Københavns Universitet
Introduktion til diskret matematik og algoritmer - Problem set 3

Victor Vangkilde Jørgensen - kft410
kft410@alumni.ku.dk

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1 **Recall that a standard deck of cards has 52 cards partitioned into four suits (hearts, spades, clubs, and diamonds) with 13 ranks each (2-10 plus jack, queen, king, and ace). In this problem, we assume that you are dealt 5 cards from a perfectly shuffled deck of cards.**

1.a **What is the probability that you get a flush, i.e., 5 cards of the same suit but not all in sequence with respect to rank? (Because five cards of the same suit in sequential rank would be a straight flush.)**

1.b **What is the probability that you get a straight, i.e., 5 cards of sequential rank but not all of the same suit? (Because if the latter condition also held, we would again have a straight flush.)**

2 **Prove mathematically that among all numbers on the form $11...100...0$, i.e., numbers consisting of m ones followed by n zeros for some $m, n \in N^+$ (sometimes notation like 1^m0^n is used to describe text strings constructed in such a way), there is some number that is divisible by 2025. Hint: Look at all numbers $1^m = 11...1$ and consider what their remainders can be modulo 2025.**

3 **Let $a \in R^+$ be any positive real number. Show that for any integer $n \geq 2$ there is a rational number $\frac{c}{d}, c, d \in \mathbb{Z}, d \leq n$, that approximates a to within error $\frac{1}{dn}$, ie., $|a - \frac{c}{d}| \leq \frac{1}{dn}$. Hint: Consider the numbers $a, 2a, \ldots, n \cdot a$ and show that one of these numbers is at distance at most $\frac{1}{n}$ from some integer.**

4 **In this problem we focus on relations. Suppose that $A = \{e_0, e_1, \ldots, e_5\}$ is a set of 6 elements and consider the relation R on A represented by the matrix**

$$M_R = \left(\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \right)$$

(where element e_i corresponds to the row and column $i+1$).

4.a **Let us write S to denote the symmetric closure of the relation R. What is the matrix representation of S? Can you explain in words what the relation S is by describing how it can be interpreted?**

4.b **Now let T be the transitive closure of the relation S. What is the matrix representation of T? Can you explain in words what the relation T is by describing how it can be interpreted?**

4.c **Suppose that we instead let T' be the transitive closure of the relation R, and then let S' be the symmetric closure of T'. Are S' and T the same relation? If they are not the same, show some way in which they differ. If they are the same, is it true that S' and T constructed in this way from some relation R on a set A will always be the same? Please make sure to motivate your answers clearly.**

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