Københavns Universitet
LinAlgDat - Project C

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Hold 13 Mach

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Indhold

1 Opgave

1.a

Vi definerer de 3 søjler som vektoererne:

$$u_1 = \begin{bmatrix} 3 \\ 0 \\ -2 \\ 6 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 1 \\ 2 \\ 6 \\ -2 \end{bmatrix}, \quad u_3 = \begin{bmatrix} 19 \\ 14 \\ -15 \\ 10 \end{bmatrix}$$

$$r_{11} = norm(u_1) = \sqrt{3^2 + 0^2 + (-2)^2 + 6^2} = \sqrt{9 + 4 + 36} = \sqrt{49} = 7$$

$$q_1 = \frac{u_1}{r_{11}} = \begin{bmatrix} \frac{3}{7} \\ 0 \\ \frac{-2}{7} \\ \frac{6}{7} \end{bmatrix}$$

$$r_{12} = q_1 \cdot u_2 = \begin{bmatrix} \frac{3}{7} \\ 0 \\ -\frac{2}{7} \\ \frac{6}{7} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 6 \\ -2 \end{bmatrix} = \frac{3}{7} \cdot 1 + 0 \cdot 2 + (-\frac{2}{7} \cdot 6) + \frac{6}{7} \cdot (-2) = \frac{3}{7} - \frac{12}{7} - \frac{12}{7} = -\frac{21}{7} = -3$$

$$q_2' = u_2 - r_{12} q_1 = \begin{bmatrix} 1 \\ 2 \\ 6 \\ -2 \end{bmatrix} - 3 \begin{bmatrix} \frac{3}{7} \\ 0 \\ -\frac{2}{7} \\ \frac{6}{7} \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 6 \\ -2 \end{bmatrix} - \begin{bmatrix} \frac{9}{7} \\ 0 \\ -\frac{6}{7} \\ \frac{18}{7} \end{bmatrix} = \begin{bmatrix} -\frac{2}{7} \\ 2 \\ \frac{48}{7} \\ -\frac{32}{7} \end{bmatrix}$$

$$r_{22} = norm(q_2') = \sqrt{\left(-\frac{2}{7}\right)^2 + 2^2 + \left(\frac{48}{7}\right)^2 + \left(-\frac{32}{7}\right)^2} = \sqrt{\frac{3528}{49}} = \sqrt{72}$$

$$q_2 = \frac{q_2'}{r_{22}} =$$

$$A = QR = \left[\begin{array}{c|c|c} q_1 & q_2 & q_3 \end{array}\right] \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \\ 0 & 0 & r_{33} \\ 0 & 0 & 0 \end{bmatrix}$$

1.b

1.c

1.d

1.e

2 Opgave

2.a

Vi bestemmer først $\lambda I - M$:

$$\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -6 & -3 \\ 3 & 7 & 3 \\ -6 & -12 & -5 \end{bmatrix} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} -2 & -6 & -3 \\ 3 & 7 & 3 \\ -6 & -12 & -5 \end{bmatrix} = \begin{bmatrix} \lambda+2 & 6 & 3 \\ -3 & \lambda-7 & -3 \\ 6 & 12 & \lambda+5 \end{bmatrix}$$

Vi udfører de rækkeoperationer, som er givet i opgavens vink:

$$\begin{bmatrix} \lambda+2 & 6 & 3 \\ -3 & \lambda-7 & -3 \\ 6 & 12 & \lambda+5 \end{bmatrix} \begin{matrix} \\ \\ +r_2 \end{matrix} \rightsquigarrow \begin{bmatrix} \lambda-1 & \lambda-1 & 0 \\ -3 & \lambda-7 & -3 \\ 6 & 12 & \lambda+5 \end{bmatrix} \begin{matrix} \\ \\ +2r_2 \end{matrix} \rightsquigarrow \begin{bmatrix} \lambda-1 & \lambda-1 & 0 \\ -3 & \lambda-7 & -3 \\ 0 & 2\lambda-2 & \lambda-1 \end{bmatrix}$$

2.b

2.c

2.d

2.e

3 Opgave

3.a

3.b

3.c

4 Opgave

Se vedhæftede python-fil.