

Københavns Universitet

LinAlgDat - Project A

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Hold 13 Mach

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1 Opgave

1.a

Vi omskriver ligningssystemet til totalmatrix-form:

$$\left[\begin{array}{ccc|c} 1 & 2 & 8 & a \\ a & a & 4a & a \\ 2 & 2 & 2a^2 & 0 \end{array} \right]$$

Vi benytter Gauss-Jordan elimination til at omskrive totalmatrix'en til en reduceret rækkeechelonform.

$$\begin{aligned} \left[\begin{array}{ccc|c} 1 & 2 & 8 & a \\ a & a & 4a & a \\ 2 & 2 & 2a^2 & 0 \end{array} \right] & \xrightarrow{-ar_1} \left[\begin{array}{ccc|c} 1 & 2 & 8 & a \\ 0 & -a & -4a & a-a^2 \\ 2 & 2 & 2a^2 & 0 \end{array} \right] \xrightarrow{-2r_1} \\ \left[\begin{array}{ccc|c} 1 & 2 & 8 & a \\ 0 & -a & -4a & a-a^2 \\ 0 & -2 & 2a^2-16 & -2a \end{array} \right] & \xrightarrow{r_2 \cdot \frac{2}{-a}} \left[\begin{array}{ccc|c} 1 & 2 & 8 & a \\ 0 & -a & -4a & a-a^2 \\ 0 & 0 & 2a^2-8 & -2a-\frac{2(a-a^2)}{a} = -2 \end{array} \right] \xrightarrow{r_2 \cdot \frac{2}{a}} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2-a \\ 0 & -a & -4a & a-a^2 \\ 0 & 0 & 2a^2-8 & -2 \end{array} \right] & \xrightarrow{r_3 \cdot \frac{2a}{a^2-4}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2-a \\ 0 & -a & 0 & a-a^2-\frac{4a}{(a^2-4)} \\ 0 & 0 & 2a^2-8 & -2 \end{array} \right] \xrightarrow{\cdot \frac{1}{2a^2-8}} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2-a \\ 0 & -a & 0 & a-a^2-\frac{4a}{(a^2-4)} \\ 0 & 0 & 1 & -\frac{2}{2a^2-8} = -\frac{2}{2(a^2-4)} = \frac{1}{(4-a^2)} \end{array} \right] & \xrightarrow{\cdot \frac{1}{-a}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2-a \\ 0 & 1 & 0 & \frac{(a^3-a^2-4a+8)}{(a^2-4)} \\ 0 & 0 & 1 & \frac{1}{(4-a^2)} \end{array} \right] \square \end{aligned}$$

Vi har nu fået den løsning vi ledte efter, så vi er dermed færdige.

1.b

Vi opskriver igen vores ligningssystem som en totalmatrix, og erstatter denne gang a med 0:

$$\begin{aligned} \left[\begin{array}{ccc|c} 1 & 2 & 8 & 0 \\ 0 & 0 & 4 \cdot 0 & 0 \\ 2 & 2 & 2 \cdot 0^2 & 0 \end{array} \right] & \xrightarrow{reducer} \left[\begin{array}{ccc|c} 1 & 2 & 8 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \end{array} \right] \xrightarrow{> byttes} \left[\begin{array}{ccc|c} 1 & 2 & 8 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{-2r_1} \\ \left[\begin{array}{ccc|c} 1 & 2 & 8 & 0 \\ 0 & -2 & -16 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] & \xrightarrow{\cdot (-\frac{1}{2})} \left[\begin{array}{ccc|c} 1 & 2 & 8 & 0 \\ 0 & 1 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{-2r_1} \left[\begin{array}{ccc|c} 1 & 0 & -8 & 0 \\ 0 & 1 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \end{aligned}$$

Vi kan nu aflæse løsningerne til:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} 8 \\ -8 \\ 1 \end{bmatrix}$$

Vi ser nu, hvad vi får, når vi bruger den rækkereducerede totalmatrix fra tidligere, når vi erstatter a med 0:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{2-a}{(a^3-a^2-4a+8)} \\ 0 & 1 & 0 & \frac{(a^2-4)}{1} \\ 0 & 0 & 1 & \frac{1}{(4-a^2)} \end{array} \right] a=0 \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{2-0}{(0^3-0^2-4\cdot 0+8)} \\ 0 & 1 & 0 & \frac{(0^2-4)}{1} \\ 0 & 0 & 1 & \frac{1}{(4-0^2)} \end{array} \right] \text{reducer} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & \frac{1}{4} \end{array} \right]$$

Denne matrix antyder en unik løsning, hvilket ikke afspejler hvad vi fandt lige før, hvor vi fandt uendelig mange løsninger. Dette skete sandsynligvis fordi at den rækkereducerede totalmatrix er lavet ud fra antagelsen, at $a \neq 0$.

1.c

Vi opskriver igen vores ligningssystem som en totalmatrix, og erstatter denne gang a med 2:

$$\begin{aligned} & \left[\begin{array}{ccc|c} 1 & 2 & 8 & 2 \\ 0 & 0 & 4 \cdot 2 & 2 \\ 2 & 2 & 2 \cdot 2^2 & 0 \end{array} \right] \text{reducer} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 2 & 8 & 2 \\ 0 & 0 & 8 & 2 \\ 2 & 2 & 8 & 0 \end{array} \right] \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 2 & 8 & 2 \\ 2 & 2 & 8 & 0 \\ 0 & 0 & 8 & 2 \end{array} \right] \begin{matrix} -2r_1 \\ +r_2 \end{matrix} \rightsquigarrow \\ & \left[\begin{array}{ccc|c} 1 & 2 & 8 & 2 \\ 0 & -2 & -8 & -4 \\ 0 & 0 & 8 & 2 \end{array} \right] \begin{matrix} -r_3 \\ +r_3 \end{matrix} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & -2 & -8 & -4 \\ 0 & 0 & 8 & 2 \end{array} \right] \begin{matrix} +r_2 \\ +r_3 \end{matrix} \rightsquigarrow \\ & \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & -2 & 0 & -2 \\ 0 & 0 & 8 & 2 \end{array} \right] \cdot (-\frac{1}{2}) \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 8 & 2 \end{array} \right] \cdot \frac{1}{8} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \frac{1}{4} \end{array} \right] \end{aligned}$$

Vi kan nu aflæse løsningen til:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ \frac{1}{4} \end{bmatrix}$$

Lad os nu se, hvad vi får, når vi bruger den rækkereducerede totalmatrix fra tidligere, når vi erstatter a med 2:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{2-2}{(2^3-2^2-4\cdot 2+8)} \\ 0 & 1 & 0 & \frac{(2^2-4)}{1} \\ 0 & 0 & 1 & \frac{1}{(4-2^2)} \end{array} \right] \text{reducer} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{4}{0} \\ 0 & 0 & 1 & \frac{1}{0} \end{array} \right]$$

Vi ser nu, at vi får division med 0, hvilket må betyde, at der ikke er nogen løsninger, når $a = 2$.

1.d

Vi omskriver ligningssystemet til koefficientmatrice-form på venstre side, hvor $a = 1$, og sætter identitetsmatricen på højre side:

$$\begin{aligned} & \left[\begin{array}{ccc|ccc} 1 & 2 & 8 & 1 & 0 & 0 \\ 1 & 1 & 4 \cdot 1 & 0 & 1 & 0 \\ 2 & 2 & 2 \cdot 1^2 & 0 & 0 & 1 \end{array} \right] \text{reducer} \rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 8 & 1 & 0 & 0 \\ 1 & 1 & 4 & 0 & 1 & 0 \\ 2 & 2 & 2 & 0 & 0 & 1 \end{array} \right] \begin{matrix} -1r_1 \\ -2r_1 \end{matrix} \rightsquigarrow \\ & \left[\begin{array}{ccc|ccc} 1 & 2 & 8 & 1 & 0 & 0 \\ 0 & -1 & -4 & -1 & 1 & 0 \\ 2 & 2 & 2 & 0 & 0 & 1 \end{array} \right] \rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 8 & 1 & 0 & 0 \\ 0 & -1 & -4 & -1 & 1 & 0 \\ 0 & -2 & -14 & -2 & 0 & 1 \end{array} \right] \begin{matrix} -2r_2 \\ -2r_2 \end{matrix} \rightsquigarrow \end{aligned}$$

$$\begin{aligned}
& \left[\begin{array}{ccc|ccc} 1 & 2 & 8 & 1 & 0 & 0 \\ 0 & -1 & -4 & -1 & 1 & 0 \\ 0 & 0 & -6 & 0 & -2 & 1 \end{array} \right] \begin{array}{l} \cdot 3 \\ \cdot (-3) \end{array} \rightsquigarrow \left[\begin{array}{ccc|ccc} 3 & 6 & 24 & 3 & 0 & 0 \\ 0 & 3 & 12 & 3 & -3 & 0 \\ 0 & 0 & -6 & 0 & -2 & 1 \end{array} \right] \begin{array}{l} +4r_3 \\ \\ \end{array} \rightsquigarrow \\
& \left[\begin{array}{ccc|ccc} 3 & 6 & 0 & 3 & -8 & 4 \\ 0 & 3 & 12 & 3 & -3 & 0 \\ 0 & 0 & -6 & 0 & -2 & 1 \end{array} \right] \begin{array}{l} \\ +2r_3 \end{array} \rightsquigarrow \left[\begin{array}{ccc|ccc} 3 & 6 & 0 & 3 & -8 & 4 \\ 0 & 3 & 0 & 3 & -7 & 2 \\ 0 & 0 & -6 & 0 & -2 & 1 \end{array} \right] \begin{array}{l} \\ -2r_2 \\ \end{array} \rightsquigarrow \\
& \left[\begin{array}{ccc|ccc} 3 & 0 & 0 & -3 & 6 & 0 \\ 0 & 3 & 0 & 3 & -7 & 2 \\ 0 & 0 & -6 & 0 & -2 & 1 \end{array} \right] \begin{array}{l} \cdot \frac{1}{3} \\ \cdot \frac{1}{3} \\ \cdot (-\frac{1}{6}) \end{array} \rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & 1 & -\frac{7}{3} & \frac{2}{3} \\ 0 & 0 & 1 & 0 & \frac{1}{3} & -\frac{1}{6} \end{array} \right]
\end{aligned}$$

2 Opgave

2.a

2.b

2.c

3 Opgave

3.a

Vi sætter et 1-tal i hver e_{uv} -element i matricen for hver edge $E(u, v)$ i grafen G_N :

$$N = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

For at finde antallet af veje fra knude 4 til knude 1 med netop længde 8, skal vi kigge på element $(4, 1)$ i N^8 . Vi kan finde $(N^8)_{4,1}$ ved:

$$69 \cdot 0 + 45 \cdot 1 + 45 \cdot 1 + 18 \cdot 1 + 29 \cdot 1 = 137$$

unikke veje fra knude 4 til knude 1 med længde 8.

3.b

Linkmatricen A findes ved at tage N^T og dividere hvert element med antallet af udgående edges i dens column.

$$N^T = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & \frac{1}{2} & 1 & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 \end{bmatrix}$$

3.c

Vi opskriver vores ligningssystem som en totalmatrix:

$$\begin{bmatrix} -1 & \frac{1}{2} & 1 & 0 & 0 & 0 \\ \frac{1}{2} & -1 & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -1 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 & -1 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{4} & -1 & 0 \end{bmatrix} \begin{matrix} r_1 \cdot 2 \\ r_2 \cdot 4 \\ r_3 \cdot 4 \\ r_4 \cdot 4 \\ r_5 \cdot 4 \end{matrix} \rightsquigarrow \begin{bmatrix} -2 & 1 & 2 & 0 & 0 & 0 \\ 2 & -4 & 0 & 0 & 2 & 0 \\ 2 & 0 & -4 & 0 & 0 & 0 \\ 2 & 2 & 4 & -4 & 2 & 0 \\ 2 & 0 & 0 & 1 & -4 & 0 \end{bmatrix} \begin{matrix} +r_1 \\ +r_1 \\ +r_1 \\ +r_1 \end{matrix} \rightsquigarrow$$

$$\begin{bmatrix} -2 & 1 & 2 & 0 & 0 & 0 \\ 0 & -3 & 2 & 0 & 2 & 0 \\ 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 3 & 6 & -4 & 2 & 0 \\ 0 & 1 & 2 & 1 & -4 & 0 \end{bmatrix} \begin{matrix} r_3 \cdot 3 \\ r_5 \cdot 3 \end{matrix} \rightsquigarrow \begin{bmatrix} -2 & 1 & 2 & 0 & 0 & 0 \\ 0 & -3 & 2 & 0 & 2 & 0 \\ 0 & 3 & -6 & 0 & 0 & 0 \\ 0 & 3 & 6 & -4 & 2 & 0 \\ 0 & 3 & 6 & 3 & -12 & 0 \end{bmatrix} \begin{matrix} +r_2 \\ +r_2 \\ +r_2 \end{matrix} \rightsquigarrow$$

$$\begin{bmatrix} -2 & 1 & 2 & 0 & 0 & 0 \\ 0 & -3 & 2 & 0 & 2 & 0 \\ 0 & 0 & -4 & 0 & 2 & 0 \\ 0 & 0 & 8 & -4 & 4 & 0 \\ 0 & 0 & 8 & 3 & -10 & 0 \end{bmatrix} \begin{matrix} \cdot 2 \\ \cdot 2 \end{matrix} \rightsquigarrow \begin{bmatrix} -2 & 1 & 2 & 0 & 0 & 0 \\ 0 & -3 & 2 & 0 & 2 & 0 \\ 0 & 0 & -8 & 0 & 4 & 0 \\ 0 & 0 & 8 & -4 & 4 & 0 \\ 0 & 0 & 8 & 3 & -10 & 0 \end{bmatrix} \begin{matrix} +r_3 \\ +r_3 \end{matrix} \rightsquigarrow$$

$$\begin{bmatrix} -2 & 1 & 2 & 0 & 0 & 0 \\ 0 & -3 & 2 & 0 & 2 & 0 \\ 0 & 0 & -8 & 0 & 4 & 0 \\ 0 & 0 & 0 & -4 & 8 & 0 \\ 0 & 0 & 0 & 3 & -6 & 0 \end{bmatrix} \begin{matrix} \rightsquigarrow \\ \cdot 3 \\ \cdot 4 \end{matrix} \rightsquigarrow \begin{bmatrix} -2 & 1 & 2 & 0 & 0 & 0 \\ 0 & -3 & 2 & 0 & 2 & 0 \\ 0 & 0 & -8 & 0 & 4 & 0 \\ 0 & 0 & 0 & -12 & 24 & 0 \\ 0 & 0 & 0 & 12 & -24 & 0 \end{bmatrix} \begin{matrix} \rightsquigarrow \\ +r_4 \\ +r_4 \end{matrix} \rightsquigarrow$$

$$\begin{bmatrix} -2 & 1 & 2 & 0 & 0 & 0 \\ 0 & -3 & 2 & 0 & 2 & 0 \\ 0 & 0 & -8 & 0 & 4 & 0 \\ 0 & 0 & 0 & -12 & 24 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \rightsquigarrow \\ \cdot 4 \\ \cdot 4 \end{matrix} \rightsquigarrow \begin{bmatrix} -8 & 4 & 8 & 0 & 0 & 0 \\ 0 & -12 & 8 & 0 & 8 & 0 \\ 0 & 0 & -8 & 0 & 4 & 0 \\ 0 & 0 & 0 & -12 & 24 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \rightsquigarrow \\ +r_3 \\ +r_3 \end{matrix} \rightsquigarrow$$

$$\begin{bmatrix} -8 & 4 & 0 & 0 & 4 & 0 \\ 0 & -12 & 0 & 0 & 12 & 0 \\ 0 & 0 & -8 & 0 & 4 & 0 \\ 0 & 0 & 0 & -12 & 24 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \cdot 3 \\ \rightsquigarrow \end{matrix} \rightsquigarrow \begin{bmatrix} -24 & 12 & 0 & 0 & 12 & 0 \\ 0 & -12 & 0 & 0 & 12 & 0 \\ 0 & 0 & -8 & 0 & 4 & 0 \\ 0 & 0 & 0 & -12 & 24 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} +r_2 \\ \rightsquigarrow \end{matrix} \rightsquigarrow$$

$$\begin{bmatrix} -24 & 0 & 0 & 0 & 24 & 0 \\ 0 & -12 & 0 & 0 & 12 & 0 \\ 0 & 0 & -8 & 0 & 4 & 0 \\ 0 & 0 & 0 & -12 & 24 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \cdot (-\frac{1}{24}) \\ \cdot (-\frac{1}{12}) \\ \cdot (-\frac{1}{8}) \\ \cdot (-\frac{1}{12}) \end{matrix} \rightsquigarrow \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Vi kan nu aflæse løsningerne til:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \\ \frac{1}{2} \\ 2 \\ 0 \end{bmatrix}$$

4 Opgave

Se vedhæftede python-fil.