

# Københavns Universitet

## LinAlgDat - Project B

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## 1 Opgave

### 1.a

Vi kan aflæse  $M_a$  til:

$$\begin{bmatrix} a & -1 & -1 \\ 0 & (a-1) & -1 \\ 0 & 2 & (a+2) \end{bmatrix}$$

### 1.b

$$\begin{aligned} & \left[ \begin{array}{ccc|c} a & -1 & -1 & 0 \\ 0 & a-1 & -1 & 0 \\ 0 & 2 & a+2 & 0 \end{array} \right] \cdot \frac{1}{a-1} \rightsquigarrow \left[ \begin{array}{ccc|c} a & -1 & -1 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 \\ 0 & 2 & a+2 & 0 \end{array} \right] \xrightarrow{-2r_2} \\ & \left[ \begin{array}{ccc|c} a & -1 & -1 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 \\ 0 & 0 & \frac{a^2+a}{a-1} & 0 \end{array} \right] \cdot \frac{a-1}{a^2+a} \rightsquigarrow \left[ \begin{array}{ccc|c} a & -1 & -1 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{+r_3} \\ & \left[ \begin{array}{ccc|c} a & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{+r_2} \left[ \begin{array}{ccc|c} a & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \cdot \frac{1}{a} \rightsquigarrow \\ & \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \end{aligned}$$

$T_a$  er altså injektiv.

$T_a$  er surjektiv, da vi har 3 vektorer.  $T_a$  er dermed bijektiv, da den både er injektiv og surjektiv.

Vi bestemmer nu  $T_a^{-1}$ :

$$\begin{aligned} & \left[ \begin{array}{ccc|ccc} a & -1 & -1 & 1 & 0 & 0 \\ 0 & a-1 & -1 & 0 & 1 & 0 \\ 0 & 2 & a+2 & 0 & 0 & 1 \end{array} \right] \cdot \frac{1}{a-1} \rightsquigarrow \left[ \begin{array}{ccc|ccc} a & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 & \frac{1}{a-1} & 0 \\ 0 & 2 & a+2 & 0 & 0 & 1 \end{array} \right] \xrightarrow{-2r_2} \\ & \left[ \begin{array}{ccc|ccc} a & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 & \frac{1}{a-1} & 0 \\ 0 & 0 & \frac{a^2+a}{a-1} & 0 & -\frac{2}{a-1} & 1 \end{array} \right] \cdot \frac{a-1}{a^2+a} \rightsquigarrow \left[ \begin{array}{ccc|ccc} a & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 & \frac{1}{a-1} & 0 \\ 0 & 0 & 1 & 0 & -\frac{2}{a^2+a} & \frac{a-1}{a^2+a} \end{array} \right] \xrightarrow{+r_3} \\ & \left[ \begin{array}{ccc|ccc} a & -1 & 0 & 1 & -\frac{2}{a^2+a} & \frac{a-1}{a^2+a} \\ 0 & 1 & 0 & 0 & \frac{a+2}{a^2+a} & \frac{1}{a^2+a} \\ 0 & 0 & 1 & 0 & -\frac{2}{a^2+a} & \frac{a-1}{a^2+a} \end{array} \right] \xrightarrow{+r_2} \left[ \begin{array}{ccc|ccc} a & 0 & 0 & 1 & \frac{a+1}{a^2+a} & \frac{a+1}{a^2+a} \\ 0 & 1 & 0 & 0 & \frac{a+2}{a^2+a} & \frac{1}{a^2+a} \\ 0 & 0 & 1 & 0 & -\frac{2}{a^2+a} & \frac{a-1}{a^2+a} \end{array} \right] \cdot \frac{1}{a} \rightsquigarrow \\ & \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{a} & \frac{1}{a^2+a} & \frac{1}{a^2+a} \\ 0 & 1 & 0 & 0 & \frac{a+2}{a^2+a} & \frac{1}{a^2+a} \\ 0 & 0 & 1 & 0 & -\frac{2}{a^2+a} & \frac{a-1}{a^2+a} \end{array} \right] \end{aligned}$$

### 1.c

Vi opstill igen  $T_a$ , hvor  $a = -1$ :

$$\begin{aligned}
& \left[ \begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 2 & -1+2 & 0 \end{array} \right] \text{ reducer } \rightsquigarrow \left[ \begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & -2 & -1 & 0 \\ 0 & 2 & 1 & 0 \end{array} \right] \rightsquigarrow -r_2 \\
& \left[ \begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & -2 & -1 & 0 \\ 0 & 0 & 2 & 0 \end{array} \right] \rightsquigarrow \left[ \begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & -2 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \begin{array}{l} +r_3 \\ +r_3 \end{array} \rightsquigarrow \\
& \left[ \begin{array}{ccc|c} -1 & -1 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \begin{array}{l} \cdot(-1) \\ \cdot(-\frac{1}{2}) \end{array} \rightsquigarrow \left[ \begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \begin{array}{l} -r_2 \\ \\ \end{array} \rightsquigarrow \\
& \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]
\end{aligned}$$

1.d

1.e

## 2 Opgave

2.a

Vi opstiller et ligningssystem i form af en totalmatrix, hvor vi sætter  $u_1, u_2, u_3$  lig hhv.  $v_1, v_2, v_3$ , og finder løsningerne til disse, ved brug af Gauss-Jordan elimination.

$$u_1 + u_2 + u_3 = v_1 \Leftrightarrow$$

$$\begin{aligned}
& \left[ \begin{array}{ccc|c} 2 & 0 & 1 & 7 \\ 1 & 1 & -1 & -2 \\ -1 & -1 & 2 & 3 \\ 1 & 1 & 2 & 1 \end{array} \right] \begin{array}{l} \cdot 2 \\ \cdot(-2) \\ \cdot 2 \end{array} \rightsquigarrow \left[ \begin{array}{ccc|c} 2 & 0 & 1 & 7 \\ 2 & 2 & -2 & -4 \\ 2 & 2 & -4 & -6 \\ 2 & 2 & 4 & 2 \end{array} \right] \begin{array}{l} -r_1 \\ -r_1 \\ -r_1 \end{array} \rightsquigarrow \\
& \left[ \begin{array}{ccc|c} 2 & 0 & 1 & 7 \\ 0 & 2 & -3 & -11 \\ 0 & 2 & -5 & -13 \\ 0 & 2 & 3 & -5 \end{array} \right] \begin{array}{l} \\ -r_2 \\ -r_2 \end{array} \rightsquigarrow \left[ \begin{array}{ccc|c} 2 & 0 & 1 & 7 \\ 0 & 2 & -3 & -11 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 6 & 6 \end{array} \right] \begin{array}{l} \\ \\ +3r_3 \end{array} \rightsquigarrow \\
& \left[ \begin{array}{ccc|c} 2 & 0 & 1 & 7 \\ 0 & 2 & -3 & -11 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} \\ \cdot(-\frac{1}{2}) \\ \cdot(-\frac{1}{2}) \end{array} \rightsquigarrow \left[ \begin{array}{ccc|c} 2 & 0 & 1 & 7 \\ 0 & 2 & -3 & -11 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} -1r_3 \\ +3r_3 \\ \\ \end{array} \rightsquigarrow \\
& \left[ \begin{array}{ccc|c} 2 & 0 & 0 & 6 \\ 0 & 2 & 0 & -8 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} \cdot\frac{1}{2} \\ \cdot\frac{1}{2} \\ \\ \end{array} \rightsquigarrow \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]
\end{aligned}$$

Vores første kolonne i  $P_{B \leftarrow C}$  er dermed:  $\begin{bmatrix} 3 \\ -4 \\ 1 \\ 0 \end{bmatrix}$

$$u_1 + u_2 + u_3 = v_2 \Leftrightarrow$$

$$\begin{aligned}
& \left[ \begin{array}{ccc|c} 2 & 0 & 1 & -1 \\ 1 & 1 & -1 & 0 \\ -1 & -1 & 2 & -1 \\ 1 & 1 & 2 & -3 \end{array} \right] \begin{array}{l} \cdot 2 \\ \cdot (-2) \\ \cdot 2 \end{array} \rightsquigarrow \left[ \begin{array}{ccc|c} 2 & 0 & 1 & -1 \\ 2 & 2 & -2 & 0 \\ 2 & 2 & -4 & 2 \\ 2 & 2 & 4 & -6 \end{array} \right] \begin{array}{l} -r_1 \\ -r_1 \\ -r_1 \end{array} \rightsquigarrow \\
& \left[ \begin{array}{ccc|c} 2 & 0 & 1 & -1 \\ 0 & 2 & -3 & 1 \\ 0 & 2 & -5 & 3 \\ 0 & 2 & 3 & -5 \end{array} \right] \begin{array}{l} -r_2 \\ -r_2 \end{array} \rightsquigarrow \left[ \begin{array}{ccc|c} 2 & 0 & 1 & -1 \\ 0 & 2 & -3 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 6 & -6 \end{array} \right] \begin{array}{l} +3r_3 \\ -1r_3 \\ +3r_3 \end{array} \rightsquigarrow \\
& \left[ \begin{array}{ccc|c} 2 & 0 & 1 & -1 \\ 0 & 2 & -3 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right] \cdot (-\tfrac{1}{2}) \rightsquigarrow \left[ \begin{array}{ccc|c} 2 & 0 & 1 & -1 \\ 0 & 2 & -3 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} -1r_3 \\ +3r_3 \end{array} \rightsquigarrow \\
& \left[ \begin{array}{ccc|c} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} \cdot \frac{1}{2} \\ \cdot \frac{1}{2} \end{array} \rightsquigarrow \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right] \\
& \text{Vores anden kolonne i } P_{B \leftarrow C} \text{ er dermed: } \begin{bmatrix} 0 \\ -1 \\ -1 \\ 0 \end{bmatrix}
\end{aligned}$$

$$u_1 + u_2 + u_3 = v_3 \Leftrightarrow$$

$$\begin{aligned}
& \left[ \begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 2 & 2 \\ 1 & 1 & 2 & 2 \end{array} \right] \begin{array}{l} \cdot 2 \\ \cdot (-2) \\ \cdot 2 \end{array} \rightsquigarrow \left[ \begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 2 & 2 & -2 & -2 \\ 2 & 2 & -4 & -4 \\ 2 & 2 & 4 & 4 \end{array} \right] \begin{array}{l} -r_1 \\ -r_1 \\ -r_1 \end{array} \rightsquigarrow \\
& \left[ \begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 0 & 2 & -3 & -5 \\ 0 & 2 & -5 & -7 \\ 0 & 2 & 3 & 1 \end{array} \right] \begin{array}{l} -r_2 \\ -r_2 \end{array} \rightsquigarrow \left[ \begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 0 & 2 & -3 & -5 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 6 & 6 \end{array} \right] \begin{array}{l} +3r_3 \\ -1r_3 \\ +3r_3 \end{array} \rightsquigarrow \\
& \left[ \begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 0 & 2 & -3 & -5 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right] \cdot (-\tfrac{1}{2}) \rightsquigarrow \left[ \begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 0 & 2 & -3 & -5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} -1r_3 \\ +3r_3 \end{array} \rightsquigarrow \\
& \left[ \begin{array}{ccc|c} 2 & 0 & 0 & 2 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} \cdot \frac{1}{2} \\ \cdot \frac{1}{2} \end{array} \rightsquigarrow \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right] \\
& \text{Vores sidste kolonne i } P_{B \leftarrow C} \text{ er dermed: } \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}
\end{aligned}$$

Sammensætter vi nu vores tre kolonner til en matrix, får vi:

$$P_{B \leftarrow C} = \begin{bmatrix} 3 & 0 & 1 \\ -4 & -1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

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**2.b**

$$x = \begin{bmatrix} 7 \\ -2 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ -1 \\ -1 \end{bmatrix} + \begin{bmatrix} 3 \\ -1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 9 \\ -3 \\ 4 \\ 0 \end{bmatrix}$$

Da konstanterne foran  $v$  i hvert led er 1, og  $v_1, v_2, v_3 \in \mathcal{C}$ , er koordinaterne for  $x$  med henhold til  $\mathcal{C}$ :

$$[x]_{\mathcal{C}} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Vi benytter vores basisskriftmatrice til at transformere vores koordinater til basen  $\mathcal{B}$  fra  $\mathcal{C}$ :

$$[x]_{\mathcal{B}} = \begin{bmatrix} 3 & 0 & 1 \\ -4 & -1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ 1 \end{bmatrix}$$

**2.c**

Vi ganger kolonne 2 i vores basisskriftmatrice på  $u_1$  og  $u_2$ :

$$-1 \cdot u_1 + (-1) \cdot u_2 = -1 \cdot \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} + (-1) \cdot \begin{bmatrix} 1 \\ -1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -1 \\ -3 \end{bmatrix}$$

Vi får  $v_2$ , så  $v_2$  må dermed række spannet af  $u_2, u_3$ .

Mangler at lave resten af opgaven

**2.d****2.e**