

$$F = kq_1q_2$$

#### Potential energy of 2 charge system

$$U = kq_1q_2$$

#### Derivation (Imp)

let charge  $q_1$  and  $q_2$  be at a distance  $n$

$$\text{Electrostatic force on } q_2 = F_e = kq_1q_2 \quad (F_e \text{ is positive})$$

$$\text{External force to move } q_2 \text{ is } F_{ext} = (-kq_1q_2)/x^2$$

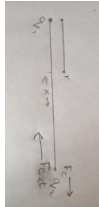
External work done to bring  $q_2$  from infinity to  $r$  is:

$$\begin{aligned} W &= \int F_{ext} dx = \int (-kq_1q_2 \cdot dx)/x^2 \\ &= -kq_1q_2 \int dx/x^2 \\ &= \int (1/x^2) dx = \int x^{-2} dx \\ &= \int x^{-2} dx = x^{-2}/(-2+1) \\ &= x^{-1}/-1 \\ &= -1/x \end{aligned}$$

$$\begin{aligned} W &= -kq_1q_2 [-1/x]_{\infty}^r \\ &= kq_1q_2 [-1/x]_{\infty}^r \\ &= kq_1q_2 [1/r - 1/\infty] \quad [1/\infty = 0] \\ &\Rightarrow W = kq_1q_2/r \end{aligned}$$

This work is stored as potential energy of system

$$\therefore U = kq_1q_2/r$$



#### Potential energy of a charge in external $E_{ext}$

$$U = qV$$

#### Potential energy of 2 charges in external $E_{ext}$

$$U = q_1V_1 + q_2V_2 + kq_1q_2/r$$

#### Potential due to dipole

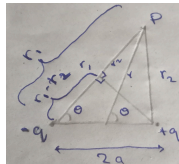
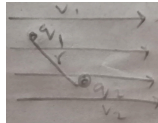
Potential @ P

$$V = V_{+q} + V_{-q}$$

$$\begin{aligned} &= K \cdot q/r_1 + k \cdot q/r_2 \\ &= Kq[1/r_1 + 1/r_2] \Rightarrow Kq[1/r_2 - 1/r_1] \\ &= Kq[r_1 - r_2 / r_1r_2] \quad [r_1r_2 \approx r^2] \\ &= Kq[r_1 - r_2 / r^2] \\ &= Kq[2a \cos \theta] / r^2 \quad [2a = p] \\ &\therefore V = kP \cos \theta / r^2 \end{aligned}$$

If 'p' is on equatorial line  
 $\cos \theta = b/h = r_1 - r_2 / 2a$   
 $r_1r_2 \approx 2a^2$   
 $\theta = 90^\circ$

$$V = 0 \quad [\cos \theta = 0]$$



#### Potential difference

$$W_{\infty \rightarrow p} = U$$

$$W = U_p - U_i$$

$$W = \Delta U$$

#### Potential energy of multiple charge system

$$U = k[(q_1q_2/r_{12}) + (q_1q_3/r_{13}) + (q_1q_4/r_{14}) + (q_2q_3/r_{23}) + (q_2q_4/r_{24}) + (q_3q_4/r_{34})]$$

imagine  $U = 500J$ , work done to bring charge from infinity to configuration,  $W_{\infty \rightarrow \text{configuration}}$

#### Potential

Potential energy per unit charge =  $V$

$$\therefore W = qV \quad (\text{from infinity to } p)$$

$$\therefore W = q\Delta V \quad (\text{for 2 points})$$

#### Electric potential due to a single charge

$$V = kq/r \quad \text{Graph of } E_{elec} \& \text{ Potential due to a point charge}$$

$$V = kq/r; \quad V \propto 1/r$$

$$E = kq/r^2; \quad E \propto 1/r^2$$

#### Unit and DF of potential

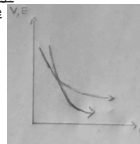
SI unit =  $J/C$  = volt

$$\text{Dimensional formula} = [ML^2T^{-3}A^{-1}]$$

#### Electric potential due to a system of charges

$$V = k[q_1/r_1 + q_2/r_2 + q_3/r_3]$$

Sum of potential of potential of all charges



#### Capacitor

Charge storing device

Capacitance ( $C$ ) is the ability to store charge

Defined as charge stored per unit potential

$$C = q/V$$

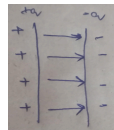
SI unit:  $C/V$  or Farad

#### Parallel Plate Capacitance

$$C = q/V = q/Ed = q \cdot \epsilon_0 q A / qd \quad [\sigma = q/A]$$

$$C = \epsilon_0 A/d$$

$A$  = plate area;  $d$  = distance between plates



#### Effect of inserting dielectric

$$C_m = k\epsilon_0 A / d$$

$$C_m = KC_{air}$$

#### Parallel combination

$$C = C_1 + C_2 + C_3$$

#### Series combination

$$1/C = 1/C_1 + 1/C_2 + 1/C_3$$

#### Potential energy of a capacitor

$$W = Q^2/2C; \quad V = Q^2/2C$$

$$U = Q^2/2C$$

$$U = \frac{1}{2} QV$$

$$U = \frac{1}{2} QV$$

#### Relation between E and V

$$E = -dv/dr$$

Electrical field is the negative of potential gradient

$$V = Ed$$

If  $E_{field}$  is 0, i.e. potential is constant

#### Equipotential surface

A surface where electric potential is same @ all points is called an equipotential surface

#### Properties

- Potential difference between 2 points = 0 [ $\Delta V = 0$ ]
- Work done to move any charge = 0
- Two  $E_s$  surfaces can never intersect. (If they could, there will be 2 values of potential @ that point, which is more possible) [ $W = q\Delta V = 0$ ]
- $E_{field}$  must be proportional to  $E_{surface}$  @ any point

#### Potential energy of a dipole in uniform $E_{ext}$

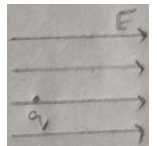
Work done to rotate dipole by angle  $d\theta$ :

$$dw = Fdx \Rightarrow Td\theta \quad [T = \text{torque}]$$

$$W = -PE[\cos \theta_2 - \cos \theta_1]$$

$$V_{\theta_2} - V_{\theta_1} = -PE[\cos \theta_2 - \cos \theta_1]$$

$$\therefore V_{\theta} = -P \cdot E$$



#### Electrical energy (energy per unit volume)

$$U = \frac{1}{2} \epsilon_0 E^2 Ad$$

$$M = \frac{1}{2} \epsilon_0 E^2$$

#### Electrostatics of conductor

- Fields inside a conductor material = 0
- $E$ -field is always proportional to conductor
- Excess charge always reside on surface of conductor
- Potential inside a conductor @ any point is same as surface
- $E$ -field inside a cavity of a conductor is always 0, electro static shielding
- $E$ -field @ surface of conductor =  $E = \sigma / \epsilon_0$