Computational Fluid Dynamics
(AE 706)

Computer Assignment 2:

Numerical Solution of Laplace Equation for Steady-State Temperature Distribution

By

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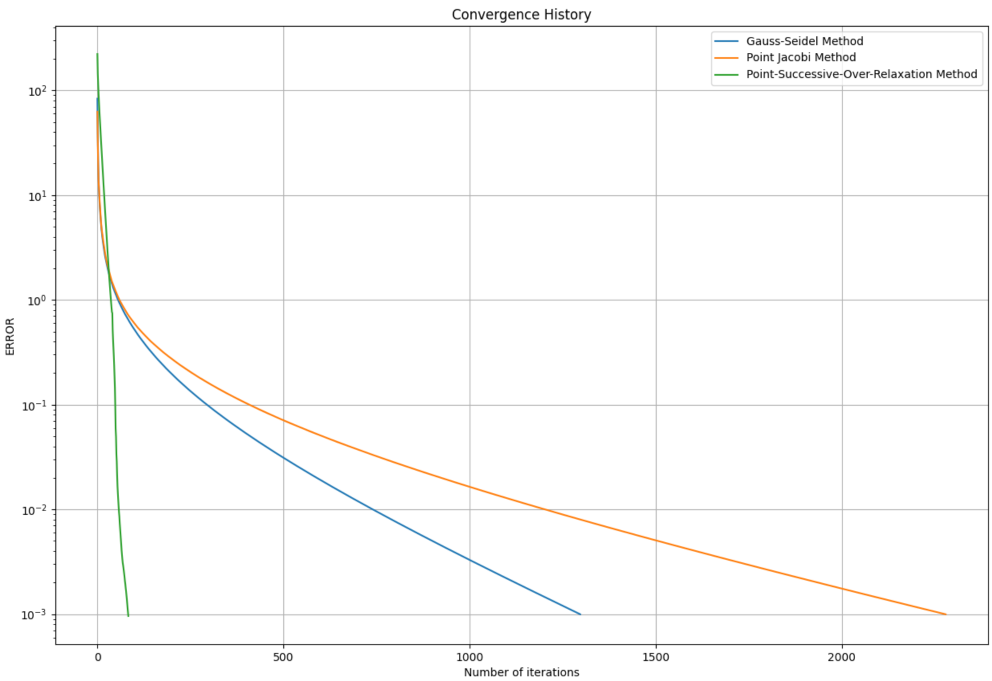
Abstract:

This computer assignment focuses on solving the Laplace equation, representing steady-state temperature distribution on a 2D rectangular plate. The discretization of the domain using a grid with 41x81 points enables the application of numerical methods - Point Jacobi, Point Gauss-Seidel, and Point Successive Over Relaxation. The convergence history is examined, and the number of iterations required for each method to meet the convergence criterion is recorded. Temperature variations along midlines (x=0.5 and y=1.0) are plotted and compared with exact solutions. Finally, a contour plot is generated for visualizing the temperature distribution. The assignment emphasizes code readability, adherence to given class notations, and comprehensive documentation. Submissions are expected to include a flowchart, well-documented code, a README file detailing code execution, and a concise report presenting results and observations.

Convergence:

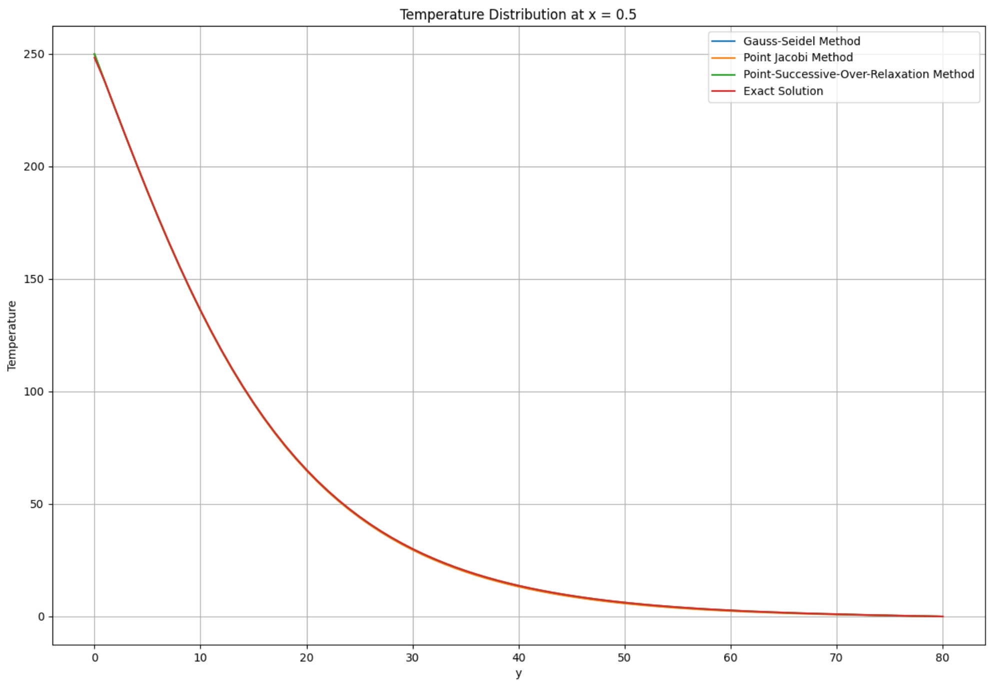


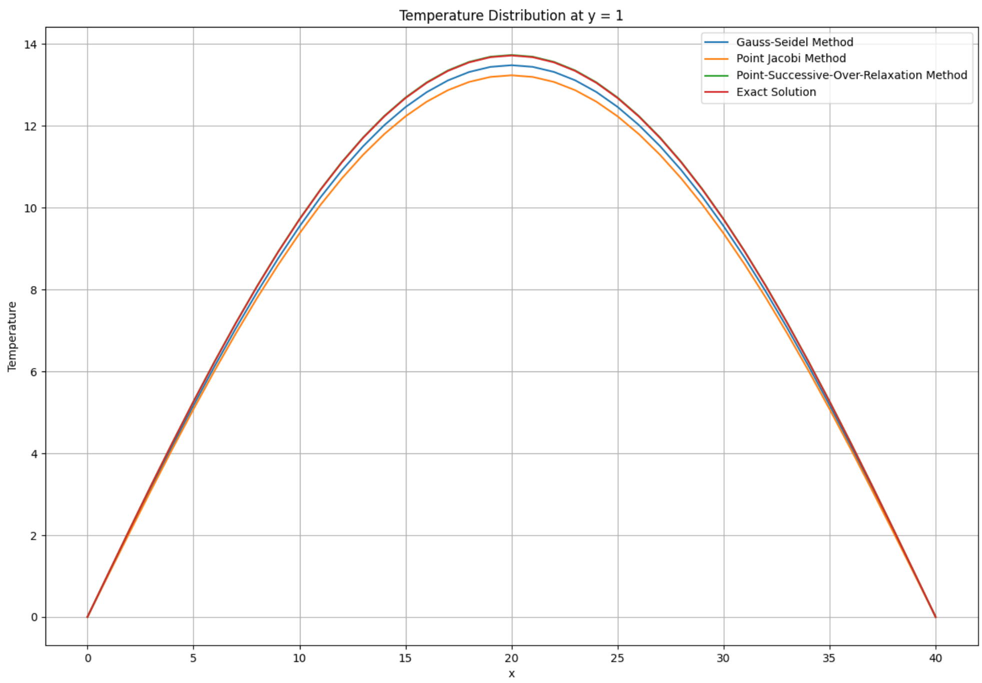
Number of iterations for convergence using Point Jacobi:	2280
Number of iterations for convergence using Point Gauss Seidel:	1298
Number of iterations for convergence using Point Successive Over Relaxation:	84

Observations:

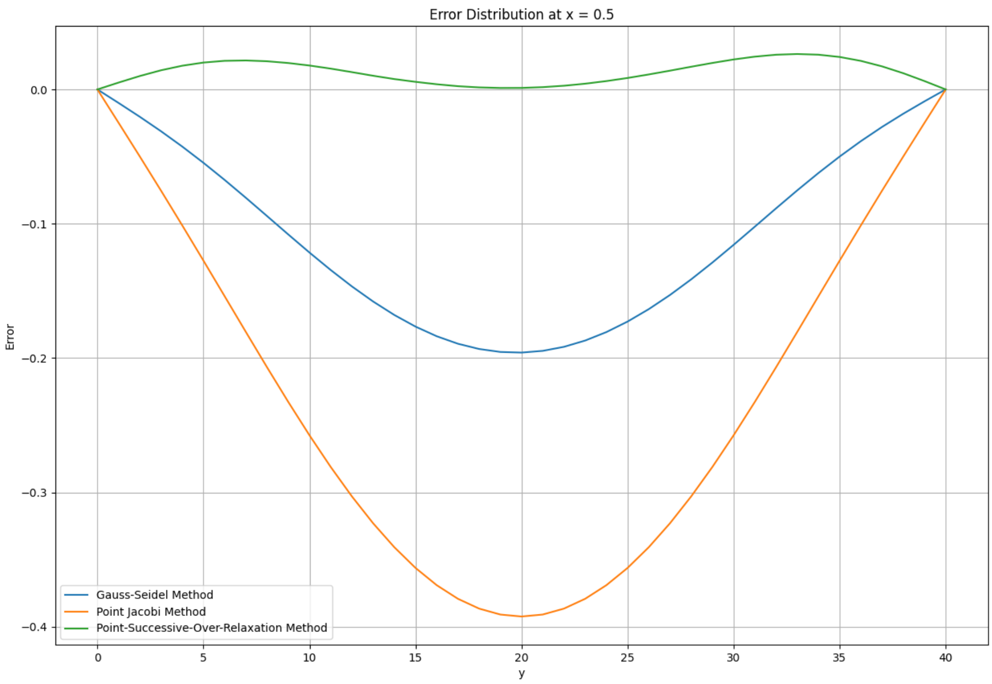
The Point Successive Over Relaxation method exhibits the most rapid convergence, substantiating theoretical predictions, while the Point Jacobi method requires the greatest number of iterations for convergence, aligning with theoretical expectations. This observation correlates with the calculated theoretical truncation error; specifically, the steeper slope in the convergence history plot for PSOR suggests a higher-order truncation error, whereas Point Jacobi demonstrates a gentler slope indicative of a lower-order truncation error. This alignment with theory is attributed to the fact that Point Jacobi solely utilizes data from the previous time step, while Point Gauss-Seidel employs a combination of information from both the previous and current time steps. Notably, Point Successive Over Relaxation employs a weighted addition of the new time step with the previous one, representing an additional refinement in the convergence strategy.

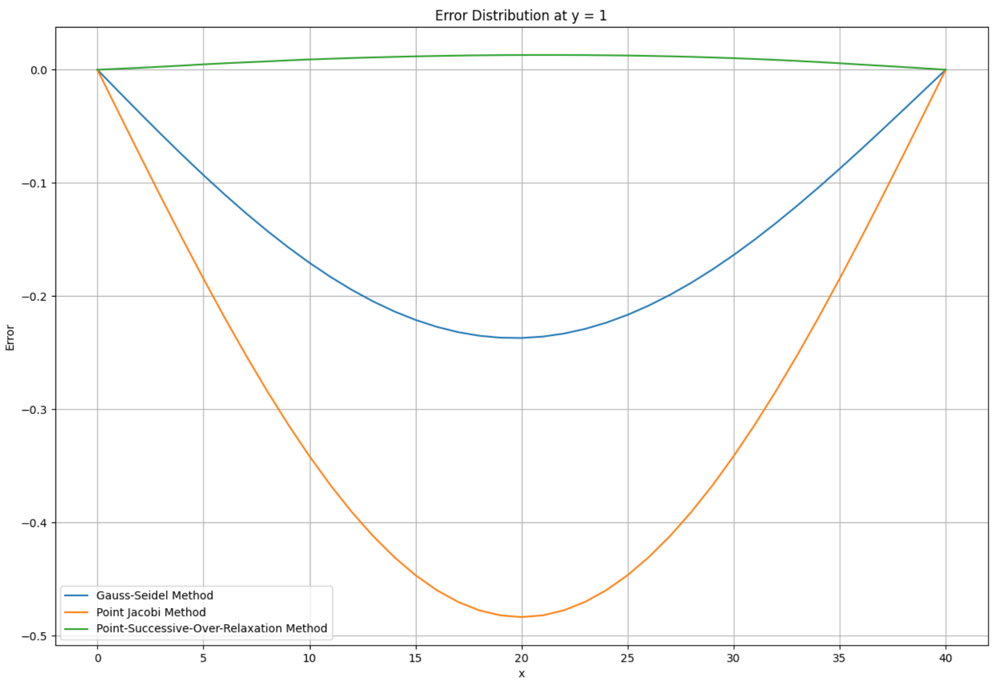
Temperature Distribution at Midlines:



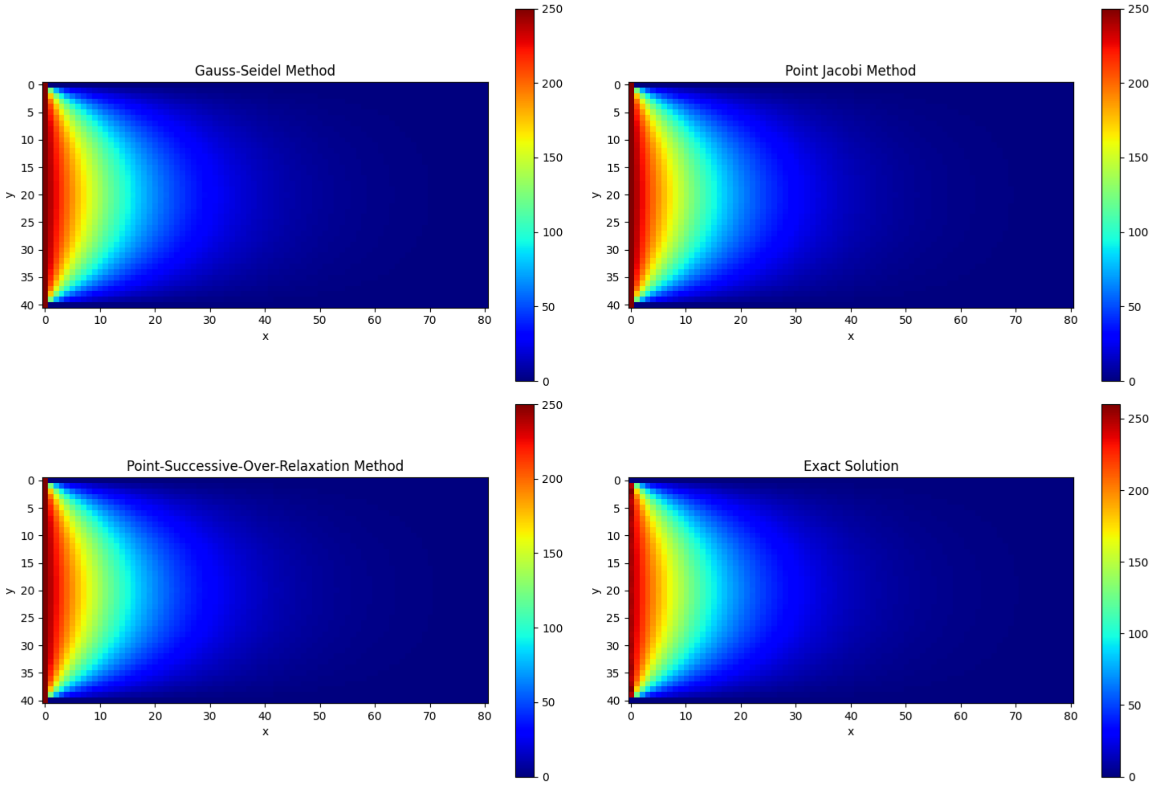


Temperature Difference (Error) from Exact solution at Midlines:





Contour Plots:



Conclusion:

The analysis of errors across various methods reveals a consistent pattern, with Point Successive Over Relaxation consistently exhibiting the least deviation from the exact solution, followed by Point Gauss-Seidel. Notably, Point Jacobi method demonstrates the highest deviation from the exact solution. This empirical observation aligns with our theoretical expectations, affirming that PSOR possesses the highest-order truncation error, while Point Jacobi method has the lowest-order truncation error.

Furthermore, the contour plots depicting temperature distributions generated by different methods closely coincide with each other and with the exact solutions. This agreement underscores the reliability and accuracy of the employed numerical methods in capturing the underlying physical behavior, reinforcing their effectiveness in simulating the heat equation dynamics.