Advanced Programming Exam
December 13, 2021.   8.15–13.15

Instructions: Treat at most one question per sheet, write only on one side of each sheet, number and mark each page with your initials. **Do not use red ink!** Write legibly and comment your work extensively, this can give you extra marks even if the end result is wrong. The exam contains 5 question and each question is worth 10 points.

Allowed Material: The course books (Dasgupta, Papadimitriou, Vazirani. *Algorithms*; Skiena, Revilla. *Programming Challenges*; Bentley. *Programming Pearls*), course notes and other printed material, pen or pencil and paper.

Hand-in: Answers should arrive in my email at bengt.nilsson.TS@mau.se as *one* document at the latest 13:15. Allowed document formats are: word, .txt (if you trust me) or .jpg, .png, PDF/A (if you don't). If you have questions during the exam, you can contact me via email.   **Good Luck!**

1. Your job is to arrange n ill-behaved children $c_1, c_2, \ldots, c_n$ in a straight line, facing front. You are given a list of m reports from teachers of the type "c_i hates c_j". If c_i hates c_j, you do not want to put c_i anywhere before c_j, because then c_i can throw things at c_j. Design an algorithm that places the children in the line or reports that no placement is possible. Your algorithm should take $O(n + m)$ time. (10p)

2. Given an unordered sequence of $2n$ numbers. Design an algorithm that partitions the numbers into n pairs, with the property that the partition minimizes the maximum sum of a pair. For example, the sequence $[1, 3, 2, 6, 4, 9]$ has 20 such partitions, three of them being $[(1, 3), (2, 6), (4, 9)]$, $[(1, 6), (3, 4), (2, 9)]$, and $[(1, 9), (2, 6), (3, 4)]$. The third of these has 10 as its maximum pair sum which happens to be the minimum over all 20 partitions. Your algorithm should take $O(n \log n)$ time. (10p)

3. Suppose we are given three strings of characters, X, Y, and Z, where $|X| = n$, $|Y| = m$, and $|Z| = n + m$. Z is called a *shuffle* of X and Y if and only if Z can be formed by interleaving the characters from X and Y in such a way that the left-to-right ordering of the characters from each string X and Y is maintained.

   For example, `cchocohilaptes` is a shuffle of `chocolate` and `chips` but `cchocohilatspe` is not.
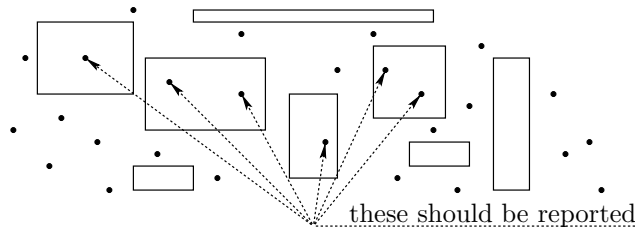
   (a) Design a recurrence for shuffle testing given the three words X, Y, and Z. (4p) (Hint: The recurrence returns boolean values.)

   (b) Construct an efficient algorithm for shuffle testing based on the recurrence from (a), using dynamic programming. (4p)

   (c) Analyze the complexity of your algorithm in terms of n and m. (2p)

4. You own a movie distribution company that transmits movies over the net for screening in customers' homes. You have k movies $M_1, M_2, \ldots, M_k$ and n customers $c_1, c_2, \ldots, c_n$. Each customer indicates which two movies they would like to see this weekend. Movies are shown at 9pm on Saturdays and Sundays. Multiple movies may be transmitted at the same time.

   You would like to partition the movies into those only shown on the Saturday and those only shown on the Sunday in such a way that every customer gets to see the two movies they desire. Showing a movie both on Saturday and Sunday is considered wasting bandwidth. Design an efficient algorithm to construct such a partition or report that no partition exists. Your algorithm should take $O(n + k)$ time. (10p)

5. Let S_1 be a set of n_1 points in the plane and let S_2 a set of n_2 non-intersecting axis parallel rectangles. Construct an $O(n \log n)$ algorithm, where $n = n_1 + n_2$, that reports the points of S_1 that lie inside the rectangles in S_2. (10p)

these should be reported

(Hint: Use plane sweep.)