Københavns Universitet
LinAlgDat - Project A

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29. april 2025

Indhold

1 Opgave

1.a

Vi omskriver ligningssystemet til totalmatrix-form:

$$\left[\begin{array}{ccc|c} 1 & 2 & 8 & a \\ a & a & 4a & a \\ 2 & 2 & 2a^2 & 0 \end{array}\right]$$

Vi benytter Gauss-Jordan elimination til at omskrive totalmatrix'en til en reduceret rækkeechelonform.

$$\left[\begin{array}{ccc|c} 1 & 2 & 8 & a \\ a & a & 4a & a \\ 2 & 2 & 2a^2 & 0 \end{array}\right] \begin{array}{l} \cdot 2a \\ \cdot 2 \\ \cdot a \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 2a & 4a & 16a & 2a^2 \\ 2a & 2a & 8a & 2a \\ 2a & 2a & 2a^3 & 0 \end{array}\right] \begin{array}{l} \\ -r_1 \\ -r_1 \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} 2a & 4a & 16a & 2a^2 \\ 0 & -2a & -8a & 2a-2a^2 \\ 0 & -2a & 2a^3-16a & -2a^2 \end{array}\right] \begin{array}{l} \\ \\ -r_2 \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 2a & 4a & 16a & 2a^2 \\ 0 & -2a & -8a & 2a-2a^2 \\ 0 & 0 & 2a^3-8a & -2a \end{array}\right] \begin{array}{l} +2r_2 \\ \\ \\ \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} 2a & 0 & 0 & 4a-2a^2 \\ 0 & -2a & -8a & 2a-2a^2 \\ 0 & 0 & 2a^3-8a & -2a \end{array}\right] \begin{array}{l} \\ \cdot\frac{1}{-8a} \\ \cdot\frac{1}{2a^3-8a} \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 2a & 0 & 0 & 4a-2a^2 \\ 0 & \frac{1}{4} & 1 & \frac{2a-2a^2}{-8a} \\ 0 & 0 & 1 & -\frac{2a}{2a^3-8a} \end{array}\right] -r_3 \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} 2a & 0 & 0 & 4a-2a^2 \\ 0 & \frac{1}{4} & 0 & \frac{a^3-a^2-4a+8}{4a^2-16} \\ 0 & 0 & 1 & -\frac{2a}{2a^3-8a} \end{array}\right] \begin{array}{l} \cdot\frac{1}{2a} \\ \cdot 4 \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{4a-2a^2}{2a} \\ 0 & 1 & 0 & \frac{4a^3-4a^2-16a+32}{4a^2-16} \\ 0 & 0 & 1 & -\frac{2a}{2a^3-8a} \end{array}\right] reducer \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2-a \\ 0 & 1 & 0 & \frac{a^3-a^2-4a+8}{a^2-4} \\ 0 & 0 & 1 & \frac{1}{4-a^2} \end{array}\right]$$

Har nu reduceret til echelonform, så vi er dermed færdige.

1.b

Vi opskriver igen vores ligningssystem som en totalmatrix, og erstatter denne gang a med 0:

$$\left[\begin{array}{ccc|c} 1 & 2 & 8 & 0 \\ 0 & 0 & 4\cdot 0 & 0 \\ 2 & 2 & 2\cdot 0^2 & 0 \end{array}\right] reducer \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 2 & 8 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \end{array}\right] > byttes \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 2 & 8 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] -2r_1 \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 8 & 0 \\ 0 & -2 & -16 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \cdot\left(-\tfrac{1}{2}\right) \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 2 & 8 & 0 \\ 0 & 1 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} -2r_1 \\ \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & -8 & 0 \\ 0 & 1 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Vi kan nu aflæse løsningerne til:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} 8 \\ -8 \\ 1 \end{bmatrix}$$

Vi ser nu, hvad vi får, når vi bruger den rækkereducerede totalmatrix fra tidligere, når vi erstatter a med 0:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2-a \\ 0 & 1 & 0 & \frac{(a^3-a^2-4a+8)}{(a^2-4)} \\ 0 & 0 & 1 & \frac{1}{(4-a^2)} \end{array}\right] a = 0 \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2-0 \\ 0 & 1 & 0 & \frac{(0^3-0^2-4\cdot 0+8)}{(0^2-4)} \\ 0 & 0 & 1 & \frac{1}{(4-0^2)} \end{array}\right] reducer \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & \frac{1}{4} \end{array}\right]$$

Denne matrix antyder en unik løsning, hvilket ikke afspejler hvad vi fandt lige før, hvor vi fandt uendelig mange løsninger. Dette skete sandstnligvis fordi at den rækkereducerede totalmatrix er lavet ud fra antagelsen, at $a \neq 0$.

1.c

Vi opskriver igen vores ligningssystem som en totalmatrix, og erstatter denne gang a med 2:

$$\left[\begin{array}{ccc|c} 1 & 2 & 8 & 2 \\ 0 & 0 & 4\cdot 2 & 2 \\ 2 & 2 & 2\cdot 2^2 & 0 \end{array}\right] reducer \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 2 & 8 & 2 \\ 0 & 0 & 8 & 2 \\ 2 & 2 & 8 & 0 \end{array}\right] \begin{array}{c} \\ \\ +r_2 \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 2 & 8 & 2 \\ 2 & 2 & 8 & 0 \\ 0 & 0 & 8 & 2 \end{array}\right] \begin{array}{c} \\ -2r_1 \\ \\ \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 8 & 2 \\ 0 & -2 & -8 & -4 \\ 0 & 0 & 8 & 2 \end{array}\right] \begin{array}{c} -r_3 \\ \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & -2 & -8 & -4 \\ 0 & 0 & 8 & 2 \end{array}\right] \begin{array}{c} \\ +r_3 \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & -2 & 0 & -2 \\ 0 & 0 & 8 & 2 \end{array}\right] \begin{array}{c} +r_2 \\ \\ \\ \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & -2 & 0 & -2 \\ 0 & 0 & 8 & 2 \end{array}\right] \begin{array}{c} \\ \cdot(-\frac{1}{2}) \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 8 & 2 \end{array}\right] \begin{array}{c} \\ \cdot\frac{1}{8} \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \frac{1}{4} \end{array}\right]$$

Vi kan nu aflæse løsningen til:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ \frac{1}{4} \end{bmatrix}$$

Lad os nu se, hvad vi får, når vi bruger den rækkereducerede totalmatrix fra tidligere, når vi erstatter a med 2:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2-2 \\ 0 & 1 & 0 & \frac{(2^3-2^2-4\cdot 2+8)}{(2^2-4)} \\ 0 & 0 & 1 & \frac{1}{(4-2^2)} \end{array}\right] reducer \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{4}{0} \\ 0 & 0 & 1 & \frac{1}{0} \end{array}\right]$$

Vi ser nu, at vi får division med 0, hvilket må betyde, at der ikke er nogen løsninger, når $a = 2$.

1.d

Vi omskriver ligningssystemet til koefficientmatrice-form på venstre side, hvor $a = 1$, og sættr identitetmatricen på højre side:

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 8 & 1 & 0 & 0 \\ 1 & 1 & 4\cdot 1 & 0 & 1 & 0 \\ 2 & 2 & 2\cdot 1^2 & 0 & 0 & 1 \end{array}\right] reducer \rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 8 & 1 & 0 & 0 \\ 1 & 1 & 4 & 0 & 1 & 0 \\ 2 & 2 & 2 & 0 & 0 & 1 \end{array}\right] \begin{array}{c} \\ -1r_1 \\ \\ \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 8 & 1 & 0 & 0 \\ 0 & -1 & -4 & -1 & 1 & 0 \\ 2 & 2 & 2 & 0 & 0 & 1 \end{array}\right] \begin{array}{c} \\ \\ -2r_1 \end{array} \rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 8 & 1 & 0 & 0 \\ 0 & -1 & -4 & -1 & 1 & 0 \\ 0 & -2 & -14 & -2 & 0 & 1 \end{array}\right] \begin{array}{c} \\ \\ -2r_2 \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 8 & 1 & 0 & 0 \\ 0 & -1 & -4 & -1 & 1 & 0 \\ 0 & 0 & -6 & 0 & -2 & 1 \end{array}\right] \begin{array}{c} \cdot 3 \\ \cdot(-3) \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|ccc} 3 & 6 & 24 & 3 & 0 & 0 \\ 0 & 3 & 12 & 3 & -3 & 0 \\ 0 & 0 & -6 & 0 & -2 & 1 \end{array}\right] \begin{array}{c} +4r_3 \\ \\ \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|ccc} 3 & 6 & 0 & 3 & -8 & 4 \\ 0 & 3 & 12 & 3 & -3 & 0 \\ 0 & 0 & -6 & 0 & -2 & 1 \end{array}\right] \begin{array}{c} \\ +2r_3 \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|ccc} 3 & 6 & 0 & 3 & -8 & 4 \\ 0 & 3 & 0 & 3 & -7 & 2 \\ 0 & 0 & -6 & 0 & -2 & 1 \end{array}\right] \begin{array}{c} -2r_2 \\ \\ \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|ccc} 3 & 0 & 0 & -3 & 6 & 0 \\ 0 & 3 & 0 & 3 & -7 & 2 \\ 0 & 0 & -6 & 0 & -2 & 1 \end{array}\right] \begin{array}{c} \cdot\frac{1}{3} \\ \cdot\frac{1}{3} \\ \cdot(-\frac{1}{6}) \end{array} \rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & 1 & -\frac{7}{3} & \frac{2}{3} \\ 0 & 0 & 1 & 0 & \frac{1}{3} & -\frac{1}{6} \end{array}\right]$$

2 Opgave

2.a

2.b

2.c

3 Opgave

3.a

Vi sætter et 1-tal i hver e_{uv}-element i matricen for hver edge $E(u, v)$ i grafen G_N:

$$N = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

For at finde antallet af veje fra knude 4 til knude 1 med netop længde 8, skal vi kigge på element $(4, 1)$ i N^8. Vi kan finde $(N^8)_{4,1}$ ved:

$$69 \cdot 0 + 45 \cdot 1 + 45 \cdot 1 + 18 \cdot 1 + 29 \cdot 1 = 137$$

unikke veje fra knude 4 til knude 1 med længde 8.

3.b

Linkmatricen A findes ved at tage N^T og dividere hvert element med antallet af udgående edges i dens column.

$$N^T = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & \frac{1}{2} & 1 & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 \end{bmatrix}$$

3.c

Vi kan lave et ligningssystem ud fra N, hvor alle elementer nu divideres med mængden af udgående edges den vertex i kolonnen er connected med:

$x_1 = \frac{1}{2}x_2 + x_3$
$x_2 = \frac{1}{2}x_1 + \frac{1}{2}x_5$
$x_3 = \frac{1}{2}x_1$
$x_4 = \frac{1}{2}x_1 + \frac{1}{2}x_2 + x_3 + \frac{1}{2}x_5$
$x_5 = \frac{1}{2}x_1 + \frac{1}{4}x_4$

Vi opskriver vores ligningssystem som en totalmatrix:

$$\left[\begin{array}{ccccc|c} -1 & \frac{1}{2} & 1 & 0 & 0 & 0 \\ \frac{1}{2} & -1 & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -1 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 & -1 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{4} & -1 & 0 \end{array}\right] \begin{array}{l} r_1 \cdot 2 \\ r_2 \cdot 4 \\ r_3 \cdot 4 \\ r_4 \cdot 4 \\ r_5 \cdot 4 \end{array} \rightsquigarrow \left[\begin{array}{ccccc|c} -2 & 1 & 2 & 0 & 0 & 0 \\ 2 & -4 & 0 & 0 & 2 & 0 \\ 2 & 0 & -4 & 0 & 0 & 0 \\ 2 & 2 & 4 & -4 & 2 & 0 \\ 2 & 0 & 0 & 1 & -4 & 0 \end{array}\right] \begin{array}{l} \\ +r_1 \\ +r_1 \rightsquigarrow \\ +r_1 \\ +r_1 \end{array}$$

$$\left[\begin{array}{ccccc|c} -2 & 1 & 2 & 0 & 0 & 0 \\ 0 & -3 & 2 & 0 & 2 & 0 \\ 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 3 & 6 & -4 & 2 & 0 \\ 0 & 1 & 2 & 1 & -4 & 0 \end{array}\right] \begin{array}{l} \\ \\ r_3 \cdot 3 \\ \\ r_5 \cdot 3 \end{array} \rightsquigarrow \left[\begin{array}{ccccc|c} -2 & 1 & 2 & 0 & 0 & 0 \\ 0 & -3 & 2 & 0 & 2 & 0 \\ 0 & 3 & -6 & 0 & 0 & 0 \\ 0 & 3 & 6 & -4 & 2 & 0 \\ 0 & 3 & 6 & 3 & -12 & 0 \end{array}\right] \begin{array}{l} \\ \\ +r_2 \rightsquigarrow \\ +r_2 \\ +r_2 \end{array}$$

$$\left[\begin{array}{ccccc|c} -2 & 1 & 2 & 0 & 0 & 0 \\ 0 & -3 & 2 & 0 & 2 & 0 \\ 0 & 0 & -4 & 0 & 2 & 0 \\ 0 & 0 & 8 & -4 & 4 & 0 \\ 0 & 0 & 8 & 3 & -10 & 0 \end{array}\right] \begin{array}{l} \\ \\ \cdot 2 \\ \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccccc|c} -2 & 1 & 2 & 0 & 0 & 0 \\ 0 & -3 & 2 & 0 & 2 & 0 \\ 0 & 0 & -8 & 0 & 4 & 0 \\ 0 & 0 & 8 & -4 & 4 & 0 \\ 0 & 0 & 8 & 3 & -10 & 0 \end{array}\right] \begin{array}{l} \\ \\ \rightsquigarrow \\ +r_3 \\ +r_3 \end{array}$$

$$\left[\begin{array}{ccccc|c} -2 & 1 & 2 & 0 & 0 & 0 \\ 0 & -3 & 2 & 0 & 2 & 0 \\ 0 & 0 & -8 & 0 & 4 & 0 \\ 0 & 0 & 0 & -4 & 8 & 0 \\ 0 & 0 & 0 & 3 & -6 & 0 \end{array}\right] \begin{array}{l} \\ \\ \\ \cdot 3 \\ \cdot 4 \end{array} \rightsquigarrow \left[\begin{array}{ccccc|c} -2 & 1 & 2 & 0 & 0 & 0 \\ 0 & -3 & 2 & 0 & 2 & 0 \\ 0 & 0 & -8 & 0 & 4 & 0 \\ 0 & 0 & 0 & -12 & 24 & 0 \\ 0 & 0 & 0 & 12 & -24 & 0 \end{array}\right] \begin{array}{l} \\ \\ \rightsquigarrow \\ \\ +r_4 \end{array}$$

$$\left[\begin{array}{ccccc|c} -2 & 1 & 2 & 0 & 0 & 0 \\ 0 & -3 & 2 & 0 & 2 & 0 \\ 0 & 0 & -8 & 0 & 4 & 0 \\ 0 & 0 & 0 & -12 & 24 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \cdot 4 \\ \cdot 4 \\ \\ \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccccc|c} -8 & 4 & 8 & 0 & 0 & 0 \\ 0 & -12 & 8 & 0 & 8 & 0 \\ 0 & 0 & -8 & 0 & 4 & 0 \\ 0 & 0 & 0 & -12 & 24 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} +r_3 \\ +r_3 \\ \rightsquigarrow \\ \\ \\ \end{array}$$

$$\left[\begin{array}{ccccc|c} -8 & 4 & 0 & 0 & 4 & 0 \\ 0 & -12 & 0 & 0 & 12 & 0 \\ 0 & 0 & -8 & 0 & 4 & 0 \\ 0 & 0 & 0 & -12 & 24 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{c} \cdot 3 \\ \\ \rightsquigarrow \\ \\ \\ \end{array} \left[\begin{array}{ccccc|c} -24 & 12 & 0 & 0 & 12 & 0 \\ 0 & -12 & 0 & 0 & 12 & 0 \\ 0 & 0 & -8 & 0 & 4 & 0 \\ 0 & 0 & 0 & -12 & 24 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{c} +r_2 \\ \\ \rightsquigarrow \\ \\ \\ \end{array}$$

$$\left[\begin{array}{ccccc|c} -24 & 0 & 0 & 0 & 24 & 0 \\ 0 & -12 & 0 & 0 & 12 & 0 \\ 0 & 0 & -8 & 0 & 4 & 0 \\ 0 & 0 & 0 & -12 & 24 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{c} \cdot(-\frac{1}{24}) \\ \cdot(-\frac{1}{12}) \\ \cdot(-\frac{1}{8}) \\ \cdot(-\frac{1}{12}) \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Vi kan nu aflæse løsningerne til:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \\ \frac{1}{2} \\ 2 \\ 0 \end{bmatrix}$$

4 Opgave

Se vedhæftede python-fil.