Code Figures

Sept 2016
T. Hales.

Some figures that help to document the computer code.

Some calculations that were too minor to include in the article.

$\boxed{\text{Notation updates}}$

$\varepsilon_\mu = \varepsilon_0'$

$\varepsilon_\nu = \varepsilon_0$.

$m_+ = m_1 \quad m_- = m_2 \quad n_+ = n_1 \quad n_- = n_2$.

$a_{cut} = a_K$

$a_{min} = a_0$

$area_\eta = areta$.

$\bar{l} = l$

$K = c = \rho$.

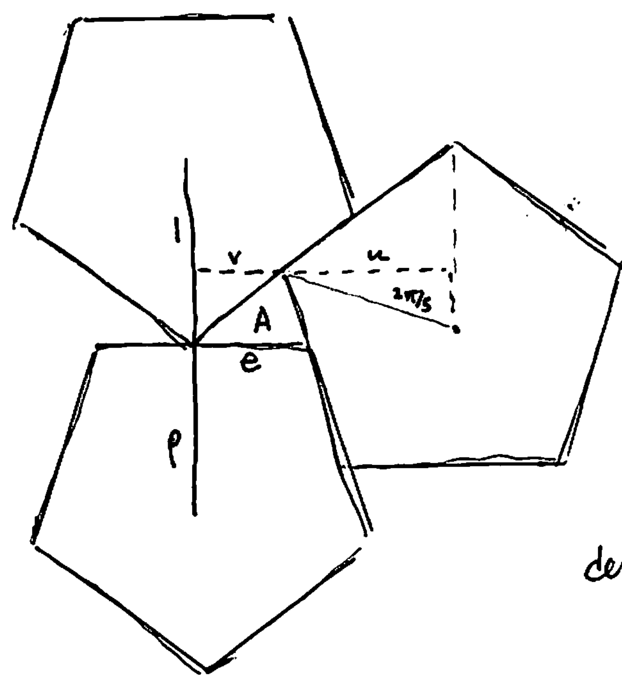
Area of Delaunay triangle in a double lattice.

$\kappa = \rho = \cos \pi/5 = 0.809\ldots.$

$\sigma = e = \sin \pi/5. = 0.587785\ldots$

$u = \sin 2\pi/5 = 2\rho e$

$u + v = 3\rho e.$



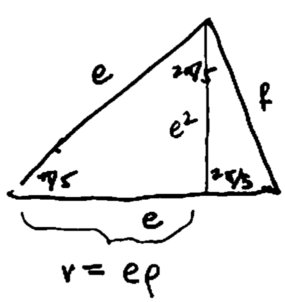
$$\boxed{\begin{aligned} \text{area} &= \tfrac{1}{2}(1+\rho)(3\rho e) \\ &= 1.29036\ldots. \end{aligned}}$$

$$\text{density} = \frac{5\rho e/2}{(1+\rho)(3\rho e)/2}$$

$$= \frac{5}{3(1+\rho)}$$

$$= \frac{1}{3}(5-\sqrt{5}). = 0.921311\ldots$$

A:



isosceles triangle.

Note: $\sin 2\pi/5 = e^2/f \Rightarrow f = \dfrac{e}{2\rho} = 0.3632$

ff

$= 0.618\ldots e$

$= \left(\frac{\sqrt{5}-1}{2}\right)e.$

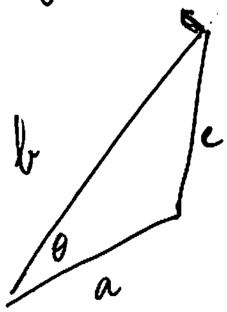
3
obtuse

11/26/2015.

<u>SAVE</u>
code document for pent
iloc, iarc functions

ocaml code for iloc.

b-obtuse.

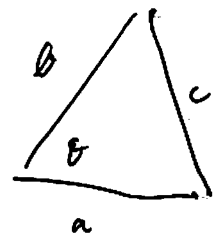


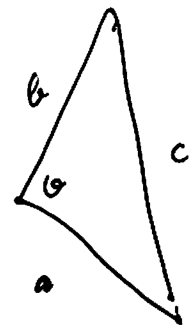
$c\nearrow$: $\boxed{\begin{array}{l} a\nearrow\ c\searrow \\ a\searrow\ , \end{array}}$; $b\nearrow c\nearrow$; $\theta\nearrow c\nearrow$; $\cos\theta\nearrow c\searrow$

& $b\nearrow$ $\quad$ $\cos\theta\searrow$

a switches sign.

ab-acute.





$a\nearrow c\nearrow$; $b\nearrow c\nearrow$; $\theta\nearrow c\nearrow$; $\cos\theta\nearrow c\searrow$

$c\nearrow$: $a\nearrow$ $b\nearrow$ $\cos\theta\searrow$

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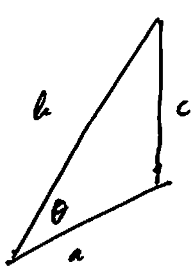
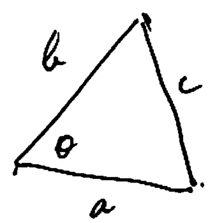
ocaml code for iarc.

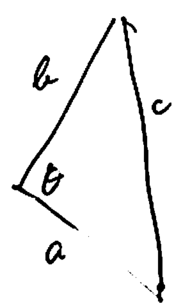
b-obtuse.



$a\nearrow \theta\nearrow$; $b\nearrow \theta\searrow$; $c\nearrow \theta\nearrow$

$\theta\nearrow$: $a\nearrow$ $b\searrow$ $c\nearrow$

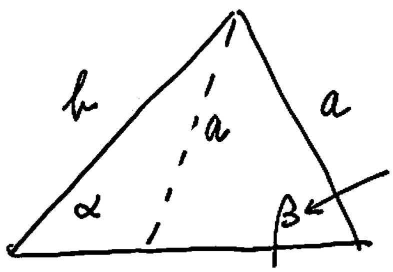




$a\nearrow \theta\searrow$; $b\nearrow \theta\searrow$; $c\nearrow \theta\nearrow$.

$\theta\nearrow$: $a\searrow$ $b\searrow$ $c\nearrow$.

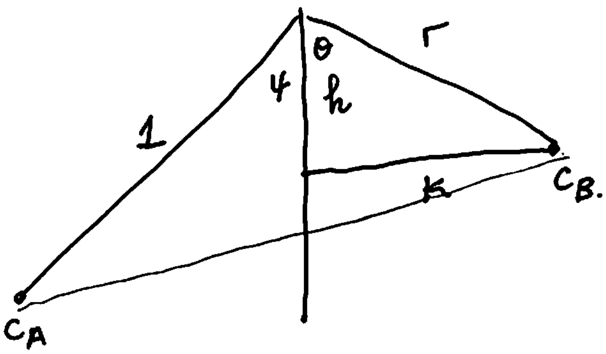
lawbeta α a b $= \beta < \pi/2$.



Pentagons in contact.

ell-function. h, ψ given $\quad \cancel{0 \le \pi \le}\ \ 0 \le \psi \le \pi.$

h positive or negative



$$r = \sqrt{h^2 + k^2}$$

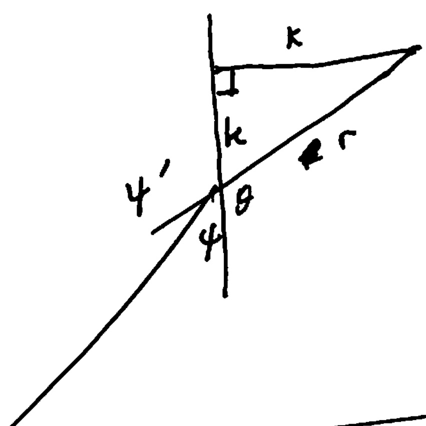
$$\cos\theta = h/r$$

$$d_{AB} = \mathrm{loc}\ 1\ r\ (\theta + \psi)$$
$$= \ell(h, \psi)$$

---

symmetry.

$$\psi \to \psi' = \pi - \psi \qquad r = r'$$
$$h \to k' = -h$$
$$\theta \to \theta' = \pi - \theta$$

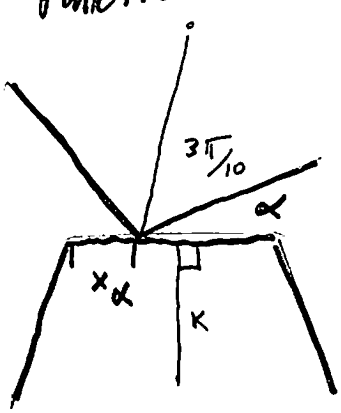


$$d_{AB} = \mathrm{loc}\ 1\ r'\ (\theta' + \psi')$$
$$= \mathrm{loc}\ 1\ r\ (\theta + \psi)$$

---

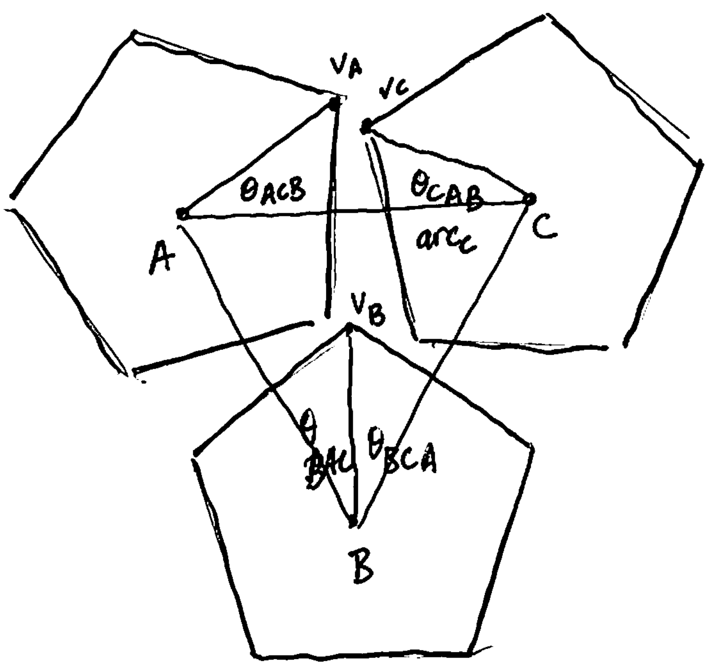
ell$_\pi$ function.

$$ell_\pi(x_\alpha, \alpha) = \ell\left(x_\alpha - \sigma, \tfrac{7\pi}{10} - \alpha\right) = \ell\left(\sigma - x_{\alpha'}, \tfrac{3\pi}{10} + \alpha\right)$$



invariant $\theta_{CAB} + arc_C + \theta_{CBA} \equiv 0 \mod 2\pi/5$.

Sign of θ, θ'

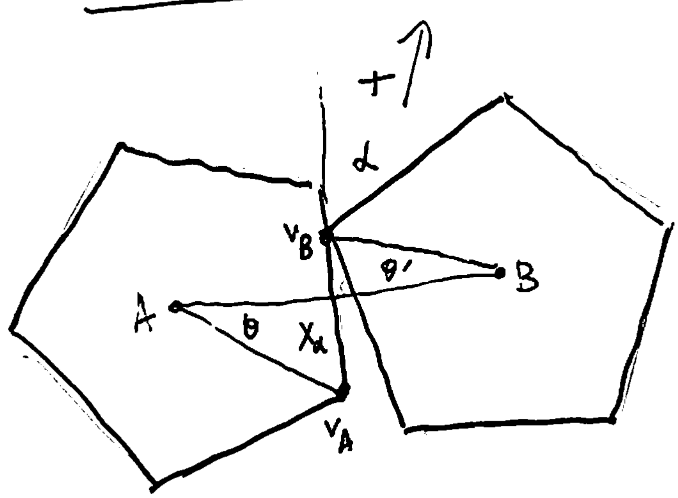


When part of a Delaunay triangle, the angles $\theta_{ACB}, \theta_{CAB}$ etc. are positive when v_A, v_C etc. are outside the Delaunay triangle.

In this example

$\theta_{ACB} > 0 \quad \theta_{CAB} > 0$

$\theta_{BCA} < 0 \quad \theta_{BAC} < 0$.



When using coordinates (x_α, α) to calculate $(d_{AB}, \theta, \theta')$,

θ' is the angle $\angle(\underline{c_B}, c_A, v_B)$,

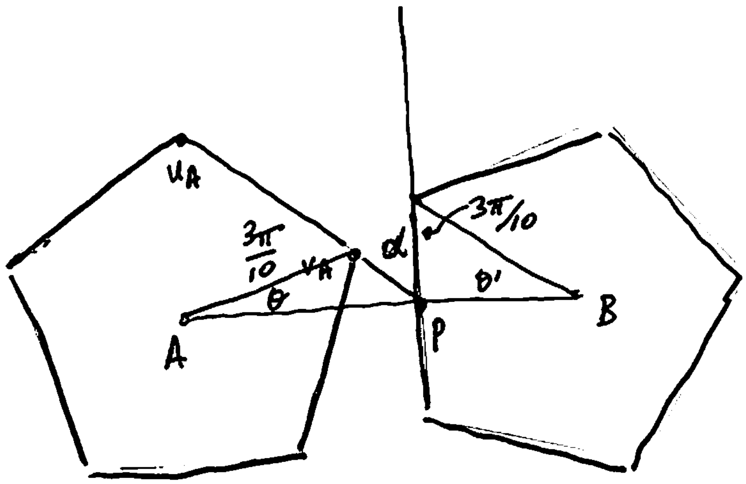
~~tot~~ $\theta' > 0$ here because v_B is in the same halfplane as the unbounded region of α

θ is the angle $\angle(c_A, \underline{c_B}, v_A)$

$\theta < 0$ here because v_A is in the opposite halfplane....

<u>Lemma</u> $\theta + \theta' \equiv \alpha \mod 2\pi/5$. (pentagons need not touch)

θ, θ' in any order.

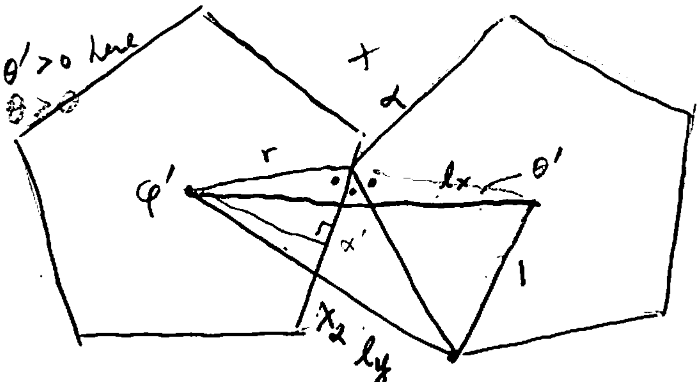


<u>Proof</u> Rotate at p

$$\alpha + \frac{3\pi}{10} - \theta - \theta' - \frac{3\pi}{10} \equiv 0.$$

Start with (x_α, α). 3/2016.

$\ell_x, \theta_x, \theta_x'$ coordinates



$h := x_\alpha - \sigma \geq 0$

$r := \sqrt{h^2 + k^2}$

$\sin\varphi' := h/r$ $\quad \varphi' \geq 0.$ (see below for $\varphi' < 0$)

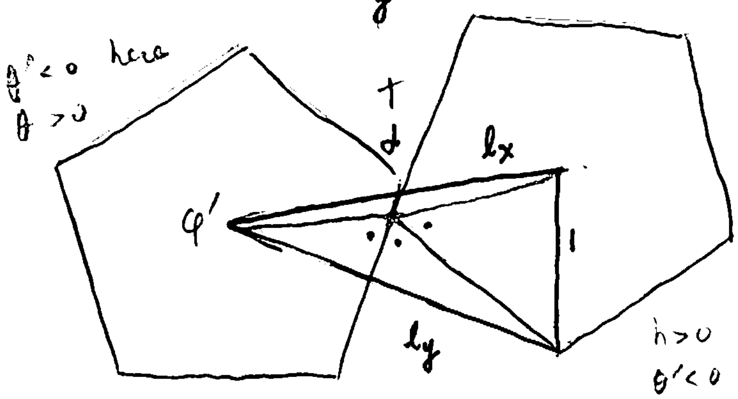
$\delta := \frac{6\pi}{5} - \alpha - \varphi' = \left(\frac{\pi}{2} - \varphi'\right) + \left(\frac{3\pi}{5} - \alpha\right) + \frac{3\pi}{10}$ (3 dot angles)

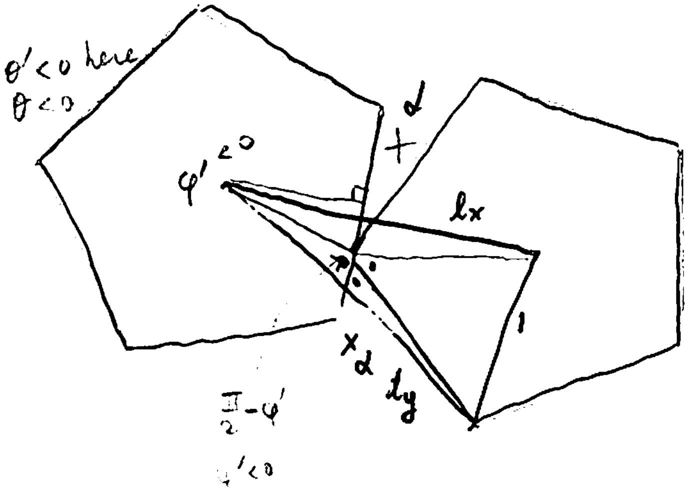
$\ell_x := \mathrm{loc}\ r\ 1\ \delta$

$\ell_y := \mathrm{loc}\ r\ (2\sigma)\ (\delta - 3\pi/10)$

$\theta' := 2\pi/5 - (\mathrm{arc}\ 1\ \ell_x\ \underset{\mathrm{opp}}{\ell_y})$

$\underset{\sim}{\theta} + \theta' := \alpha \bmod (2\pi/5).$





$h := x_\alpha - \sigma \leq 0.$ $\sin\varphi' = h/r$ $\quad \varphi' \leq 0.$

same formulas hold.

Standard conventions

- We use $|\theta'| \leq |\theta|$. So θ' is the pointer vertex.
- The cone over α opens into the $+$ side (for θ')

acute

29-δ

2/2016.

Reversing Coordinates $\bar{l}, \bar{\theta}$!

Given $l, l_x^{\text{Drilled}}, \theta'$ find α, x_α, θ

start here.

$l_x = l. \in [2\kappa, 2]$

$l_y^2 = l_x^2 + 1^2 - 2 l_x \cos(2\pi/5 - \theta')$. — Note $\theta' < 0$ in figure.

$r = \mathrm{loc}\ 1\ l_x\ \theta'$

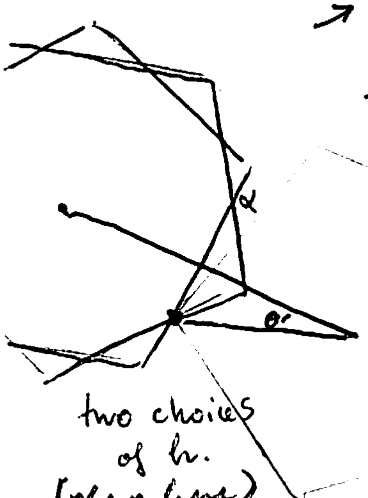
~~$\Delta(r, 1, 2\sigma, 1, l_y, l_x) = 0$~~

$\nwarrow r$

$r^2 = h^2 + \frac{K^2}{2}$

$\nwarrow h$ up to sign. both can occur. two solutions. use both.

$x_\alpha = h + \sigma$.

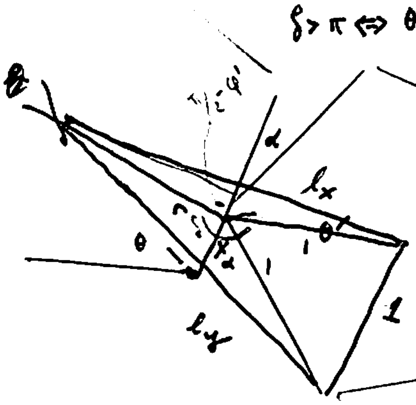


two choices of h. ($\theta' < 0$ here)

$\sin\varphi' = h/r$

$\delta > \pi \Leftrightarrow \theta' < 0$ $\begin{cases} l_x = \mathrm{loc}\ r\ 1\ \delta \\ l_y = \mathrm{loc}\ \mp (2\sigma)(\delta - \frac{3\pi}{10}) \end{cases}$ Alternate: Use this if $l_x \approx 1+r$. $\delta \approx \pi$.

$\delta = \frac{6\pi}{5} - \alpha - \varphi'$



{$\theta' < 0$ here in figure.} $\theta + \theta' = \alpha \mod (2\pi/5)$

$\underline{l}$

$\underline{l_y}$

$\underline{r}$ $\in [\kappa, 1]$

$\pm \underline{h}$ (2 choices)

$\underline{x_\alpha} \in [0, 2\sigma]$

* $\underline{\varphi'}$ (2 choices) dependencies.

$\underline{\delta}$

$\underline{\alpha} \in [0, 2\pi/5]$

$\underline{\theta}$.

acute $2\varphi - \delta$!

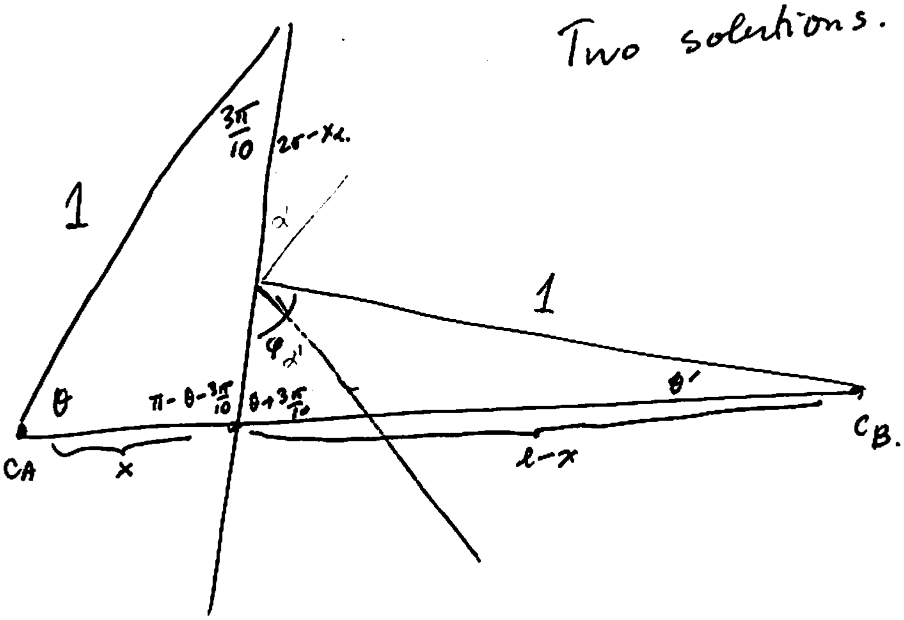
two-contact-coord-all-theta!

From (θ, ℓ) to coordinates

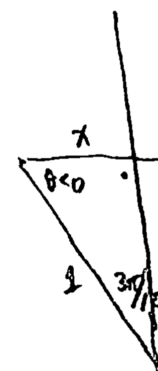
$(\theta' \geq 0$ case$)$

$(\theta'$ pointer$)$

Two solutions.



if $\theta < 0$



$\circ = \pi - \left(\frac{3\pi}{10} + |\theta|\right)$

$= \frac{7\pi}{10} - \theta$

$= \frac{3\pi}{10} - \theta + \frac{2\pi}{5}$

jump by $\frac{2\pi}{5}$

Renormalize $\tilde{\theta} \in [0, 2\pi/5]$.

if $\tilde{\theta} = 0$ then $x = 1$, if $\theta = \frac{2\pi}{5}$ then $x = 1$

(x) los: $\dfrac{x}{\sin\left(\frac{3\pi}{10}\right)} = \dfrac{1}{\sin\left(\pi - \tilde{\theta} - \frac{3\pi}{10}\right)}$

$\parallel$

(φ) los: $\dfrac{\ell - x}{\sin\varphi} = \dfrac{1}{\sin\left(\tilde{\theta} + \frac{3\pi}{10}\right)}$ two solutions φ, $\pi - \varphi$. Use both.

$\varphi = \alpha' + \frac{3\pi}{10}$. φ obtuse: bool.

$\sin\varphi = (\ell - x)\sin\left(\tilde{\theta} + \frac{3\pi}{10}\right)$.

Then continue with (l, θ') case.

(θ') $\theta' + \varphi + \left(\tilde{\theta} + \frac{3\pi}{10}\right) = \pi$.

$\varphi = \alpha' + \frac{3\pi}{10}$ $\varphi > \frac{\pi}{2}$ iff $\alpha' + \frac{3\pi}{10} > \frac{5\pi}{10}$

iff $\alpha' > \frac{\pi}{5}$.

$\alpha + \varphi \equiv \underline{\theta'} + \varphi + \underline{\tilde{\theta}} = \frac{7\pi}{10}$ $\varphi + \alpha \equiv \frac{3\pi}{10} \bmod \frac{2\pi}{5}$ $\frac{4\pi}{10}$

$0 \leq x \leq \ldots$

$\left.\begin{array}{l}\text{constraint}\\ \text{binds}\end{array}\right.$

(α) $\alpha \equiv \tilde{\theta} + \theta' \bmod \left(\frac{2\pi}{5}\right)$

φ obtuse $\Leftrightarrow \frac{7\pi}{10} - \alpha \gg \frac{\pi}{2} \Leftrightarrow \frac{\pi}{5} > \alpha \Leftrightarrow \alpha' \geq \frac{\pi}{5}$.

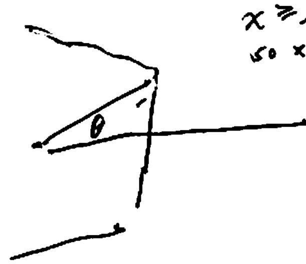
$x \geq \ell$ means C_B inside pentagon $\ast$!?

so $x \leq \ell$. $\Rightarrow \ell - x \geq 0$. $\Rightarrow \sin\varphi \geq 0 \Rightarrow \varphi \geq 0$.

$-\pi/5 \leq \theta \leq \pi/5$.

$\frac{\pi}{10} \leq \theta + \frac{3\pi}{10} \leq \frac{\pi}{2}$

so $x \geq 0$.



φ code acute 29-XX

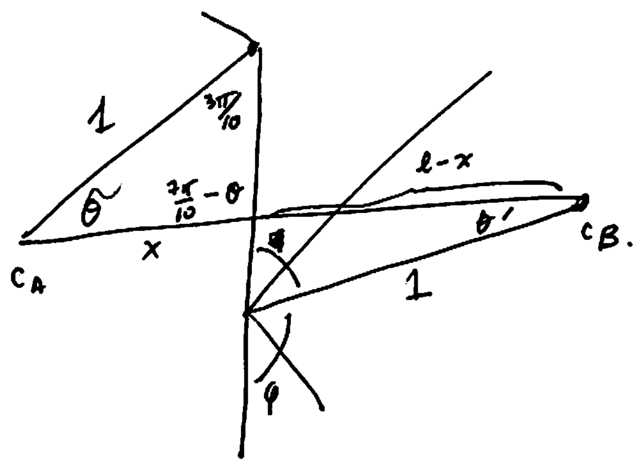
3/2016.

From (ℓ, θ) to coordinates $\tilde{\theta} \in [0, 2\pi/5]$

$\theta' \le 0$ case.

θ' = pointer

Two solutions



ⓧ. los $\dfrac{x}{\sin(\frac{3\pi}{10})} = \dfrac{1}{\sin(\frac{7\pi}{10} - \tilde{\theta})}$

ⓟ los : $\dfrac{\ell - x}{\sin\varphi} = \dfrac{1}{\sin(\frac{7\pi}{10} - \tilde{\theta})}$ φ and $\pi - \varphi$ <u>two solutions</u>.

ⓣ $(\pi - \varphi) + (\frac{7\pi}{10} - \tilde{\theta}) + (-\theta') = \pi.$

$\frac{7\pi}{10}. = \varphi + \tilde{\theta} + \theta'$ Then continue with θ'!

all equations identical to $\theta' \ge 0$ case.

φ code acute

29-XX

3C coordinates

" " shared variables. $\bar{\alpha}, \bar{\beta}, \bar{x}_\beta$ input order

<u>nonshared</u>:

pinwheeledge α β x_γ

pintedge-extended α β x_α

deltajedge α β x_α'

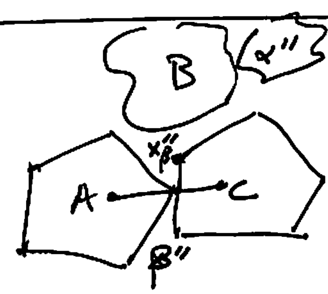
ljedge α β x_α

tjedge α β x_γ

<u>Shared</u> systems (α, β, x_β)

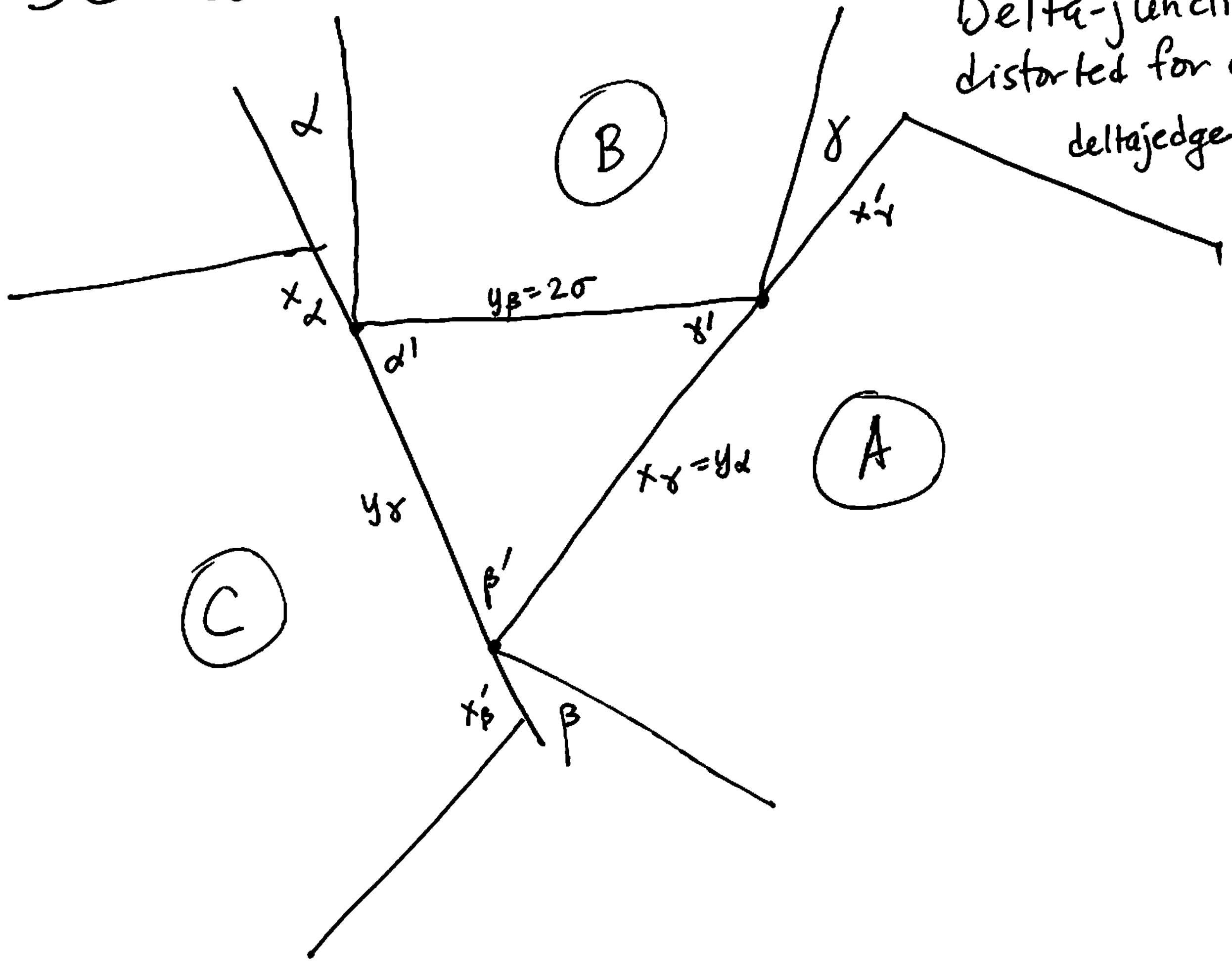
$A \to C$ B outer.

return dBC, dAC, dAB.

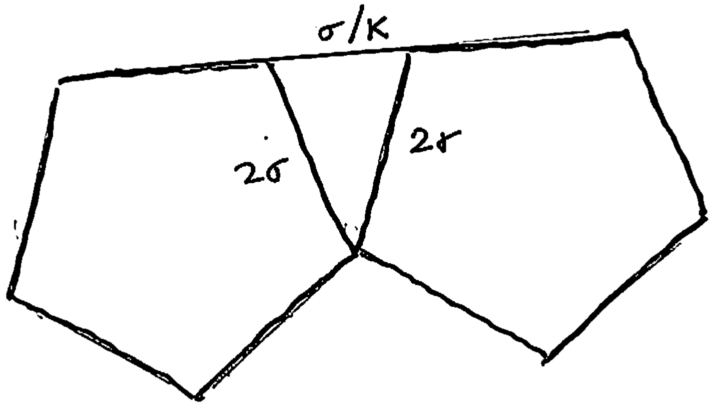


3C coordinates

Delta-junction
distorted for clarity

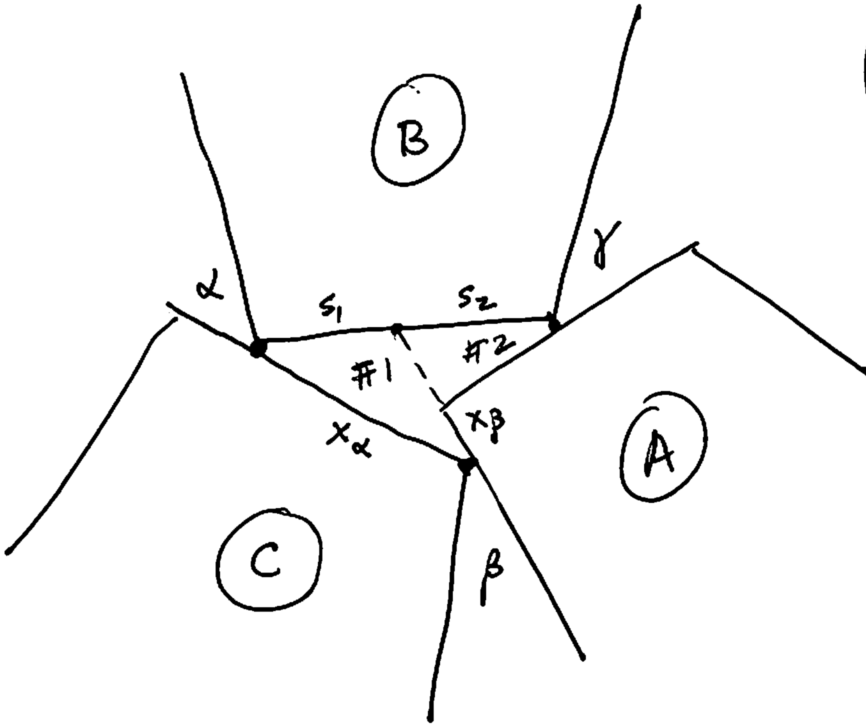


Delta junction inequality:



$0 \leq x \leq 2\sigma - \sigma/K$

L-junction coordinates.

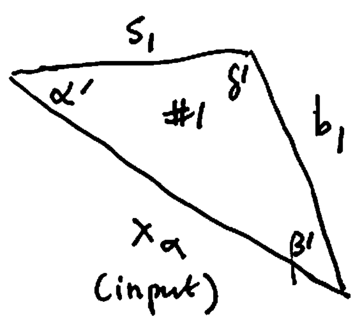


$\alpha+\beta+\gamma = 3\pi/5$

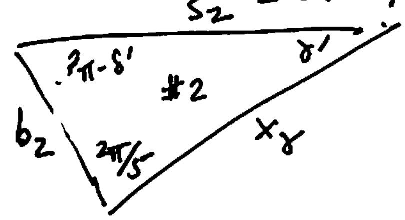
$\pi/5 \le \alpha+\beta \le 3\pi/5$

$\alpha, \beta, \gamma \in [0, 2\pi/5]$

$\boxed{\text{Coords } \alpha, \beta, x_\alpha}$



$s_2 = 2\sigma - s_1$ (input)



$\bar{l}(x_\alpha, \alpha)$

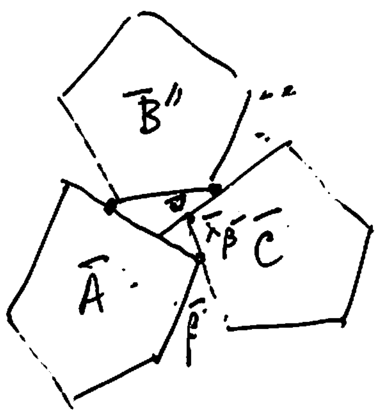
$\bar{l}(x_\beta, \beta)$ $\qquad x_\beta = b_1 - b_2.$

$\bar{l}(x_\gamma, \gamma).$

LJ_1 - coordinates.
(shared)

$\bar{\alpha}$ $\bar{\beta}$ $\bar{x}_\beta$

shared

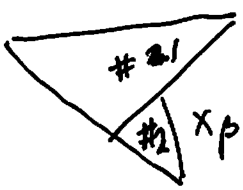


Drop, ~~as~~ basis

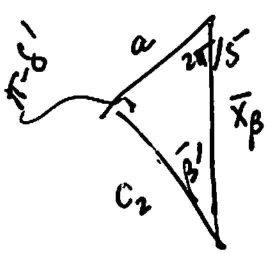
$\bar{\alpha} = \gamma'$.

input (α' β x_β)

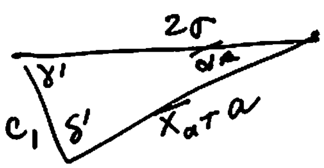
use prime here.



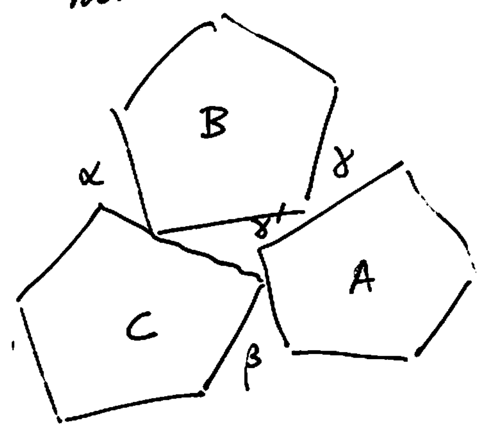
#2



#1



nonshared



(not used).

$\pi/5$ $2\pi/5$

$\le$ $\le$

$\alpha + \beta + \gamma = 3\pi/5.$

$\pi/5 \le \alpha + \beta \le 3\pi/5.$

$\pi/5 \le \beta + \gamma \le 3\pi/5$

$\beta \le 3\pi/5 - \gamma'$

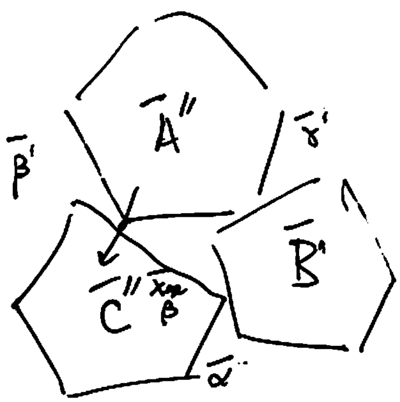
$\beta \le \frac{\pi}{5} + \gamma'$

if $\beta > \pi/5$ then $\gamma' > 0$.

$\boxed{LJ_2}$

$\bar\alpha \quad \bar\beta \quad \bar x_\beta$

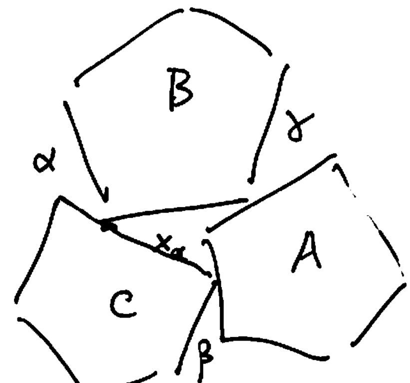
<u>shared</u>. $\quad$ " $\quad$ " $\quad$ "



LJ_2 - coordinates $\quad \bar\beta' \quad \bar\alpha'' \quad \bar x_\beta'$

$\qquad\qquad\qquad\qquad\qquad\quad \| \quad\;\; \| \quad\;\; \|$

nonshared. $\quad \alpha \quad \beta \quad x_\alpha$



output $\quad d_{BC},\ d_{AC},\ d_{AB}$

$\qquad\qquad\quad \| $

$\qquad\qquad\quad d''_{AC} \quad d''_{BC} \quad d''_{AB}$

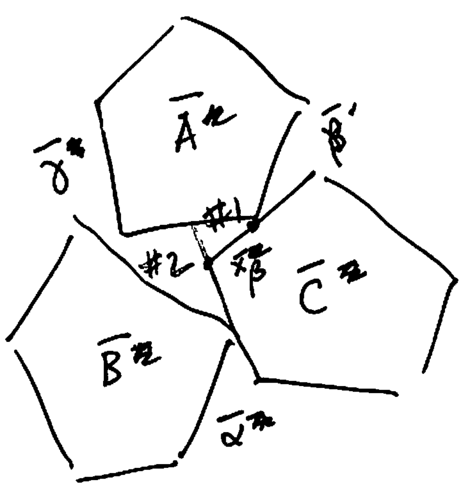
$\overline{(\alpha, x_\alpha)\beta}$

$$\begin{cases} \text{if } \alpha < \pi/5. \\ \pi/5 \le \alpha+\beta \Rightarrow \beta > 0. \end{cases}$$

LJ_3

LJ_3.

shared $\bar{\alpha} \cdot \bar{\beta} \cdot \bar{x}_\beta$

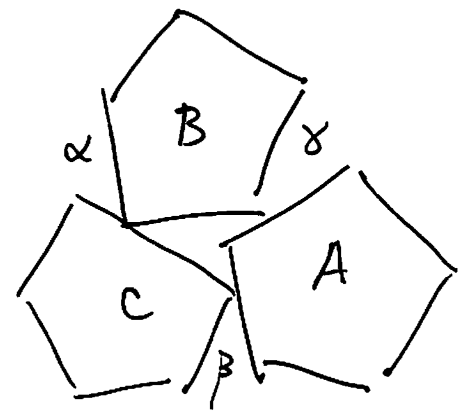


LJ_3-coords.

$\bar{\gamma} \cdot \bar{\alpha} \cdot \bar{x}_\gamma$

$\parallel$

nonshared α β x_α



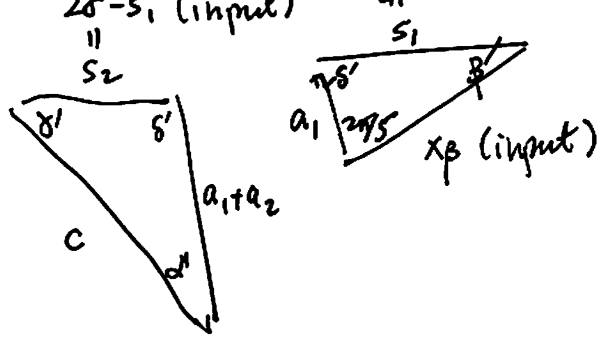
Drop i · 3 bars

Input α β x_β

$2\sigma - s_1$ (input) #1

$\parallel$

s_2



$X_\gamma = c$

X_β (input)

$X_\alpha = a_2$

$\bar{\alpha}$ has stability constraints o.7. (one-ljx-constant)

$\parallel$

β

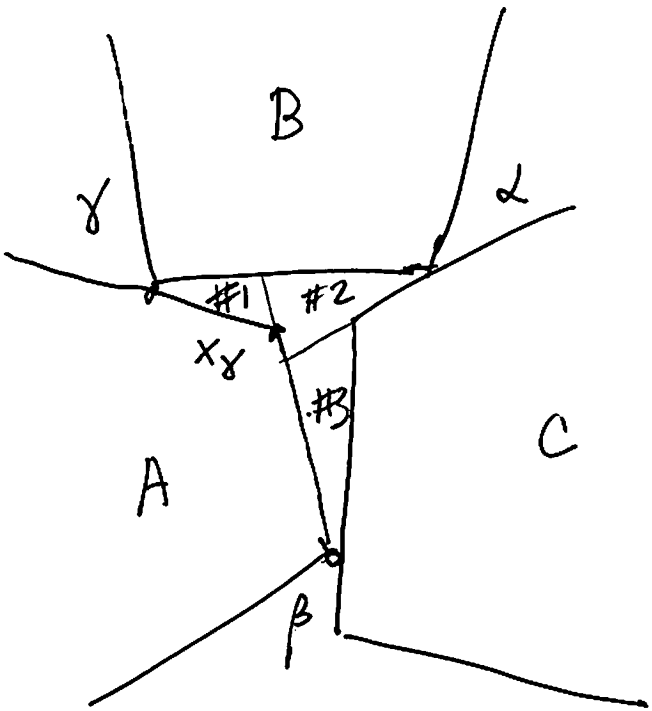
T-junction

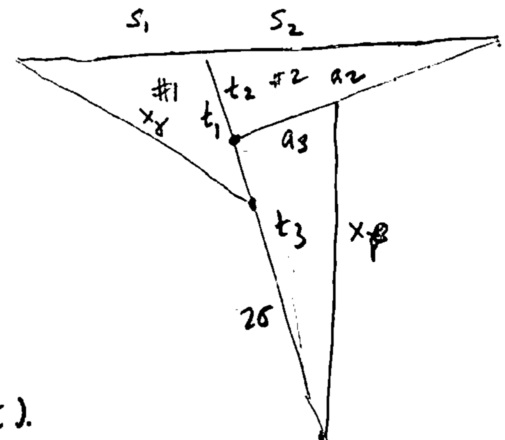
t_j edge-extended

input α, β, x_γ

$\alpha, \beta, \gamma \in [\pi/5, 2\pi/5]$

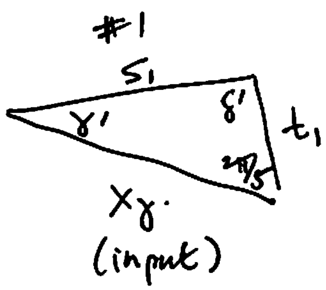
$\alpha + \beta + \gamma = \pi$.

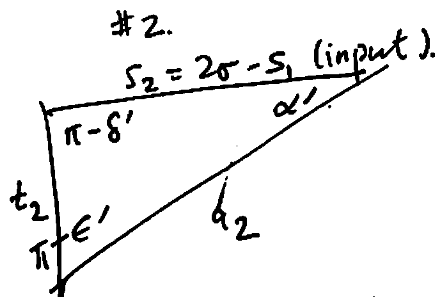




$t_3 + t_2 - t_1 = 2\sigma$

$t_3 = 2\sigma + t_1 - t_2$

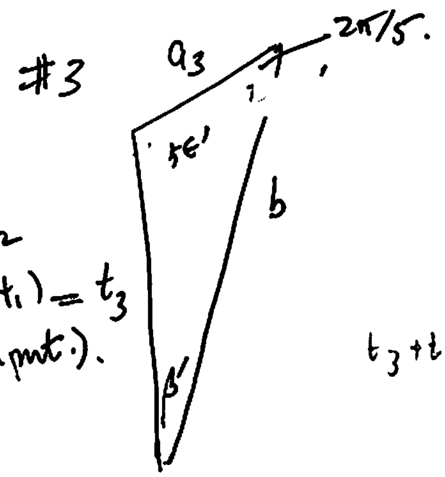




$x_\alpha = a_2 - a_3$

$x_\beta = b$

$x_\gamma =$ (input)



$2\sigma + t_1 - t_2$

$2\sigma - (t_2 - t_1) = t_3$

(input).

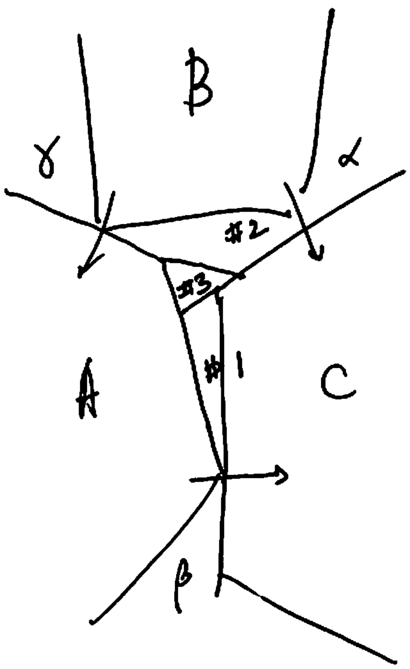
$t_3 + t_2 - t_1 = 2\sigma$.

extended x_α x_β

$\boxed{TJ_1}$

TJ_1 -coordinates
tj_1 edge
shared same
pentagon labels.
$\bar{\xi} = \xi$.

nonshared



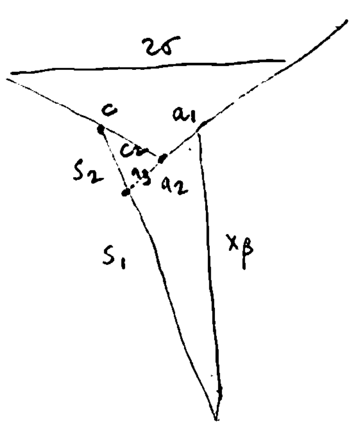
input α β x_β

$\alpha, \beta, \gamma \in [\pi/5, 2\pi/5]$

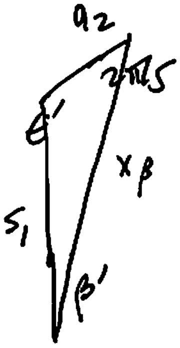
$\frac{3\pi}{5} \le \alpha + \beta \le \frac{4\pi}{5}$.

$\alpha + \beta + \gamma = \pi$.

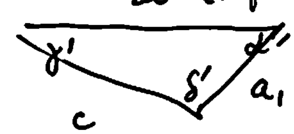
β ... ≥ 1 (one-tjx-constraint)



#1

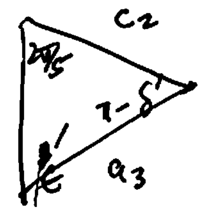


#2 arms



#3 mid.

$2\sigma - s_1 = s_2$
(input)



$x_\alpha = a_1 + a_3 - a_2$

x_β (input)

$x_\gamma = c - c_2$

$\boxed{TJ_2}$

shared
tj2 edge.

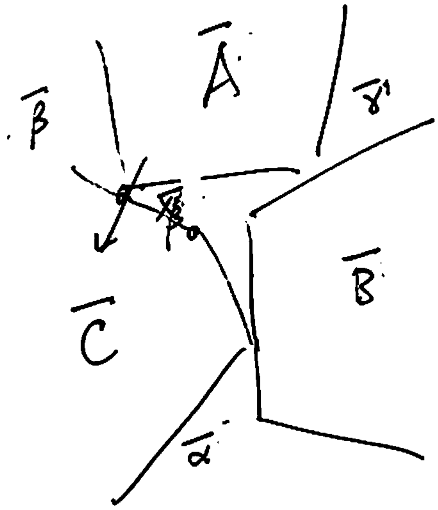
~~shar~~ nonshared

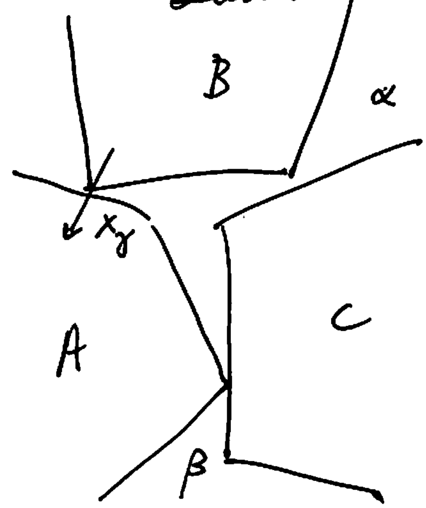
$\bar{\gamma}^{*}\ \bar{\alpha}^{*}\ \bar{x}_{\beta}^{*}$

$\shortparallel\ \ \shortparallel\ \ \shortparallel$

$\swarrow\ \beta\ x_{\gamma}$

shared. $\bar{\alpha}'$ $\bar{\beta}$ $\bar{x}_{\beta}^{2}$





$d_{BC},\ d_{AC},\ d_{AB}$

$\shortparallel$

$d''_{AB}\ \ d''_{BC}\ \ d''_{AC}$

"

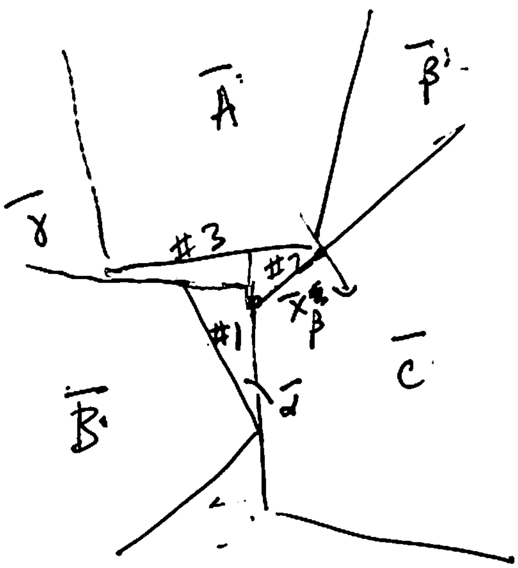
tj2 midedge output

extended x''_{γ} x''_{α}

TJ_3

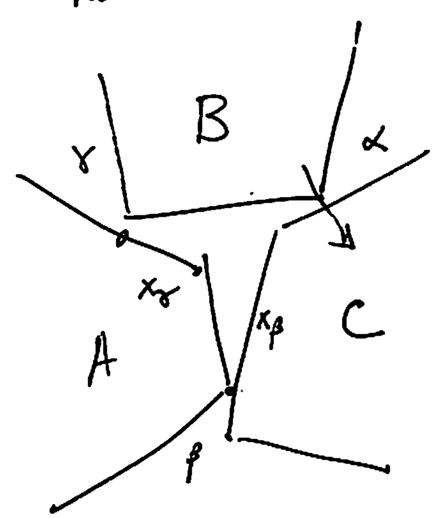
shared $\bar{\alpha}'$ $\bar{\beta}'$ $\bar{x}_\beta$

shared —
t_j 3edge
$B \to C$



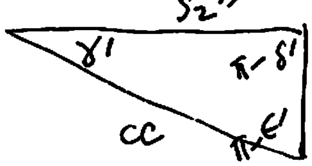
$\beta' = \bar{\alpha}$ $\bar{\beta}_1$ $\bar{\alpha}'$

nonshared α β x_γ

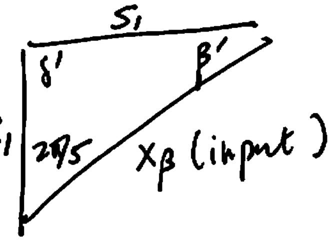


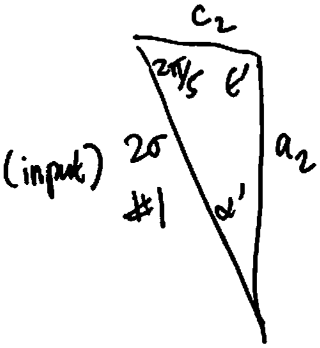
drop $\bar{\beta}'$ bars. ... (α' β x_β)

#3 $s_2 = 2\sigma - s_1$ (input)



#2 s_1





② $x_\gamma = cc - c_2$ $= x_\gamma$ nonsh

$x_\beta = $ input $= x_\alpha$ nonsh

① $x_\alpha = a_2 + a_3 - a_1$ $= x_\beta$ nonshd

near ice-ray nonshared
$\alpha \approx \pi/5$.
$\beta' \approx 0$ $\beta \approx 2\pi/5$.

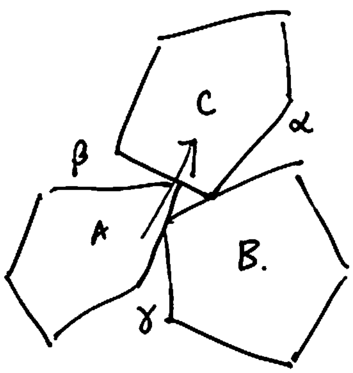
$\frac{3\pi}{5} \le \alpha + \beta$,

$\frac{\pi}{5} + \beta' \le \alpha$ so if $\beta' > 0$ then $\alpha > \pi/5$.

$\frac{\pi}{5} + \bar{\alpha} \le \bar{\beta}$ if $\bar{\alpha} > 0$ then $\bar{\beta} > \pi/5$

$\boxed{\text{shared pinwheel}}$

nonshared. α, β, x_γ

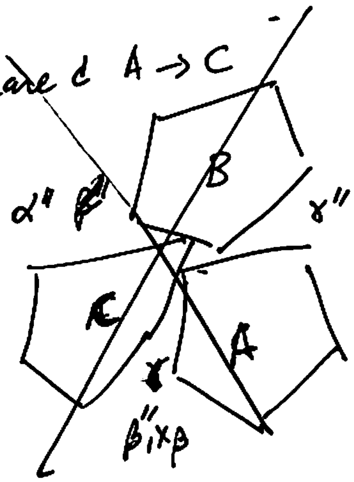


out $\underset{\alpha}{d_{BC}}, \underset{\beta}{d_{AC}}, \underset{\gamma}{d_{AB}}$

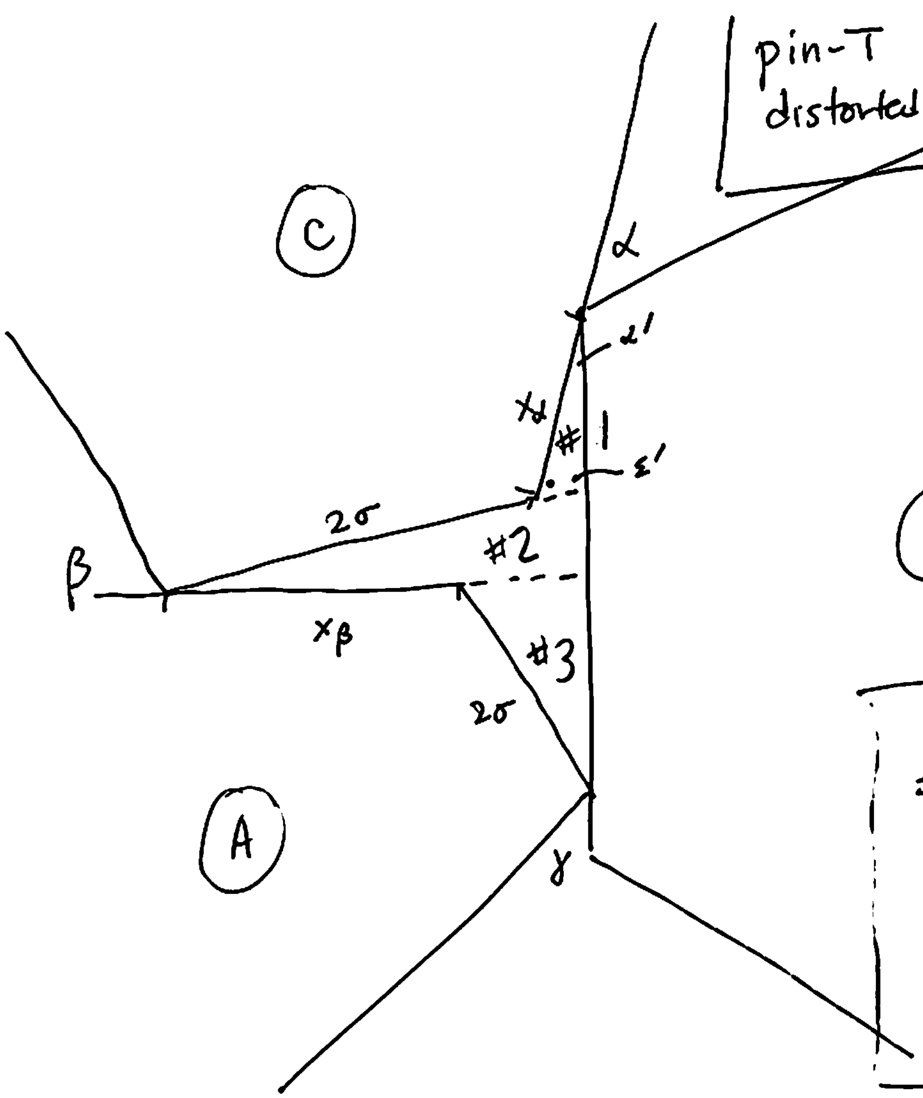
shared $\underset{\gamma}{\alpha}\ \underset{\alpha}{\beta}\ x_\beta =$

pinwheel $\therefore\ \tilde{}\ \ x_\beta = \underset{\gamma}{d_{AB}}, \underset{\alpha}{d_{BC}}, \underset{\beta}{d_{AC}}.$

shared $A \to C$



pin-T coordinates. distorted for clarity



$0 \le X_\alpha \le 0.0605$

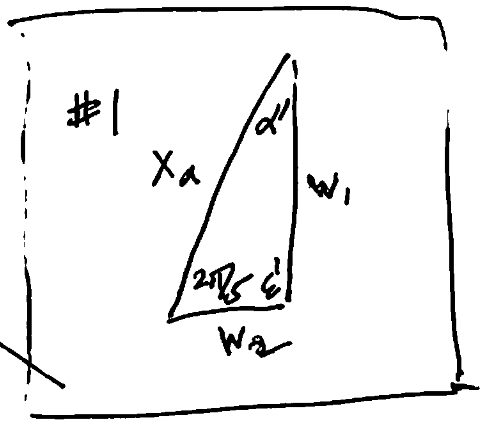
$\alpha+\beta+\gamma = \pi$

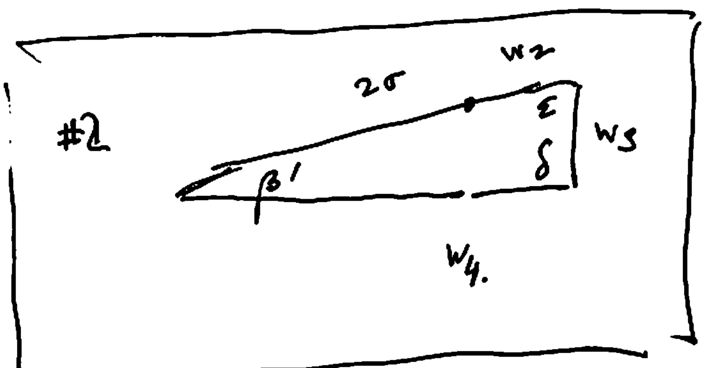
$\frac{\pi}{5} \le \alpha \le \frac{2\pi}{5}$ γ

$\frac{\pi}{5} \le \beta \le \frac{2\pi}{5}.$

$\frac{3\pi}{5} \le \alpha+\beta$

$\frac{\pi}{5} \le \gamma \le \frac{2\pi}{5}$

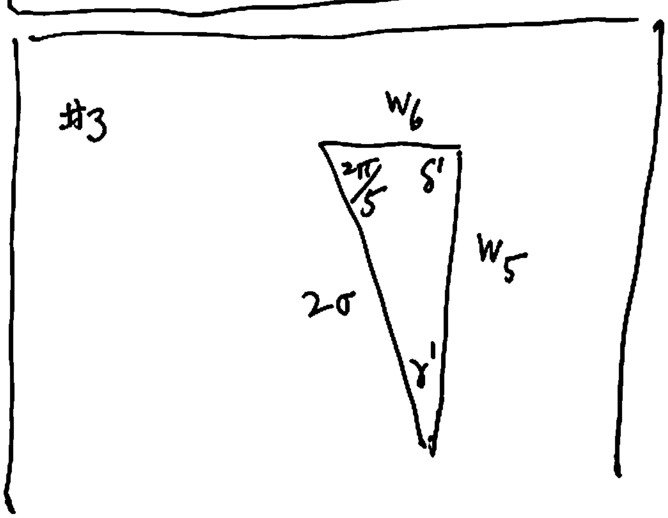




$$d_{BC} = \bar{\ell}(x_\alpha, \alpha)$$

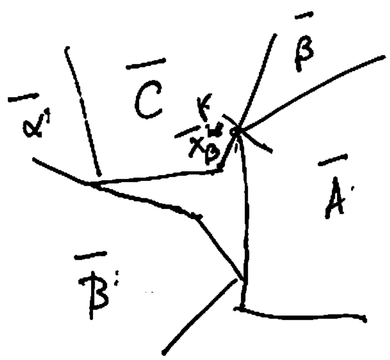
$$d_{AC} = \bar{\ell}(w_4 - w_6, \beta)$$

$$d_{AB} = \bar{\ell}(w_1 + w_3 + w_5, \gamma)$$



Shared Pin-T coordinates

Shared pin-T.



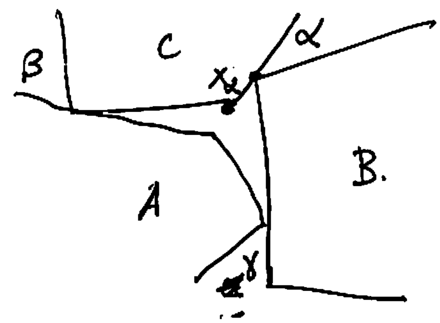
$3\pi/5 + \bar{\beta} + \frac{3\pi}{5} + \frac{\pi}{2} \approx 2\pi.$

$\bar{\beta} \approx \frac{(20-5-6-6)\pi}{10} \approx \frac{3\pi}{10}$

$\bar{x}_\beta \leq 0.0605.$

$\bar{\beta}\ \ \bar{\alpha}\ \ \bar{x}_\beta$
$\ \ \shortparallel\ \ \shortparallel\ \ \shortparallel$

non share $d(\alpha\ \ \beta\ \ x_\alpha.)$



out d_{BC}, d_{AC}, d_{AB}

$= \bar{d}_{AC}, \bar{d}_{BC}, \bar{d}_{AB}$

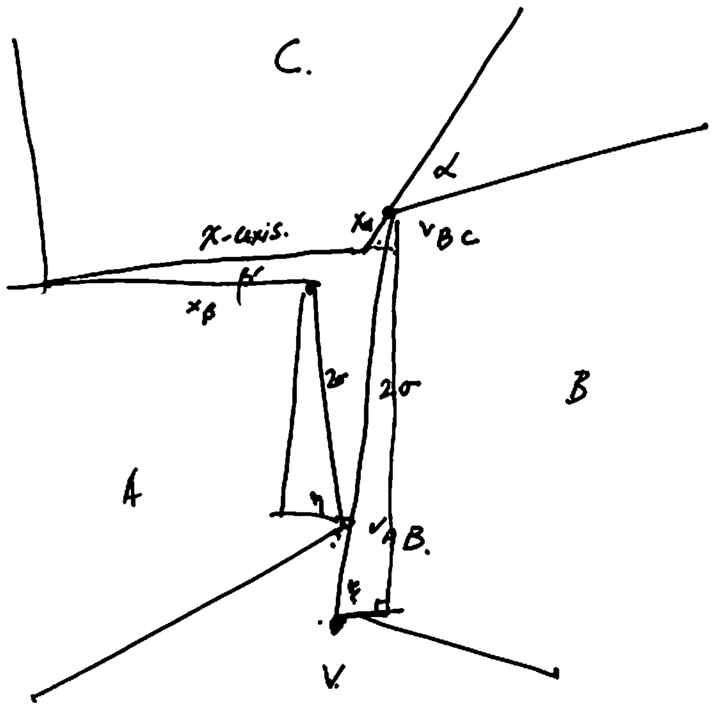
pin-T-0605

Lemma: 0605.

$\alpha \approx 2\pi - \frac{23\pi}{5} - \frac{\pi}{2}$.

$= \frac{20-12-5}{10} \approx \frac{3\pi}{10}$

$\alpha + \frac{\pi}{5} \approx \frac{\pi}{2}$.



$\xi - \frac{2\pi}{5} + \alpha - \frac{2\pi}{5} = 0$

$\xi + \alpha = \frac{4\pi}{5}$.

$\alpha + \frac{\pi}{5} = \pi - \xi$.

$\sin(\alpha + \frac{\pi}{5}) = \sin(\pi - \xi) = \sin(\xi)$.

$X_\alpha \sin(2\pi/5) - 2\sigma \sin(\alpha + \pi/5) \leq -x_\beta \sin\beta' - 2\sigma \sin(\beta + \pi/5)$

$\geq$

$\leq -$

$\eta - (\frac{2\pi}{5} + \beta') = 0$

$\eta = (\frac{2\pi}{5} + \beta')$

$\eta = \frac{4\pi}{5} - \beta$

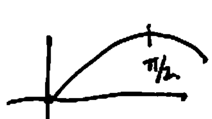
$= \pi - (\frac{\pi}{5} + \beta)$

$\sin\eta = \sin(\frac{\pi}{5} + \beta)$.

$X_\alpha \sin(2\pi/5) \leq 2\sigma(1 - \sin(2\pi/5))$

$\sin 2\pi/5$. $\sin \frac{3\pi}{5}$.

π



acute

33 for. 4

The path P to Γ

auto diff close to ice-ray: pinwheel, LJ_1, LJ_2, TJ_3.

	top $\beta_0 < \pi/5$.	$\beta_0 = \pi/5$	bottom. $\beta_0 > \pi/5$.
pinwheel.^{topA}	β_0+s, $\alpha_0 - s$ $\quad \alpha_0 > 0$	rigid Γ ice-ray.	X
pinwheel^{topB}	$\beta_0 + s$ $\quad \alpha_0 \equiv 0$		
		β_0, $\alpha_0 - s$.	$\beta_0 - s$ $\quad \alpha_0 - s$ $\quad (\alpha_0 > 0)$. X
LJ_1 topA	$\beta_0 + s$ $\quad \alpha_0 - s$ $\quad \alpha_0 > 0$.	twist_chain_lj1	
$\beta_0 < \pi/5$ $\alpha_0 \geq 0$. topB	$\beta_0 + s$ $\quad \alpha_0 \equiv 0$		$\beta_0 - s$ $\quad \alpha_0 - s$. bottom$_A$
		β_0, $\alpha_0 - s$.	$\beta_0 - s$ $\quad \alpha_0 \equiv 0$ bottom$_B$.
LJ_2	$\beta_0 + s$ $\quad \alpha_0 - s$ $\quad \alpha_0 > 0$. ($\alpha_0 > 0$ forced) X	twist_chain_lj2	
			$\beta_0 - s$ $\quad \alpha_0 - s$ bottom$_A$
TJ_3.	X	rigid. Γ ice-ray	$\beta_0 - s$ $\quad \alpha_0 \equiv 0$ bottom$_B$. Note $\pi/5 + \bar\alpha \leq \bar\beta$

domain. $\alpha^B_1, \beta^B_1, x^B_{\beta_1}, \alpha^D . \begin{pmatrix} [0,\varepsilon] \\ [0] \end{pmatrix}$

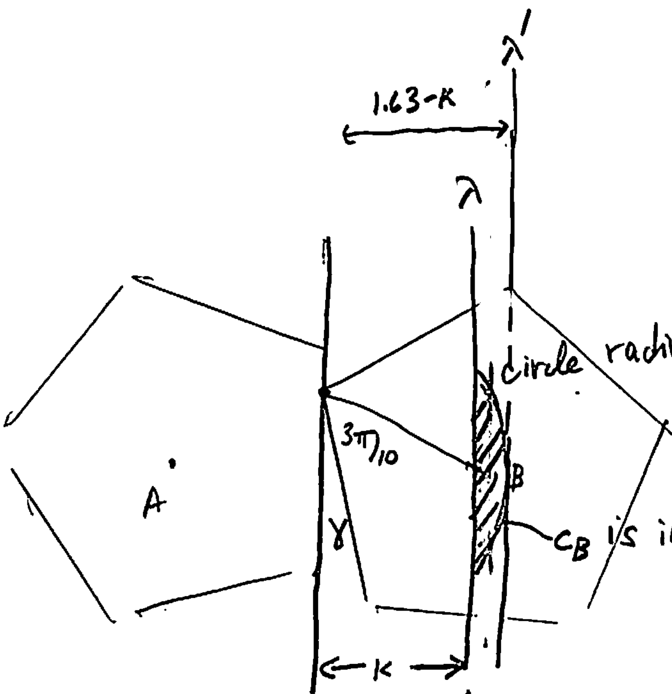
x^B_{β}: $[\sigma-\varepsilon, \sigma+\varepsilon]$

α^B: $\begin{pmatrix} [0,\varepsilon] \\ [0] \end{pmatrix}$

β^B: $\begin{pmatrix} [\pi/5 - \varepsilon, \pi/5] \text{ top} \\ [\pi/5, \pi/5+\varepsilon] \text{ bottom.} \end{pmatrix}$

$$\boxed{1.63 \text{ lemma}}$$

$\text{area}(1.8, 1.8, 1.63) > a_{crit} + 2\varepsilon_\mu$



circle radius 1.63 at c_A tangent to λ'

c_B is in this region.

c_B on λ when $\gamma = 0$ and slider contact.

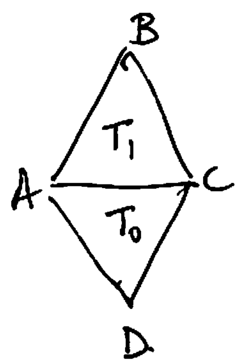
So $\sin\left(\gamma + \frac{3\pi}{10}\right) \leq 1.63 - K$.

dimer code (ΨD condition)

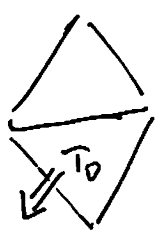
The test for ΨD in the dimer code is

$AD > AC + 0.03$ and $AD > 1.8$. $(*)$

The dimer code disregards configurations that satisfy $(*)$.



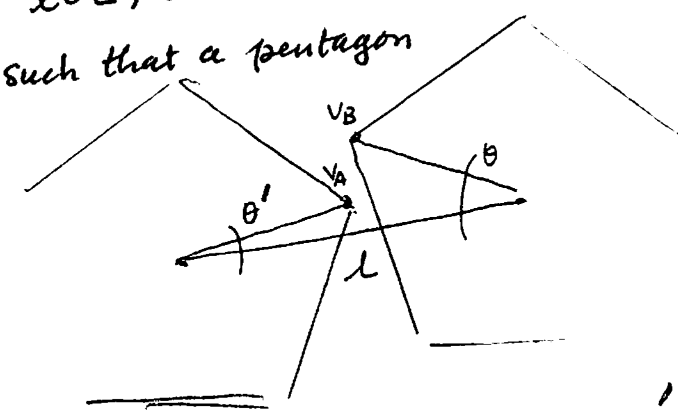
If $(*)$ holds, then longest edge of T_0 is AD or DC so it is a ΨD, and ~~it is~~ disregarding it is justified



4/2015.

PET
Pent Existence Test.

<u>Question</u> Given interval bounds $\ell \in L$, $\theta \in I$, $\theta' \in I'$ does there exist $(\ell, \theta, \theta') \in L \times I \times I'$ such that a pentagon arrangement exists?



<u>Assumptions</u>

$L \subseteq [2\rho, 2.1]$

$I, I' \subseteq [-\pi/5, \pi/5]$

WARNING θ' = pointer. as of 3/2016

<u>Initial setup and prep</u>.

- subdivide I, I' as needed so that they have fixed signs
- wlog take $\ell = \ell_{max}$.
- wlog by symmetry take $0 \leq |\theta'| \leq |\theta|$.
  (if $|I| \cap |I'| \neq \emptyset$, ~~#~~ break into two subcases according to which variable $|\theta|$ or $|\theta'|$ is larger).
- reset I so that $\theta'_{max} \leq |\theta|_{max}$.
- reset I' so that $|\theta|_{min} \geq \theta'_{min}$

<u>Test 1</u> if $loc(1, \ell_{max}, \theta'_{max}) \geq 1$, then <u>yes</u> it exists
(because v_A is the closest point to $c_{A'}$)

<u>Test 2</u> if $loc(1, \ell_{max}, \theta'_{max}) < \rho$, then <u>no</u> it doesn't exist.
(because $loc(1, \ell_{max}, \theta') \leq loc(1, \ell_{max}, \theta'_{max}) < \rho$, so v_A is definitely overlapping pentagon B).

acute 58

PET Case 1 $0 \le \theta' \le \theta$

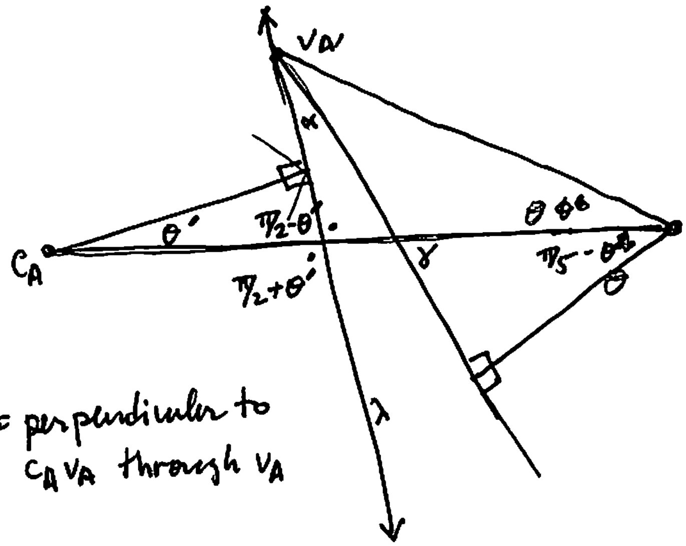
4/2015

Figure 1

$$\gamma = \frac{\pi}{2} - \left(\frac{\pi}{5} - \theta\right) = \theta + \frac{3\pi}{10}$$

$$\alpha + \left(\frac{\pi}{2} + \theta'\right) + \left(\theta' + \frac{3\pi}{10}\right) = \pi$$

$$\alpha + \theta + \theta' = \pi/5$$



λ = perpendicular to $C_A V_A$ through V_A

Case 1 $0 \le \theta' \le \theta$

- wlog, take $\theta \doteq \theta'_{max}$.

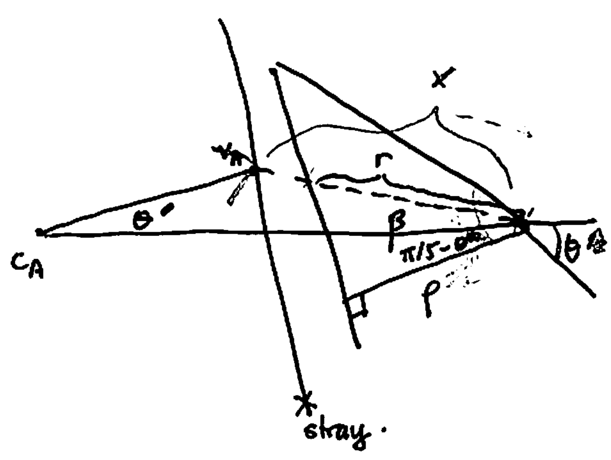
We rotate V_A away from V_A'. Direction of rotation depends on α vs 0.

The configuration exists $\iff$ it exists for $\theta' = \theta_{min}$ or $\theta' = \theta'_{max}$

Test if $loc(\ell, 1, \theta'_{min}) \ge r$ or $loc(\ell, 1, \theta'_{max}) \ge r$ then <u>yes</u>

else <u>no</u>.

Figure 2



x : loc ℓ 1 θ'

β : lawofbeta $\underbrace{\theta'}_{op}\ x\ \underbrace{1\ \beta}_{op}$

r : $\rho/r = \cos\left(\beta + \left(\frac{\pi}{5} - \theta\right)\right)$

Test

Must have $\beta \le \theta$

(else $\theta < \theta'$ ✗)

acute

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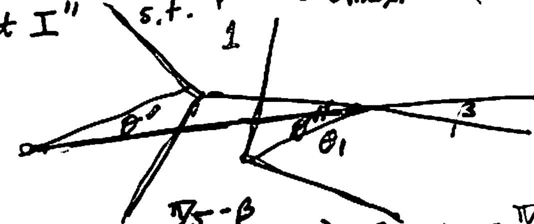
4/2015.

$\theta_1 = -\theta$

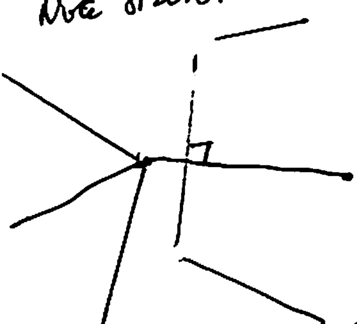
$\boxed{\text{PET} \quad \text{Case 2} \qquad 0 \le \theta' \le \theta_1}$

$\theta_{1\,min}$ s.t. $\ge \theta'_{max}$.

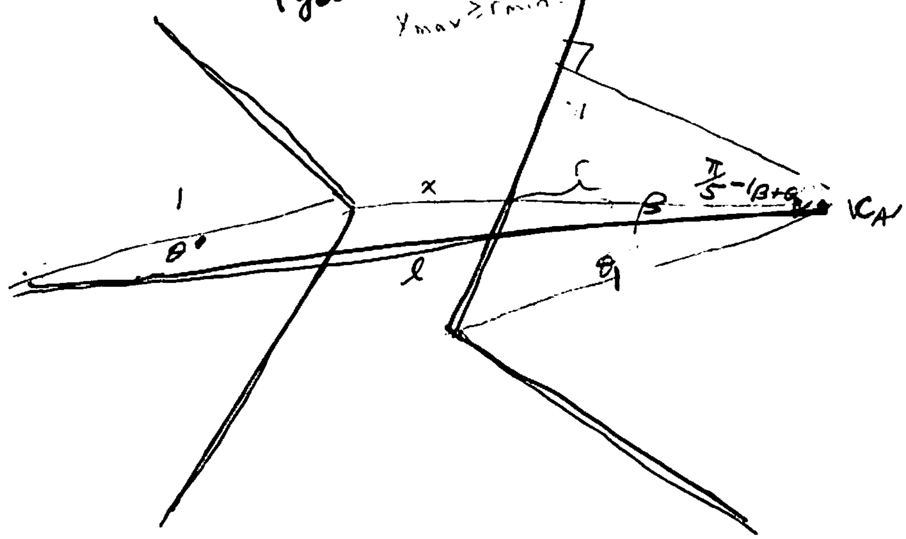


- rotate θ' until $\theta' = \theta'_{max}$. reset I''
- set $x = loc\ 1\ \ell\ \theta''$
  $\beta = lambeta\ \theta'' \times 1$
- $\underline{test}$ $\underline{if}$ (CASE A) $\exists_{\theta_1}$: $\frac{\pi}{5} - (\beta + \theta_1) = 0$ (i.e. $\pi/5 - \beta \in I_1$). Set $\theta_1 = \pi/5 - \beta$.
  $\rho/\cos(\frac{\pi}{5} - (\beta+\theta_1)) = \rho/\cos 0 = \rho$.
  then ~~post~~ (yes $\Longleftrightarrow$ $x \ge \rho$).
- $\underline{test\ CASE\ B}$ $\theta_{1\,max} < \pi/5 - \beta$. Take $\theta_1 = \theta_{1\,max}$
  r: $\rho/r = \cos(\frac{\pi}{5} - (\beta + \theta_1))$
  (yes $\Leftrightarrow$ $x \ge r$).
  $x_{max} \ge r_{min}$

Note 8/2016.



If we can make "receptor" pentagon A' perpendicular, then apply existence test at the perpendicular. Otherwise rotate pentagon A' until as close to perp as possible. Then test that.



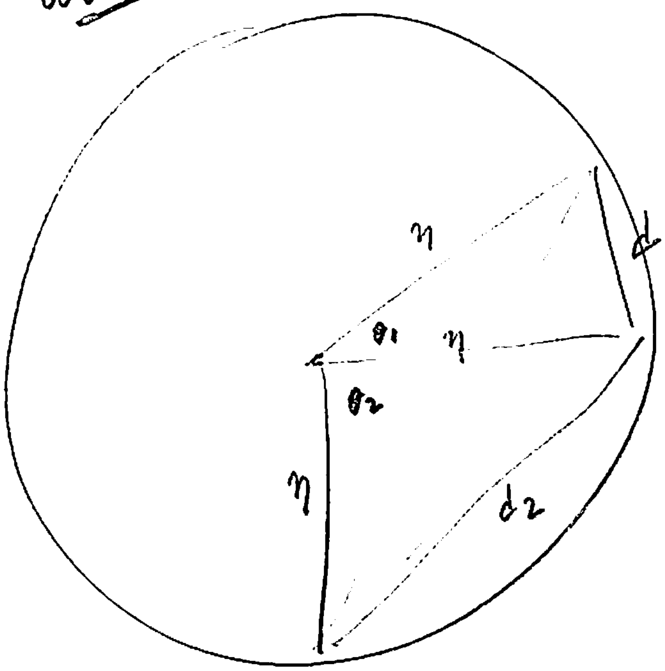
- $\underline{test\ CASE\ C}$ $\theta_{1\,min} > \pi/5 - \beta$. Take $\theta_1 = \theta_{1\,min}$.
  r: $\rho/r = \cos(\pi/5 - (\beta + \theta_1))$
  (yes $\Leftrightarrow$ $x \ge r$).
  $x_{max} \ge r_{min}$

(• empty remainder $\theta_{1\,min} \le \pi/5 - \beta \le \theta_{1\,max}$.)
$\pi/5 - \beta \notin I_1$.

acute.

9/2016.

arela



$\theta' = arc(\eta, \eta, d_1) \pm arc(\eta, \eta, d_2)$;

$d_3 = loc\ \eta\ \eta\ \theta'$ // max.

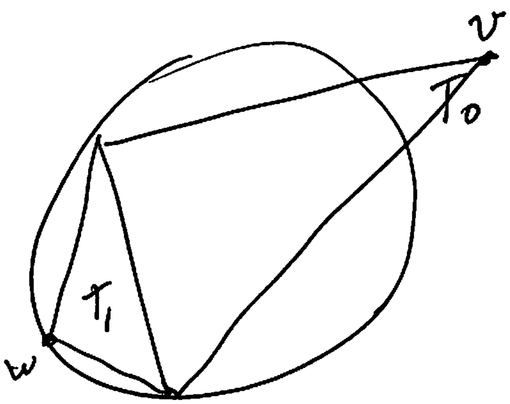
$test\theta = arc(\eta, \eta, d_1) + arc(\eta, \eta, 2r)$

$test\theta \geq \pi$ or $d_2 < 2\eta \sin(\theta/2)$

obtuse
14

Case 1 – Obtuse.

Assume f.a.c. that ① cocircular Q, ② min d_i all i: do not occur.



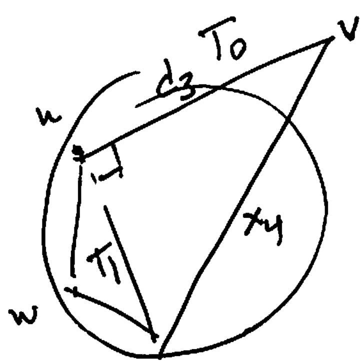
- Move ~~T_1~~ w first (fixing the diagonal) until • d_1, d_2 bounds are met.

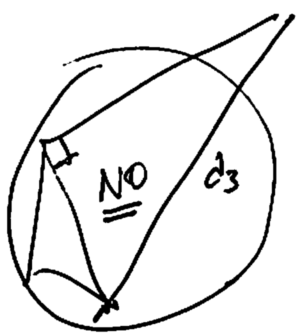
  OR • $\eta(T_1) = 2$. and d_2 met.

  (Note T_1 stays obtuse)

- Move ~~T_0~~ v next until • d_3 d_4 bounds are met.

  OR • T_0 a right triangle at an angle at ~~??~~ diagonal. and d_3 bound is met.





$\in \{2k, 1.72\}$.

Claim d_3 not hypotenuse.

Else $d_3 \geq K\sqrt{8} > 2 > d_3$ ✗.

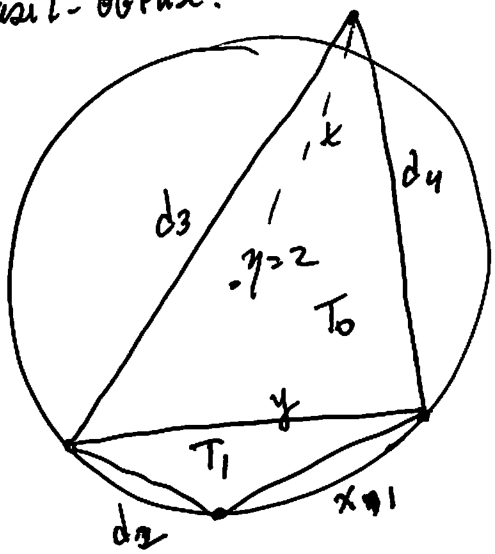
So we can move v fixing $|u-v|$ decreasing x_4, making T_0 acute again.

(Note $x_4 \geq K\sqrt{8} > 2 > d_4$ when right.)

So we move ~~T_0~~ v until d_3, d_4 bounds are met.

We get $d_2 d_3, d_4$ met, and $\eta(T_1) = 2$. not cocircular.

Case 1 - obtuse.



$\eta(T_1)=2$

~~Can assume $d_2 \le x_1$ else shrink. T_1 move.~~

(i.e. $arc(2,t,d) < \pi/2$.).

Have $arc(2,t,d_3)+arc(2,t,d_4)$

$\le arc(2,2,1.72)\cdot 2 \approx 1.77 < \pi$.

So this does not exist.

T_0 is not acute.

So we always reduce to ① or ②

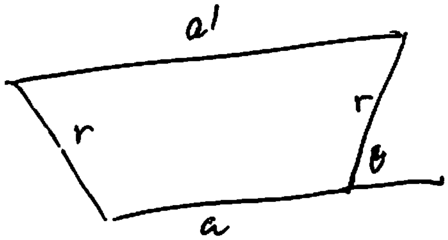
Obtuse - Case 1 - concavity.

Obtuse concavity.

isosceles trapezoid.

wlog $a' \geq a$ so $\theta \leq \pi/2$.

fix r, θ. Let $a' = a'(\theta)$



$$A(\theta) = \text{area}_\theta = a r \sin\theta + (r\cos\theta)(r\sin\theta)$$

$$= ar\sin\theta + r^2 \cos\theta \sin\theta.$$

$$= ar\sin\theta + \frac{1}{2} r^2 \sin 2\theta.$$

~~Case~~

$$A'' = \underbrace{-ar\sin\theta}_{\leq 0} - \underbrace{2r^2\sin 2\theta}_{\leq 0.} \leq 0 \quad \text{for } \theta \in [0, \pi/2]$$

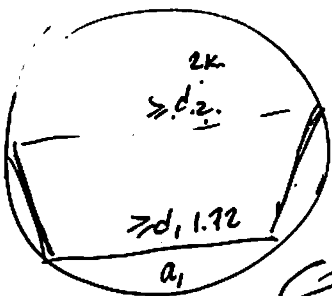
So min area is at an endpoint.

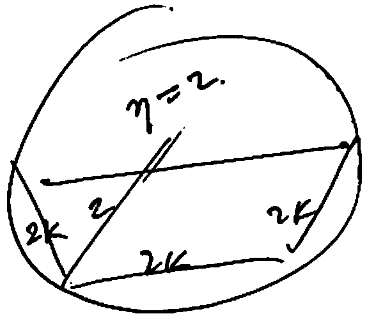
OBTUSE

2/2016.

Final Reading Case 1. of obtuse case

cocircular case





$> 2a_K + 4\varepsilon_0$. $(*_1)$.

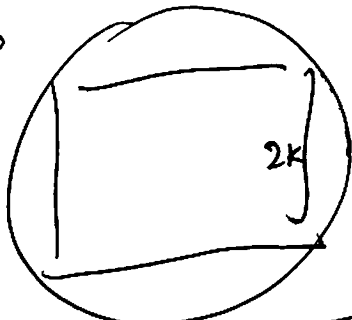
$\theta_3 = 3\,\mathrm{arc}(2,2,2K)$

$\theta_2 = 2\,\mathrm{arc}(2,2,2K)$

$\mathrm{area}(2K, 2K, 2\sin(\theta_2/2)) + \mathrm{area}(2K, x_2, 2\sin(\theta_3/2))$

x_2

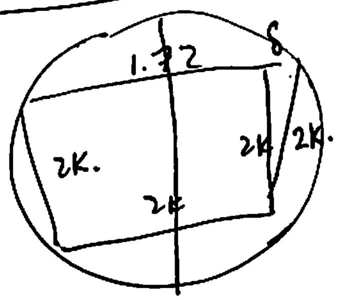
$n' = 0$



$\cancel{2K^2} = \frac{(2K)^2}{\cancel{2}} > 2a_K$. ✓

cocircular is isosceles. Use formula from concavity calc.

$n' = 1$.



$2\delta + 2K = 1.72$.

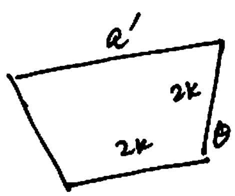
$h^2 = (2K)^2 - \delta^2$.

$\mathrm{area} = (2K)h + \delta h = 2.699.. > 2a_K + \varepsilon_0$ ✓

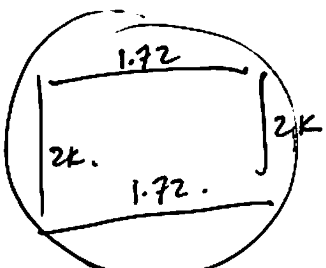
$a = 2K$

$a' = 1.72 = 2K + 2(2K)\cos\theta$ ✓

$\mathrm{area} = (2K\sin\theta)\left(\frac{a+a'}{2}\right)$

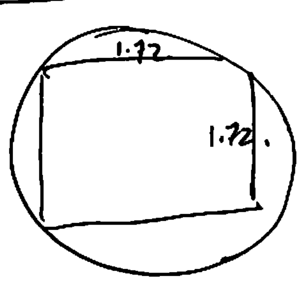


$n' = 2, 3$.



$2K(1.72) > 2a_K + 3\varepsilon_0$. ✓

$n' = 4$.



$1.72^2 > 2a_K + 4\varepsilon_0$.

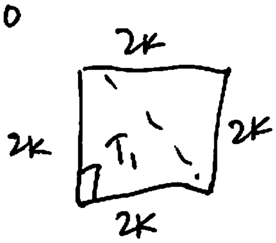
calc.ml obtuse-case1-cocircular.

OBTUSE

Final reading <u>case 1</u> obtuse theorem.

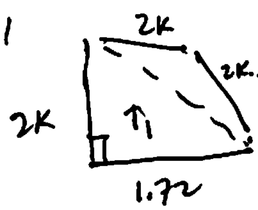
noncocirular.

$n' = 0$

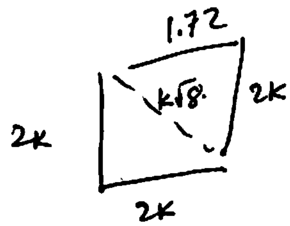


✓ done before in cocirular case. ok.

$n' = 1$

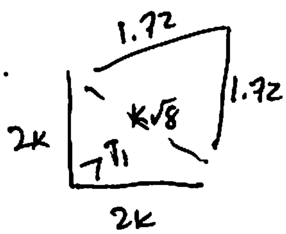


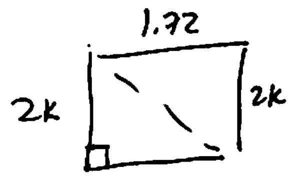
← bad Delauny. So ignore



$> 2a_k + \varepsilon_0$ ✓ 2/18/2016.

$n' = 2.$

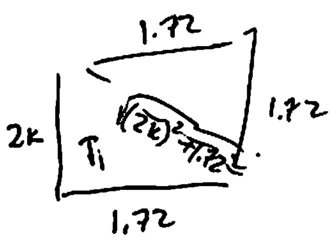




cocirular so ok.

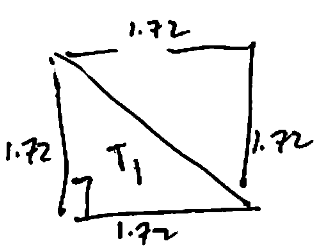
$> 2a_k + 2\varepsilon_0$ ✓ 2/8/2016.

$n' = 3$



$> 2a_k + 3\varepsilon_0$ ✓ 2/8/2016.

$n' = 4.$



cocirular, so ok.

9/2016 ✓
calcs.ml
obtuse - case1 - d4.

obtuse.
23.