IEEE Working Group P3109 Interim Report on 8-bit Binary Floating-point Formats

Questions and comments via GitHub issues at https://github.com/P3109/Public

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1 Introduction

This document represents the results of discussions and decisions made by the IEEE Working Group P3109, "Standard for Arithmetic Formats for Machine Learning". The Project Authorization Request (PAR) for P3109 defines the scope, need, and stakeholders as follows:

Scope of proposed standard: This standard defines a binary arithmetic and data format for machine learning-optimized domains. It also specifies the default handling of exceptions that occur in this arithmetic. This standard provides a consistent and flexible arithmetic framework optimized for Machine Learning Systems (MLS) in hardware and/or software implementations to minimize the work required to make MLS interoperable with each other, as well as other dependent systems. This standard is aligned with IEEE Std 754-2019 for Floating-Point Arithmetic.

Need for the Work: Machine Learning Systems have different arithmetic requirements from most other domains. Precisions tend to be lower, and accuracy is measured in dimensions other than just numerical (e.g. inference accuracy). Furthermore, machine learning systems are often integrated into mission-critical and safety-critical systems. With no standards specifically addressing these needs, Machine Learning Systems are built with inconsistent expectations and assumptions that hinder the compatibility and reuse of machine learning hardware, software, and training data.

Stakeholders for the Standard: System developers, vendors, and users of machine learning applications across many industries and interests including but not limited to computation, storage, medical, telecommunications, e-commerce, fleet management, automotive, robotics, and security.

The scope of this interim release is interchange formats only. The working group continues to deliberate on the specification of operations.

1.1 Typographical conventions and notation

Bold text describes the decisions and specifications of this document.

Text that is not bold is background material, typically providing rationale and arguments that represent discussions of the working group leading to a decision and specification.

This document specifies 8-bit floating-point interchange formats (binary formats) and associated operations. Binary formats are parameterized by their width, the number of bits spanned in memory (here, 8); and their precision (p), the number of bits spanned by the true significand (this is one more than the bits of the significand that are stored explicitly).

The formats defined herein shall be referred to as "binary8" formats, and further qualified by precision yielding names "binary8pp".

For example, "binary 8p3" is a format with 3 bits of precision (one implied – "the hidden bit" – and two explicit),

2 Values

This section describes the set of values that a binary8 format shall represent. The universe of values in existing floating point usage encompasses some finite real numerical values, the non-finite numerical values positive and negative infinity $(-\ln f, +\ln f)$, the non-numeric not-a-number values $(NaN, NaN_1, ...)$, and negative zero (-0.0). The value set for each binary8 format specifies the set of values that are available in that format.

Each binary format shall be associated with a unique encoding. An 8-bit binary encoding is a mapping from 8-bit strings to values. Some of these mappings are included in Appendix C.

The special values have encodings that are shared by all binary8 formats, as shown in table 2.

The set of finite floating-point numbers representable with a binary format is determined by two *format-defining* parameters:

- Precision p, the number of digits in the significand including the implicit leading bit.
- Maximum exponent emax, the exponent of the largest finite value.

IEEE-754 2019 includes the radix b and the minimum exponent emin in the list of format-defining parameters, while this document excludes them with the following rationale:

- This document covers binary (radix 2) formats only, so b is not a format parameter.
- The parameter *emin* is determinable from other parameters (, so is also not a format-defining parameter.

P3109 formats shall define emax(p) to be $\lceil 2^{8-p-1} - 1 \rceil$. In IEEE-754, emax was consistently chosen across formats to be $2^{w-1} - 1$, where w is the exponent field width in bits. In this report, this convention is formalized: emax is a fixed function of p, written emax(p), with the formula as given above.

This choice of formula yields the following properties:

- the binary8pp value sets are subsets of the IEEE-754 binary16 value set for p > 2
- values are distributed close to symmetrically below and above the value 1.

For p=8, the IEEE-754 formula yields emax $=-\frac{1}{2}$, meaning all non-special values are irrational. Rounding the computation upward yields emax(8) = 0, with the consequence that the value sets and encodings for binary8p7 and binary8p8 are identical.

The choice of emax for a given format then determines the exponent bias for that format. The bias is chosen so that the exponent of the largest finite value is emax. For IEEE-754 formats, the largest finite value corresponds to an exponent field which has all but the zeroth bit set (e.g. 11110 for binary16), because all of the values with all-bits-one exponents (ABOE values) are occupied by non-finite values (Not-a-Numbers or Infinities). Thus, the unbiased exponent of the largest finite value is $2^w - 2$, from which bias is computed as

$$emax = (2^w - 2) - bias \implies bias = (2^w - 2) - (2^{w-1} - 1) = 2^{w-1} - 2 + 1 = emax$$

For the binary8 formats in this document where p > 1, only one of the ABOE values is non-finite (\pm Infinity), so the unbiased exponent of the largest finite value is $2^w - 1$. Hence the bias calculation becomes

bias =
$$(2^w - 1) - (2^{w-1} - 1) = 2^{w-1} - 1 + 1 = \text{emax} + 1$$

For p = 1, there are zero trailing significand bits, so all ABOE values are special, so again bias = emax.

Parameter	binary $8p\{p\}$	binary8p5	binary8p4	binary8p3	binary16	binary32	binary64
Storage width in bits k	8	8	8	8	16	32	64
Precision in bits p	р	5	4	3	11	24	53
Max exponent emax	$\lceil 2^{8-p-1} - 1 \rceil$	3	7	15	15	127	1023
Sign bit	1	1	1	1	1	1	1
Exponent field width w	8-p	3	4	5	5	8	11
Exponent bias, bias	emax + (p > 1)	4	8	16	15	127	1023
Trailing significand field width in bits t	p-1	4	3	2	10	23	52

Table 1: Parameters for binary formats. Format-defining parameters in bold, derived parameters in normal font. Adapted from Table 3.5 of IEEE-754 (2019), and extended to include proposed binary8pp formats. Concepts are explained in detail in section 2.

2.1 Subnormals

Binary8 value sets shall include subnormals.

IEEE-754 value sets include subnormals. A value with trailing significand field m and exponent e is interpreted as $1.m \times 2^{e-b}$ except when all bits of the exponent bitfield are 0, in which case the value is $0.m \times 2^{e-b}$.

When training models, it is common to represent near-zero values for gradients. Subnormal numbers induce equal quantization steps around zero; this expands the reach of binary8 trainable models. In statistical applications, the subnormal range is useful for uniform and similar distributions; subnormals are uniformly spaced around zero. They also support working with Gaussian-like distributions, where numbers around zero are more probable.

2.2 Not a number (NaN)

Binary8 value sets shall include exactly one NaN, which shall not signal.

Other floating-point formats define several NaN values, denoted (NaN, NaN₁, ...). NaNs are returned from operations with results outside the set of values. For example, DIV(0,0), or ADD(Inf, -Inf). Multiple NaN encodings are used in other formats to allow different exceptional conditions to be distinguished.

In the context of machine learning systems, uses of NaN include:

- Debugging of code running on accelerator hardware. In AI accelerators, exceptions may be difficult or expensive
 to convey back to user code, so it is common practice to allow NaN values to propagate through calculations to
 indicate that an error has occurred.
- Use as a 'notable value' indicator. In some datasets, for example, tabular data, values may be missing. It is useful to use a value outside the normal numeric range to indicate the position of these values. Particularly when memory usage is a concern, as may be expected in applications where 8-bit formats are being considered, the use of a separate "mask" array, or a list of indices, imposes additional memory overhead. In some cases, Inf can be used as a missing value, but given the restricted range of binary8 formats, it is likely that infinity shall be used as a separate indicator of rounding from values outside of the finite range.
- The use of multiple NaN payloads is known in statistical code (e.g. the R system has NaN and N/A), but it is not widely used, and in the context of binary8, multiple NaNs impose either additional hardware complexity (using

Value	Hexadecimal Encoding	Bit Sequence
Zero	0x00	0000 0000
Positive Infinity (+Inf)	0x7F	0111 1111
Negative Infinity (-Inf)	OxFF	1111 1111
Not a Number (NaN)	0x80	1000 0000

Table 2: Mappings from special values to encodings, common to all binary8 formats.

only a subset of the significand range), or a large reduction in encoding space (e.g. 8 codes for E5, 16 codes for E4, 32 codes for E3).

2.3 Zero

Binary8 formats shall have exactly one zero. This zero value is nonnegative.

The inclusion of negative zero would incur the cost of an additional code point. Given the decision to encode only a single NaN, placing that NaN at the negative zero code point enables the strictly positive and strictly negative number ranges to be symmetric.

A key rationale for including -0 in IEEE-754 was the consistent implementation of branch cuts in the atan2 function [4, 5]. Although the atan function is common in deep learning, it is generally used as an activation function, rather than a trigonometric operation, and the atan2 function is rare, if not unknown, in deep learning applications. Furthermore, it is not expected that this standard shall define either atan or atan2.

A secondary reason for providing -0 is the hardware simplification offered by its presence in the implementation of sign/magnitude arithmetic. However, the existence of in-market implementations is evidence that the small hardware simplification has not been sufficient to balance the loss of one code point.

It might be considered that the use of integer comparisons in sorting would argue against placing NaN at the negative zero code point. For example, the JAX machine learning framework is known to sort using integer comparison [3]. However, such sorting still requires O(n) preprocessing and postprocessing steps to enable the use of twos-complement integer comparison, and already has special treatment of NaN and -0, so eliminating -0 and placing NaN in the -0 position imposes negligible additional burden. Sorting using comparison operations, as typically implemented, is undefined in the presence of NaNs. However, existing practice is to sort NaNs using totalOrder.

2.4 Infinities

Binary8 formats shall include positive and negative infinities.

This decision causes a reduction in dynamic range (252 values rather than 254), while offering improved numerical robustness in important machine learning use cases.

Two generic classes of such usage are:

- Mask values, for example, in Transformer models in machine learning [1].
- Representation of overflow.

As illustrated in Appendix A, both usages are facilitated by the presence of infinity.

2.5 Extremal Values

Format	minSubnormal	maxSubnormal	minNormal	maxNormal	maxFinite
p2	1×2^{-32}	1×2^{-32}	1×2^{-31}	1×2^{31}	1×2^{31}
p3	1×2^{-17}	$3/2 \times 2^{-16}$	1×2^{-15}	$3/2 \times 2^{15}$	$3/2 \times 2^{15}$
p4	1×2^{-10}	$7/4 \times 2^{-8}$	1×2^{-7}	$7/4 \times 2^{7}$	$7/4 \times 2^{7}$
p5	1×2^{-7}	$15/8 \times 2^{-4}$	1×2^{-3}	$15/8 \times 2^{3}$	$15/8 \times 2^{3}$
p6	1×2^{-6}	$31/16 \times 2^{-2}$	1×2^{-1}	$31/16 \times 2^{1}$	$31/16 \times 2^{1}$
p7	1×2^{-6}	$63/32 \times 2^{-1}$	1×2^0	$63/32 \times 2^{0}$	$63/32 \times 2^{0}$

Table 3: Extremal values

Table 3 shows these values in binary8 formats for 1 .

3 Classification operators

Conforming implementations shall provide these classification predicates and the classifier function. The classification predicates and the classifier function shall not signal exceptions.

The classification operators comprise: 1) a set of functions with a boolean return value, taking a single binary8 value as input; 2) a function class(x) that returns a single value of enumeration type, describing the input value's properties.

Predicates shall behave as follows:

Predicate	Definition
isZero isNaN isInfinite isFinite isNormal isSubnormal isSignMinus isCanonical isSignaling	iff ^a x is 0 iff x is NaN iff x is infinite iff x is zero, subnormal or normal iff x is normal, hence finite iff x is subnormal iff x has a negative sign ^b True ^c False ^d

Table 4: Classification Predicates

The Classifier function class(x) shall return enumeration values as follows:

Enumeration	Condition
NaN	isNaN(x)
Zero	isZero(x)
positiveInfinity	<pre>isInfinite(x) and not(isSignMinus(x))</pre>
positiveNormal	<pre>isNormal(x) and not(isSignMinus(x))</pre>
positiveSubnormal	<pre>isSubnormal(x) and not(isSignMinus(x))</pre>
negativeInfinity	isInfinite(x) and isSignMinus(x)
negativeNormal	isNormal(x) and isSignMinus(x)
negativeSubnormal	isSubnormal(x) and isSignMinus(x)

Table 5: Classifier Logic

aiff abbreviates "if and only if"

^bisSignMinus(NaN) is True: NaN is 0x80 (0b10000000).

^cThere are no non-canonical binary8 interchange formats.

^dAll binary8 formats have one NaN; it does not signal.

4 Comparison operators

Conforming implementations shall provide the following comparison operators and the totalOrder(x,y) function.

Comparison operators are two argument predicates and their negations that return True or False. Comparisons shall not raise exceptions. Comparisons are ordered or unordered. A comparison is unordered iff either argument is NaN. All other comparisons are ordered.

For $\{=, >, \geq, <, \leq, \leq\}$, if any argument is NaN, the result is False.

For $\{\neq, \not>, \not\geq, \not\leq, \not\leq, \not\leq\}$, if any argument is NaN, the result is True.

Otherwise, the result of a comparison shall match the mathematical result.

math symbol	predicate true relations	math symbol	negation true relations
=	CompareEqual equal	\neq , not $=$	CompareNotEqual less than, greater than, unordered
>	CompareGreater greater than	≯, NOT >	CompareNotGreater less than, equal, unordered
<u> </u>	CompareGreaterEqual equal, greater than	$\not\geq$, not \geq	CompareLessUnordered less than, unordered
<	CompareLess less than	$\not<$, NOT $<$	CompareNotLess greater than, equal, unordered
<u>≤</u>	CompareLessEqual less than, equal	$\not\leq$, not \leq	CompareGreaterUnordered greater than, unordered
\$	CompareOrdered less than, equal, greater than	≸, NOT ≶	CompareUnordered unordered

Table 6: Comparison Predicates and Negations

4.1 The totalOrder predicate

totalOrder(x, y) provides a total ordering over each binary8 format's value set.

The predicate totalOrder(x, y) shall return { True, False } in accord with the logic given below. It shall not raise any exceptions.

```
boolean totalOrder(x, y)
    if isNaN(x): return True
    if isNaN(y): return False
    return compareLessEqual(x, y)
end
```

Note: Following 754's definition of totalOrder(x, y), binary8 NaNs (0x80) compare as the most negative value. The most significant bit of NaN is set, so to be consistent with 754, NaN is ordered before all numerical values.

A Numerical Examples

A.1 Mask Values

A common use for ∞ is to create masks, for example, in Transformer models in machine learning, [1].

These values, assembled in mask matrix M with values $M_{ij}in\{0, -\infty\}$ are typically added to computed values A, in a computation such as:

$$\log(\sup(\exp(\tau * (A + M))))$$

where τ is a "temperature" or "base" parameter [2]. This calculation depends on the property $\exp(\tau * (A_{ij} - \infty)) = 0$.

If a floating point encoding does not provide infinity, then instead M_{ij} will be replaced by a large float (e.g. 480). This is not in itself a difficulty: if all the A values are bounded (e.g. the results of a softmax operation), then $\exp(1.0-480)$ is an extremely small number, which will certainly round to zero. Therefore, an explicit representation of infinity is *not* needed in order for this computation to yield its desired value.

However, careful implementations do not execute the calculation as written, and instead fuse the $\log(\text{sum}(\exp(v)))$ operation into a single operation $\log \text{sum}(\exp(v))$, whose implementation makes use of the identity transformation

$$logsumexp(v) \rightarrow logsumexp(v - max(v)) + max(v)$$

Without the "sticky" properties of Inf, this would produce incorrect answers. For example, in a format where MaxFloat=240 without Inf, and MaxFloat=224 with Inf:

$$logsumexp(\tau * [-224, -\infty]) \rightarrow logsumexp(\tau * [0, -\infty])$$

while

$$logsumexp(\tau * [-224, -240]) \rightarrow logsumexp(\tau * [0, -16])$$

If $\tau=1$ and all calculations are done in 8-bit floating point, then the answer will be the same, because $\exp(-16)\approx 1.1\times 10^{-7}$, which will round to zero in all precisions p>2; but if τ is small, or calculations are done in mixed precision, as is common with 8-bit floating point, the loss of "stickiness" will silently yield unexpected answers. It is not expected that the full calculation shall be done in 8-bit floating point, but the subtraction of the maximum value (and computation of the maximum) might reasonably be in 8-bit floating point.

A.2 Overflow to Infinity

A second use of infinity is to indicate overflow on conversion to the binary8 type. Existing implementations offer several behaviors on overflow: overflow to infinity, saturation to MaxFloat, and overflow to NaN. The existence of a code point for infinity allows any of these options to be implemented in a given instantiation, while removing the code point removes the possibility of implementing the first.

B Comparison table

This table summarizes the points of difference and agreement between the formats proposed in this document and a number of existing formats, some of which have hardware implementations.

OCP: Open Compute Platform [6], describing hardware implementations including nVidia, Intel, and ARM.

AGQ: AMD, Graphcore, Qualcomm[7], implemented in Graphcore's C600 product, and AMD's gfx940.

TSL: Tesla Dojo Technology [8], A Guide to Tesla's Configurable Floating Point Formats & Arithmetic

Format		P3109)	O	CP	A(GQ .	TS	SL
Subformat	P3	P4	P5	E5	E4	E5	E4	E4	E5
Special values shared by all subformats		Y		N	1	7	Y	1	1
Exactly one NaN		Y		N	1	,	Y	,	Y
Positive and negative infinity	Y		N	Y	N		1	1	
Include negative zero		N		N	1	7	Y	1	1
Max exponent emax	15	7	3	15	8	15	7	N/A	N/A

C Value Tables

Value tables mapping 8-bit strings to value sets are provided in this section.

A typical entry is of the form:

```
HEX BINARY = BINARY_FLOAT = DECIMAL

0x01 = 0_00000_01 = +0b0.01 x 2^-15 = 7.62939453125E-06
```

Where the fields are interpreted as follows:

HEX Hexadecimal encoding of the code point

BINARY Binary expansion of the code point, with underscores separating sign_exponent_significand

BINARY_FLOAT The precise float value as a binary fraction followed by 2^e with decimal exponent e

DECIMAL The decimal expansion of the value. If the decimal expansion is not an exact representation of the precise

float value, the preceding equals sign is replaced by "approximately equals" \approx .

C.1 Value Table: P3

0.00. 0.00000.00. 0.0	0.40 0.40000.00 1.014.0014.000 4.0	0.00 4.00000.00 11.11	0 0 4 40000 00 014 00 400
$0x00 = 0.00000_00 = 0.0$	$0x40 = 0_10000_00 = +0b1.00 \times 2^{\circ} = 1.0$	$0x80 = 1_00000_00 = NaN$	$0xc0 = 1.10000.00 = -0b1.00 \times 2^{\circ}0 = -1.0$
$0x01 = 0.00000.01 = +0b0.01 \times 2^{-15} \approx 7.6293945E-06$	$0x41 = 0_10000_01 = +0b1.01 \times 2^{\circ} = 1.25$	$0x81 = 1_00000_01 = -0b0.01 \times 2^-15 \approx -7.6293945E-06$	$0xc1 = 1_10000_01 = -0b1.01 \times 2^0 = -1.25$
$0x02 = 0.00000.10 = +0b0.10 \times 2^{-15} \approx 1.5258789E-05$	$0x42 = 0.10000.10 = +0b1.10 \times 20 = 1.5$	$0x82 = 1.00000.10 = -0b0.10 \times 2^{-15} \approx -1.5258789E-05$	$0xc2 = 1.10000.10 = -0b1.10 \times 2^{\circ}0 = -1.5$
$0x03 = 0.00000_{-11} = +0b0.11 \times 2^{-15} \approx 2.2888184E-05$	$0x43 = 0.10000_11 = +0b1.11 \times 2^{\circ} = 1.75$	$0x83 = 1.00000.11 = -0b0.11 \times 2^{-15} \approx -2.2888184E-05$	$0xc3 = 1.10000.11 = -0b1.11 \times 2^{\circ}0 = -1.75$
•			
$0x04 = 0.00001.00 = +0b1.00 \times 2^{-15} \approx 3.0517578E-05$	$0x44 = 0_10001_00 = +0b1.00 \times 2^1 = 2.0$	$0x84 = 1_00001_00 = -0b1.00 \times 2^-15 \approx -3.0517578E-05$	$0xc4 = 1_10001_00 = -0b1.00 \times 2^1 = -2.0$
$0x05 = 0.00001.01 = +0b1.01 \times 2^{-15} \approx 3.8146973E-05$	$0x45 = 0_10001_01 = +0b1.01 \times 2^1 = 2.5$	$0x85 = 1_00001_01 = -0b1.01 \times 2^-15 \approx -3.8146973E-05$	$0xc5 = 1_10001_01 = -0b1.01 \times 2^1 = -2.5$
$0x06 = 0.00001.10 = +0b1.10 \times 2^{-15} \approx 4.5776367E-05$	$0x46 = 0.10001.10 = +0b1.10 \times 2^1 = 3.0$	$0x86 = 1.00001.10 = -0b1.10 \times 2^{-15} \approx -4.5776367E-05$	$0xc6 = 1.10001.10 = -0b1.10 \times 2^1 = -3.0$
$0x07 = 0.00001_{-}11 = +0b1.11 \times 2^{-}15 \approx 5.3405762E_{-}05$	$0x47 = 0.10001.11 = +0b1.11 \times 2^1 = 3.5$	$0x87 = 1.00001.11 = -0b1.11 \times 2^-15 \approx -5.3405762E-05$	$0xc7 = 1.10001.11 = -0b1.11 \times 2^1 = -3.5$
$0x08 = 0.00010.00 = +0b1.00 \times 2^{-14} \approx 6.1035156E-05$	$0x48 = 0_10010_00 = +0b1.00 \times 2^2 = 4.0$	$0x88 = 1_00010_00 = -0b1.00 \times 2^-14 \approx -6.1035156E-05$	$0xc8 = 1_10010_00 = -0b1.00 \times 2^2 = -4.0$
$0x09 = 0.00010.01 = +0b1.01 \times 2^{-14} \approx 7.6293945E-05$	$0x49 = 0_10010_01 = +0b1.01 \times 2^2 = 5.0$	$0x89 = 1_00010_01 = -0b1.01 \times 2^-14 \approx -7.6293945E-05$	$0xc9 = 1_10010_01 = -0b1.01 \times 2^2 = -5.0$
$0x0a = 0.00010.10 = +0b1.10 \times 2^{-14} \approx 9.1552734E-05$	$0x4a = 0.10010.10 = +0b1.10 \times 2^2 = 6.0$	$0x8a = 1.00010.10 = -0b1.10 \times 2^{-14} \approx -9.1552734E-05$	$0xca = 1.10010.10 = -0b1.10 \times 2^2 = -6.0$
$0x0b = 0.00010.11 = +0b1.11 \times 2^-14 \approx 0.00010681152$	$0x4b = 0_10010_11 = +0b1.11 \times 2^2 = 7.0$	$0x8b = 1_{-}00010_{-}11 = -0b1.11 \times 2^{-}14 \approx -0.00010681152$	$0xcb = 1_10010_11 = -0b1.11 \times 2^2 = -7.0$
$0x0c = 0.00011.00 = +0b1.00 \times 2^{-13} \approx 0.00012207031$	$0x4c = 0_10011_00 = +0b1.00 \times 2^3 = 8.0$	$0x8c = 1_00011_00 = -0b1.00 \times 2^{-13} \approx -0.00012207031$	$0xcc = 1_10011_00 = -0b1.00 \times 2^3 = -8.0$
$0x0d = 0.00011.01 = +0b1.01 \times 2^{-13} \approx 0.00015258789$	$0x4d = 0_10011_01 = +0b1.01 \times 2^3 = 10.0$	$0x8d = 1_00011_01 = -0b1.01 \times 2^-13 \approx -0.00015258789$	$0xcd = 1_10011_01 = -0b1.01 \times 2^3 = -10.0$
$0x0e = 0.00011.10 = +0b1.10 \times 2^{-13} \approx 0.00018310547$	$0x4e = 0.10011.10 = +0b1.10 \times 2^3 = 12.0$	$0x8e = 1.00011.10 = -0b1.10 \times 2^{-13} \approx -0.00018310547$	$0xce = 1.10011.10 = -0b1.10 \times 2^3 = -12.0$
$0x0f = 0.00011.11 = +0b1.11 \times 2^-13 \approx 0.00021362305$	$0x4f = 0_10011_11 = +0b1.11 \times 2^3 = 14.0$	$0x8f = 1_{-}00011_{-}11 = -0b1.11 \times 2^{-}-13 \approx -0.00021362305$	$0xcf = 1_10011_11 = -0b1.11 \times 2^3 = -14.0$
$0x10 = 0.00100.00 = +0b1.00 \times 2^{-12} = 0.000244140625$	$0x50 = 0.10100.00 = +0b1.00 \times 2^4 = 16.0$	$0x90 = 1.00100.00 = -0b1.00 \times 2^{-12} \approx -0.00024414062$	$0xd0 = 1_10100_00 = -0b1.00 \times 2^4 = -16.0$
$0x11 = 0.00100.01 = +0b1.01 \times 2^{-12} \approx 0.00030517578$	$0x51 = 0_10100_01 = +0b1.01 \times 2^4 = 20.0$	$0x91 = 1_00100_01 = -0b1.01 \times 2^-12 \approx -0.00030517578$	$0xd1 = 1_10100_01 = -0b1.01 \times 2^4 = -20.0$
$0x12 = 0.00100_{-}10 = +0b1.10 \times 2^{-}-12 \approx 0.00036621094$	$0x52 = 0.10100.10 = +0b1.10 \times 2^4 = 24.0$	$0x92 = 1.00100.10 = -0b1.10 \times 2^{-12} \approx -0.00036621094$	$0xd2 = 1.10100.10 = -0b1.10 \times 2^4 = -24.0$
$0x13 = 0.00100.11 = +0b1.11 \times 2^{-12} \approx 0.00042724609$	$0x53 = 0.10100.11 = +0b1.11 \times 2^4 = 28.0$	$0x93 = 1.00100.11 = -0b1.11 \times 2^-12 \approx -0.00042724609$	$0xd3 = 1.10100.11 = -0b1.11 \times 2^4 = -28.0$
$0x14 = 0.00101.00 = +0b1.00 \times 2^{-11} = 0.00048828125$	$0x54 = 0_10101_00 = +0b1.00 \times 2^5 = 32.0$	$0x94 = 1.00101.00 = -0b1.00 \times 2^{-11} = -0.00048828125$	$0xd4 = 1_10101_00 = -0b1.00 \times 2^5 = -32.0$
$0x15 = 0.00101.01 = +0b1.01 \times 2^{-11} \approx 0.00061035156$	$0x55 = 0_10101_01 = +0b1.01 \times 2^5 = 40.0$	$0x95 = 1_00101_01 = -0b1.01 \times 2^-11 \approx -0.00061035156$	$0xd5 = 1_10101_01 = -0b1.01 \times 2^5 = -40.0$
$0x16 = 0.00101.10 = +0b1.10 \times 2^{-11} = 0.000732421875$	$0x56 = 0_{-}10101_{-}10 = +0b1.10 \times 2^{5} = 48.0$	$0x96 = 1.00101.10 = -0b1.10 \times 2^{-11} \approx -0.00073242188$	$0xd6 = 1.10101.10 = -0b1.10 \times 2^5 = -48.0$
$0x17 = 0.00101.11 = +0b1.11 \times 2^-11 \approx 0.00085449219$	$0x57 = 0.10101.11 = +0b1.11 \times 2^5 = 56.0$	$0x97 = 1.00101.11 = -0b1.11 \times 2^{-1}1 \approx -0.00085449219$	$0xd7 = 1.10101.11 = -0b1.11 \times 2^5 = -56.0$
$0x18 = 0.00110.00 = +0b1.00 \times 2^{-10} = 0.0009765625$	$0x58 = 0.10110.00 = +0b1.00 \times 2^{6} = 64.0$	$0x98 = 1.00110.00 = -0b1.00 \times 2^{-10} = -0.0009765625$	$0xd8 = 1.10110.00 = -0b1.00 \times 2^6 = -64.0$
$0x19 = 0.00110.01 = +0b1.01 \times 2^{-10} = 0.001220703125$	$0x59 = 0.10110.01 = +0b1.01 \times 2^6 = 80.0$	$0x99 = 1_00110_01 = -0b1.01 \times 2^-10 \approx -0.0012207031$	$0xd9 = 1_10110_01 = -0b1.01 \times 2^6 = -80.0$
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$0x1a = 0.00110.10 = +0b1.10 \times 2^-10 = 0.00146484375$	$0x5a = 0_{-}10110_{-}10 = +0b1.10 \times 2^{6} = 96.0$	$0x9a = 1.00110.10 = -0b1.10 \times 2^{-1}0 = -0.00146484375$	$0xda = 1.10110.10 = -0b1.10 \times 2^6 = -96.0$
$0x1b = 0.00110.11 = +0b1.11 \times 2^{-10} = 0.001708984375$	$0x5b = 0_{-}10110_{-}11 = +0b1.11 \times 2^{6} = 112.0$	$0x9b = 1_00110_11 = -0b1.11 \times 2^-10 \approx -0.0017089844$	$0xdb = 1.10110.11 = -0b1.11 \times 2^6 = -112.0$
$0x1c = 0.00111.00 = +0b1.00 \times 2^{-9} = 0.001953125$	$0x5c = 0_10111_00 = +0b1.00 \times 27 = 128.0$	$0x9c = 1.00111.00 = -0b1.00 \times 2^-9 = -0.001953125$	$0xdc = 1_10111_00 = -0b1.00 \times 27 = -128.0$
$0x1d = 0.00111.01 = +0b1.01 \times 2^{-9} = 0.00244140625$	$0x5d = 0.10111.01 = +0b1.01 \times 2.7 = 160.0$	$0x9d = 1.00111.01 = -0b1.01 \times 2^{-9} = -0.00244140625$	$0xdd = 1_10111_101 = -0b1.01 \times 2^7 = -160.0$
$0x1e = 0.00111.10 = +0b1.10 \times 2^-9 = 0.0029296875$	$0x5e = 0_{-}10111_{-}10 = +0b1.10 \times 27 = 192.0$	$0x9e = 1.00111.10 = -0b1.10 \times 2^-9 = -0.0029296875$	$0xde = 1_10111_10 = -0b1.10 \times 27 = -192.0$
$0x1f = 0.00111.11 = +0b1.11 \times 2^{-9} = 0.00341796875$	$0x5f = 0.10111.11 = +0b1.11 \times 27 = 224.0$	$0x9f = 1.00111.11 = -0b1.11 \times 2^{-9} = -0.00341796875$	$0xdf = 1.10111.11 = -0b1.11 \times 27 = -224.0$
$0x20 = 0_01000_00 = +0b1.00 \times 2^-8 = 0.00390625$	$0x60 = 0_11000_00 = +0b1.00 \times 28 = 256.0$	$0xa0 = 1_01000_00 = -0b1.00 \times 2^-8 = -0.00390625$	$0xe0 = 1.11000.00 = -0b1.00 \times 2^8 = -256.0$
$0x21 = 0.01000.01 = +0b1.01 \times 2^{-8} = 0.0048828125$	$0x61 = 0.11000.00 = 001.00 \times 2.0 = 200.0$ $0x61 = 0.11000.01 = +0b1.01 \times 2.8 = 320.0$	$0xa1 = 1.01000.01 = -0b1.01 \times 2^{-8} = -0.0048828125$	$0xe1 = 1.11000.00 = 0b1.00 \times 2.0 = 200.0$ $0xe1 = 1.11000.01 = -0b1.01 \times 2.8 = -320.0$
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$0x22 = 0.01000.10 = +0b1.10 \times 2^{-8} = 0.005859375$	$0x62 = 0.11000.10 = +0b1.10 \times 2^8 = 384.0$	$0xa2 = 1_01000_10 = -0b1.10 \times 2^-8 = -0.005859375$	$0xe2 = 1.11000.10 = -0b1.10 \times 2^8 = -384.0$
$0x23 = 0.01000.11 = +0b1.11 \times 2^{-8} = 0.0068359375$	$0x63 = 0.11000.11 = +0b1.11 \times 2\% = 448.0$	$0xa3 = 1_01000_11 = -0b1.11 \times 2^-8 = -0.0068359375$	$0xe3 = 1.11000.11 = -0b1.11 \times 2^8 = -448.0$
$0x24 = 0.01001.00 = +0b1.00 \times 2^{-7} = 0.0078125$	$0x64 = 0_11001_00 = +0b1.00 \times 2^9 = 512.0$	$0xa4 = 1_01001_00 = -0b1.00 \times 2^-7 = -0.0078125$	$0xe4 = 1_11001_00 = -0b1.00 \times 2^9 = -512.0$
$0x25 = 0.01001.01 = +0b1.01 \times 2^{-7} = 0.009765625$	$0x65 = 0_11001_01 = +0b1.01 \times 2^9 = 640.0$	$0xa5 = 1_01001_01 = -0b1.01 \times 2^-7 = -0.009765625$	$0xe5 = 1_11001_01 = -0b1.01 \times 2^9 = -640.0$
$0x26 = 0.01001.10 = +0b1.10 \times 2^{-7} = 0.01171875$	$0x66 = 0.11001.10 = +0b1.10 \times 2^9 = 768.0$	$0xa6 = 1.01001.10 = -0b1.10 \times 2^-7 = -0.01171875$	$0xe6 = 1_11001_10 = -0b1.10 \times 2^9 = -768.0$
$0x27 = 0.01001.11 = +0b1.11 \times 2^{-7} = 0.013671875$	$0x67 = 0.11001.11 = +0b1.11 \times 2^9 = 896.0$	$0xa7 = 1.01001.11 = -0b1.11 \times 2^{-7} = -0.013671875$	$0xe7 = 1.11001.11 = -0b1.11 \times 2^9 = -896.0$
$0x28 = 0.01010.00 = +0b1.00 \times 2^{-6} = 0.015625$	$0x68 = 0_11010_00 = +0b1.00 \times 2^10 = 1024.0$	$0xa8 = 1_01010_00 = -0b1.00 \times 2^-6 = -0.015625$	$0xe8 = 1_11010_00 = -0b1.00 \times 2^10 = -1024.0$
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$0x29 = 0_01010_01 = +0b1.01 \times 2^-6 = 0.01953125$	$0x69 = 0_11010_01 = +0b1.01 \times 2^10 = 1280.0$	$0xa9 = 1_01010_01 = -0b1.01 \times 2^-6 = -0.01953125$	$0xe9 = 1_11010_01 = -0b1.01 \times 2^10 = -1280.0$
$0x2a = 0.01010.10 = +0b1.10 \times 2^{-6} = 0.0234375$	$0x6a = 0_{-}11010_{-}10 = +0b1.10 \times 2^{-}10 = 1536.0$	$0xaa = 1_01010_10 = -0b1.10 \times 2^-6 = -0.0234375$	$0xea = 1_{-}11010_{-}10 = -0b1.10 \times 2^{1}0 = -1536.0$
$0x2b = 0.01010.11 = +0b1.11 \times 2^{-6} = 0.02734375$	$0x6b = 0.11010.11 = +0b1.11 \times 2^10 = 1792.0$	$0xab = 1_01010_11 = -0b1.11 \times 2^-6 = -0.02734375$	$0xeb = 1_11010_11 = -0b1.11 \times 2^10 = -1792.0$
$0x2c = 0.01011.00 = +0b1.00 \times 2^{-5} = 0.03125$	$0x6c = 0_11011_00 = +0b1.00 \times 2^11 = 2048.0$	$0xac = 1_01011_00 = -0b1.00 \times 2^{-5} = -0.03125$	$0 \text{xec} = 1.11011.00 = -0b1.00 \times 2^11 = -2048.0$
			$0 \text{xed} = 1.11011.00 = 001.00 \times 2.11 = 2010.00$ $0 \text{xed} = 1.11011.01 = -001.01 \times 2^{1}1 = -2560.00$
$0x2d = 0.01011.01 = +0b1.01 \times 2^{-5} = 0.0390625$	$0x6d = 0_11011_01 = +0b1.01 \times 2^11 = 2560.0$	$0xad = 1_01011_01 = -0b1.01 \times 2^-5 = -0.0390625$	
$0x2e = 0.01011.10 = +0b1.10 \times 2^{-5} = 0.046875$	$0x6e = 0_11011_10 = +0b1.10 \times 2^11 = 3072.0$	$0xae = 1_01011_10 = -0b1.10 \times 2^5 = -0.046875$	$0xee = 1.11011.10 = -0b1.10 \times 2^11 = -3072.0$
$0x2f = 0.01011.11 = +0b1.11 \times 2^{-5} = 0.0546875$	$0x6f = 0.11011.11 = +0b1.11 \times 2^11 = 3584.0$	$0xaf = 1_01011_11 = -0b1.11 \times 2^5 = -0.0546875$	$0xef = 1_11011_11 = -0b1.11 \times 2^11 = -3584.0$
$0x30 = 0.01100.00 = +0b1.00 \times 2^{-4} = 0.0625$	$0x70 = 0.11100.00 = +0b1.00 \times 2^12 = 4096.0$	$0xb0 = 1_01100_00 = -0b1.00 \times 2^-4 = -0.0625$	$0xf0 = 1_11100_00 = -0b1.00 \times 2^12 = -4096.0$
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$0x31 = 0.01100.01 = +0b1.01 \times 2^{-4} = 0.078125$	$0x71 = 0_11100_01 = +0b1.01 \times 2^12 = 5120.0$	$0xb1 = 1_01100_01 = -0b1.01 \times 2^-4 = -0.078125$	$0xf1 = 1_11100_01 = -0b1.01 \times 2^12 = -5120.0$
$0x32 = 0.01100.10 = +0b1.10 \times 2^{-4} = 0.09375$	$0x72 = 0_{-}11100_{-}10 = +0b1.10 \times 2^{-}12 = 6144.0$	$0xb2 = 1_01100_10 = -0b1.10 \times 2^4 = -0.09375$	$0xf2 = 1.11100.10 = -0b1.10 \times 2^12 = -6144.0$
$0x33 = 0.01100.11 = +0b1.11 \times 2^{-4} = 0.109375$	$0x73 = 0_11100_11 = +0b1.11 \times 2^12 = 7168.0$	$0xb3 = 1_01100_11 = -0b1.11 \times 2^-4 = -0.109375$	$0xf3 = 1_11100_11 = -0b1.11 \times 2^12 = -7168.0$
$0x34 = 0_01101_00 = +0b1.00 \times 2^{-3} = 0.125$	$0x74 = 0_11101_00 = +0b1.00 \times 2^13 = 8192.0$	$0xb4 = 1_01101_00 = -0b1.00 \times 2^{-3} = -0.125$	$0xf4 = 1_11101_00 = -0b1.00 \times 2^13 = -8192.0$
$0x35 = 0_01101_01 = +0b1.01 \times 2^-3 = 0.15625$	$0x75 = 0_11101_01 = +0b1.01 \times 2^13 = 10240.0$	$0xb5 = 1_01101_01 = -0b1.01 \times 2^-3 = -0.15625$	$0xf5 = 1_11101_01 = -0b1.01 \times 2^13 = -10240.0$
$0x36 = 0.01101.10 = +0b1.10 \times 2^{-3} = 0.1875$	$0x76 = 0.11101.10 = +0b1.10 \times 2^13 = 12288.0$	$0xb6 = 1_01101_10 = -0b1.10 \times 2^3 = -0.1875$	$0xf6 = 1_{-}11101_{-}10 = -0b1.10 \times 2^{-}13 = -12288.0$
$0x37 = 0_01101_11 = +0b1.11 \times 2^3 = 0.21875$	$0x77 = 0.11101.11 = +0b1.11 \times 2^13 = 14336.0$	$0xb7 = 1_01101_11 = -0b1.11 \times 2^{-3} = -0.21875$	$0xf7 = 1.11101.11 = -0b1.11 \times 2^13 = -14336.0$
$0x38 = 0.01110.00 = +0b1.00 \times 2^{-2} = 0.25$	$0x78 = 0.11110_00 = +0b1.00 \times 2^14 = 16384.0$	$0xb8 = 1_01110_00 = -0b1.00 \times 2^2 = -0.25$	$0xf8 = 1_11110_00 = -0b1.00 \times 2^14 = -16384.0$
$0x39 = 0_01110_01 = +0b1.01 \times 2^-2 = 0.3125$	$0x79 = 0_11110_01 = +0b1.01 \times 2^14 = 20480.0$	$0xb9 = 1_01110_01 = -0b1.01 \times 2^2 = -0.3125$	$0xf9 = 1_11110_01 = -0b1.01 \times 2^14 = -20480.0$
$0x3a = 0.01110.10 = +0b1.10 \times 2^{-2} = 0.375$	$0x7a = 0.11110.10 = +0b1.10 \times 2^14 = 24576.0$	$0xba = 1_01110_10 = -0b1.10 \times 2^2 = -0.375$	$0xfa = 1_11110_10 = -0b1.10 \times 2^14 = -24576.0$
$0x3b = 0.01110.11 = +0b1.11 \times 2^{-2} = 0.4375$	$0x7b = 0_11110_11 = +0b1.11 \times 2^14 = 28672.0$	$0xbb = 1_01110_11 = -0b1.11 \times 2^2 = -0.4375$	$0xfb = 1.11110.11 = -0b1.11 \times 2^14 = -28672.0$
$0x3c = 0.01111.00 = +0b1.00 \times 2^{-1} = 0.5$	$0x7c = 0.11111.00 = +0b1.00 \times 2^{15} = 32768.0$	$0xbc = 1_01111_00 = -0b1.00 \times 2^-1 = -0.5$	$0xfc = 1.11111.00 = -0b1.00 \times 2^15 = -32768.0$
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$0x3d = 0_01111_01 = +0b1.01 \times 2^-1 = 0.625$	$0x7d = 0_11111_01 = +0b1.01 \times 2^15 = 40960.0$	$0xbd = 1_01111_01 = -0b1.01 \times 2^-1 = -0.625$	$0xfd = 1_11111_01 = -0b1.01 \times 2^15 = -40960.0$
$0x3e = 0_01111_10 = +0b1.10 \times 2^-1 = 0.75$	$0x7e = 0_11111_10 = +0b1.10 \times 215 = 49152.0$	$0xbe = 1_01111_10 = -0b1.10 \times 2^-1 = -0.75$	$0xfe = 1_11111_10 = -0b1.10 \times 2^15 = -49152.0$
$0x3f = 0_01111_11 = +0b1.11 \times 2^-1 = 0.875$	$0x7f = 0_11111_11 = +Inf$	$0xbf = 1_01111_11 = -0b1.11 \times 2^-1 = -0.875$	$0xff = 1_11111_11 = -Inf$
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C.2 Value Table: P4

0x00 = 0.0000.000 = 0.0	$0x40 = 0.1000.000 = +0b1.000 \times 20 = 1.0$	$0x80 = 1_0000_000 = NaN$	$0xc0 = 1_1000_000 = -0b1.000 \times 2^0 = -1.0$
$0x01 = 0.0000.001 = +0b0.001 \times 2^{-7} = 0.0009765625$	$0x41 = 0_1000_001 = +0b1.001 \times 20 = 1.125$	$0x81 = 1.0000.001 = -0b0.001 \times 2^{-7} = -0.0009765625$	$0xc1 = 1.1000.001 = -0b1.001 \times 2^{\circ}0 = -1.125$
$0x02 = 0.0000.010 = +0b0.010 \times 2^{-7} = 0.001953125$	$0x42 = 0.1000.010 = +0b1.010 \times 2\% = 1.25$	$0x82 = 1.0000.010 = -0b0.010 \times 2^{-7} = -0.001953125$	$0xc2 = 1.1000.010 = -0b1.010 \times 2^{\circ} = -1.25$
$0x03 = 0.0000.011 = +0b0.011 \times 2^{-7} = 0.0029296875$	$0x43 = 0.1000.011 = +0b1.011 \times 20 = 1.375$	$0x83 = 1.0000.011 = -0b0.011 \times 2^{-7} = -0.0029296875$	$0xc3 = 1.1000.011 = -0b1.011 \times 2^{\circ}0 = -1.375$
$0x04 = 0.0000 \cdot 100 = +0b0 \cdot 100 \times 2^{-7} = 0.00390625$	$0x44 = 0.1000.100 = +0b1.100 \times 20 = 1.5$	$0x84 = 1.0000.100 = -0b0.100 \times 2^{-7} = -0.00390625$	$0xc4 = 1_1000_100 = -0b1.100 \times 2^{\circ}0 = -1.5$
$0x05 = 0.0000_101 = +0b0.101 \times 2^{-7} = 0.0048828125$	$0x45 = 0_1000_101 = +0b1.101 \times 2^{\circ} = 1.625$	$0x85 = 1_0000_101 = -0b0.101 \times 2^{-7} = -0.0048828125$	$0xc5 = 1_1000_101 = -0b1.101 \times 2^{\circ} = -1.625$
$0x06 = 0.0000.110 = +0b0.110 \times 2^{-7} = 0.005859375$	$0x46 = 0.1000.110 = +0b1.110 \times 2^{\circ} = 1.75$	$0x86 = 1.0000.110 = -0b0.110 \times 2^{-7} = -0.005859375$	$0xc6 = 1.1000.110 = -0b1.110 \times 2^{\circ} = -1.75$
$0x07 = 0.0000.111 = +0b0.111 \times 2^{-7} = 0.0068359375$	$0x47 = 0.1000.111 = +0b1.111 \times 2^{\circ} = 1.875$	$0x87 = 1.0000.111 = -0b0.111 \times 2^{-7} = -0.0068359375$	$0xc7 = 1.1000.111 = -0b1.111 \times 2^{\circ} = -1.875$
$0x08 = 0.0001.000 = +0b1.000 \times 2^{-7} = 0.0078125$	$0x48 = 0.1001.000 = +0b1.000 \times 2^{2} = 2.0$	$0x88 = 1_0001_000 = -0b1.000 \times 2^-7 = -0.0078125$	$0xc8 = 1_1001_000 = -0b1.000 \times 2^1 = -2.0$
$0x09 = 0.0001.001 = +0b1.001 \times 2^{-7} = 0.0087890625$	$0x49 = 0_1001_001 = +0b1.001 \times 2^1 = 2.25$	$0x89 = 1_0001_001 = -0b1.001 \times 2^{-7} = -0.0087890625$	$0xc9 = 1_1001_001 = -0b1.001 \times 2^1 = -2.25$
$0x0a = 0.0001.010 = +0b1.010 \times 2^{-7} = 0.009765625$	$0x4a = 0.1001.010 = +0b1.010 \times 2^1 = 2.5$	$0x8a = 1.0001.010 = -0b1.010 \times 2^{-7} = -0.009765625$	$0xca = 1.1001.010 = -0b1.010 \times 2^1 = -2.5$
$0x0b = 0.0001.011 = +0b1.011 \times 2^{-7} = 0.0107421875$	$0x4b = 0_1001_011 = +0b1.011 \times 2^1 = 2.75$	$0x8b = 1.0001.011 = -0b1.011 \times 2^{-7} = -0.0107421875$	$0xcb = 1.1001.011 = -0b1.011 \times 2^1 = -2.75$
$0x0c = 0.0001.100 = +0b1.100 \times 2^{-7} = 0.01171875$	$0x4c = 0_1001_100 = +0b1.100 \times 2^1 = 3.0$	$0x8c = 1_0001_100 = -0b1.100 \times 2^-7 = -0.01171875$	$0xcc = 1.1001.100 = -0b1.100 \times 2^1 = -3.0$
$0x0d = 0.0001.101 = +0b1.101 \times 2^{-7} = 0.0126953125$	$0x4d = 0_1001_101 = +0b1.101 \times 2^1 = 3.25$	$0x8d = 1.0001.101 = -0b1.101 \times 2^{-7} = -0.0126953125$	$0xcd = 1_1001_101 = -0b1.101 \times 2^1 = -3.25$
$0x0e = 0.0001.110 = +0b1.110 \times 2^{-7} = 0.013671875$	$0x4e = 0.1001.110 = +0b1.110 \times 2^1 = 3.5$	$0x8e = 1.0001.110 = -0b1.110 \times 2^{-7} = -0.013671875$	$0xce = 1.1001.110 = -0b1.110 \times 2^1 = -3.5$
$0x0f = 0_0001_111 = +0b1.111 \times 2^{-7} = 0.0146484375$	$0x4f = 0.1001.111 = +0b1.111 \times 2^1 = 3.75$	$0x8f = 1.0001.111 = -0b1.111 \times 2^{-7} = -0.0146484375$	$0xcf = 1.1001.111 = -0b1.111 \times 2^1 = -3.75$
$0x10 = 0.0010.000 = +0b1.000 \times 2^{-6} = 0.015625$	$0x50 = 0_1010_000 = +0b1.000 \times 2^2 = 4.0$	$0x90 = 1.0010.000 = -0b1.000 \times 2^{-6} = -0.015625$	$0xd0 = 1_1010_000 = -0b1.000 \times 2^2 = -4.0$
$0x11 = 0_0010_001 = +0b1.001 \times 2^-6 = 0.017578125$	$0x51 = 0_1010_001 = +0b1.001 \times 2^2 = 4.5$	$0x91 = 1_0010_001 = -0b1.001 \times 2^-6 = -0.017578125$	$0xd1 = 1_1010_001 = -0b1.001 \times 2^2 = -4.5$
$0x12 = 0.0010.010 = +0b1.010 \times 2^{-6} = 0.01953125$	$0x52 = 0.1010.010 = +0b1.010 \times 2^2 = 5.0$	$0x92 = 1.0010.010 = -0b1.010 \times 2^{-6} = -0.01953125$	$0xd2 = 1.1010.010 = -0b1.010 \times 2^2 = -5.0$
$0x13 = 0.0010.011 = +0b1.011 \times 2^{-6} = 0.021484375$	$0x53 = 0_1010_011 = +0b1.011 \times 2^2 = 5.5$	$0x93 = 1.0010.011 = -0b1.011 \times 2^{-6} = -0.021484375$	$0xd3 = 1.1010.011 = -0b1.011 \times 2^2 = -5.5$
$0x14 = 0.0010.100 = +0b1.100 \times 2^{-6} = 0.0234375$	$0x54 = 0_1010_100 = +0b1.100 \times 2^2 = 6.0$	$0x94 = 1_0010_100 = -0b1.100 \times 2^-6 = -0.0234375$	$0xd4 = 1.1010.100 = -0b1.100 \times 2^2 = -6.0$
$0x15 = 0.0010.101 = +0b1.101 \times 2^{-6} = 0.025390625$	$0x55 = 0_1010_101 = +0b1.101 \times 2^2 = 6.5$	$0x95 = 1_0010_101 = -0b1.101 \times 2^-6 = -0.025390625$	$0xd5 = 1_1010_101 = -0b1.101 \times 2^2 = -6.5$
$0x16 = 0.0010.110 = +0b1.110 \times 2^{-6} = 0.02734375$	$0x56 = 0.1010.110 = +0b1.110 \times 2^2 = 7.0$	$0x96 = 1.0010.110 = -0b1.110 \times 2^{-6} = -0.02734375$	$0xd6 = 1.1010.110 = -0b1.110 \times 2^2 = -7.0$
$0x17 = 0.0010.111 = +0b1.111 \times 2^{-6} = 0.029296875$	$0x57 = 0_1010_111 = +0b1.111 \times 2^2 = 7.5$	$0x97 = 1.0010.111 = -0b1.111 \times 2^{-6} = -0.029296875$	$0xd7 = 1_1010_111 = -0b1.111 \times 2^2 = -7.5$
$0x18 = 0_0011_000 = +0b1.000 \times 2^5 = 0.03125$	$0x58 = 0_1011_000 = +0b1.000 \times 2^3 = 8.0$	$0x98 = 1_0011_000 = -0b1.000 \times 2^{-5} = -0.03125$	$0xd8 = 1_1011_000 = -0b1.000 \times 2^3 = -8.0$
$0x19 = 0_0011_001 = +0b1.001 \times 2^5 = 0.03515625$	$0x59 = 0_1011_001 = +0b1.001 \times 2^3 = 9.0$	$0x99 = 1_0011_001 = -0b1.001 \times 2^{-5} = -0.03515625$	$0xd9 = 1_1011_001 = -0b1.001 \times 2^3 = -9.0$
$0x1a = 0.0011.010 = +0b1.010 \times 2^{-5} = 0.0390625$	$0x5a = 0.1011.010 = +0b1.010 \times 2^3 = 10.0$	$0x9a = 1.0011.010 = -0b1.010 \times 2^{-5} = -0.0390625$	$0xda = 1.1011.010 = -0b1.010 \times 2^3 = -10.0$
$0x1b = 0.0011.011 = +0b1.011 \times 2^{-5} = 0.04296875$	$0x5b = 0_1011_011 = +0b1.011 \times 2^3 = 11.0$	$0x9b = 1.0011.011 = -0b1.011 \times 2^{-5} = -0.04296875$	$0xdb = 1.1011.011 = -0b1.011 \times 2^3 = -11.0$
$0x1c = 0.0011.100 = +0b1.100 \times 2^{-5} = 0.046875$	$0x5c = 0_1011_100 = +0b1.100 \times 2^3 = 12.0$	$0x9c = 1.0011.100 = -0b1.100 \times 2^{-5} = -0.046875$	$0xdc = 1_{-1}011_{-1}00 = -0b1.100 \times 2^{3} = -12.0$
$0x1d = 0.0011.101 = +0b1.101 \times 2^{-5} = 0.05078125$	$0x5d = 0_1011_101 = +0b1.101 \times 2^3 = 13.0$	$0x9d = 1.0011.101 = -0b1.101 \times 2^{-5} = -0.05078125$	$0xdd = 1_{-}1011_{-}101 = -0b1.101 \times 2^{3} = -13.0$
$0x1e = 0.0011.110 = +0b1.110 \times 2^{-5} = 0.0546875$	$0x5e = 0.1011.110 = +0b1.110 \times 2^3 = 14.0$	$0x9e = 1.0011.110 = -0b1.110 \times 2^{-5} = -0.0546875$	$0xde = 1.1011.110 = -0b1.110 \times 2^3 = -14.0$
$0x1f = 0.0011.111 = +0b1.111 \times 2^{-5} = 0.05859375$	$0x5f = 0.1011.111 = +0b1.111 \times 2^3 = 15.0$	$0x9f = 1.0011.111 = -0b1.111 \times 2^{-5} = -0.05859375$	$0xdf = 1.1011.111 = -0b1.111 \times 2^3 = -15.0$
$0x20 = 0.0100.000 = +0b1.000 \times 2^{-4} = 0.0625$	$0x60 = 0.1100.000 = +0b1.000 \times 2^4 = 16.0$	$0xa0 = 1.0100.000 = -0b1.000 \times 2^{-4} = -0.0625$	$0 \times 0 = 1.1100.000 = -0b1.000 \times 2^4 = -16.0$
$0x21 = 0.0100.001 = +0b1.001 \times 2^{-4} = 0.0703125$	$0x61 = 0.1100.001 = +0b1.001 \times 2^4 = 18.0$	$0xa1 = 1.0100.001 = -0b1.001 \times 2^{-4} = -0.0703125$	$0xe1 = 1_1100_001 = -0b1.001 \times 2^4 = -18.0$
$0x22 = 0.0100.010 = +0b1.010 \times 2^{-4} = 0.078125$	$0x62 = 0.1100.010 = +0b1.010 \times 2^4 = 20.0$	$0xa2 = 1.0100.010 = -0b1.010 \times 2^{-4} = -0.078125$	$0 \times 2 = 1.1100.010 = -0b1.010 \times 2^4 = -20.0$
$0x23 = 0.0100_{-}011 = +0b1.011 \times 2^{-}4 = 0.0859375$ $0x24 = 0.0100_{-}100 = +0b1.100 \times 2^{-}4 = 0.09375$	$0x63 = 0.1100.011 = +0b1.011 \times 2^4 = 22.0$ $0x64 = 0.1100.100 = +0b1.100 \times 2^4 = 24.0$	$0xa3 = 1.0100.011 = -0b1.011 \times 2^-4 = -0.0859375$ $0xa4 = 1.0100.100 = -0b1.100 \times 2^-4 = -0.09375$	$0xe3 = 1.1100.011 = -0b1.011 \times 2^4 = -22.0$ $0xe4 = 1.1100.100 = -0b1.100 \times 2^4 = -24.0$
$0x24 = 0.0100 \cdot 100 = +0b1.100 \times 2^{-4} = 0.09375$ $0x25 = 0.0100 \cdot 101 = +0b1.101 \times 2^{-4} = 0.1015625$	$0x64 = 0.1100.100 = +0b1.100 \times 24 = 24.0$ $0x65 = 0.1100.101 = +0b1.101 \times 24 = 26.0$	$0xa4 = 1.0100.100 = -001.100 \times 2 - 4 = -0.09375$ $0xa5 = 1.0100.101 = -001.101 \times 2^{-4} = -0.1015625$	$0xe4 = 1.1100.100 = -0b1.100 \times 24 = -24.0$ $0xe5 = 1.1100.101 = -0b1.101 \times 24 = -26.0$
$0x25 = 0.0100 \cdot 101 = +0b1.101 \times 2^{-4} = 0.1013025$ $0x26 = 0.0100 \cdot 110 = +0b1.110 \times 2^{-4} = 0.109375$	$0x66 = 0.1100.101 = +0b1.101 \times 2^4 = 28.0$ $0x66 = 0.1100.110 = +0b1.110 \times 2^4 = 28.0$	$0xa6 = 1.0100.101 = -0b1.101 \times 2^{-4} = -0.1013023$ $0xa6 = 1.0100.110 = -0b1.110 \times 2^{-4} = -0.109375$	$0xe6 = 1.1100.101 = -0b1.101 \times 24 = -28.0$ $0xe6 = 1.1100.110 = -0b1.110 \times 24 = -28.0$
$0x20 = 0.0100.110 = +0b1.110 \times 2^{-4} = 0.109373$ $0x27 = 0.0100.111 = +0b1.111 \times 2^{-4} = 0.1171875$	$0x67 = 0.1100.111 = +0b1.111 \times 2^4 = 20.0$ $0x67 = 0.1100.111 = +0b1.111 \times 2^4 = 30.0$	$0xa7 = 1.0100.111 = -0b1.111 \times 2^{-4} = -0.1171875$ $0xa7 = 1.0100.111 = -0b1.111 \times 2^{-4} = -0.1171875$	$0xe0 = 1.1100.110 = -0b1.110 \times 24 = -20.0$ $0xe7 = 1.1100.111 = -0b1.111 \times 24 = -30.0$
$0x28 = 0.0101.000 = +0b1.000 \times 2^{-3} = 0.125$	$0x68 = 0.1101.000 = +0b1.000 \times 2^5 = 32.0$	$0xa8 = 1.0101.000 = -0b1.000 \times 2^{-3} = -0.125$	$0xe8 = 1.1100.111 = 0b1.111 \times 2.4 = 30.0$ $0xe8 = 1.1101.000 = -0b1.000 \times 2.5 = -32.0$
$0x29 = 0.0101.000 = +0b1.000 \times 2^{-3} = 0.140625$	$0x69 = 0.1101.000 = +0b1.000 \times 2.5 = 36.0$	$0xa9 = 1.0101.000 = 001.000 \times 2^{-3} = 0.140625$	$0xe9 = 1.1101.001 = -0b1.001 \times 2^5 = -36.0$
$0x2a = 0.0101.010 = +0b1.010 \times 2^{-3} = 0.15625$	$0x6a = 0.1101.010 = +0b1.010 \times 2^5 = 40.0$	$0xaa = 1.0101.010 = -0b1.010 \times 2^{-3} = -0.15625$	$0 = 1.1101.001 = 0.01.001 \times 2.5 = 0.001 \times $
$0x2b = 0.0101.011 = +0b1.011 \times 2^{-3} = 0.171875$	$0x6b = 0.1101.011 = +0b1.011 \times 2^5 = 44.0$	$0xab = 1.0101.011 = -0b1.011 \times 2^{-3} = -0.171875$	$0 \text{xeb} = 1.1101.011 = -0 \text{b} 1.011 \times 2^5 = -44.0$
$0x2c = 0.0101.100 = +0b1.100 \times 2^{-3} = 0.1875$	$0x6c = 0.1101.100 = +0b1.100 \times 2^5 = 48.0$	$0 \text{xac} = 1.0101.100 = -0b1.100 \times 2^{-3} = -0.1875$	$0 \text{xec} = 1.1101.100 = -0b1.100 \times 25 = -48.0$
$0x2d = 0.0101.101 = +0b1.101 \times 2^{-3} = 0.203125$	$0x6d = 0_11101_101 = +0b1.101 \times 2^5 = 52.0$	$0xad = 1_0101_101 = -0b1.101 \times 2^-3 = -0.203125$	$0 \text{xed} = 1.1101.101 = -0b1.101 \times 2^5 = -52.0$
$0x2e = 0.0101.110 = +0b1.110 \times 2^{-3} = 0.21875$	$0x6e = 0.1101.110 = +0b1.110 \times 25 = 56.0$	$0xae = 1.0101.110 = -0b1.110 \times 2^{-3} = -0.21875$	$0 \text{xee} = 1.1101.110 = -0 \text{b} 1.110 \times 2^5 = -56.0$
$0x2f = 0.0101.111 = +0b1.111 \times 2^{-3} = 0.234375$	$0x6f = 0.1101.111 = +0b1.111 \times 25 = 60.0$	$0xaf = 1.0101.111 = -0b1.111 \times 2^{-3} = -0.234375$	$0xef = 1.1101.111 = -0b1.111 \times 2^5 = -60.0$
$0x30 = 0_0110_000 = +0b1.000 \times 2^2 = 0.25$	$0x70 = 0_1110_000 = +0b1.000 \times 26 = 64.0$	$0xb0 = 1_0110_000 = -0b1.000 \times 2^2 = -0.25$	$0xf0 = 1_1110_000 = -0b1.000 \times 2^6 = -64.0$
$0x31 = 0_0110_001 = +0b1.001 \times 2^2 = 0.28125$	$0x71 = 0_1110_001 = +0b1.001 \times 26 = 72.0$	$0xb1 = 1_0110_001 = -0b1.001 \times 2^2 = -0.28125$	$0xf1 = 1_1110_001 = -0b1.001 \times 2^6 = -72.0$
$0x32 = 0.0110.010 = +0b1.010 \times 2^{-2} = 0.3125$	$0x72 = 0.1110.010 = +0b1.010 \times 26 = 80.0$	$0xb2 = 1.0110.010 = -0b1.010 \times 2^{-2} = -0.3125$	$0xf2 = 1_1110_010 = -0b1.010 \times 2^6 = -80.0$
$0x33 = 0_0110_011 = +0b1.011 \times 2^2 = 0.34375$	$0x73 = 0_1110_011 = +0b1.011 \times 26 = 88.0$	$0xb3 = 1_0110_011 = -0b1.011 \times 2^2 = -0.34375$	$0xf3 = 1_1110_011 = -0b1.011 \times 2^6 = -88.0$
$0x34 = 0_0110_100 = +0b1.100 \times 2^2 = 0.375$	$0x74 = 0_1110_100 = +0b1.100 \times 26 = 96.0$	$0xb4 = 1_0110_100 = -0b1.100 \times 2^2 = -0.375$	$0xf4 = 1_1110_100 = -0b1.100 \times 2^6 = -96.0$
$0x35 = 0_0110_101 = +0b1.101 \times 2^2 = 0.40625$	$0x75 = 0_1110_101 = +0b1.101 \times 26 = 104.0$	$0xb5 = 1_0110_101 = -0b1.101 \times 2^2 = -0.40625$	$0xf5 = 1_1110_101 = -0b1.101 \times 2^6 = -104.0$
$0x36 = 0_0110_110 = +0b1.110 \times 2^2 = 0.4375$	$0x76 = 0.1110.110 = +0b1.110 \times 2^{6} = 112.0$	$0xb6 = 1.0110.110 = -0b1.110 \times 2^{-2} = -0.4375$	$0xf6 = 1.1110.110 = -0b1.110 \times 2^6 = -112.0$
$0x37 = 0_0110_111 = +0b1.111 \times 2^2 = 0.46875$	$0x77 = 0_1110_111 = +0b1.111 \times 26 = 120.0$	$0xb7 = 1_0110_111 = -0b1.111 \times 2^2 = -0.46875$	$0xf7 = 1_1110_111 = -0b1.111 \times 2^6 = -120.0$
$0x38 = 0_0111_000 = +0b1.000 \times 2^-1 = 0.5$	$0x78 = 0_11111_000 = +0b1.000 \times 27 = 128.0$	$0xb8 = 1_0111_000 = -0b1.000 \times 2^-1 = -0.5$	$0xf8 = 1_1111_000 = -0b1.000 \times 27 = -128.0$
$0x39 = 0_0111_001 = +0b1.001 \times 2^-1 = 0.5625$	$0x79 = 0_1111_001 = +0b1.001 \times 27 = 144.0$	$0xb9 = 1_0111_001 = -0b1.001 \times 2^-1 = -0.5625$	$0xf9 = 1_1111_001 = -0b1.001 \times 27 = -144.0$
$0x3a = 0.0111.010 = +0b1.010 \times 2^{-1} = 0.625$	$0x7a = 0.1111.010 = +0b1.010 \times 27 = 160.0$	$0xba = 1.0111.010 = -0b1.010 \times 2^-1 = -0.625$	$0xfa = 1.1111.010 = -0b1.010 \times 27 = -160.0$
$0x3b = 0.0111.011 = +0b1.011 \times 2^{-1} = 0.6875$	$0x7b = 0.1111.011 = +0b1.011 \times 27 = 176.0$	$0xbb = 1.0111.011 = -0b1.011 \times 2^-1 = -0.6875$	$0xfb = 1_{-1111}_{-011} = -0b1.011 \times 27 = -176.0$
$0x3c = 0.0111.100 = +0b1.100 \times 2^{-1} = 0.75$	$0x7c = 0_{-1}111_{-1}00 = +0b1.100 \times 27 = 192.0$	$0xbc = 1.0111.100 = -0b1.100 \times 2^{-1} = -0.75$	$0xfc = 1_{-1111_{-100}} = -0b1_{-100} \times 27 = -192_{-100}$
$0x3d = 0.0111.101 = +0b1.101 \times 2^{-1} = 0.8125$	$0x7d = 0_1111_101 = +0b1.101 \times 27 = 208.0$	$0xbd = 1_0111_101 = -0b1.101 \times 2^{-1} = -0.8125$	$0xfd = 1_1111_101 = -0b1.101 \times 27 = -208.0$
$0x3e = 0.0111.110 = +0b1.110 \times 2^{-1} = 0.875$	$0x7e = 0.1111.110 = +0b1.110 \times 2^7 = 224.0$	$0xbe = 1.0111.110 = -0b1.110 \times 2^{-1} = -0.875$	$0xfe = 1.1111.110 = -0b1.110 \times 27 = -224.0$
$0x3f = 0_0111_111 = +0b1.111 \times 2^-1 = 0.9375$	0x7f = 0.1111.111 = +Inf	$0xbf = 1_0111_111 = -0b1.111 \times 2^-1 = -0.9375$	0xff = 1.1111.111 = -Inf

C.3 Value Table: P5

```
0x00 = 0.000_0000 = 0.0
                                                            0x40 = 0.100.0000 = +0b1.0000 \times 2^{\circ} = 1.0
                                                                                                                  0x80 = 1.000.0000 = NaN
                                                                                                                                                                                0xc0 = 1.100.0000 = -0b1.0000 \times 2^{\circ}0 = -1.0
0x01 = 0.000.0001 = +0b0.0001 \times 2^{-3} = 0.0078125
                                                           0x41 = 0.100.0001 = +0b1.0001 \times 2^{\circ}0 = 1.0625
                                                                                                                  0x81 = 1_000_0001 = -0b0.0001 \times 2^{-3} = -0.0078125
                                                                                                                                                                               0xc1 = 1_100_0001 = -0b1.0001 \times 2^{\circ} = -1.0625
0x02 = 0.000.0010 = +0b0.0010 \times 2^{-3} = 0.015625
                                                            0x42 = 0.100.0010 = +0b1.0010 \times 2^{\circ}0 = 1.125
                                                                                                                  0x82 = 1.000.0010 = -0b0.0010 \times 2^{-3} = -0.015625
                                                                                                                                                                                0xc2 = 1.100.0010 = -0b1.0010 \times 2^{\circ}0 = -1.125
0x03 = 0.000.0011 = +0b0.0011 \times 2^{-3} = 0.0234375
                                                           0x43 = 0.100.0011 = +0b1.0011 \times 2^{\circ} = 1.1875
                                                                                                                  0x83 = 1.000 \cdot 0011 = -0b0 \cdot 0011 \times 2^{-3} = -0.0234375
                                                                                                                                                                               0xc3 = 1.100.0011 = -0b1.0011 \times 2^{\circ}0 = -1.1875
0x04 = 0.000.0100 = +0b0.0100 \times 2^{-3} = 0.03125
                                                            0x44 = 0.100.0100 = +0b1.0100 \times 2^{\circ} = 1.25
                                                                                                                  0x84 = 1.000.0100 = -0b0.0100 \times 2^{-3} = -0.03125
                                                                                                                                                                                0xc4 = 1.100.0100 = -0b1.0100 \times 2^{\circ} = -1.25
0x05 = 0.000_0101 = +0b0.0101 \times 2^{-3} = 0.0390625
                                                           0x45 = 0.100.0101 = +0b1.0101 \times 2^{\circ} = 1.3125
                                                                                                                  0x85 = 1_000_0101 = -0b0.0101 \times 2^{-3} = -0.0390625
                                                                                                                                                                               0xc5 = 1_100_0101 = -0b1.0101 \times 2^{\circ} = -1.3125
0x06 = 0.000.0110 = +0b0.0110 \times 2^{-3} = 0.046875
                                                           0x46 = 0.100.0110 = +0b1.0110 \times 2^{\circ}0 = 1.375
                                                                                                                  0x86 = 1.000.0110 = -0b0.0110 \times 2^{-3} = -0.046875
                                                                                                                                                                               0xc6 = 1.100.0110 = -0b1.0110 \times 2^{\circ} = -1.375
0x07 = 0.000.0111 = +0b0.0111 \times 2^{-3} = 0.0546875
                                                           0x47 = 0.100.0111 = +0b1.0111 \times 2^{\circ} = 1.4375
                                                                                                                  0x87 = 1.000_0111 = -0b0.0111 \times 2^{-3} = -0.0546875
                                                                                                                                                                               0xc7 = 1.100.0111 = -0b1.0111 \times 2^{\circ} = -1.4375
0x08 = 0.000_1000 = +0b0.1000 \times 2^{-3} = 0.0625
                                                                                                                  0x88 = 1.000.1000 = -0b0.1000 \times 2^{-3} = -0.0625
                                                                                                                                                                                0xc8 = 1_100_1000 = -0b1.1000 \times 2^{\circ} = -1.5
                                                            0x48 = 0_100_1000 = +0b1.1000 \times 2^0 = 1.5
                                                           0x49 = 0_100_1001 = +0b1.1001 \times 2^{\circ}0 = 1.5625
0x09 = 0.000_1001 = +0b0.1001 \times 2^{-3} = 0.0703125
                                                                                                                  0x89 = 1_000_1001 = -0b0.1001 \times 2^{-3} = -0.0703125
                                                                                                                                                                               0xc9 = 1_100_1001 = -0b1.1001 \times 20 = -1.5625
0x0a = 0.000_{-}1010 = +0b0.1010 \times 2^{-}3 = 0.078125
                                                            0x4a = 0.100.1010 = +0b1.1010 \times 2^{\circ}0 = 1.625
                                                                                                                  0x8a = 1.000.1010 = -0b0.1010 \times 2^{-3} = -0.078125
                                                                                                                                                                                0xca = 1.100.1010 = -0b1.1010 \times 2^{\circ}0 = -1.625
0x0b = 0.000 \cdot 1011 = +0b0 \cdot 1011 \times 2^{-3} = 0.0859375
                                                            0x4b = 0.100.1011 = +0b1.1011 \times 2^{\circ} = 1.6875
                                                                                                                  0x8b = 1.000.1011 = -0b0.1011 \times 2^{-3} = -0.0859375
                                                                                                                                                                                0xcb = 1.100.1011 = -0b1.1011 \times 2^{\circ}0 = -1.6875
0x0c = 0.000_{-}1100 = +0b0.1100 \times 2^{-}3 = 0.09375
                                                            0x4c = 0_100_1100 = +0b1.1100 \times 2^{\circ} = 1.75
                                                                                                                  0x8c = 1_000_1100 = -0b0.1100 \times 2^{-3} = -0.09375
                                                                                                                                                                                0xcc = 1.100.1100 = -0b1.1100 \times 2^{\circ} = -1.75
0x0d = 0.000_1101 = +0b0.1101 \times 2^{-3} = 0.1015625
                                                                                                                  0x8d = 1_000_1101 = -0b0.1101 \times 2^{-3} = -0.1015625
                                                                                                                                                                               0xcd = 1_100_1101 = -0b1.1101 \times 2^0 = -1.8125
                                                           0x4d = 0.100.1101 = +0b1.1101 \times 2^{\circ} = 1.8125
0x0e = 0.000_{-}1110 = +0b0.1110 \times 2^{-}3 = 0.109375
                                                            0x4e = 0.100.1110 = +0b1.1110 \times 2^{\circ}0 = 1.875
                                                                                                                  0x8e = 1.000.1110 = -0b0.1110 \times 2^{-3} = -0.109375
                                                                                                                                                                                0xce = 1.100.1110 = -0b1.1110 \times 2^{\circ} = -1.875
0x0f = 0.000.1111 = +0b0.1111 \times 2^{-3} = 0.1171875
                                                           0x4f = 0.100.1111 = +0b1.1111 \times 2^{\circ} = 1.9375
                                                                                                                  0x8f = 1.000_11111 = -0b0.1111 \times 2^{-3} = -0.1171875
                                                                                                                                                                               0xcf = 1.100.1111 = -0b1.1111 \times 2^{\circ} = -1.9375
0x10 = 0.001.0000 = +0b1.0000 \times 2^{-3} = 0.125
                                                            0x50 = 0.101.0000 = +0b1.0000 \times 2^1 = 2.0
                                                                                                                  0x90 = 1.001.0000 = -0b1.0000 \times 2^{-3} = -0.125
                                                                                                                                                                                0xd0 = 1.101.0000 = -0b1.0000 \times 2^1 = -2.0
0x11 = 0.001.0001 = +0b1.0001 \times 2^{-3} = 0.1328125
                                                           0x51 = 0_101_0001 = +0b1.0001 \times 2^1 = 2.125
                                                                                                                  0x91 = 1_001_0001 = -0b1.0001 \times 2^{-3} = -0.1328125
                                                                                                                                                                               0xd1 = 1_101_0001 = -0b1.0001 \times 2^1 = -2.125
0x12 = 0.001.0010 = +0b1.0010 \times 2^{-3} = 0.140625
                                                            0x52 = 0.101.0010 = +0b1.0010 \times 2^1 = 2.25
                                                                                                                  0x92 = 1.001.0010 = -0b1.0010 \times 2^{-3} = -0.140625
                                                                                                                                                                                0xd2 = 1.101.0010 = -0b1.0010 \times 2^1 = -2.25
0x13 = 0.001.0011 = +0b1.0011 \times 2^{-3} = 0.1484375
                                                           0x53 = 0.101.0011 = +0b1.0011 \times 2^1 = 2.375
                                                                                                                  0x93 = 1.001.0011 = -0b1.0011 \times 2^{-3} = -0.1484375
                                                                                                                                                                               0xd3 = 1.101.0011 = -0b1.0011 \times 2^1 = -2.375
0x14 = 0_001_0100 = +0b1.0100 \times 2^{-3} = 0.15625
                                                            0x54 = 0.101.0100 = +0b1.0100 \times 2^1 = 2.5
                                                                                                                  0x94 = 1_001_0100 = -0b1.0100 \times 2^{-3} = -0.15625
                                                                                                                                                                                0xd4 = 1.101.0100 = -0b1.0100 \times 2^1 = -2.5
0x15 = 0.001.0101 = +0b1.0101 \times 2^{-3} = 0.1640625
                                                           0x55 = 0_101_0101 = +0b1.0101 \times 2^1 = 2.625
                                                                                                                  0x95 = 1_001_0101 = -0b1.0101 \times 2^{-3} = -0.1640625
                                                                                                                                                                                0xd5 = 1.101.0101 = -0b1.0101 \times 2^1 = -2.625
0x16 = 0.001.0110 = +0b1.0110 \times 2^{-3} = 0.171875
                                                            0x56 = 0.101.0110 = +0b1.0110 \times 2^1 = 2.75
                                                                                                                  0x96 = 1.001.0110 = -0b1.0110 \times 2^{-3} = -0.171875
                                                                                                                                                                                0xd6 = 1.101.0110 = -0b1.0110 \times 2^1 = -2.75
0x17 = 0.001.0111 = +0b1.0111 \times 2^{-3} = 0.1796875
                                                           0x57 = 0.101.0111 = +0b1.0111 \times 2^1 = 2.875
                                                                                                                  0x97 = 1.001.0111 = -0b1.0111 \times 2^{-3} = -0.1796875
                                                                                                                                                                               0xd7 = 1.101.0111 = -0b1.0111 \times 2^1 = -2.875
0x18 = 0.001.1000 = +0b1.1000 \times 2^{-3} = 0.1875
                                                            0x58 = 0_101_1000 = +0b1.1000 \times 2^1 = 3.0
                                                                                                                  0x98 = 1.001.1000 = -0b1.1000 \times 2^{-3} = -0.1875
                                                                                                                                                                                0xd8 = 1_101_1000 = -0b1.1000 \times 2^1 = -3.0
0x19 = 0.001_1001 = +0b1.1001 \times 2^{-3} = 0.1953125
                                                           0x59 = 0_101_1001 = +0b1.1001 \times 2^1 = 3.125
                                                                                                                  0x99 = 1_001_1001 = -0b1.1001 \times 2^{-3} = -0.1953125
                                                                                                                                                                               0xd9 = 1_101_1001 = -0b1.1001 \times 2^1 = -3.125
0x1a = 0.001.1010 = +0b1.1010 \times 2^{-3} = 0.203125
                                                            0x5a = 0.101.1010 = +0b1.1010 \times 2^1 = 3.25
                                                                                                                  0x9a = 1.001.1010 = -0b1.1010 \times 2^{-3} = -0.203125
                                                                                                                                                                               0xda = 1.101.1010 = -0b1.1010 \times 2^1 = -3.25
0x1b = 0.001.1011 = +0b1.1011 \times 2^{-3} = 0.2109375
                                                           0x5b = 0.101.1011 = +0b1.1011 \times 2^1 = 3.375
                                                                                                                  0x9b = 1.001.1011 = -0b1.1011 \times 2^{-3} = -0.2109375
                                                                                                                                                                               0xdb = 1.101.1011 = -0b1.1011 \times 2^1 = -3.375
0x1c = 0.001.1100 = +0b1.1100 \times 2^{-3} = 0.21875
                                                            0x5c = 0_101_1100 = +0b1.1100 \times 2^1 = 3.5
                                                                                                                  0x9c = 1_001_1100 = -0b1.1100 \times 2^3 = -0.21875
                                                                                                                                                                               0xdc = 1_101_1100 = -0b1.1100 \times 2^1 = -3.5
0 \texttt{x1d} = 0\_001\_1101 = +0 \texttt{b1.1101} \times 2 \hat{\ \ } -3 = 0.2265625
                                                                                                                  0x9d = 1_001_1101 = -0b1.1101 \times 2^-3 = -0.2265625
                                                           0x5d = 0_101_1101 = +0b1.1101 \times 2^1 = 3.625
                                                                                                                                                                               0xdd = 1_101_1101 = -0b1.1101 \times 2^1 = -3.625
0x1e = 0.001.1110 = +0b1.1110 \times 2^{-3} = 0.234375
                                                           0x5e = 0.101.1110 = +0b1.1110 \times 2^1 = 3.75
                                                                                                                  0x9e = 1.001.1110 = -0b1.1110 \times 2^{-3} = -0.234375
                                                                                                                                                                                0xde = 1.101.1110 = -0b1.1110 \times 2^1 = -3.75
0x1f = 0.001.1111 = +0b1.1111 \times 2^{-3} = 0.2421875
                                                           0x5f = 0.101.1111 = +0b1.1111 \times 2^1 = 3.875
                                                                                                                  0x9f = 1.001.1111 = -0b1.1111 \times 2^{-3} = -0.2421875
                                                                                                                                                                                0xdf = 1.101.1111 = -0b1.1111 \times 2^1 = -3.875
0x20 = 0.010.0000 = +0b1.0000 \times 2^{-2} = 0.25
                                                            0x60 = 0_110_0000 = +0b1.0000 \times 2^2 = 4.0
                                                                                                                  0xa0 = 1.010.0000 = -0b1.0000 \times 2^{-2} = -0.25
                                                                                                                                                                                0xe0 = 1.110.0000 = -0b1.0000 \times 2^2 = -4.0
0x21 = 0_010_0001 = +0b1.0001 \times 2^2 = 0.265625
                                                            0x61 = 0_110_0001 = +0b1.0001 \times 2^2 = 4.25
                                                                                                                  0xa1 = 1_010_0001 = -0b1.0001 \times 2^2 = -0.265625
                                                                                                                                                                                0xe1 = 1_110_0001 = -0b1.0001 \times 2^2 = -4.25
0x22 = 0.010.0010 = +0b1.0010 \times 2^{-2} = 0.28125
                                                            0x62 = 0.110.0010 = +0b1.0010 \times 2^2 = 4.5
                                                                                                                  0xa2 = 1.010.0010 = -0b1.0010 \times 2^{-2} = -0.28125
                                                                                                                                                                                0xe2 = 1.110.0010 = -0b1.0010 \times 2^2 = -4.5
0x23 = 0.010.0011 = +0b1.0011 \times 2^{-2} = 0.296875
                                                            0x63 = 0.110.0011 = +0b1.0011 \times 2^2 = 4.75
                                                                                                                  0xa3 = 1.010.0011 = -0b1.0011 \times 2^{-2} = -0.296875
                                                                                                                                                                               0xe3 = 1.110.0011 = -0b1.0011 \times 2^2 = -4.75
0x24 = 0_010_0100 = +0b1.0100 \times 2^2 = 0.3125
                                                            0x64 = 0_110_0100 = +0b1.0100 \times 2^2 = 5.0
                                                                                                                  0xa4 = 1_010_0100 = -0b1.0100 \times 2^2 = -0.3125
                                                                                                                                                                                0xe4 = 1_110_0100 = -0b1.0100 \times 2^2 = -5.0
0x25 = 0.010.0101 = +0b1.0101 \times 2^{-2} = 0.328125
                                                           0x65 = 0_110_0101 = +0b1.0101 \times 2^2 = 5.25
                                                                                                                  0xa5 = 1_010_0101 = -0b1.0101 \times 2^2 = -0.328125
                                                                                                                                                                               0xe5 = 1_110_0101 = -0b1.0101 \times 2^2 = -5.25
0x26 = 0.010.0110 = +0b1.0110 \times 2^{-2} = 0.34375
                                                            0x66 = 0.110.0110 = +0b1.0110 \times 2^2 = 5.5
                                                                                                                  0xa6 = 1.010.0110 = -0b1.0110 \times 2^{-2} = -0.34375
                                                                                                                                                                               0xe6 = 1_110_0110 = -0b1.0110 \times 2^2 = -5.5
0x27 = 0_010_0111 = +0b1.0111 \times 2^2 = 0.359375
                                                           0x67 = 0.110.0111 = +0b1.0111 \times 2^2 = 5.75
                                                                                                                  0xa7 = 1.010.0111 = -0b1.0111 \times 2^{-2} = -0.359375
                                                                                                                                                                               0xe7 = 1.110.0111 = -0b1.0111 \times 2^2 = -5.75
0x28 = 0_010_1000 = +0b1.1000 \times 2^-2 = 0.375
                                                            0x68 = 0_110_1000 = +0b1.1000 \times 2^2 = 6.0
                                                                                                                  0xa8 = 1_010_1000 = -0b1.1000 \times 2^2 = -0.375
                                                                                                                                                                                0xe8 = 1_110_1000 = -0b1.1000 \times 2^2 = -6.0
0x29 = 0_010_1001 = +0b1.1001 \times 2^2 = 0.390625
                                                           0x69 = 0_110_1001 = +0b1.1001 \times 2^2 = 6.25
                                                                                                                  0xa9 = 1_010_1001 = -0b1.1001 \times 2^2 = -0.390625
                                                                                                                                                                                0xe9 = 1_110_1001 = -0b1.1001 \times 2^2 = -6.25
                                                            0x6a = 0.110.1010 = +0b1.1010 \times 2^2 = 6.5
0x2a = 0.010.1010 = +0b1.1010 \times 2^{-2} = 0.40625
                                                                                                                  0xaa = 1.010.1010 = -0b1.1010 \times 2^{-2} = -0.40625
                                                                                                                                                                                0xea = 1.110.1010 = -0b1.1010 \times 2^2 = -6.5
0x2b = 0.010.1011 = +0b1.1011 \times 2^{-2} = 0.421875
                                                           0x6b = 0.110.1011 = +0b1.1011 \times 2^2 = 6.75
                                                                                                                  0xab = 1.010.1011 = -0b1.1011 \times 2^{-2} = -0.421875
                                                                                                                                                                                0xeb = 1.110.1011 = -0b1.1011 \times 2^2 = -6.75
0x2c = 0_010_1100 = +0b1.1100 \times 2^2 = 0.4375
                                                            0x6c = 0_110_1100 = +0b1.1100 \times 2^2 = 7.0
                                                                                                                  0xac = 1_010_1100 = -0b1.1100 \times 2^2 = -0.4375
                                                                                                                                                                                0 \text{xec} = 1.110.1100 = -0b1.1100 \times 2^2 = -7.0
0x2d = 0.010.1101 = +0b1.1101 \times 2^{-2} = 0.453125
                                                                                                                  0xad = 1_010_1101 = -0b1.1101 \times 2^2 = -0.453125
                                                                                                                                                                               0xed = 1_110_1101 = -0b1.1101 \times 2^2 = -7.25
                                                           0x6d = 0_110_1101 = +0b1.1101 \times 2^2 = 7.25
0x2e = 0.010.1110 = +0b1.1110 \times 2^{-2} = 0.46875
                                                            0x6e = 0.110.1110 = +0b1.1110 \times 2^2 = 7.5
                                                                                                                  0xae = 1.010.1110 = -0b1.1110 \times 2^2 = -0.46875
                                                                                                                                                                                0xee = 1.110.1110 = -0b1.1110 \times 2^2 = -7.5
0x2f = 0.010.1111 = +0b1.1111 \times 2^{-2} = 0.484375
                                                           0x6f = 0_110_1111 = +0b1.1111 \times 2^2 = 7.75
                                                                                                                  0xaf = 1.010.1111 = -0b1.1111 \times 2^{-2} = -0.484375
                                                                                                                                                                               0xef = 1.110.1111 = -0b1.1111 \times 2^2 = -7.75
0x30 = 0.011.0000 = +0b1.0000 \times 2^{-1} = 0.5
                                                           0x70 = 0_111_0000 = +0b1.0000 \times 2^3 = 8.0
                                                                                                                  0xb0 = 1_011_0000 = -0b1.0000 \times 2^{-1} = -0.5
                                                                                                                                                                               0xf0 = 1_111_0000 = -0b1.0000 \times 2^3 = -8.0
0x31 = 0_011_0001 = +0b1.0001 \times 2^-1 = 0.53125
                                                           0x71 = 0_111_0001 = +0b1.0001 \times 2^3 = 8.5
                                                                                                                  0xb1 = 1_011_0001 = -0b1.0001 \times 2^-1 = -0.53125
                                                                                                                                                                               0xf1 = 1_111_0001 = -0b1.0001 \times 2^3 = -8.5
0x32 = 0.011.0010 = +0b1.0010 \times 2^-1 = 0.5625
                                                           0x72 = 0.111.0010 = +0b1.0010 \times 2^3 = 9.0
                                                                                                                  0xb2 = 1.011.0010 = -0b1.0010 \times 2^{-1} = -0.5625
                                                                                                                                                                               0xf2 = 1.111.0010 = -0b1.0010 \times 2^3 = -9.0
0x33 = 0.011.0011 = +0b1.0011 \times 2^{-1} = 0.59375
                                                           0x73 = 0_111_0011 = +0b1.0011 \times 2^3 = 9.5
                                                                                                                  0xb3 = 1_011_0011 = -0b1.0011 \times 2^-1 = -0.59375
                                                                                                                                                                               0 {\tt xf3} = 1\_111\_0011 = -0 {\tt b1.0011} {\times} 2 {\tt \^{-}3} = -9.5
0x34 = 0.011.0100 = +0b1.0100 \times 2^{-1} = 0.625
                                                            0x74 = 0.111.0100 = +0b1.0100 \times 2^3 = 10.0
                                                                                                                  0xb4 = 1_011_0100 = -0b1.0100 \times 2^-1 = -0.625
                                                                                                                                                                                0xf4 = 1_111_0100 = -0b1.0100 \times 2^3 = -10.0
0x35 = 0.011.0101 = +0b1.0101 \times 2^{-1} = 0.65625
                                                            0x75 = 0.111_0101 = +0b1.0101 \times 2^3 = 10.5
                                                                                                                  0xb5 = 1_011_0101 = -0b1.0101 \times 2^-1 = -0.65625
                                                                                                                                                                                0xf5 = 1_111_0101 = -0b1.0101 \times 2^3 = -10.5
0x36 = 0.011.0110 = +0b1.0110 \times 2^{-1} = 0.6875
                                                            0x76 = 0.111.0110 = +0b1.0110 \times 2^3 = 11.0
                                                                                                                  0xb6 = 1.011.0110 = -0b1.0110 \times 2^{-1} = -0.6875
                                                                                                                                                                                0xf6 = 1_111_0110 = -0b1.0110 \times 2^3 = -11.0
0x37 = 0.011.0111 = +0b1.0111 \times 2^-1 = 0.71875
                                                            0x77 = 0.111.0111 = +0b1.0111 \times 2^3 = 11.5
                                                                                                                  0xb7 = 1_011_0111 = -0b1.0111 \times 2^-1 = -0.71875
                                                                                                                                                                                0xf7 = 1.111.0111 = -0b1.0111 \times 2^3 = -11.5
                                                                                                                  0xb8 = 1_011_1000 = -0b1.1000 \times 2^-1 = -0.75
0x38 = 0_011_1000 = +0b1.1000 \times 2^-1 = 0.75
                                                                                                                                                                                0xf8 = 1_111_1000 = -0b1.1000 \times 2^3 = -12.0
                                                            0x78 = 0.111.1000 = +0b1.1000 \times 2^3 = 12.0
0x39 = 0_011_1001 = +0b1.1001 \times 2^-1 = 0.78125
                                                            0x79 = 0_111_1001 = +0b1.1001 \times 2^3 = 12.5
                                                                                                                  0xb9 = 1_011_1001 = -0b1.1001 \times 2^-1 = -0.78125
                                                                                                                                                                                0xf9 = 1_111_1001 = -0b1.1001 \times 2^3 = -12.5
0x3a = 0.011.1010 = +0b1.1010 \times 2^{-1} = 0.8125
                                                            0x7a = 0.111.1010 = +0b1.1010 \times 2^3 = 13.0
                                                                                                                  0xba = 1.011.1010 = -0b1.1010 \times 2^{-1} = -0.8125
                                                                                                                                                                                0xfa = 1.111.1010 = -0b1.1010 \times 2^3 = -13.0
0x3b = 0_011_1011 = +0b1.1011 \times 2^-1 = 0.84375
                                                           0x7b = 0_111_1011 = +0b1.1011 \times 2^3 = 13.5
                                                                                                                  0xbb = 1_011_1011 = -0b1.1011 \times 2^-1 = -0.84375
                                                                                                                                                                               0xfb = 1_111_1011 = -0b1.1011 \times 2^3 = -13.5
0x3c = 0_011_1100 = +0b1.1100 \times 2^{-1} = 0.875
                                                            0x7c = 0_111_1100 = +0b1.1100 \times 2^3 = 14.0
                                                                                                                  0xbc = 1_011_1100 = -0b1.1100 \times 2^{-1} = -0.875
                                                                                                                                                                               0xfc = 1_111_1100 = -0b1.1100 \times 2^3 = -14.0
0x3d = 0_011_1101 = +0b1.1101 \times 2^-1 = 0.90625
                                                           \mathtt{0x7d} = \mathtt{0\_111\_1101} = +\mathtt{0b1.1101} {\times} \mathtt{2^3} = \mathtt{14.5}
                                                                                                                  0 \texttt{xbd} = 1\_011\_1101 = -0 \texttt{b}1.1101 \times 2 \hat{} -1 = -0.90625
                                                                                                                                                                               0xfd = 1_111_1101 = -0b1.1101 \times 2^3 = -14.5
                                                           0x7e = 0.111.1110 = +0b1.1110 \times 2^3 = 15.0
                                                                                                                  0 \texttt{xbe} = 1 \text{-} 011 \text{-} 1110 = -0 \text{b} 1.1110 \times 2 \text{-} 1 = -0.9375
                                                                                                                                                                               0xfe = 1.111.1110 = -0b1.1110 \times 2^3 = -15.0
0x3e = 0.011.1110 = +0b1.1110 \times 2^{-1} = 0.9375
0x3f = 0_011_1111 = +0b1.1111 \times 2^-1 = 0.96875
                                                           0x7f = 0_{-}111_{-}1111 = +Inf
                                                                                                                  0xbf = 1_011_1111 = -0b1.1111 \times 2^-1 = -0.96875
                                                                                                                                                                               0xff = 1_111_1111 = -Inf
```

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