

# Københavns Universitet

## LinAlgDat - Project C

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Hold 13 Mach

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## 1 Opgave

### 1.a

Vi definerer de 3 søjler som vektorer:

$$u_1 = \begin{bmatrix} 3 \\ 0 \\ -2 \\ 6 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 1 \\ 2 \\ 6 \\ -2 \end{bmatrix}, \quad u_3 = \begin{bmatrix} 19 \\ 14 \\ -15 \\ 10 \end{bmatrix}$$

$$r_{11} = \text{norm}(u_1) = \sqrt{3^2 + 0^2 + (-2)^2 + 6^2} = \sqrt{9 + 4 + 36} = \sqrt{49} = 7$$

$$q_1 = \frac{u_1}{r_{11}} = \begin{bmatrix} \frac{3}{7} \\ 0 \\ -\frac{2}{7} \\ \frac{6}{7} \end{bmatrix}$$

$$r_{12} = q_1 \cdot u_2 = \begin{bmatrix} \frac{3}{7} \\ 0 \\ -\frac{2}{7} \\ \frac{6}{7} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 6 \\ -2 \end{bmatrix} = \frac{3}{7} \cdot 1 + 0 \cdot 2 + \left(-\frac{2}{7}\right) \cdot 6 + \frac{6}{7} \cdot (-2) = \frac{3}{7} - \frac{12}{7} - \frac{12}{7} = -\frac{21}{7} = -3$$

$$q'_2 = u_2 - r_{12}q_1 = \begin{bmatrix} 1 \\ 2 \\ 6 \\ -2 \end{bmatrix} - 3 \begin{bmatrix} \frac{3}{7} \\ 0 \\ -\frac{2}{7} \\ \frac{6}{7} \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 6 \\ -2 \end{bmatrix} - \begin{bmatrix} \frac{9}{7} \\ 0 \\ -\frac{6}{7} \\ \frac{18}{7} \end{bmatrix} = \begin{bmatrix} -\frac{2}{7} \\ 2 \\ \frac{48}{7} \\ -\frac{32}{7} \end{bmatrix}$$

$$r_{22} = \text{norm}(q'_2) = \sqrt{\left(-\frac{2}{7}\right)^2 + 2^2 + \left(\frac{48}{7}\right)^2 + \left(-\frac{32}{7}\right)^2} = \sqrt{\frac{3528}{49}} = \sqrt{72}$$

$$q_2 = \frac{q'_2}{r_{22}} =$$

$$A = QR = \left[ \begin{array}{c|c|c} q_1 & q_2 & q_3 \end{array} \right] \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \\ 0 & 0 & r_{33} \\ 0 & 0 & 0 \end{bmatrix}$$

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1.b

1.c

1.d

1.e

## 2 Opgave

2.a

Vi bestemmer først  $\lambda I - M$ :

$$\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -6 & -3 \\ 3 & 7 & 3 \\ -6 & -12 & -5 \end{bmatrix} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} -2 & -6 & -3 \\ 3 & 7 & 3 \\ -6 & -12 & -5 \end{bmatrix} = \begin{bmatrix} \lambda+2 & 6 & 3 \\ -3 & \lambda-7 & -3 \\ 6 & 12 & \lambda+5 \end{bmatrix}$$

Vi udfører de rækkeoperationer, som er givet i opgavens vink:

$$\begin{bmatrix} \lambda+2 & 6 & 3 \\ -3 & \lambda-7 & -3 \\ 6 & 12 & \lambda+5 \end{bmatrix} \xrightarrow{+r_2} \begin{bmatrix} \lambda-1 & 6-\lambda & 0 \\ -3 & \lambda-7 & -3 \\ 6 & 12 & \lambda+5 \end{bmatrix} \xrightarrow{+2r_2} \begin{bmatrix} \lambda-1 & 6-\lambda & 0 \\ -3 & \lambda-7 & -3 \\ 0 & 2\lambda-2 & \lambda-1 \end{bmatrix}$$

2.b

2.c

2.d

2.e

## 3 Opgave

3.a

3.b

3.c

## 4 Opgave

Se vedhæftede python-fil.