Training Assignment 05a

NUMA01: Computational Programming with Python

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The purpose of this training assignment is to repeat list comprehensions, to give some examples for slicing, work more with functions and investigate sets in Python.
This assignment has 14 tasks.

Warming-up Exercises

Consult the lecture notes for solutions to the warming-up exercises. You should *not* run the code before you have written down what it will do when executed.

Task 1

Assume we have stored the following values in a list:

```
L = [0,1,2,1,0,-1,-2,-1,0]
```

What is the outcome of the following commands?

```
L[0]
L[-1]
L[:-1]
L + L[1:-1] + L
L[2:2]=[-3]
L[3:4]=[]
L[2:5]=[-5]
```

Task 2

What parts of the code below generate an error, and why?

```
def f(x):
  return sin(x)

# does this work?
x = 3.
print(f())

# and this?
print(f)
```

```
# what about:
y = 2*pi
print(f(y))
```

Task 3

What is the return value of this function?

```
def f(m):
  L = [n-m/2 for n in range(m)]
  return 1 + L[0] + L[-1]
```

Can you write another function which has the same output?

What happens if you instead use integer division? That is, replacing in your code `/` by `//`.

Exercises

Task 4

The Fibonacci numbers may be defined by the recurrence relation

$$\begin{aligned} F_0 &= 0, \\ F_1 &= 1 \\ F_n &= F_{n-1} + F_{n-2} \quad \forall\ n = 2, \ldots \end{aligned}$$

Compute F_{19} by using a `list` and in particular the `append` functionality.

Task 5

Write a function `fibonacci` that takes one parameter `n` and returns F_n. Don't use `list` in this task. Compare with your previous result.

Task 6

Program the following function:

$$f(x) = 3x^3/100. - 5x^2 + x - 3.$$

And then plot the function for $x \in [0, 5]$. To do that create a list with equally spaced x-values in the interval $[0, 5]$ and then a list that contains all $f(x)$ for these x-values.

Task 7

A positive number $\epsilon \in \mathbb{R}$ is called machine epsilon if the following holds:

$$(1.0 + \epsilon) - 1.0 = 0.$$

ϵ is usually small, but how small? Write a function that iteratively computes the machine epsilon.
Hint: Use a while loop and start with $\epsilon = 0.1$.

Task 8

Change your function to such that an arbitrary number can be, not just 1.0.

Task 9

Let

$$x_n = \frac{(\sin n)^2}{n} \quad \forall n > 0.$$

It is easy to show that $\lim_{n\to\infty} x_n = 0$. Create a list containing all the elements of this sequence x_n for all n until $x_n < 10^{-9}$. How long is this list?

Task 10

Consider the sequence:

$$x_{n+1} = 0.2\, x_n - \alpha(x_n^2 - 5) \text{ with } x_0 = 1$$

for α successively equal to -0.5, +0.5, -0.25, 0.25.

- Check the convergence; if the sequence converges, print the message

  `Sequence converged to x= <the value you got>`

  otherwise print

  `No convergence detected.`
- Check whether there are negative elements in the sequence

Hint: If $|x_n - x_{n-1}| < 10^{-9}$ consider a sequence to be convergent.

Task 11

For $\alpha = 0.5$ in the last task you got positive as well as negative elements. Construct two lists, one which contains the positive elements of the sequence and one which contains the negative elements.

Task 12

Write a function which has α as input. The function should perform the convergence test of the sequence given above. It should return `True` if the sequence converged within 30 iterations and return `False` if it didn't.

Test your function with the same values of α as in Task 4. Try it also with $\alpha = 1$. What happens?

Task 13

Give your function a second input argument `x0` so that it allows you to test convergence for different α and different starting values x_0.

Task 14

Give your functions two additional output parameters: `pos` and `neg`. One of them should contain the positive elements of the sequence and the other the negative ones. If there are only positive elements in the sequence `neg` should be an empty list and vice versa.