

## Written Problem

Sunday, February 2, 2020

12:00 PM

### Problem 1

Given,

	$X=0$	$X=1$
$Y=0$	$1/4$	$1/4$
$Y=1$	$1/6$	$1/3$

$$\begin{aligned} a) \quad P(X=1) &= P(X=1|Y=0) + P(X=1|Y=1) \\ &= 1/4 + 1/3 \\ &= 7/12 \end{aligned}$$

$$b) \quad P(X=1|Y=1) = \frac{1/3}{1/3 + 1/6} = \frac{1/3}{3/6} = \frac{1}{3} \times \frac{6}{3} = \frac{2}{3}$$

$$\begin{aligned} c) \quad \text{Var}[X] &= E[X^2] - (E[X])^2 \\ E[X^2] &= 0^2 \cdot \frac{5}{12} + 1^2 \cdot \frac{7}{12} = \frac{7}{12} \\ E[X] &= 0 \cdot \frac{5}{12} + 1 \cdot \frac{7}{12} = \frac{7}{12} \end{aligned}$$

$$\text{so, } \text{Var}[X] = \frac{7}{12} - \left(\frac{7}{12}\right)^2 = \frac{7}{12} - \frac{49}{144} = \frac{35}{144}$$

$$\begin{aligned} d) \quad \text{Var}[X|Y=1] &= E[X^2|Y=1] - (E[X|Y=1])^2 \\ &= (0^2 \cdot 1/6 + 1^2 \cdot 1/3) - (0 \cdot 1/6 + 1 \cdot 1/3)^2 \\ &= 1/3 - 1/9 \\ &= \frac{2}{9} \end{aligned}$$

$$\begin{aligned} e) \quad E[X^3 + X^2 + 3Y^7] | Y=1 &= (0^3 + 0^2 + 3 \cdot 1^7) \cdot 1/6 + (1^3 + 1^2 + 3 \cdot 1^7) \cdot 1/3 \\ &= 3 \cdot 1/6 + 5/3 \\ &= 1/2 + 5/3 = \frac{3+10}{6} = \frac{13}{6} \end{aligned}$$