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To move a point by  $(a_x, a_y, a_z, a_w)$ ,

$$\begin{pmatrix} 1 & 0 & 0 & 0 & a_x \\ 0 & 1 & 0 & 0 & a_y \\ 0 & 0 & 1 & 0 & a_z \\ 0 & 0 & 0 & 1 & a_w \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (1)$$

To scale an object by  $(c_x, c_y, c_z, c_w)$ ,

$$\begin{pmatrix} c_x & 0 & 0 & 0 & 0 \\ 0 & c_y & 0 & 0 & 0 \\ 0 & 0 & c_z & 0 & 0 \\ 0 & 0 & 0 & c_w & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (2)$$

Assume camera position is  $(c_x, c_y, c_z, c_w)$  and the normalized viewing vector is  $(v_x, v_y, v_z, v_w)$  and up vectors are  $(u_{1x}, u_{1y}, u_{1z}, u_{1w})$ ,  $(u_{2x}, u_{2y}, u_{2z}, u_{2w})$ , and assuming each are orthogonal, then the view matrix is

$$\begin{pmatrix} l_x & l_y & l_z & l_w & 0 \\ u_{1x} & u_{1y} & u_{1z} & u_{1w} & 0 \\ u_{2x} & u_{2y} & u_{2z} & u_{2w} & 0 \\ v_x & v_y & v_z & v_w & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & -c_x \\ 0 & 1 & 0 & 0 & -c_y \\ 0 & 0 & 1 & 0 & -c_z \\ 0 & 0 & 0 & 1 & -c_w \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (3)$$

where  $(l_x, l_y, l_z, l_w) = \text{cross}(v, u_1, u_2)$ .

The orthogonal projection is

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (4)$$

In other words,  $(x, y, z) = \frac{f}{d} (v_x, v_y, v_z)$

Assume the front clipping plane is  $w = d$  and for a vertex coordinate after applying ModelView matrix  $(v_x, v_y, v_z, v_w)$ , let  $f = v_w$ , then The perspective projection is

$$\begin{pmatrix} \frac{f}{d} & 0 & 0 & 0 & 0 \\ 0 & \frac{f}{d} & 0 & 0 & 0 \\ 0 & 0 & \frac{f}{d} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (5)$$

In other words,  $(x, y, z) = \frac{f}{d} (v_x, v_y, v_z)$

To do stereographic projection, we first need to push each vertex to the 4d sphere,  $S^3$ , so for each vertex  $v$ , let  $v' = \frac{v}{\|v\|}$ . (Assume that  $\|v\| \neq 0$ .) then

$$(x, y, z) = \left( \frac{x'}{\frac{1}{2} - w'}, \frac{y'}{\frac{1}{2} - w'}, \frac{z'}{\frac{1}{2} - w'} \right). \quad (6)$$

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Note that for our hypercube, it has center as  $c = (1/2, 1/2, 1/2, 1/2)$ , so to do correct stereographic projection, we should consider  $v' = \frac{v-c}{\|v-c\|}$  and take stereographic projection, and translate it by  $c$  in 3D. (in fact, 4D homogeneous coordinate.)