Strategic game: Rainbow Ruse

Gameplay Description

On a very rowdy Saint Patrick's Day, a leprechaun got very drunk, created a game, and decided to go around the world and let lucky humans play and win gold from his pot of gold at the end of the rainbow. In the 3-player game, each player start with 6 cards, each representing a different colour of the rainbow: Red, Orange, Yellow, Green, Blue, and Purple (the leprechaun forgot Indigo due to their inebriation). During a round, each player plays 3 cards in front (called front cards) and 1 card in the back (called the back card), all face down. While doing so, they are allowed to talk and bluff about what they may be playing.

After all of them have played, they each flip over their back card. Any players who have the same colour back card as a different player loses the round, and cannot win any gold this round. Then, all the players reveal their forward 3 cards. For each front card a player plays, they win 1 gold for each other player that played that colour front card. Then, each player wins 2 gold for each front card played that matches their back card.

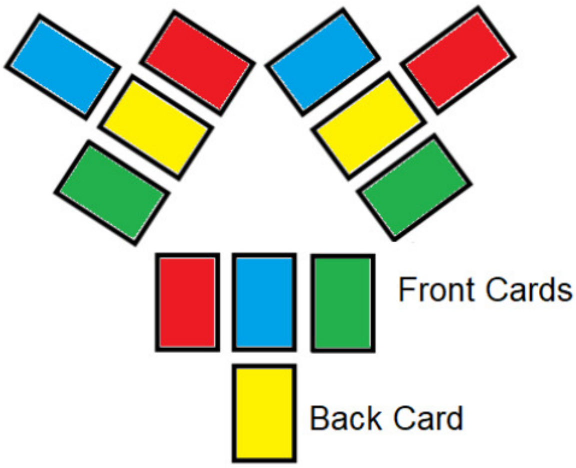
After points have been scored, the players will pick up all the cards they have played, and players who have the least gold that round or are tied for least gold for the round lose one of their cards at random, setting it aside face up so everyone can see. (Players who lost due to having the same back card as another player have zero gold for the round, and thus will always lose one of their cards). Then the round ends, and players continue playing. If a player does not have enough cards to play (i.e. at least 4 cards), they are eliminated from the game. The last remaining player wins the game and wins an extra 42 gold, and all players keep the gold that they won throughout the game.

Analysis

We notice that each round of the leprechaun's game is a strategic game. We assume all players are rational and selfish, wanting to maximise their utility (which is the amount of gold they get), and all players have knowledge of all game parameters. Players are also moving simultaneously, revealing their played cards all at the same time. A player plays a strategy of 1 back card and 3 front cards. The utility for the game is the amount of gold won. We can ignore the fact that players lose a card when they have the least points or are tied for the least points, and that the overall winner wins 42 extra gold, as these two rules are irrelevant for analysis of a single round.

The set of strategies for a player is all possible combinations of 1 back card and 3 front cards. This means there are $\binom{n}{1}\binom{n-1}{3} = n\frac{(n-1)(n-2)(n-3)}{6}$ possible strategies if a player has $4 \leq n \leq 6$ cards. Thus, if a player has 6 cards, there are 60 possible strategies, 5 cards means 20 possible strategies, and 4 cards means 4 possible strategies.

We notice that for some rounds, players may not have cards in common with other players due to lost cards, but the knowledge of what cards are lost is public. If a player were to play a card that a different player lost, they would not be able to win any gold from that player for that card, but they would also be able to play it in the back without fear of losing due to having the same colour back card as that player. If we consider the first round where every player has 6 cards, we can easily determine some pure Nash Equilibria.



In this example, every player wins 4+1+1+2=8 gold. However, they are also all tied for least amount of gold won, so they would all lose a card randomly after the round.

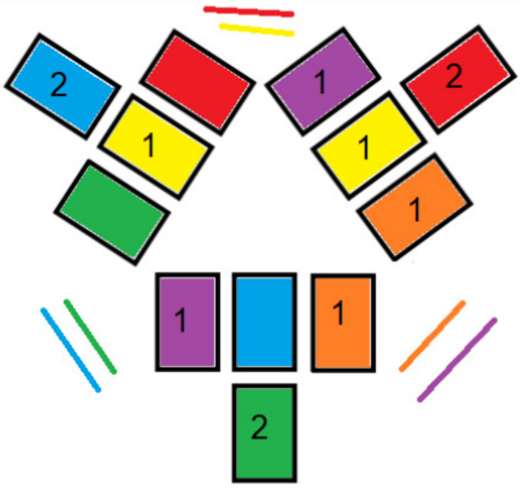
This is a pure NE, since if any player played a different card, they would have lower utility (i.e. gold won).

We notice that there are other pure NE that are similar to the one above, and we can find them by switching from one colour to another. However, we also notice that every player in this equilibrium would lose a card. This would mean that the overall game will be much shorter, which means less gold to win overall the rounds. Thus, for the overall game, a player may actually be incentivized to play a different card and lose gold intentionally, so that instead of 3 cards being lost overall, only 1 card is lost, and the game may go longer, possibly resulting in a greater social welfare for the overall game and more total gold for themselves at the end. However, as we are analyzing just individual rounds as a strategic game, we will not account for this possibility and assume players will maximise gold for a single round only.

As long as all players have at least 4 of the same cards, they can feasibly create a Nash Equilibrium of a similar form of the example above. We notice that any two players must have at least two cards in common by the pigeonhole principle. If we suppose that the three players don't share a card, and we know that any two players must have at least two cards in common, this means that the players must have exactly 4 cards to play. We claim that this means a Nash Equilibrium will exist as long as no two players play the same back card. Suppose not, i.e. two players don't play the same back card and it is not an NE. Then, some player should be able to switch a back card and a front card for greater utility. Suppose this is player i. Since all players share at least 2 cards, no 3 players share a card, and none of the cards in the back are the same for any two players, then player i's card in the back must match some card in the front of a different player. Player i cannot switch their back card for a card that has already been played in the back by another player, or else they will get zero utility. There must be at least one card that player i has played in the front that has not already been played in the back (by pigeonhole principle). However, this card in the front must match some other player's card in the front (again by pigeonhole principle). Thus, their utility does not change with this switch. The players only have 4 cards, and switching doesn't increase utility, which is a contradiction. Thus, our claim holds.

In this example, every player wins 2+1+1=4 gold. However, they are also all tied for least amount of gold won, so they would all lose a card randomly after the round.

This is a pure NE, as the players cannot achieve a greater utility. Lines show shared colour cards between players.



In this example, we see that the player who played blue in the back has the least utility, and thus will lose a card and be eliminated after this round.

This is a pure NE. Notice that switching any back card with a front card will result in equal utility or zero utility for that player, as explained above.

If players do share one or more cards, these cards should be played to maximise utility, and we can again find other possible NE's. In the end, all the players won a lot of gold and had a great Saint Patrick's Day with the little drunk leprechaun.