

Fusion-Based Variational Image Dehazing

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Abstract—We propose a novel image-dehazing technique based on the minimization of two energy functionals and a fusion scheme to combine the output of both optimizations. The proposed fusion-based variational image-dehazing (FVID) method is a spatially varying image enhancement process that first minimizes a previously proposed variational formulation that **maximizes contrast and saturation** on the hazy input. The iterates produced by this minimization are kept, and a second energy that **shrinks faster intensity values of well-contrasted regions is minimized, allowing to generate a set of difference-of-saturation (DiffSat) maps by observing the shrinking rate**. The iterates produced in the first minimization are then fused with these DiffSat maps to produce a haze-free version of the degraded input. The FVID method does not rely on a physical model from which to estimate a depth map, nor it needs a training stage on a database of human-labeled examples. Experimental results on a wide set of hazy images demonstrate that FVID better preserves the image structure on nearby regions that are less affected by fog, and it is successfully compared with other current methods in the task of removing haze degradation from faraway regions.

Index Terms—Color correction, contrast enhancement, image dehazing, image fusion, variational image processing.

I. INTRODUCTION

IMAGES acquired in outdoor scenarios often suffer from **degradation produced by the atmosphere that lies between the observer and the objects in the scene**. This phenomenon, known as haze, distorts contrast and color in the image, decreasing the visibility of contents in the scene and reducing visual quality.

The task of removing haze degradation from an image is known as image dehazing and has recently given rise to an

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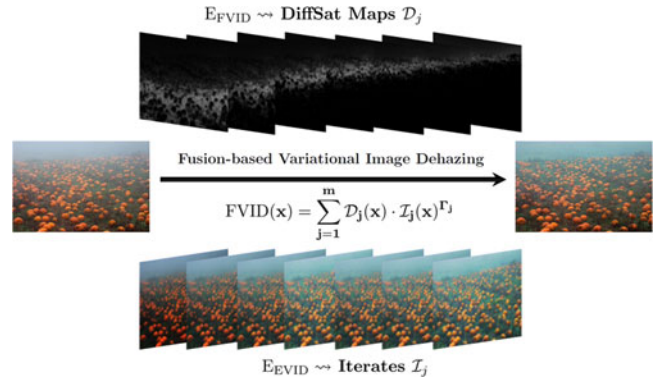


Fig. 1. Schematic representation of the three steps composing the FVID technique.

important area of research. A first family of methods rely on inverting a physical model of the degradation, such as Koschmieder's atmospheric scattering model [1]:

$$I(x) = t(x)J(x) + (1 - t(x))A. \quad (1)$$

Here, x represents a pixel location, $I(x)$ is the observed intensity, $J(x)$ is the scene radiance of a haze-free image, $t(x)$ is the transmission of light in the atmosphere, inversely related to the scene's depth, and A is the airlight, a global vector quantity describing the ambient light.

When no extra information apart from the input image is considered, the problem is known as single-image dehazing. A common approach is to formulate some restriction on the visual characteristics that a reasonable solution can have (see [2]–[4]). The most popular work is the **Dark Channel Prior** [5], which states that **haze-free images locally have a low value of luminance in some color channel**. Some extensions have been further used in [6]–[12]. A second approach is based on machine-learning techniques [10], [13]–[15]. A third group of methods is based on spatially varying image-processing techniques. They study the local structure of the image to infer the presence of haze and try to compensate for this effect (see, e.g., [16] or [17]). **A fusion of white-balanced and contrast-enhanced versions of the input hazy image has also been proposed** [18]. There also exist approaches based on perceptual models, such as Retinex [19]–[21]. One drawback of these methods is that, due to the **partial lack of physical information within their formulation, they may present under or overenhanced results**. In general, a well-designed image-dehazing method should **enhance visibility in faraway areas of the scene, but regions not affected by fog should keep their original luminance and chromatic characteristics**.

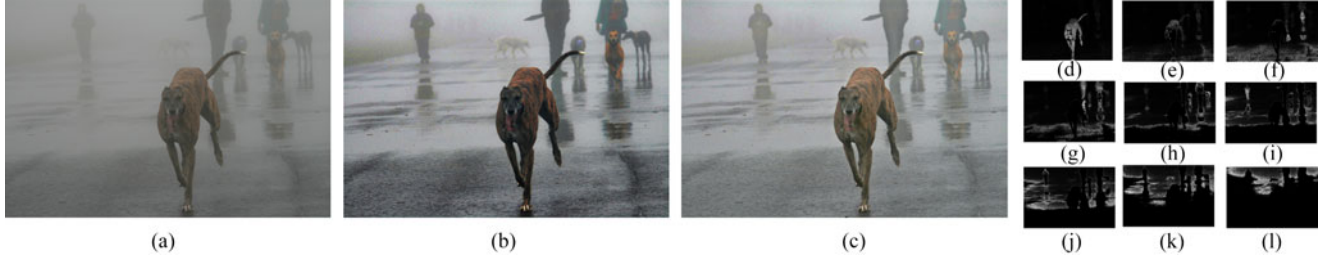


Fig. 2. (a) Hazy scene, and the result of applying (b) EVID dehazing technique and (c) the proposed FVID method. (d)–(l) Progressive DiffSat maps generated by our approach. Note the loss of details in the face of the dog produced by EVID. This overenhancement artifact is properly corrected by FVID.

The main contribution in this letter is a fusion-based variational dehazing method that improves previous works by retaining high contrast and colorfulness enhancing capabilities on faraway regions, while preserving image content on nearby regions. To that end, we design a three-step procedure. We first minimize the enhanced variational image-dehazing (EVID) energy, which was designed for image dehazing [22], but tends to generate some overenhancement on nearby areas. Instead of retaining only the last EVID iterate as the dehazed output, we store images associated with all iterates, corresponding to progressively dehazed images. We then minimize a new energy, which we call E_{FVID} . While the minimum of E_{FVID} is a dark image, we can employ its iterates to estimate the degree of degradation of each pixel by computing the difference of saturation (DiffSat) of consecutive iterations. Finally, we combine both sources of information through a fusion procedure that leads to a satisfactorily dehazed output, free of overenhancement artifacts (see Fig. 1).

II. VARIATIONAL IMAGE DEHAZING

To restore haze-degraded outdoor images, the authors of [22] proposed to minimize the following image energy:

$$E_{\text{EVID}}(I^j) = \frac{\alpha}{2} \sum_x (I^j(x) - \mu^j)^2 + \frac{\beta}{2} \sum_x (I^j(x) - I_0^j(x))^2 - \frac{\gamma}{2} \sum_{x,y} \omega(x,y) |I^j(x) - I^j(y)| - \frac{\eta}{2} \sum_{x,y} \omega(x,y) |I^j(x) - I^{j+1}(y)| - \frac{\eta}{2} \sum_{x,y} \omega(x,y) |I^j(x) - I^{j+2}(y)| \quad (2)$$

where I^j is a color channel with values in $[0, 1]$, I_0 is the input; x and y are pixel coordinates; α, β, γ , and η are positive parameters; $\omega(x, y)$ is a distance function with values decreasing as the distance between x and y increases; $\mu^j \approx 2 \text{mean}(I^j) - A^j$ is an estimate of the predicted mean of the haze-free image; and $j \in \mathbb{Z}_3$. Here, $\{R, G, B\}$ are identified with the space of integers modulo 3, \mathbb{Z}_3 , e.g., $I^2 = I^G$, and $I^3 = I^B$. This energy is termed EVID.

III. FUSION-BASED VARIATIONAL IMAGE DEHAZING (FVID)

Despite the good dehazing capabilities of (2), it sometimes produces overenhanced results (see Fig. 2). Nonetheless, each generated iterate implicitly carries useful information on the degree of enhancement each region needs. This observation leads us to design a method to extract such information, in the

form of progressive DiffSat maps. After building those maps, we use them to formulate a fusion process to blend the EVID iterates into a single image, keeping only the most useful visual information at each input layer.

A. Progressive DiffSat Map Generation

To capture the implicit depth information that EVID generates, we extend the energy of (2) as follows:

$$E_{\text{FVID}}(I^j) = E_{\text{EVID}}(I^j) + \tau \sum_x I^j(x). \quad (3)$$

This straightforward extension keeps the dehazing capabilities of EVID, while it adds a penalty term for large image intensity values. In practice, this amounts to prioritizing a dark image.

To minimize the E_{FVID} energy, the Euler–Lagrange equations for E_{FVID} can be easily derived (see [22]):

$$\nabla E_{\text{FVID}}(I^j) = \alpha(I^j - \mu_j) + \beta(I^j - I_0^j) - \gamma R(I^j, I^j) - \eta(R(I^j, I^{j+1}) + R(I^j, I^{j+2})) + \tau = 0 \quad (4)$$

for $j \in \{R, G, B\}$. Here, $R(I^i, I^j)$ is defined as follows:

$$R(I^i, I^j)(x) = \frac{\sum_y \omega(x, y) s(I^i(x) - I^j(y))}{\sum_y \omega(x, y)} \quad (5)$$

with s being a sigmoid function, and with j defined modulo 3, varying in the $\{R, G, B\}$ components of the input image.

Since (2) is not differentiable, a smooth approximation of the absolute value should be employed. Here, we follow the same approximation as in [22] to obtain (4).

Once the gradient of the energy is computed, we can advance toward one of its minima by using gradient descent:

$$I_{k+1}^j = I_k^j(1 - \Delta t(\alpha + \beta)) + \Delta t(\alpha \mu^j + \beta I_0^j) + \Delta t[\gamma R(I_k^j, I_k^j) + \eta[R(I_k^j, I_k^{j+1}) + R(I_k^j, I_k^{j+2})] - \tau]. \quad (6)$$

The additional term included in (3) penalizes large intensity values in the image. Thus, the above gradient descent shrinks intensity values of the input image. The energy minimization takes place in the space of nonnegative images. This imposes a non-linear constraint that can be handled with different approaches, e.g., the Karush–Kuhn–Tucker conditions [23]. A simpler approach employed in this work consists of, when an iteration takes a negative value, back-projecting the image into the $[0, 1]$ range by clipping.

Note that the minimization of E_{FVID} differs from that of E_{EVID} in that we introduce a shift in the Gray-World value. To

see this, we reformulate (4) as

$$\begin{aligned} \nabla E_{\text{FVID}}(I^j) &= \alpha \left(I^j - \left(\mu_j - \frac{\tau}{\alpha} \right) \right) + \beta (I^j - I_0^j) \\ -\gamma R(I^j, I^j) - \eta (R(I^j, I^{j+1}) + R(I^j, I^{j+2})) &= 0. \end{aligned} \quad (7)$$

We are not interested in the minimum of (3), but in the rate at which intensity values of individual pixels $I(x, y)$ tend to zero. Closer regions have more saturated pixels, and their values rapidly decrease driven by the interchannel contrast maximization. Faraway regions contain mostly achromatic nonsaturated pixels that take more time to vanish. We, thus, observe the difference in saturation of each iteration with respect to the previous one. Then, we generate a series of DiffSat maps to accurately reflect the depth distribution in the scene:

$$D_k = \text{Sat}(I_k) - \text{Sat}(I_{k-1}) \quad (8)$$

where I_k are the iterations from (6), and $\text{Sat}(I_k)$ is the saturation of I_k ; $\text{Sat}(I_k) = (\max_j(I_k^j) - \min_j(I_k^j)) / \max_j(I_k^j)$, for $j \in \{R, G, B\}$, and being $I_0 = I$ the input. Fig. 1(d)–(l) displays the generated DiffSat maps.

B. Fusion Procedure

After the minimization of (2) and (3) has generated the EVID iterates $\{I_j\}_{j=1}^N$ and the DiffSat maps $\{D_k\}_{k=1}^M$, respectively, there remains to fuse both information sources. The fusion process should reflect that DiffSat maps corresponding to late iterates, i.e., to those regions with higher fog, correlate with further processed haze-free iterations.

First, we either interpolate or extrapolate the set of DiffSat maps to obtain a new set of exactly N depth maps. These new maps are convolved with a Gaussian kernel in three dimensions (x , y , and temporal) in order to present smoothed transitions, and they are later normalized so that the sum in the temporal dimension for any pixel x is equal to 1. We finally compute the fused image as the combination of these normalized depth maps and the haze-free iterates as

$$\text{FVID}(x) = \sum_{j=1}^N \mathcal{D}_j(x) \cdot \mathcal{I}_j(x)^{\Gamma_j} \quad (9)$$

where $\Gamma = [\Gamma_1, \dots, \Gamma_l]$ is a set of increasing values between 0.45 and 1.2 that countereffect the fact that the original image is in linear form (i.e., it is not gamma corrected).

IV. EXPERIMENTAL EVALUATION

We implement (2) and (3) with the following reference values: $\alpha = 0.5$, $\beta = 0.5$, $\gamma = 0.2$, $\eta = 0.02$, and $\tau = 1$. Both distance functions are Gaussians with a standard deviation of 50 pixels. The time step was set to $\Delta t = 0.15$ for (2) and $\Delta t = 0.05$ for (3), to allow for a more fine-grained DiffSat map set. We consider that a steady state of the gradient descent is achieved when the difference between the images of two consecutive iterations is below 0.02. The set of Γ values for (9) is equally spaced between 0.45 (value considered for the first iterate) and 1.2 (value employed for the last one). In terms of computational load, FVID doubles the complexity of EVID, since it minimizes

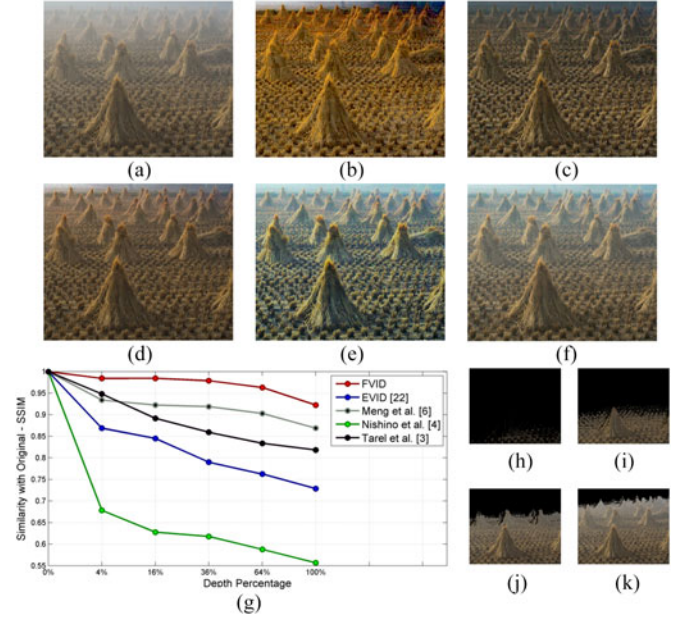


Fig. 3. Hazy scene and outputs of different dehazing methods: (b) BD [4], (c) NBP [3], (d) BCCRD [6], (e) EVID [22], and (f) FVID. (g) SSIM of partial images produced masking dehazed outputs by binarized depth maps. (h)–(k) show progressive masked images at a depth of (h) 4%, (i) 16%, (j) 36%, and (k) 64%.

both models (2) and (3). We typically observe a number of iterations between 6 and 12. The fast Fourier transform technique proposed in [24] can also be applied here, delivering an $\mathcal{O}(n)$ complexity, which, in our implementation, requires 90 s to process a 1-MP image.¹ An algorithmic analysis of the minimization of the energy in [24], which lies at the core of the proposed technique, is available in [25].

Fig. 3(a) describes a typical foggy scene. While all methods improve the visibility of the wheat piles, the method in [4] and EVID suffer from heavy overenhancing. Here, the approaches of [3] and [6] and FVID seem to recover a well-dehazed result. We confirm it by conducting a simple experiment. First, we obtain a depth map from the Dark Channel method, which provides a reliable transmission map for images degraded by homogeneous fog. We normalize the transmission and threshold it at successive depth levels to obtain cumulative depth maps and corresponding partial images [see Fig. 3(b)–(e)]. For each thresholded depth map, we measure the deviation of the original image from the dehazed results in terms of image structure, applying the SSIM index [26]. We see in Fig. 3(g) that every dehazing method produces a decreasing sequence of SSIM scores, as expected. However, the method of [4] and EVID produce rapidly decreasing scores. This implies that they aggressively modify nearby regions when dehazing faraway areas. This is confirmed by the overenhanced results shown in Fig. 4(b) and (e). Conversely, the methods of [3] and [6] and FVID produce more balanced successive SSIM scores, i.e., they better preserve nearby areas. While faraway areas of the scene seem to be satisfactorily dehazed by all the methods, the FVID approach

¹The software to reproduce the results shown in this letter is available for download at <https://sites.google.com/site/agaldran/software>

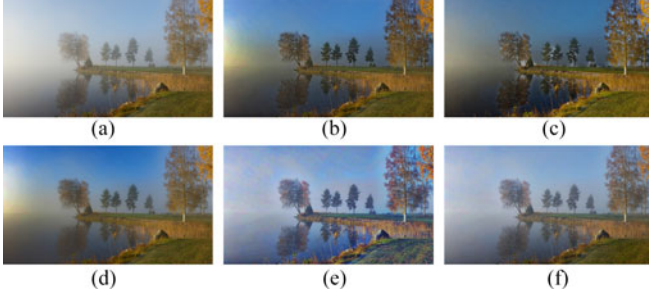


Fig. 4. Hazy scene with an uneven illumination and the outputs of (b) FADE [15], (c) no-black pixel [3], (d) dark channel [5], (e) EVID [22], and (f) FVID.

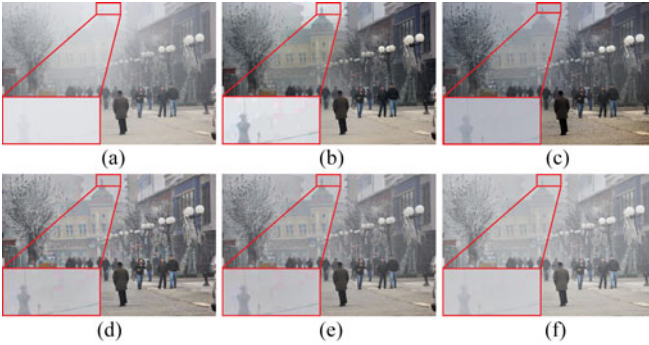


Fig. 5. Hazy scene with sky region, and results of (b) DC [5], (c) FADE [15], (d) EVID, (e) FVID, and (f) FVID with adaptive attachment to data.

produces a dehazed image that slowly separates from the input hazy scene, dehazing the scene progressively while better keeping the structural similarity in less hazy areas.

Like EVID, FVID dehazes images even when they contain uneven illumination. The sun located on the left of Fig. 4 produces a nonuniform illumination. As usual, in dehazing methods that assume a regular distribution of haze, the techniques in [3], [5], and [15] overcompensate the illumination in the left area, introducing yellowish artificial tones. In contrast, EVID and FVID increase visibility (see the central tree) without introducing color artifacts. EVID produces excessive contrast on nearby regions, as observed in the areas of grass close to the camera. These undesired artifacts are satisfactorily avoided by FVID.

Another seldom addressed problem is the appearance of chromatic artifacts in sky regions [27], [28]. Here, we take advantage of the flexibility of the variational approach to process sky areas on the image without introducing overenhancing artifacts. We consider an adaptive weighting strategy for the attachment to data term in the DiffSat map generation scheme: we modify β in (3) as an array of increasing weights, $\beta = (\beta_i) = (\beta_0 \cdot 1.1^i)$. This produces a set of DiffSat maps that do not consider sky regions as being faraway and leads to a more conservative behavior of the fusion stage therein. We compare this approach in Fig. 5 with the standard FVID, EVID, DC [5], and FADE [15] to show the artifact removal effect of the adaptive attachment-to-data term. The chromatic distortion generated by standard FVID on sky regions is mitigated by this modified version, while contrast gain is retained in other areas, including faraway

TABLE I
QUANTITATIVE RESULTS AS THE MEAN FOR 48 IMAGES

Error measure	[31]	[3]	[5]	EVID [22]	FVID
l2-color	47.96	52.03	48.93	47.18	44.42
PSNR _{lum}	16.03	16.52	15.53	17.00	17.31
PSNR _{split}	6.90	6.99	6.79	7.13	7.19
Corr _{split}	1.24	1.21	1.18	1.25	1.26
Corr _{lum}	0.70	0.69	0.66	0.71	0.72

buildings. We have zoomed-in the sky region to better observe these distortions. They are substantially alleviated with the adaptive attachment-to-data strategy.

To quantitatively evaluate FVID, we perform a similar procedure as in [22]. We simulate a dataset composed of 48 foggy images: 12 scenes from [29] are degraded with four different realistic fog layers, generated with the software provided in [30]. We compare FVID with other recent methods, namely EVID [22], and the methods of [5], [3], and [31]. Again, FVID was computed with the same fixed parameters for all scenes.

In this letter, we slightly modify the five image metrics used in [22]. Here, the first measure computes the peak-signal-to-noise ratio (PSNR) between each channel of the images and combines the results of the three channels using the l_2 norm: $\text{PSNR}_{\text{split}} = \sqrt{\sum_{c=r,g,b} \text{PSNR}(I_c^{\text{GT}}, I_c^m)^2}$, where I^{GT} is the ground-truth image, I^m is the method used, and the subindexes are each color channel. The second measure is the PSNR between the luminance of the images. The third measure is computed as $l_2 - \text{color} = \sum_{i=1}^M \sqrt{\sum_{c=r,g,b} (I_c^{\text{GT}}(i) - I_c^m(i))^2 / M}$, where the images are in the range (0,255). The last two correlation measures are defined as $\text{Corr}_{\text{split}} = \sqrt{\sum_{c=r,g,b} \text{Corr}(I_c^{\text{GT}}, I_c^m)^2}$ and $\text{Corr}_{\text{lum}} = \text{Corr}(I_{\text{lum}}^{\text{GT}}, I_{\text{lum}}^m)$, where Corr gives the correlation coefficient between the images. Results are shown in Table I. For all the evaluated measures, except for the $l_2 - \text{color}$, a higher value means a better method. The proposed FVID method outperforms all the others.

V. DISCUSSION AND CONCLUSION

We have described FVID, a novel haze removal method. It extends significantly upon a previously proposed variational dehazing scheme (EVID) by integrating information from its iterates and a hierarchical set of DiffSat maps coming from an extension of the EVID image energy. A fusion of visual information coming from both sources leads to an effective dehazing technique. FVID achieves results comparable to state-of-the-art techniques when enhancing faraway regions, while it preserves nearby regions. We also propose a variant of FVID to handle overenhancement artifacts that appear when dehazing scenes with sky regions. Moreover, the flexibility of variational methods allows us to generate families of FVID methods with different features, which shall be explored in future work.

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