Example 3-1 Equilibrium position of a spring system
Consider the simple two-spring system shown in the undeformed position in Fig. 3-1a and after deformation in Fig. 3-1b. The springs are assumed to be

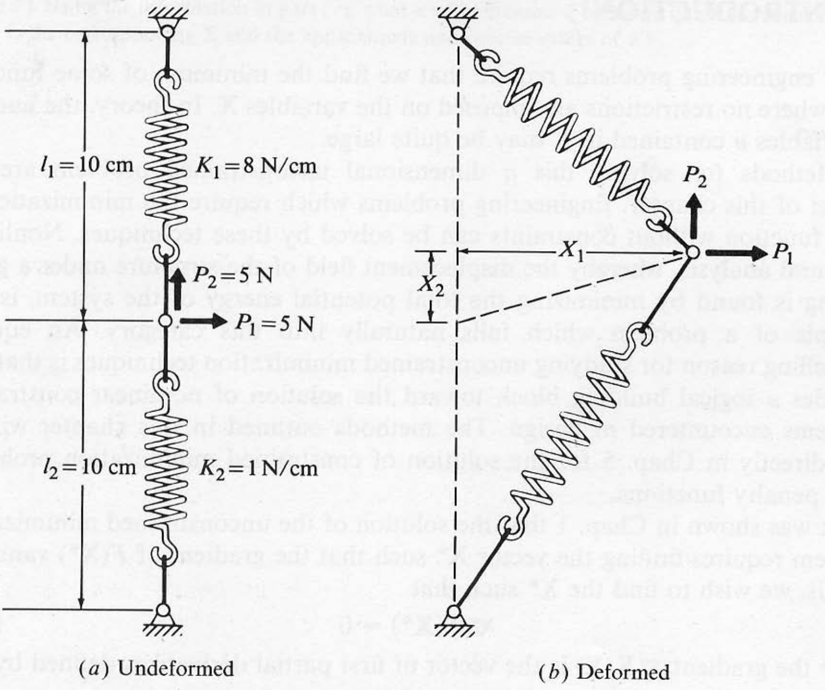

Figure 3-1 Equilibrium of a spring-force system.

linearly elastic and the loads P_1 and P_2 are constant. This is a geometrically nonlinear analysis problem because the resistance to the load is a function of the deformed position.

Defining the spring deformation as Δl_1 and Δl_2 for springs 1 and 2, respectively, we can write an expression for the total potential energy of the system as

$$\text{PE} = \tfrac{1}{2}K_1(\Delta l_1)^2 + \tfrac{1}{2}K_2(\Delta l_2)^2 - P_1X_1 - P_2X_2 \tag{3-2}$$

In terms of the original lengths l_1 and l_2 and the displacements X_1 and X_2, this becomes

$$\begin{aligned}\text{PE} = {} & \tfrac{1}{2}K_1\left[\sqrt{X_1^2 + (l_1 - X_2)^2} - l_1\right]^2 \\ & + \tfrac{1}{2}K_2\left[\sqrt{X_1^2 + (l_2 + X_2)^2} - l_2\right]^2 - P_1X_1 - P_2X_2\end{aligned} \tag{3-3}$$

Equation (3-3) can be solved for the equilibrium displacement as an unconstrained minimization problem where X_1 and X_2 are the independent design variables and PE is the objective function.

Figure 3-2 shows the two-variable function space for this example. The minimum PE $= -41.81\ N \cdot \text{cm}$ is found at $X_1 = 8.631$ cm and $X_2 = 4.533$ cm as shown on the figure. This design space will be used in the following sections to demonstrate the algorithms which can be used to solve the problem numerically. Although this is a problem in only two variables, it should be remembered that the concepts presented here are not limited to two variables. In principle, we could add many more springs and loads and incorporate material nonlinearities into the problem and still achieve a solution. In practice, we are usually limited by computational expense and by numerical ill-conditioning to perhaps 100 design variables.

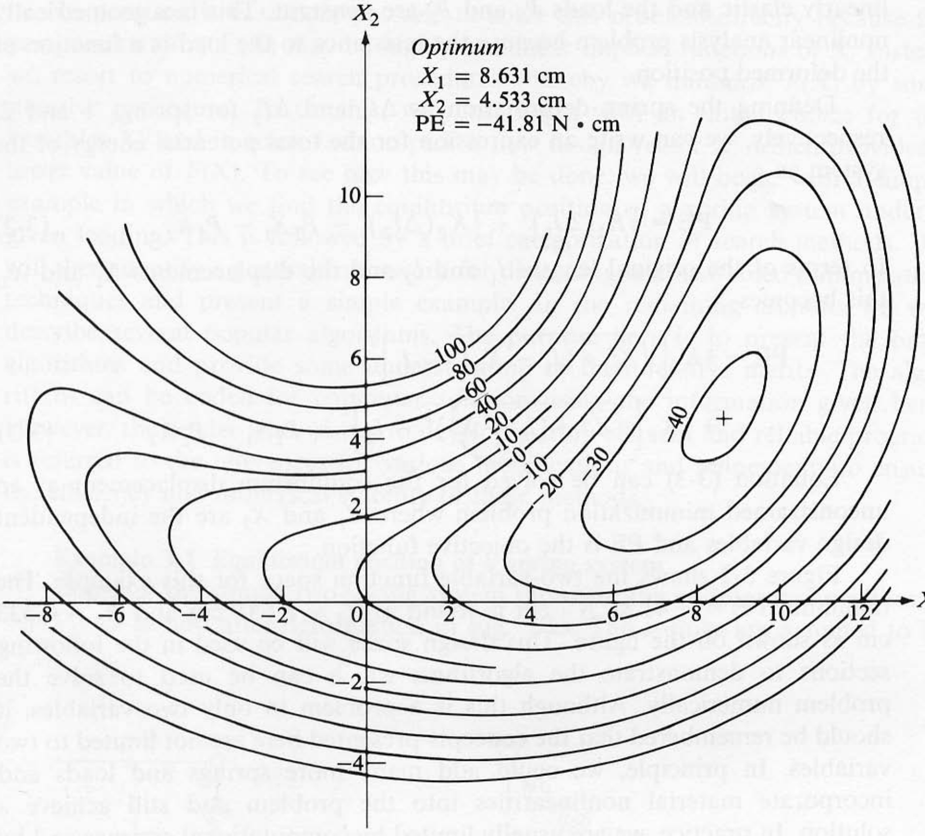

Figure 3-2 Two-Variable function space for the spring-force system.