Københavns Universitet
LinAlgDat - Project B

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Hold 13 Mach

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Indhold

1 Opgave

1.a

Vi forkaster x_1, x_2, x_3, og bruger deres konstanter til at aflæse M_a til:

$$\begin{bmatrix} a & -1 & -1 \\ 0 & a-1 & -1 \\ 0 & 2 & a+2 \end{bmatrix}$$

x_1, x_2, x_3 droppes, da disse kun indgår, når vi ganger M_a med $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$.

1.b

Vi ser først på, om T_a er injektiv. En transformation er injektiv, hvis kernen af transformationen kun er $\{0\}$. Det vil sige, at kun nulvektoren transformeret giver nulvektoren.

$$\left[\begin{array}{ccc|c} a & -1 & -1 & 0 \\ 0 & a-1 & -1 & 0 \\ 0 & 2 & a+2 & 0 \end{array}\right] \cdot \tfrac{1}{a-1} \rightsquigarrow \left[\begin{array}{ccc|c} a & -1 & -1 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 \\ 0 & 2 & a+2 & 0 \end{array}\right] -2r_2 \rightsquigarrow$$
$$\left[\begin{array}{ccc|c} a & -1 & -1 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 \\ 0 & 0 & \frac{a^2+a}{a-1} & 0 \end{array}\right] \cdot \tfrac{a-1}{a^2+a} \rightsquigarrow \left[\begin{array}{ccc|c} a & -1 & -1 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{matrix} +r_3 \\ +r_3 \cdot \frac{1}{a-1} \\ \end{matrix} \rightsquigarrow$$
$$\left[\begin{array}{ccc|c} a & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] +r_2 \rightsquigarrow \left[\begin{array}{ccc|c} a & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \cdot \tfrac{1}{a} \rightsquigarrow$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

Da ker $T_a = \{0\}$, er T_a injektiv.

T_a er surjektiv, hvis ran $T_a = \mathbb{R}^3$, da vores vektorer har 3 koordinater.

$$rank(T_a) = dim(ran(T_a)) = 3$$

da vi har 3 pivotelementer.

Da dimissionen af ran T_a er lig dimissionen af vektoerene der udgør T_a, er T_a surjektiv. T_a er dermed bijektiv, da den både er injektiv og surjektiv.

Vi bestemmer nu T_a^{-1}, ved at sætte enhedsmatricen på til højre, og reducere med Gauss-Jordan:

$$\left[\begin{array}{ccc|ccc} a & -1 & -1 & 1 & 0 & 0 \\ 0 & a-1 & -1 & 0 & 1 & 0 \\ 0 & 2 & a+2 & 0 & 0 & 1 \end{array}\right] \cdot \tfrac{1}{a-1} \rightsquigarrow \left[\begin{array}{ccc|ccc} a & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 & \frac{1}{a-1} & 0 \\ 0 & 2 & a+2 & 0 & 0 & 1 \end{array}\right] -2r_2 \rightsquigarrow$$

$$\left[\begin{array}{ccc|ccc} a & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 & \frac{1}{a-1} & 0 \\ 0 & 0 & \frac{a^2+a}{a-1} & 0 & -\frac{2}{a-1} & 1 \end{array}\right] \begin{array}{l} \\ \\ \cdot\frac{a-1}{a^2+a} \end{array} \rightsquigarrow \left[\begin{array}{ccc|ccc} a & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 & \frac{1}{a-1} & 0 \\ 0 & 0 & 1 & 0 & -\frac{2}{a^2+a} & \frac{a-1}{a^2+a} \end{array}\right] \begin{array}{l} +r_3 \\ +r_3\cdot\frac{1}{a-1} \\ \\ \end{array} \rightsquigarrow$$
$$\left[\begin{array}{ccc|ccc} a & -1 & 0 & 1 & -\frac{2}{a^2+a} & \frac{a-1}{a^2+a} \\ 0 & 1 & 0 & 0 & \frac{a+2}{a^2+a} & \frac{1}{a^2+a} \\ 0 & 0 & 1 & 0 & -\frac{2}{a^2+a} & \frac{a-1}{a^2+a} \end{array}\right] \begin{array}{l} +r_2 \\ \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|ccc} a & 0 & 0 & 1 & \frac{1}{a+1} & \frac{1}{a+1} \\ 0 & 1 & 0 & 0 & \frac{a+2}{a^2+a} & \frac{1}{a^2+a} \\ 0 & 0 & 1 & 0 & -\frac{2}{a^2+a} & \frac{a-1}{a^2+a} \end{array}\right] \begin{array}{l} \cdot\frac{1}{a} \\ \\ \\ \end{array} \rightsquigarrow$$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{a} & \frac{1}{a^2+a} & \frac{1}{a^2+a} \\ 0 & 1 & 0 & 0 & \frac{a+2}{a^2+a} & \frac{1}{a^2+a} \\ 0 & 0 & 1 & 0 & -\frac{2}{a^2+a} & \frac{a-1}{a^2+a} \end{array}\right]$$

$$T_a^{-1} = \begin{bmatrix} \frac{1}{a} & \frac{1}{a^2+a} & \frac{1}{a^2+a} \\ 0 & \frac{a+2}{a^2+a} & \frac{1}{a^2+a} \\ 0 & -\frac{2}{a^2+a} & \frac{a-1}{a^2+a} \end{bmatrix}$$

1.c

Vi opstill igen T_a, hvor $a = -1$:

$$\left[\begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & -1-1 & -1 & 0 \\ 0 & 2 & -1+2 & 0 \end{array}\right] \; reducer \rightsquigarrow \left[\begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & -2 & -1 & 0 \\ 0 & 2 & 1 & 0 \end{array}\right] \begin{array}{l} \\ \\ +r_2 \end{array} \rightsquigarrow$$
$$\left[\begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & -2 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \cdot(-1) \\ \cdot(-1) \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \cdot\frac{1}{2} \\ \\ \end{array} \rightsquigarrow$$
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} -r_2 \\ \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Vi aflæser løsningerne til:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{bmatrix}$$

Vi gør det samme for $a = 0$:

$$\left[\begin{array}{ccc|c} 0 & -1 & -1 & 0 \\ 0 & (0-1) & -1 & 0 \\ 0 & 2 & (0+2) & 0 \end{array}\right] \; reducer \rightsquigarrow \left[\begin{array}{ccc|c} 0 & -1 & -1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 2 & 2 & 0 \end{array}\right] \begin{array}{l} \\ -r_1 \\ +2r_2 \end{array} \rightsquigarrow$$
$$\left[\begin{array}{ccc|c} 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \cdot(-1) \\ \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

1.d

1.e

2 Opgave

2.a

Vi opstiller et ligningssystem i form af en totalmatrix, hvor vi sætter u_1, u_2, u_3 lig hhv. v_1, v_2, v_3, og finder løsningerne til disse, ved brug af Gauss-Jordan elimination.

$u_1 + u_2 + u_3 = v_1 \Leftrightarrow$

$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & 7 \\ 1 & 1 & -1 & -2 \\ -1 & -1 & 2 & 3 \\ 1 & 1 & 2 & 1 \end{array}\right] \begin{array}{c} \\ \cdot 2 \\ \cdot(-2) \\ \cdot 2 \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & 1 & 7 \\ 2 & 2 & -2 & -4 \\ 2 & 2 & -4 & -6 \\ 2 & 2 & 4 & 2 \end{array}\right] \begin{array}{c} \\ -r_1 \\ -r_1 \\ -r_1 \end{array} \rightsquigarrow$$
$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & 7 \\ 0 & 2 & -3 & -11 \\ 0 & 2 & -5 & -13 \\ 0 & 2 & 3 & -5 \end{array}\right] \begin{array}{c} \\ \\ -r_2 \\ -r_2 \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & 1 & 7 \\ 0 & 2 & -3 & -11 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 6 & 6 \end{array}\right] \begin{array}{c} \\ \\ \\ +3r_3 \end{array} \rightsquigarrow$$
$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & 7 \\ 0 & 2 & -3 & -11 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{c} \\ \\ \cdot(-\frac{1}{2}) \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & 1 & 7 \\ 0 & 2 & -3 & -11 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{c} -1r_3 \\ +3r_3 \\ \\ \\ \end{array} \rightsquigarrow$$
$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & 6 \\ 0 & 2 & 0 & -8 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{c} \cdot\frac{1}{2} \\ \cdot\frac{1}{2} \\ \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Vores første kolonne i $P_{B \leftarrow C}$ er dermed: $\begin{bmatrix} 3 \\ -4 \\ 1 \\ 0 \end{bmatrix}$

$u_1 + u_2 + u_3 = v_2 \Leftrightarrow$

$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & -1 \\ 1 & 1 & -1 & 0 \\ -1 & -1 & 2 & -1 \\ 1 & 1 & 2 & -3 \end{array}\right] \begin{array}{c} \\ \cdot 2 \\ \cdot(-2) \\ \cdot 2 \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & 1 & -1 \\ 2 & 2 & -2 & 0 \\ 2 & 2 & -4 & 2 \\ 2 & 2 & 4 & -6 \end{array}\right] \begin{array}{c} \\ -r_1 \\ -r_1 \\ -r_1 \end{array} \rightsquigarrow$$
$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & -1 \\ 0 & 2 & -3 & 1 \\ 0 & 2 & -5 & 3 \\ 0 & 2 & 3 & -5 \end{array}\right] \begin{array}{c} \\ \\ -r_2 \\ -r_2 \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & 1 & -1 \\ 0 & 2 & -3 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 6 & -6 \end{array}\right] \begin{array}{c} \\ \\ \\ +3r_3 \end{array} \rightsquigarrow$$
$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & -1 \\ 0 & 2 & -3 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{c} \\ \\ \cdot(-\frac{1}{2}) \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & 1 & -1 \\ 0 & 2 & -3 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{c} -1r_3 \\ +3r_3 \\ \\ \\ \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{c} \cdot\frac{1}{2} \\ \cdot\frac{1}{2} \\ \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Vores anden kolonne i $P_{B\leftarrow C}$ er dermed: $\begin{bmatrix} 0 \\ -1 \\ -1 \\ 0 \end{bmatrix}$

$u_1 + u_2 + u_3 = v_3 \Leftrightarrow$

$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 2 & 2 \\ 1 & 1 & 2 & 2 \end{array}\right] \begin{array}{c} \\ \cdot 2 \\ \cdot(-2) \\ \cdot 2 \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 2 & 2 & -2 & -2 \\ 2 & 2 & -4 & -4 \\ 2 & 2 & 4 & 4 \end{array}\right] \begin{array}{c} \\ -r_1 \\ -r_1 \\ -r_1 \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 0 & 2 & -3 & -5 \\ 0 & 2 & -5 & -7 \\ 0 & 2 & 3 & 1 \end{array}\right] \begin{array}{c} \\ \\ -r_2 \\ -r_2 \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 0 & 2 & -3 & -5 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 6 & 6 \end{array}\right] \begin{array}{c} \\ \\ \\ +3r_3 \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 0 & 2 & -3 & -5 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{c} \\ \\ \cdot(-\frac{1}{2}) \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 0 & 2 & -3 & -5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{c} -1r_3 \\ +3r_3 \\ \\ \\ \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & 2 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{c} \cdot\frac{1}{2} \\ \cdot\frac{1}{2} \\ \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Vores sidste kolonne i $P_{B\leftarrow C}$ er dermed: $\begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}$

Sammensætter vi nu vores tre kolonner til en matrix, får vi:

$$P_{B\leftarrow C} = \begin{bmatrix} 3 & 0 & 1 \\ -4 & -1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

2.b

$$x = \begin{bmatrix} 7 \\ -2 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ -1 \\ -1 \end{bmatrix} + \begin{bmatrix} 3 \\ -1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 9 \\ -3 \\ 4 \\ 0 \end{bmatrix}$$

Da konstanterne foran v i hvert led er 1, og $v_1, v_2, v_3 \in \mathcal{C}$, er koordinaterne for x med henhold til $\mathcal{C}$:

$$[x]_{\mathcal{C}} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Vi benytter vores basisskriftmatrice til at transformere vores koordinater til basen $\mathcal{B}$ fra $\mathcal{C}$:

$$[x]_{\mathcal{B}} = \begin{bmatrix} 3 & 0 & 1 \\ -4 & -1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ 1 \end{bmatrix}$$

2.c

Vi ganger kolonne 2 i vores basisskriftmatrice på u_1 og u_2:

$$-1 \cdot u_1 + (-1) \cdot u_2 = -1 \cdot \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} + (-1) \cdot \begin{bmatrix} 1 \\ -1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -1 \\ -3 \end{bmatrix}$$

Vi får v_2, så v_2 må dermed række spannet af $u2, u3$.

Mangler at lave resten af opgaven

2.d

2.e

3

3.a

Vi får givet, at koordinatforskydningen svarer til:

$$\begin{bmatrix} s_1 - c_1 \\ s_2 - c_2 \\ s_1 - c_1 \\ s_2 - c_2 \end{bmatrix}$$

Tilføjer vi forskydningen til vores nuværende koordinater, kan vi beskrive spillerens nye position som:

$$\begin{bmatrix} c_1^F \\ c_2^F \\ s_1^F \\ s_2^F \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} s_1 - c_1 \\ s_2 - c_2 \\ s_1 - c_1 \\ s_2 - c_2 \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \\ 2s_1 - c_1 \\ 2s_2 - c_2 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 2 & 0 \\ 0 & -1 & 0 & 2 \end{bmatrix}$$

3.b

Rotation mod venstre er bestemt som:

$$\begin{bmatrix} s_1^L \\ s_2^L \end{bmatrix} =$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix} \begin{bmatrix} s_1 - c_1 \\ s_2 - c_2 \end{bmatrix} =$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} (s_1 - c_1)cos(\theta) - (s_2 - c_2)sin(\theta) \\ (s_1 - c_1)sin(\theta) + (s_2 - c_2)cos(\theta) \end{bmatrix} =$$

$$\begin{bmatrix} c_1 + (s_1 - c_1)cos(\theta) - (s_2 - c_2)sin(\theta) \\ c_2 + (s_1 - c_1)sin(\theta) + (s_2 - c_2)cos(\theta) \end{bmatrix} =$$

$$\begin{bmatrix} c_1 - c_1 \cdot cos(\theta) + c_2 \cdot sin(\theta) + s_1 \cdot cos(\theta) - s_2 \cdot sin(\theta) \\ -c_1 \cdot sin(\theta) + c_2 - c_2 \cdot cos(\theta) + s_1 \cdot sin(\theta) + s_2 \cdot cos(\theta) \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 - cos(\theta) & sin(\theta) & cos(\theta) & -sin(\theta) \\ -sin(\theta) & 1 - cos(\theta) & sin(\theta) & cos(\theta) \end{bmatrix}$$

Og som der fremkommer i opgaven, er:

$$\begin{bmatrix} c_1^L \\ c_2^L \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Den endelige matrix for rotation mod venstre er dermed bestemt ved følgende variable:

$$L_\theta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 - cos(\theta) & sin(\theta) & cos(\theta) & -sin(\theta) \\ -sin(\theta) & 1 - cos(\theta) & sin(\theta) & cos(\theta) \end{bmatrix}$$

Vi mindes, at $cos(\theta) = cos(-\theta)$ og $sin(-\theta) = -sin(\theta)$.
Rotation mod højre er dermed bestemt som:

$$R_\theta = L_{-\theta} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 - cos(-\theta) & sin(-\theta) & cos(-\theta) & -sin(-\theta) \\ -sin(-\theta) & 1 - cos(-\theta) & sin(-\theta) & cos(-\theta) \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 - cos(\theta) & -sin(\theta) & cos(\theta) & sin(\theta) \\ sin(\theta) & 1 - cos(\theta) & -sin(\theta) & cos(\theta) \end{bmatrix}$$

3.c

Ved brug af matrixoperationerne fra python i *project A*, får vi følgende matricer efter vi ganger hhv. 'fremad', 'rotation til venstre' og 'rotation til højre' matricerne på til venstre:

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 2 & 0 \\ 0 & -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0.00000 \\ 0.00000 \\ 0.00000 \\ 1.00000 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 0.00000 \\ 1.00000 \\ 0.00000 \\ 2.00000 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1-cos(\theta) & -sin(\theta) & cos(\theta) & sin(\theta) \\ sin(\theta) & 1-cos(\theta) & -sin(\theta) & cos(\theta) \end{bmatrix} \begin{bmatrix} 0.00000 \\ 1.00000 \\ 0.00000 \\ 2.00000 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 0.00000 \\ 1.00000 \\ 0.91295 \\ 1.40808 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1-cos(\theta) & -sin(\theta) & cos(\theta) & sin(\theta) \\ sin(\theta) & 1-cos(\theta) & -sin(\theta) & cos(\theta) \end{bmatrix} \begin{bmatrix} 0.00000 \\ 1.00000 \\ 0.91295 \\ 1.40808 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 0.00000 \\ 1.00000 \\ 0.74511 \\ 0.33306 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 2 & 0 \\ 0 & -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0.00000 \\ 1.00000 \\ 0.74511 \\ 0.33306 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 0.74511 \\ 0.33306 \\ 1.49023 \\ -0.33388 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 2 & 0 \\ 0 & -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0.74511 \\ 0.33306 \\ 1.49023 \\ -0.33388 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1.49023 \\ -0.33388 \\ 2.23534 \\ -1.00081 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1-cos(\theta) & sin(\theta) & cos(\theta) & -sin(\theta) \\ -sin(\theta) & 1-cos(\theta) & sin(\theta) & cos(\theta) \end{bmatrix} \begin{bmatrix} 1.49023 \\ -0.33388 \\ 2.23534 \\ -1.00081 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1.49023 \\ -0.33388 \\ 2.40317 \\ 0.07421 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1-cos(\theta) & sin(\theta) & cos(\theta) & -sin(\theta) \\ -sin(\theta) & 1-cos(\theta) & sin(\theta) & cos(\theta) \end{bmatrix} \begin{bmatrix} 1.49023 \\ -0.33388 \\ 2.40317 \\ 0.07421 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1.49023 \\ -0.33388 \\ 1.49023 \\ 0.66612 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1-cos(\theta) & sin(\theta) & cos(\theta) & -sin(\theta) \\ -sin(\theta) & 1-cos(\theta) & sin(\theta) & cos(\theta) \end{bmatrix} \begin{bmatrix} 1.49023 \\ -0.33388 \\ 1.49023 \\ 0.66612 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1.49023 \\ -0.33388 \\ 0.57728 \\ 0.07421 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 2 & 0 \\ 0 & -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1.49023 \\ -0.33388 \\ 0.57728 \\ 0.07421 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 0.57728 \\ 0.07421 \\ -0.33566 \\ 0.48229 \end{bmatrix}$$

Efter alle 9 multiplikationer fra venste ender vi med postionen af spilleren og sidsen svarende til matricen:

$$\begin{bmatrix} 0.57728 \\ 0.07421 \\ -0.33566 \\ 0.48229 \end{bmatrix}$$

3.d

At gange vores 'rotation mod højre' matrice på sig selv svarer til at gange det antal gange med vinkeln θ, da:

$$R_{\theta 1} \cdot R_{\theta 2} = R_{\theta 1 + \theta 2}$$

og

$$(R_\theta)^n = \prod_{i=1}^{n} R_{\theta i} = R_{\theta 1 + \theta 2 + \ \dots \ + \theta n}$$

Vi kan dermed beregne $(R_{20})^{18}$ til:

$$(R_{20})^{18} = \prod_{i=1}^{18} R_{20} = R_{20 \cdot 18} = R_{360}$$

Med vores nye vinkel beregnet, kan vi nu indsætte 360 på θ-s plads i R_θ:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 - cos(360) & -sin(360) & cos(360) & sin(360) \\ sin(360) & 1 - cos(360) & -sin(360) & cos(360) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1-1 & 0 & 1 & 0 \\ 0 & 1-1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Således ender vi med enhedsmatricen I_4.

4 Opgave

Se vedhæftede python-fil.