroni and Holm methods. The Bonferroni method (formula 2) multiplies the p-value by the number of strategies (M) and evaluates whether these strategies are still significant, i.e., less than 0.05 (Schweder & Spjøtvoll, 1982). The statistical tests will give an overview of strategy performance with reducing the chance of false positive errors. Alternatively, Holm's method can also test adjusted p-values by following the formula 3 (Holm, 1979). Both methods give adjusted p-values that can eliminate all false discoveries regardless of the number of strategies. Harvey and Liu (2015) commented that Bonfeeroni's method is more extreme as it adjusted the original p-values compared with Holm's approach. This report uses Bonfeeroni's process due to its computational advantage. The annual SR's statistical test can be computed by simply changing SR to annual SR to compute p-value and then apply with Bonferroni's method for adjusted p-value. The investors should also be aware that increasing the number of hypotheses or tests can also raise the number of false discoveries (Harvey & Liu, 2015).

$$p_i^{Bonferroni} = min[Mp_i, 1], i = 1, ..., M. \tag{2}$$

$$p_i^{Holm} = min[max_{j<=i}(M - j + 1)p_j, 1], i = 1, ..., M. \tag{3}$$

Strategy	time_window	Strategy Return (Train)	Strategy Return (Test)	SR	Annual SR	VaR	Annual calmar ratio
BH	-	1.78%	0.13%	0.06	0.97	-0.005	0.39
TF	3	2.55%	0.38%	0.22	3.44	-0.003	8.73
TF	5	2.15%	0.39%	0.22	3.56	-0.003	7.81
TF	7	2.12%	0.37%	0.21	3.35	-0.003	6.64
MR	50	0.10%	-0.02%	-0.01	-0.27	-0.004	-0.09
ARIMA	100	1.88%	0.12%	0.06	0.96	-0.005	0.36

Table 1: Strategies' return and values of performance indicators used in the report.

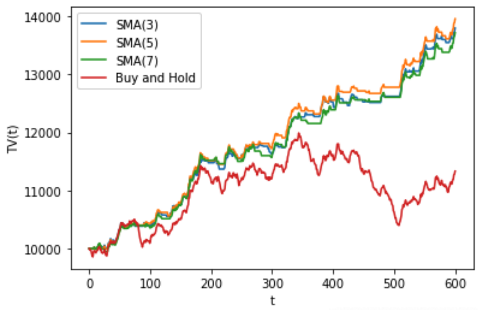
Results

Before interpreting the results obtained from both TF and mean reversion strategies, return generated from the buy and hold approach gave interesting finding that used as a performance benchmark for testing time series, i.e., the last 600 days. Over the time horizon, it gave an overall return at 0.13% with SR of 0.06 and an annual ratio at 0.97, close to one. Considering the SR annual ratio, is it almost acceptable to be used by investors. One possible reason why the value is low is the late starting time to enter the market. The negative VaR suggested that the strategy will give profit rather than loss. Moreover, the annual Calmar ratio was lower than expected, with the value of 0.39 implied that the strategy return is still at risk of maximum drawdown.

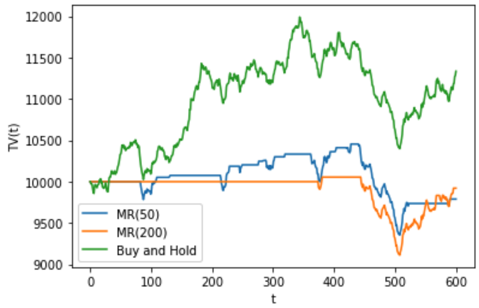
Surprisingly, a trend following (TF) strategy with simple moving average (SMA) gave the best performance among three methods for both returns and performance indicators, especially short-term MA of three, five, and seven days. All top three MAs yielded an average return at 0.38 gave annual SR greater than three, which is considered as a desirable return against market volatilities. All VaRs were negative with a value of -0.003, which showed that they have more probability of generating profit than the risk. The annual Calmar ratio also suggested strategies responsiveness maximum drawdown. MA(5) gave a higher annual SR and Calmar ratio, despite having a lower in-sample daily return than MA(3). Significantly, all MA also outperformed the buy and hold strategy, i.e., benchmark with an overall return at 0.13% (see figure 2a).

The mean reversion used the opposite logic from the TF to assume that the price will revert to the long-term average. The best time window of MA(50) obtained during the training process with positive daily return does not necessarily give the best strategy return on the testing time series. MA(200) generated better strategy return with out-sample time series with the return at -0.008% compared with -0.032% from MA(50). However, both MAs' performance indicators gave negative results, which implied that the mean reversion strategy would risk loss and strategy was sensitive to maximum drawdown. Moreover, the negative returns are not acceptable for investors as they will eventually lose capital if they do not exit the market. Furthermore, figure 2b shows that the mean reversion of MA(50) and MA(200) underperformed the simple buy and hold strategy.

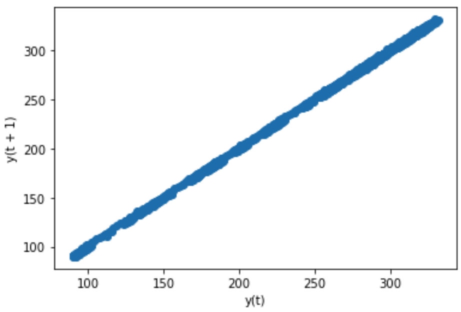
Figure 2c shows that the market price was strongly autocorrelated and can be applied with first-order of ARIMA model. With the development of the predicted price model, ARIMA improved the traditional mean reversion method with positive return and performance indicators, except VaR. The ARIMA(1,0,1) yieled the mean squared error (MSE) of 92,785.291 and gave hyperparameters of constant 0.126, AR term, and MA terms at 0.65, -0.45, respectively. This model was still not a suitable predicted model with a high MSE value, and there were other potential combinations of MA and AR terms to be considered. Despite the best time window



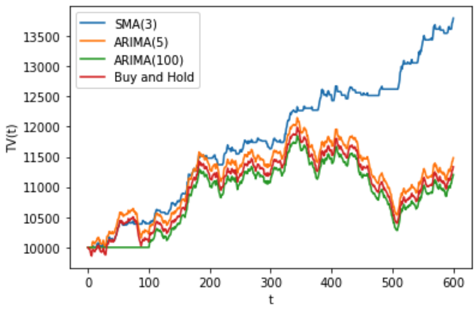
(a) The total value of top three SMA and buy and hold strategies at each time step.



(b) The total value of mean reversion with MA(50) and MA(200) and buy and hold strategies at each time step.



(c) Scatter plot of the asset price and the lag price at $t+1$



(d) The total value of ARIMA model with MA(5), MA(100), trend following of SMA(3), and buy and hold strategies at each time step.

Figure 2: Total value plot of strategies based on testing time series of 600 days of trading with additional scatter plot of the asset price.

of MA(100) obtained during the training, none of them surpassed the SMA of trend following strategy. The return was considered low at 0.12% with an annual Sharpe ratio of 0.96, which was lower than one. Similar to other strategies, the VaR was still negative, therefore this strategy was still safe from the risk of loss. The annual Calmar ratio of 0.36 also suggested that it was still sensitive to MDD, the same as the traditional mean reversion strategy. It also faced the identical issue such that the best MA(100) does not necessarily mean that it will achieve the best strategy return given the testing dataset. MA(5) surpassed MA(100) with strategy return at 0.15% and annual SR at 1.07. Overall, the ARIMA outperformed the buy and hold strategy but was still not as good as the simple moving average used in the following strategy. Therefore, the traditional and ARIMA-based mean reversion strategies were discarded. Only the top three SMA with MA(3), MA(5), MA(7) from trend following models were statistically tested with FWER.

Given the daily Sharpe ratio of MA(3), MA(5), MA(7) used in the trend following model, it can be verified against the null hypothesis of non-zero Sharpe ratio. Using the statistical test defined in the methodology section showed that all strategies had p-values lower than 0.05. Additionally, after applying Bonferroni's method with the multiplication by three, i.e., the number of tests, all adjusted p-values were still significant. Using the annual SR also yielded similar results, such that both p-value and adjusted p-value were still statistically statistical as with values lower than 0.05. Table 2 shows the p-value and adjusted p-value from statistical tests. These p-values implied that these trading strategies were significant with a low chance of false discoveries and increase investors' confidence to deploy these strategies and obtain profit.

Strategy	SR	p-value	Adjusted p-value	Annual SR	p-value	Adjusted p-value
TF MA(3)	0.22	2.54e-14	7.62e-14	3.44	6.16e-155	1.85e-154
TF MA(5)	0.22	9.89e-15	2.97e-14	3.56	1.38e-159	4.13e-159
TF MA(7)	0.21	5.00e-14	1.50e-13	3.35	1.37e-151	4.11e-151

Table 2: p-value and adjusted p-value from statistical tests of the best three trading strategies.

Conclusion and Discussion

Overall, the trend following with simple moving average signal of MA(3), MA(5), and MA(7) gave the highest return and significant performance indicators. The positive annual Sharpe ratio and Calmar ratio showed that the strategy would positively return despite the market volatility and maximum drawdown event. The statistical test of both SR and annual SR also verified that these strategies had a low probability of false discoveries with significant p-values and adjusted p-values. The value at risk was also the finding that the portfolio was more likely to return profit than loss. These top three strategies also outperformed the buy and hold portfolio that focuses on continuously remaining in the market without a short position. Meanwhile, the mean reversion yielded a negative result, opposite to the trend following approach. One possible explanation is that the market price did not or only had a few times to revert to its mean. Eventually, the mean reversion underperformed simple moving average and buy and hold strategies. Notably, the prices continuously increase over time horizon; this could imply the bull market, which explains the common issue of mean reversion against market conditions (Cunado, Gil-Alana, & de Gracia, 2010). Using the predicted price model by ARIMA also improved the performance of the mean reversion technique and slightly over the buy and hold strategy. Again, both returns and performance indicators stated that the output portfolio still underperformed the simple moving average.

This report has limitations that can be improved as part of future work. Firstly, there was no consideration of transaction costs involved in buying and selling rules over the trading period. These costs can cause investors concern, especially for the trend following strategy with a high frequency of trading as shown in figure 3 where the value of cash at each time step was varying changing. The overall return could decline due to high transaction costs. It can also decrease the other performance indicators, such as the Sharpe ratio and the Calmar ratio, which take an average return. Next is the limited choices of signal used in the trend following strategy. There are other decision weights such as exponential moving average (EMA) and crossing average of short-term and long-term moving averages. Future work could implement these signals and compare their performance against a simple moving average portfolio.

As previously mentioned, the mean reversion with the ARIMA model used only two terms at first order: AR and MA. Choosing the right value for these terms could significantly impact the price prediction result with the minimum mean square error between predicted and actual price. The hyperparameters given by the ARIMA model can also be fine tuning by applying the cross validation technique. Unlike other datasets, the time series cannot be randomly split into

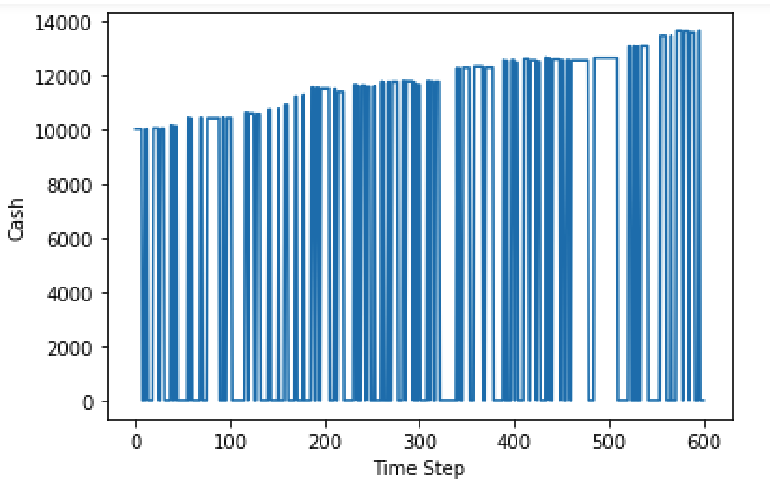


Figure 3: The capital value ($C(t)$) of the trend following strategy updated over trading time with testing sampling.

training and validating sets. Hyndman (2016) commented that the training set should consist only of prices that occurred before future observation. This process allows the forecast and cross validation based on rolling forward over time.

Recent studies have also generated signals from financial and social data availability rather than historical market prices to enhance buying and selling rules of trading strategies. For example, social media sentiment analysis, such as Twitter, represents a social signal that can cause the market impact; thus, the strategy return for cryptocurrency market (Garcia & Schweitzer, 2015). These strategies can be extended to use external information rather than historical market price to create robust portfolios.

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