Københavns Universitet
LinAlgDat - Project B

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Hold 13 Mach

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Indhold

1 Opgave

1.a

Vi kan aflæse M_a til:

$$\begin{bmatrix} a & -1 & -1 \\ 0 & (a-1) & -1 \\ 0 & 2 & (a+2) \end{bmatrix}$$

1.b

$$\left[\begin{array}{ccc|c} a & -1 & -1 & 0 \\ 0 & a-1 & -1 & 0 \\ 0 & 2 & a+2 & 0 \end{array}\right] \begin{array}{c} \\ \cdot\frac{1}{a-1} \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} a & -1 & -1 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 \\ 0 & 2 & a+2 & 0 \end{array}\right] \begin{array}{c} \\ \\ -2r_2 \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} a & -1 & -1 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 \\ 0 & 0 & \frac{a^2+a}{a-1} & 0 \end{array}\right] \begin{array}{c} \\ \\ \cdot\frac{a-1}{a^2+a} \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} a & -1 & -1 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{c} +r_3 \\ +r_3\cdot\frac{1}{a-1} \\ \\ \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} a & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{c} +r_2 \\ \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} a & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{c} \cdot\frac{1}{a} \\ \\ \\ \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

T_a er altså injektiv.
T_a er surjektiv, da vi har 3 vektoerer. T_a er dermed bijektiv, da den både er injektiv og surjektiv.

Vi bestemmer nu T_a^{-1}:

$$\left[\begin{array}{ccc|ccc} a & -1 & -1 & 1 & 0 & 0 \\ 0 & a-1 & -1 & 0 & 1 & 0 \\ 0 & 2 & a+2 & 0 & 0 & 1 \end{array}\right] \begin{array}{c} \\ \cdot\frac{1}{a-1} \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|ccc} a & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 & \frac{1}{a-1} & 0 \\ 0 & 2 & a+2 & 0 & 0 & 1 \end{array}\right] \begin{array}{c} \\ \\ -2r_2 \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|ccc} a & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 & \frac{1}{a-1} & 0 \\ 0 & 0 & \frac{a^2+a}{a-1} & 0 & -\frac{2}{a-1} & 1 \end{array}\right] \begin{array}{c} \\ \\ \cdot\frac{a-1}{a^2+a} \end{array} \rightsquigarrow \left[\begin{array}{ccc|ccc} a & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 & \frac{1}{a-1} & 0 \\ 0 & 0 & 1 & 0 & -\frac{2}{a^2+a} & \frac{a-1}{a^2+a} \end{array}\right] \begin{array}{c} +r_3 \\ +r_3\cdot\frac{1}{a-1} \\ \\ \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|ccc} a & -1 & 0 & 1 & -\frac{2}{a^2+a} & \frac{a-1}{a^2+a} \\ 0 & 1 & 0 & 0 & \frac{a+2}{a^2+a} & \frac{1}{a^2+a} \\ 0 & 0 & 1 & 0 & -\frac{2}{a^2+a} & \frac{a-1}{a^2+a} \end{array}\right] \begin{array}{c} +r_2 \\ \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|ccc} a & 0 & 0 & 1 & \frac{1}{a+1} & \frac{1}{a+1} \\ 0 & 1 & 0 & 0 & \frac{a+2}{a^2+a} & \frac{1}{a^2+a} \\ 0 & 0 & 1 & 0 & -\frac{2}{a^2+a} & \frac{a-1}{a^2+a} \end{array}\right] \begin{array}{c} \cdot\frac{1}{a} \\ \\ \\ \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{a} & \frac{1}{a^2+a} & \frac{1}{a^2+a} \\ 0 & 1 & 0 & 0 & \frac{a+2}{a^2+a} & \frac{1}{a^2+a} \\ 0 & 0 & 1 & 0 & -\frac{2}{a^2+a} & \frac{a-1}{a^2+a} \end{array}\right]$$

1.c

Vi opstill igen T_a, hvor $a = -1$:

$$\left[\begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & -1-1 & -1 & 0 \\ 0 & 2 & -1+2 & 0 \end{array}\right] \textit{reducer} \rightsquigarrow \left[\begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & -2 & -1 & 0 \\ 0 & 2 & 1 & 0 \end{array}\right] \begin{array}{c} \\ \\ -r_2 \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & -2 & -1 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right] \begin{array}{c} \\ \\ \cdot\frac{1}{2} \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & -2 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{c} +r_3 \\ +r_3 \\ \\ \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} -1 & -1 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{c} \cdot(-1) \\ \cdot(-\frac{1}{2}) \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{c} -r_2 \\ \\ \\ \end{array} \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

1.d

1.e