

Københavns Universitet

LinAlgDat - Project B

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Hold 13 Mach

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1 Opgave

1.a

Vi kan aflæse M_a til:

$$\begin{bmatrix} a & -1 & -1 \\ 0 & (a-1) & -1 \\ 0 & 2 & (a+2) \end{bmatrix}$$

1.b

$$\begin{aligned} & \left[\begin{array}{ccc|c} a & -1 & -1 & 0 \\ 0 & a-1 & -1 & 0 \\ 0 & 2 & a+2 & 0 \end{array} \right] \cdot \frac{1}{a-1} \rightsquigarrow \left[\begin{array}{ccc|c} a & -1 & -1 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 \\ 0 & 2 & a+2 & 0 \end{array} \right] \xrightarrow{-2r_2} \left[\begin{array}{ccc|c} a & -1 & -1 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 \\ 0 & 0 & \frac{a^2+a}{a-1} & 0 \end{array} \right] \\ & \xrightarrow{\cdot \frac{a-1}{a^2+a}} \left[\begin{array}{ccc|c} a & -1 & -1 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{+r_3} \left[\begin{array}{ccc|c} a & -1 & -1 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{+r_3 \cdot \frac{1}{a-1}} \left[\begin{array}{ccc|c} a & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \\ & \xrightarrow{+r_2} \left[\begin{array}{ccc|c} a & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{\cdot \frac{1}{a}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \end{aligned}$$

T_a er altså injektiv.

T_a er surjektiv, da vi har 3 vektorer. T_a er dermed bijektiv, da den både er injektiv og surjektiv.

Vi bestemmer nu T_a^{-1} :

$$\begin{aligned} & \left[\begin{array}{ccc|ccc} a & -1 & -1 & 1 & 0 & 0 \\ 0 & a-1 & -1 & 0 & 1 & 0 \\ 0 & 2 & a+2 & 0 & 0 & 1 \end{array} \right] \cdot \frac{1}{a-1} \rightsquigarrow \left[\begin{array}{ccc|ccc} a & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 & \frac{1}{a-1} & 0 \\ 0 & 2 & a+2 & 0 & 0 & 1 \end{array} \right] \xrightarrow{-2r_2} \left[\begin{array}{ccc|ccc} a & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 & \frac{1}{a-1} & 0 \\ 0 & 0 & \frac{a^2+a}{a-1} & 0 & -\frac{2}{a-1} & 1 \end{array} \right] \\ & \xrightarrow{\cdot \frac{a-1}{a^2+a}} \left[\begin{array}{ccc|ccc} a & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{a-1} & 0 & \frac{1}{a-1} & 0 \\ 0 & 0 & 1 & 0 & -\frac{2}{a-1} & \frac{a-1}{a^2+a} \end{array} \right] \xrightarrow{+r_2} \left[\begin{array}{ccc|ccc} a & 0 & 0 & 1 & \frac{1}{a-1} & \frac{1}{a-1} \\ 0 & 1 & 0 & 0 & \frac{1}{a-1} & \frac{1}{a-1} \\ 0 & 0 & 1 & 0 & -\frac{2}{a-1} & \frac{a-1}{a^2+a} \end{array} \right] \\ & \xrightarrow{\cdot \frac{1}{a}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{a} & \frac{1}{a^2+a} & \frac{1}{a^2+a} \\ 0 & 1 & 0 & 0 & \frac{1}{a^2+a} & \frac{1}{a^2+a} \\ 0 & 0 & 1 & 0 & -\frac{2}{a^2+a} & \frac{a-1}{a^2+a} \end{array} \right] \end{aligned}$$

1.c

Vi opstill igen T_a , hvor $a = -1$:

$$\begin{aligned}
& \left[\begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 2 & -1+2 & 0 \end{array} \right] \text{ reducer } \rightsquigarrow \left[\begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & -2 & -1 & 0 \\ 0 & 2 & 1 & 0 \end{array} \right] \rightsquigarrow -r_2 \\
& \left[\begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & -2 & -1 & 0 \\ 0 & 0 & 2 & 0 \end{array} \right] \rightsquigarrow \left[\begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & -2 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \begin{array}{l} +r_3 \\ +r_3 \end{array} \rightsquigarrow \\
& \left[\begin{array}{ccc|c} -1 & -1 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \begin{array}{l} \cdot \frac{1}{2} \\ \cdot (-1) \\ \cdot (-\frac{1}{2}) \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \begin{array}{l} -r_2 \\ \\ \end{array} \rightsquigarrow \\
& \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]
\end{aligned}$$

Vi gør det samme for $a = 0$:

$$\begin{aligned}
& \left[\begin{array}{ccc|c} 0 & -1 & -1 & 0 \\ 0 & (0-1) & -1 & 0 \\ 0 & 2 & (0+2) & 0 \end{array} \right] \text{ reducer } \rightsquigarrow \left[\begin{array}{ccc|c} 0 & -1 & -1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 2 & 2 & 0 \end{array} \right] \begin{array}{l} -r_1 \\ +2r_2 \end{array} \rightsquigarrow \\
& \left[\begin{array}{ccc|c} 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \cdot (-1) \rightsquigarrow \left[\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]
\end{aligned}$$

1.d

1.e

2 Opgave

2.a

Vi opstiller et ligningssystem i form af en totalmatrix, hvor vi sætter u_1, u_2, u_3 lig hhv. v_1, v_2, v_3 , og finder løsningerne til disse, ved brug af Gauss-Jordan elimination.

$$u_1 + u_2 + u_3 = v_1 \Leftrightarrow$$

$$\begin{aligned}
& \left[\begin{array}{ccc|c} 2 & 0 & 1 & 7 \\ 1 & 1 & -1 & -2 \\ -1 & -1 & 2 & 3 \\ 1 & 1 & 2 & 1 \end{array} \right] \begin{array}{l} \cdot 2 \\ \cdot (-2) \\ \cdot 2 \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & 1 & 7 \\ 2 & 2 & -2 & -4 \\ 2 & 2 & -4 & -6 \\ 2 & 2 & 4 & 2 \end{array} \right] \begin{array}{l} -r_1 \\ -r_1 \\ -r_1 \end{array} \rightsquigarrow \\
& \left[\begin{array}{ccc|c} 2 & 0 & 1 & 7 \\ 0 & 2 & -3 & -11 \\ 0 & 2 & -5 & -13 \\ 0 & 2 & 3 & -5 \end{array} \right] \begin{array}{l} \\ -r_2 \\ -r_2 \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & 1 & 7 \\ 0 & 2 & -3 & -11 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 6 & 6 \end{array} \right] \begin{array}{l} \\ \\ +3r_3 \end{array} \rightsquigarrow \\
& \left[\begin{array}{ccc|c} 2 & 0 & 1 & 7 \\ 0 & 2 & -3 & -11 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} \\ \cdot (-\frac{1}{2}) \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & 1 & 7 \\ 0 & 2 & -3 & -11 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} -1r_3 \\ +3r_3 \end{array} \rightsquigarrow
\end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & 6 \\ 0 & 2 & 0 & -8 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\substack{\cdot \frac{1}{2} \\ \cdot \frac{1}{2}}} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Vores første kolonne i $P_{B \leftarrow C}$ er dermed: $\begin{bmatrix} 3 \\ -4 \\ 1 \\ 0 \end{bmatrix}$

$$u_1 + u_2 + u_3 = v_2 \Leftrightarrow$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & -1 \\ 1 & 1 & -1 & 0 \\ -1 & -1 & 2 & -1 \\ 1 & 1 & 2 & -3 \end{array} \right] \xrightarrow{\substack{\cdot 2 \\ \cdot (-2) \\ \cdot 2}} \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & 1 & -1 \\ 2 & 2 & -2 & 0 \\ 2 & 2 & -4 & 2 \\ 2 & 2 & 4 & -6 \end{array} \right] \xrightarrow{\substack{-r_1 \\ -r_1 \\ -r_1}} \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & -1 \\ 0 & 2 & -3 & 1 \\ 0 & 2 & -5 & 3 \\ 0 & 2 & 3 & -5 \end{array} \right] \xrightarrow{\substack{-r_2 \\ -r_2}} \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & 1 & -1 \\ 0 & 2 & -3 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 6 & -6 \end{array} \right] \xrightarrow{\substack{+3r_3 \\ -1r_3 \\ +3r_3}} \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & -1 \\ 0 & 2 & -3 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\cdot (-\frac{1}{2})} \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & 1 & -1 \\ 0 & 2 & -3 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\substack{\cdot \frac{1}{2} \\ \cdot \frac{1}{2}}} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Vores anden kolonne i $P_{B \leftarrow C}$ er dermed: $\begin{bmatrix} 0 \\ -1 \\ -1 \\ 0 \end{bmatrix}$

$$u_1 + u_2 + u_3 = v_3 \Leftrightarrow$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 2 & 2 \\ 1 & 1 & 2 & 2 \end{array} \right] \xrightarrow{\substack{\cdot 2 \\ \cdot (-2) \\ \cdot 2}} \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 2 & 2 & -2 & -2 \\ 2 & 2 & -4 & -4 \\ 2 & 2 & 4 & 4 \end{array} \right] \xrightarrow{\substack{-r_1 \\ -r_1 \\ -r_1}} \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 0 & 2 & -3 & -5 \\ 0 & 2 & -5 & -7 \\ 0 & 2 & 3 & 1 \end{array} \right] \xrightarrow{\substack{-r_2 \\ -r_2}} \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 0 & 2 & -3 & -5 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 6 & 6 \end{array} \right] \xrightarrow{\substack{+3r_3 \\ -1r_3 \\ +3r_3}} \rightsquigarrow$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 0 & 2 & -3 & -5 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\cdot (-\frac{1}{2})} \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 0 & 2 & -3 & -5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & 2 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\substack{\cdot \frac{1}{2} \\ \cdot \frac{1}{2}}} \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Vores sidste kolonne i $P_{B \leftarrow C}$ er dermed:

$$\begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}$$

Sammensætter vi nu vores tre kolonner til en matrix, får vi:

$$P_{B \leftarrow C} = \begin{bmatrix} 3 & 0 & 1 \\ -4 & -1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

2.b

$$x = \begin{bmatrix} 7 \\ -2 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ -1 \\ -1 \end{bmatrix} + \begin{bmatrix} 3 \\ -1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 9 \\ -3 \\ 4 \\ 0 \end{bmatrix}$$

Da konstanterne foran v i hvert led er 1, og $v_1, v_2, v_3 \in \mathcal{C}$, er koordinaterne for x med henhold til \mathcal{C} :

$$[x]_{\mathcal{C}} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Vi benytter vores basisskriftmatrice til at transformere vores koordinater til basen \mathcal{B} fra \mathcal{C} :

$$[x]_{\mathcal{B}} = \begin{bmatrix} 3 & 0 & 1 \\ -4 & -1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ 1 \end{bmatrix}$$

2.c

Vi ganger kolonne 2 i vores basisskriftmatrice på u_1 og u_2 :

$$-1 \cdot u_1 + (-1) \cdot u_2 = -1 \cdot \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} + (-1) \cdot \begin{bmatrix} 1 \\ -1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -1 \\ -3 \end{bmatrix}$$

Vi får v_2 , så v_2 må dermed række spannet af u_2, u_3 .

Mangler at lave resten af opgaven

2.d

2.e

3

3.a

$$\begin{bmatrix} c_1^F \\ c_2^F \\ s_1^F \\ s_2^F \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} s_1 - c_1 \\ s_2 - c_2 \\ s_1 - c_1 \\ s_2 - c_2 \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \\ 2s_1 - c_1 \\ 2s_2 - c_2 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 2 & 0 \\ 0 & -1 & 0 & 2 \end{bmatrix}$$

3.b

Rotation mod venstre er bestemt som:

$$\begin{aligned} \begin{bmatrix} s_1^L \\ s_2^L \end{bmatrix} &= \\ \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} s_1 - c_1 \\ s_2 - c_2 \end{bmatrix} &= \\ \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} (s_1 - c_1)\cos(\theta) - (s_2 - c_2)\sin(\theta) \\ (s_1 - c_1)\sin(\theta) + (s_2 - c_2)\cos(\theta) \end{bmatrix} &= \\ \begin{bmatrix} c_1 + (s_1 - c_1)\cos(\theta) - (s_2 - c_2)\sin(\theta) \\ c_2 + (s_1 - c_1)\sin(\theta) + (s_2 - c_2)\cos(\theta) \end{bmatrix} &= \\ \begin{bmatrix} c_1 - c_1 \cdot \cos(\theta) + c_2 \cdot \sin(\theta) + s_1 \cdot \cos(\theta) - s_2 \cdot \sin(\theta) \\ -c_1 \cdot \sin(\theta) + c_2 - c_2 \cdot \cos(\theta) + s_1 \cdot \sin(\theta) + s_2 \cdot \cos(\theta) \end{bmatrix} &\Rightarrow \\ \begin{bmatrix} 1 - \cos(\theta) & \sin(\theta) & \cos(\theta) & -\sin(\theta) \\ -\sin(\theta) & 1 - \cos(\theta) & \sin(\theta) & \cos(\theta) \end{bmatrix} & \end{aligned}$$

Og som der fremkommer i opgaven, er:

$$\begin{bmatrix} c_1^L \\ c_2^L \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

Den endelige matrix for rotation mod venstre er dermed bestemt ved følgende variable:

$$L_\theta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 - \cos(\theta) & \sin(\theta) & \cos(\theta) & -\sin(\theta) \\ -\sin(\theta) & 1 - \cos(\theta) & \sin(\theta) & \cos(\theta) \end{bmatrix}$$

Rotation mod højre er bestemt som:

$$R_\theta = L_{-\theta} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 - \cos(\theta) & -\sin(\theta) & \cos(\theta) & \sin(\theta) \\ \sin(\theta) & 1 - \cos(\theta) & -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

3.c

Ved brug af matrixoperationerne fra python i *project A*, får vi følgende matricer efter vi ganger hhv. 'fremad', 'rotation til venstre' og 'rotation til højre' matricerne på til venstre:

$$\begin{aligned} & \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 2 & 0 \\ 0 & -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0.00000 \\ 0.00000 \\ 0.00000 \\ 1.00000 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 0.00000 \\ 1.00000 \\ 0.00000 \\ 2.00000 \end{bmatrix} \\ & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 - \cos(\theta) & -\sin(\theta) & \cos(\theta) & \sin(\theta) \\ \sin(\theta) & 1 - \cos(\theta) & -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} 0.00000 \\ 1.00000 \\ 0.00000 \\ 2.00000 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 0.00000 \\ 1.00000 \\ 0.91295 \\ 1.40808 \end{bmatrix} \\ & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 - \cos(\theta) & -\sin(\theta) & \cos(\theta) & \sin(\theta) \\ \sin(\theta) & 1 - \cos(\theta) & -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} 0.00000 \\ 1.00000 \\ 0.91295 \\ 1.40808 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 0.00000 \\ 1.00000 \\ 0.74511 \\ 0.33306 \end{bmatrix} \\ & \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 2 & 0 \\ 0 & -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0.00000 \\ 1.00000 \\ 0.74511 \\ 0.33306 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 0.74511 \\ 0.33306 \\ 1.49023 \\ -0.33388 \end{bmatrix} \\ & \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 2 & 0 \\ 0 & -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0.74511 \\ 0.33306 \\ 1.49023 \\ -0.33388 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1.49023 \\ -0.33388 \\ 2.23534 \\ -1.00081 \end{bmatrix} \\ & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 - \cos(\theta) & \sin(\theta) & \cos(\theta) & -\sin(\theta) \\ -\sin(\theta) & 1 - \cos(\theta) & \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} 1.49023 \\ -0.33388 \\ 2.23534 \\ -1.00081 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1.49023 \\ -0.33388 \\ 2.40317 \\ 0.07421 \end{bmatrix} \\ & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 - \cos(\theta) & \sin(\theta) & \cos(\theta) & -\sin(\theta) \\ -\sin(\theta) & 1 - \cos(\theta) & \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} 1.49023 \\ -0.33388 \\ 2.40317 \\ 0.07421 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1.49023 \\ -0.33388 \\ 1.49023 \\ 0.66612 \end{bmatrix} \\ & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 - \cos(\theta) & \sin(\theta) & \cos(\theta) & -\sin(\theta) \\ -\sin(\theta) & 1 - \cos(\theta) & \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} 1.49023 \\ -0.33388 \\ 1.49023 \\ 0.66612 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1.49023 \\ -0.33388 \\ 0.57728 \\ 0.07421 \end{bmatrix} \\ & \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 2 & 0 \\ 0 & -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1.49023 \\ -0.33388 \\ 0.57728 \\ 0.07421 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 0.57728 \\ 0.07421 \\ -0.33566 \\ 0.48229 \end{bmatrix} \end{aligned}$$

Efter alle 9 multiplikationer fra venste ender vi med positionen af spilleren og siden svarende til matricen:

$$\begin{bmatrix} 0.57728 \\ 0.07421 \\ -0.33566 \\ 0.48229 \end{bmatrix}$$

3.d

At gange vores 'rotation mod højre' matrice på sig selv svarer til at gange det antal gange med vinkeln θ , da:

$$R_{\theta_1} \cdot R_{\theta_2} = R_{\theta_1 + \theta_2}$$

og

$$(R_{\theta})^n = \prod_{i=1}^n R_{\theta_i} = R_{\theta_1 + \theta_2 + \dots + \theta_n}$$

Vi kan dermed beregne $(R_{20})^{18}$ til:

$$(R_{20})^{18} = \prod_{i=1}^{18} R_{20} = R_{20 \cdot 18} = R_{360}$$

Med vores nye vinkel beregnet, kan vi nu indsætte 360 på θ -s plads i R_{θ} :

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 - \cos(360) & -\sin(360) & \cos(360) & \sin(360) \\ \sin(360) & 1 - \cos(360) & -\sin(360) & \cos(360) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 - 1 & 0 & 1 & 0 \\ 0 & 1 - 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Således ender vi med enhedsmatricen I_4 .

4 Opgave

Se vedhæftede python-fil.