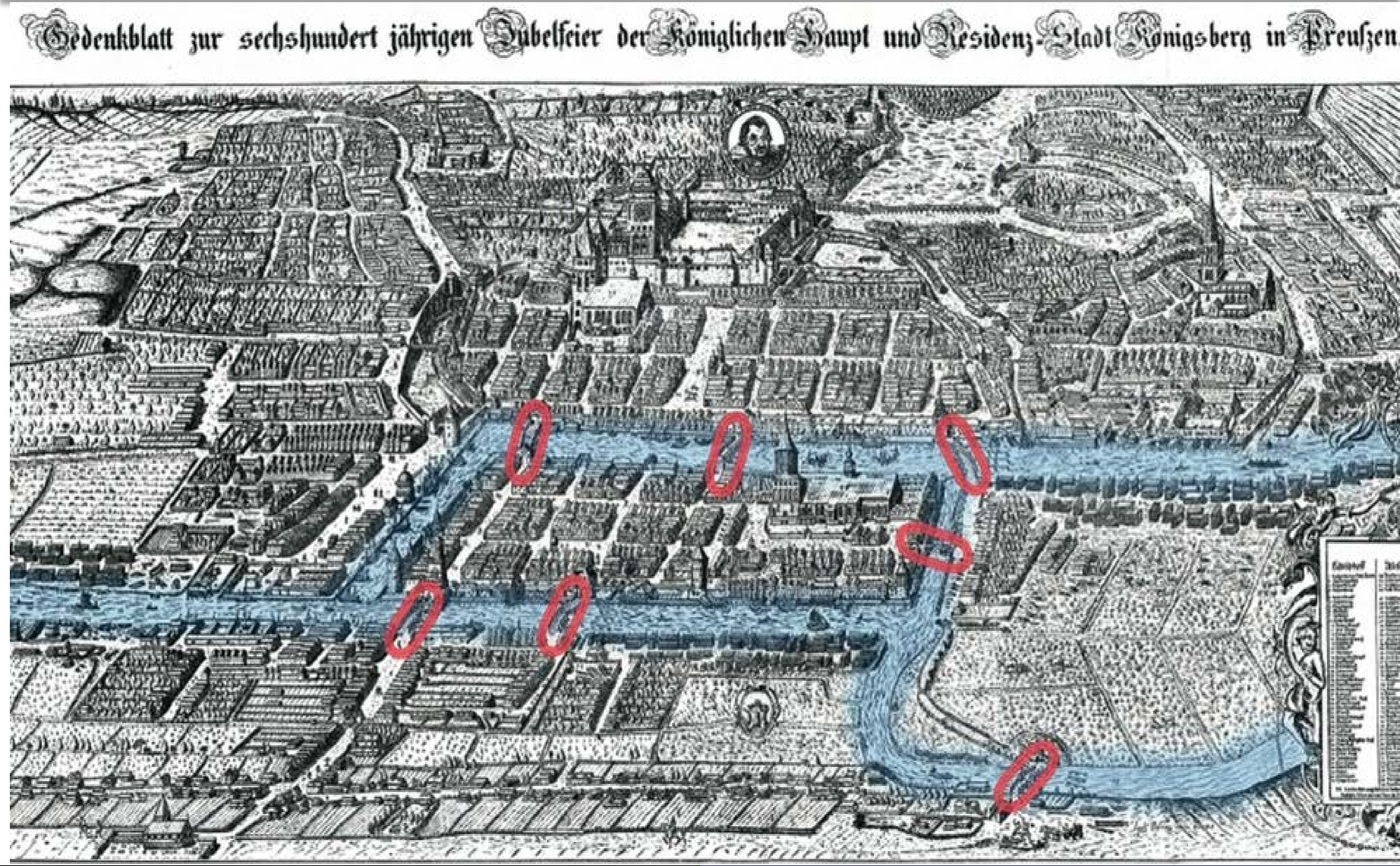
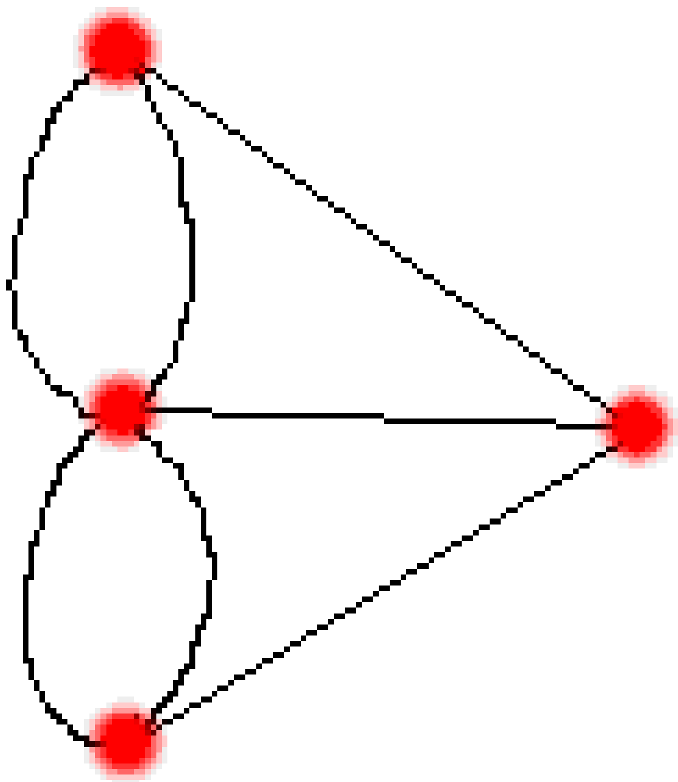
Network thinking in music

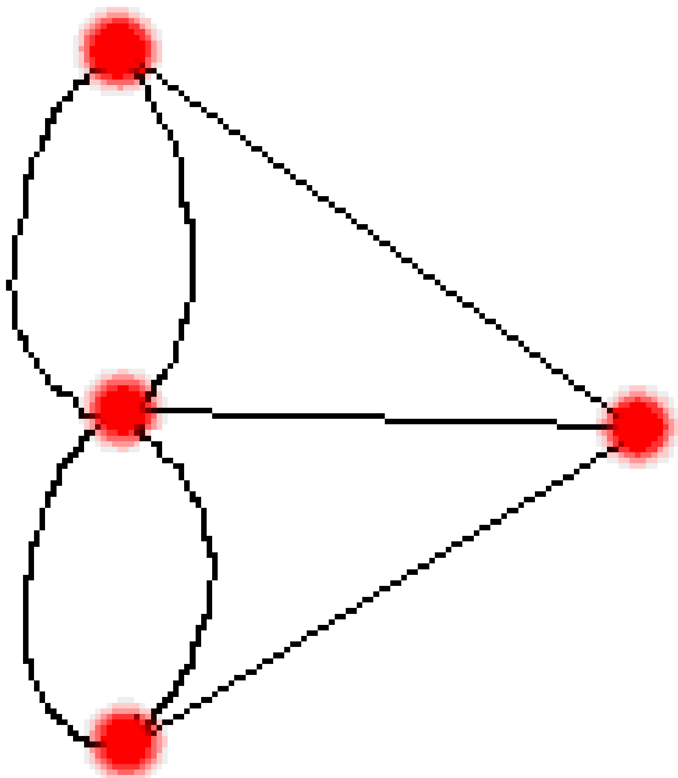
The seven bridges of Königsberg



Network thinking reduces complexity



Hidden properties of network structures



Graph or networks?

Graph and networks

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Graph and networks

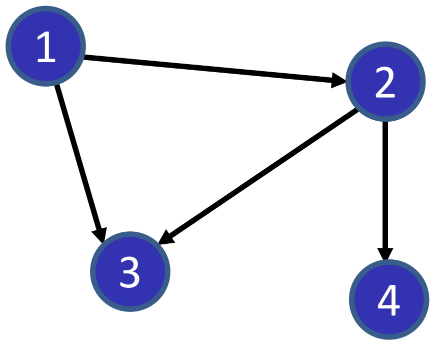
- Networks are found in nature and society
- Graphs are the mathematical representation of these networks (a map)
- In the literature, both are used interchangeably

An advice network

- Emma (1) helps Mason (2)
- Emma (1) helps William (3)
- Mason (2) helps William (3)
- Mason (2) helps Sophia (4)

A graph

- Emma (1)
- Mason (2)
- William (3)
- Sophia (4)



A graph

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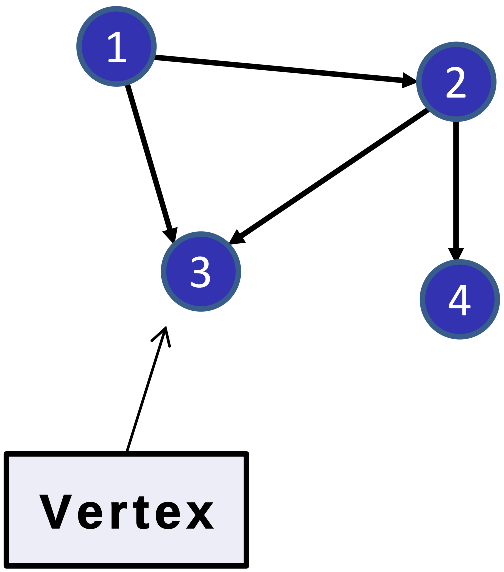

A graph

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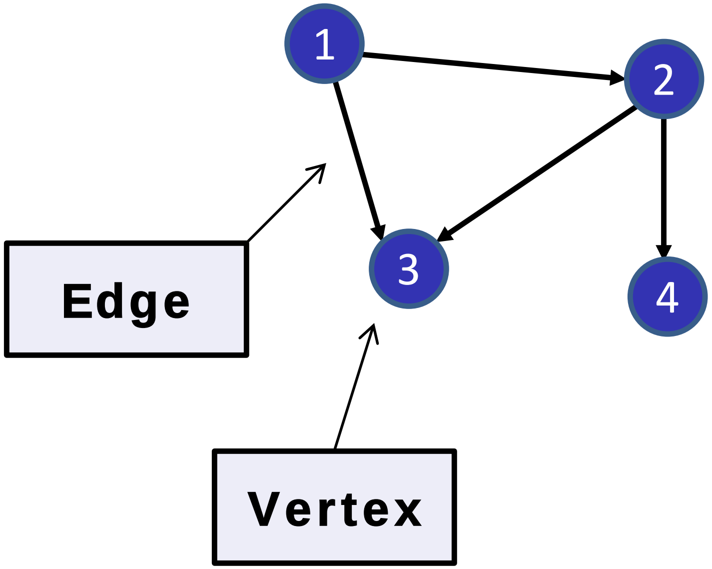

A network

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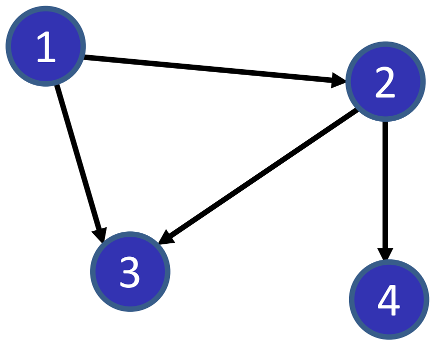
- William (3)
- Sophia (4)



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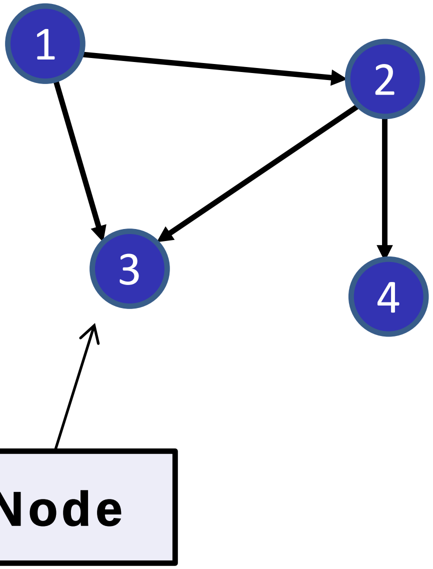

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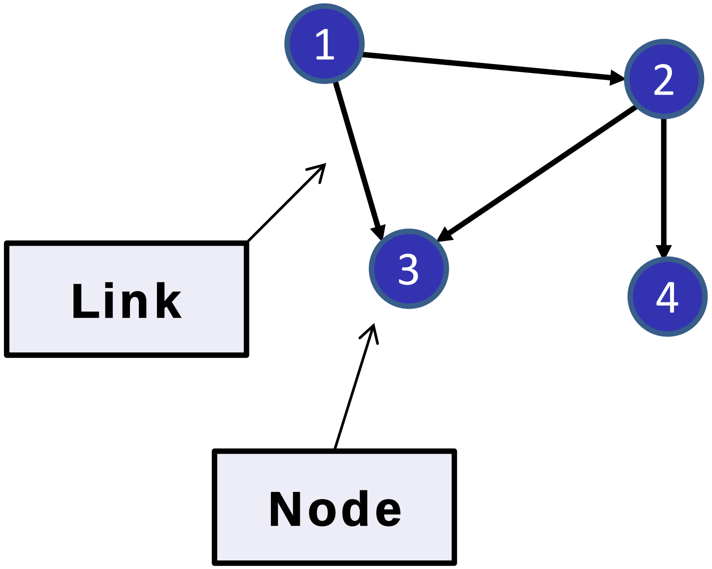
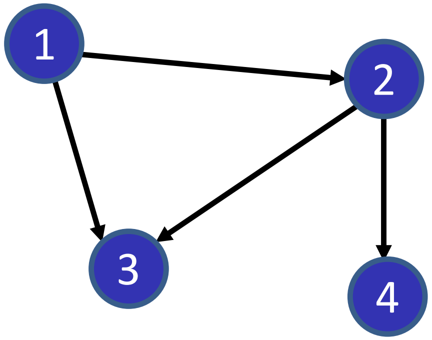


Network terms

- N = number of nodes (size of the network)
- $N = 4$
- The network is composed by the nodes $i = 1, 2, \ldots, N$
- L = number of links
- $L = 4$
- The connection between Mason and William [*Mason (2) helps William (3)*] is denoted as (2,3)
- A graph might be denoted as G, its vertex set as V(G), and its edge set as E(G)

Different types of networks

Directed network



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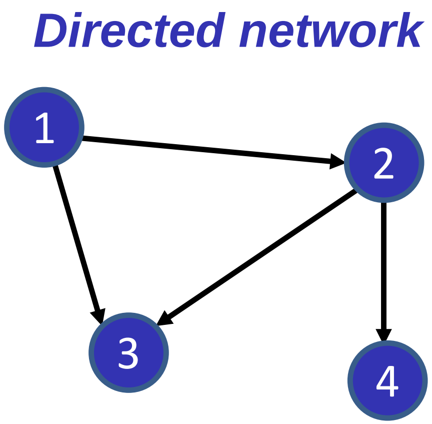
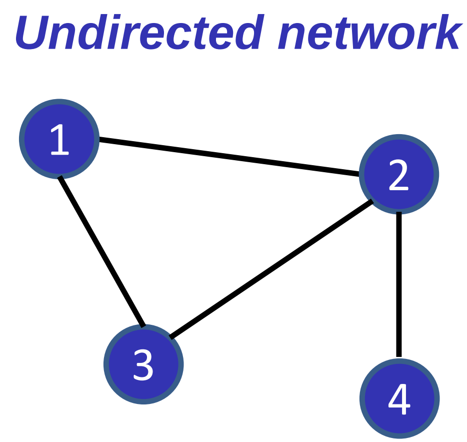
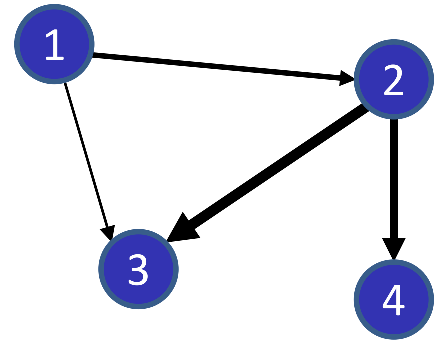


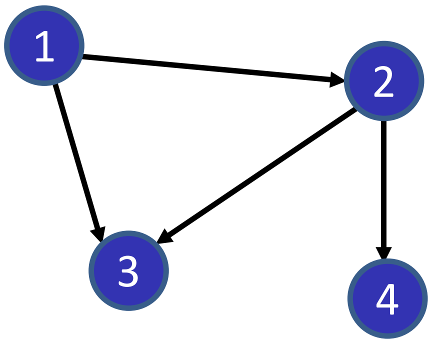


Different types of networks

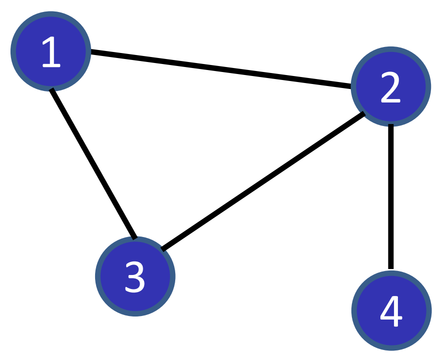
Weighted network



Directed network



Undirected network

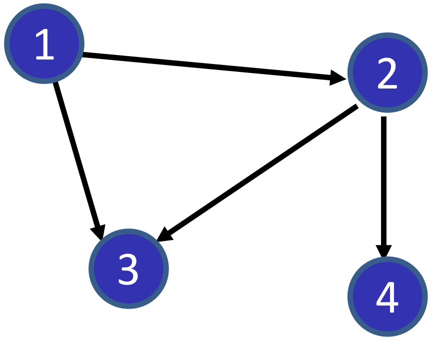


Real world networks

Network name	Nodes	Links	Direction of ties	N
Internet	Routers	Internet connections	No	200,000
WWW	WebPages	Hyperlinks	Yes	500,000
Friendship network	Individuals	Friendship	No	200
Actor network	Actors	Co-acting	No	200,000
Patent citations network	Patent documents	Citations	Yes	7,000,000
Co-invention network	Inventors	Co-patenting	No	200,000

Network representations

Directed graph (digraph)



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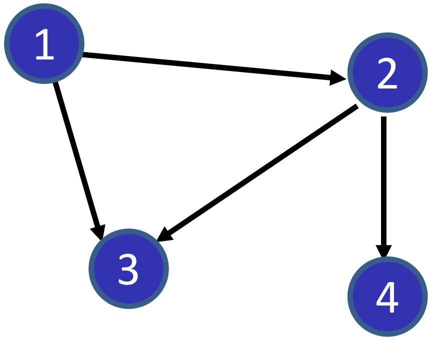

Edge list

Vertex	Vertex
1	2
1	3
2	3
2	4

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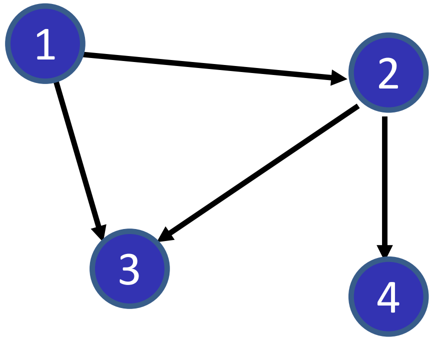

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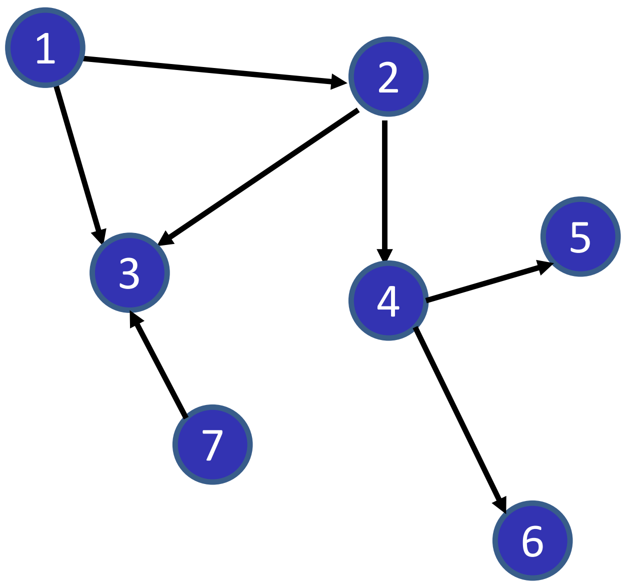
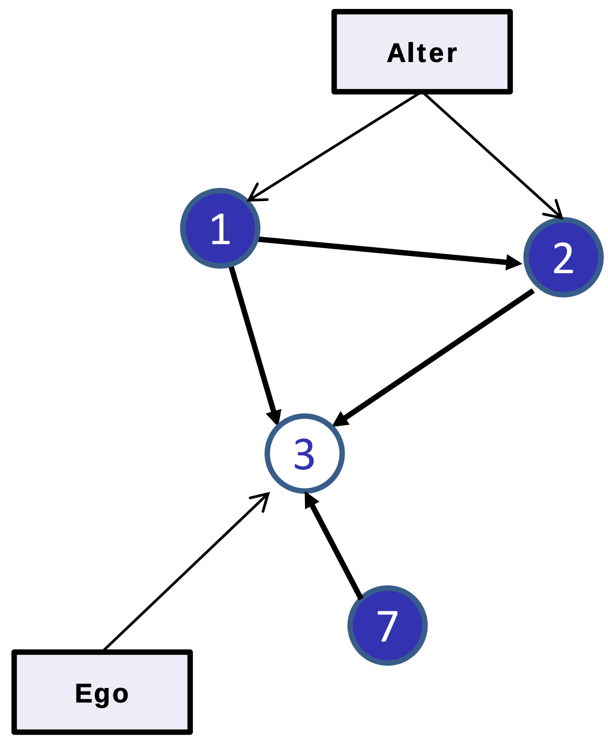
Adjacency matrix

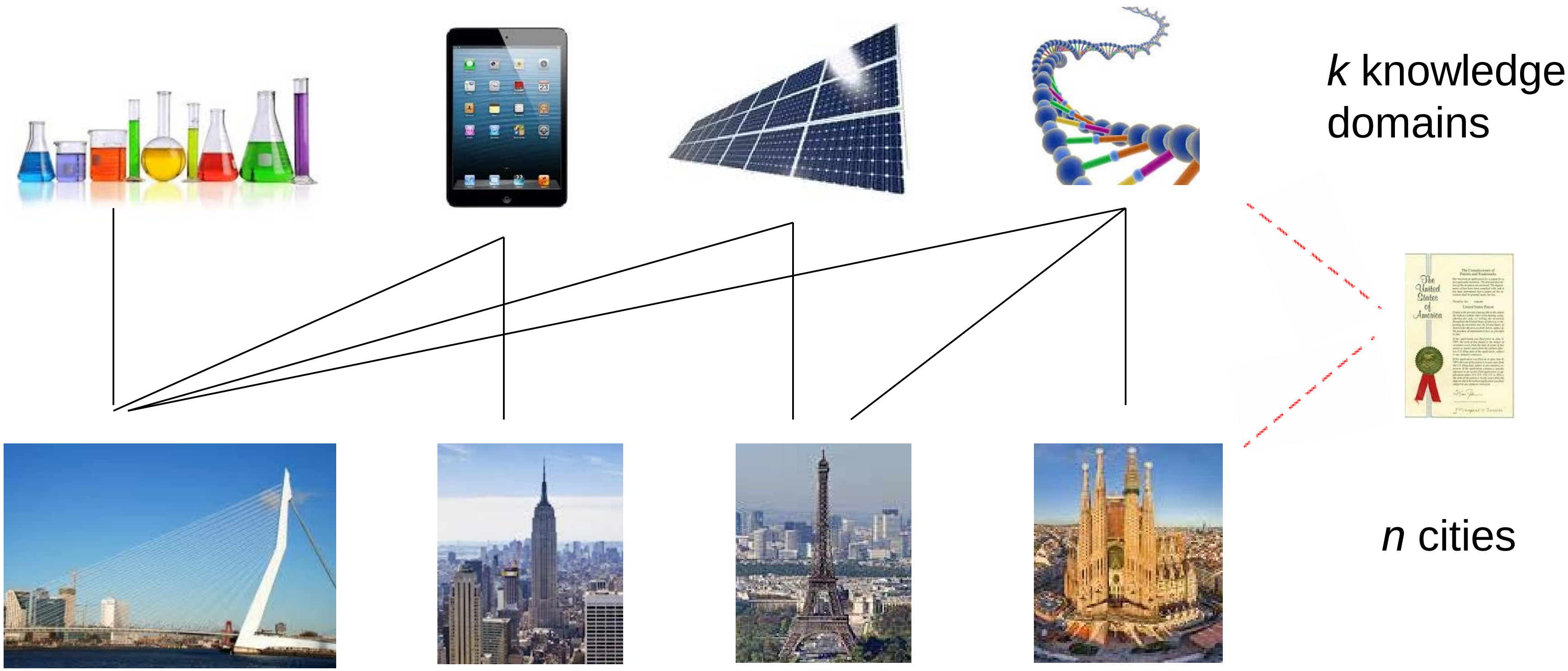
Vertex	1	2	3	4
1	-	1	1	0
2	0	-	1	1
3	0	0	-	0
4	0	0	0	-

Ego networks and whole networks





Bipartite network (2-mode)

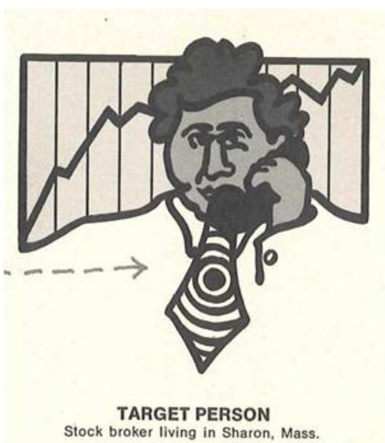


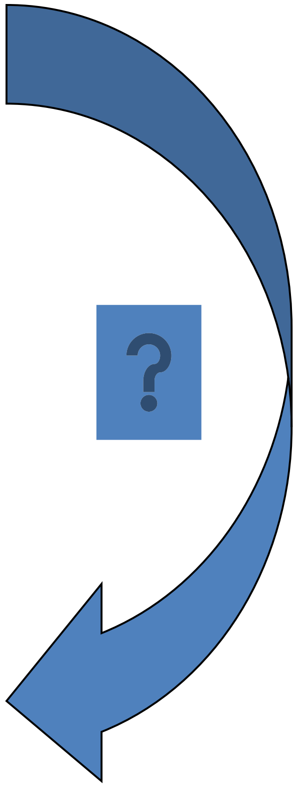
It is possible to formalize the data that connect **cities** to the **knowledge** they produce as a *n* by *k* bipartite **network** (two different sets of nodes)
A **link** between a city *i* and a knowledge domain *j* means that *i* produces knowledge in category *j*

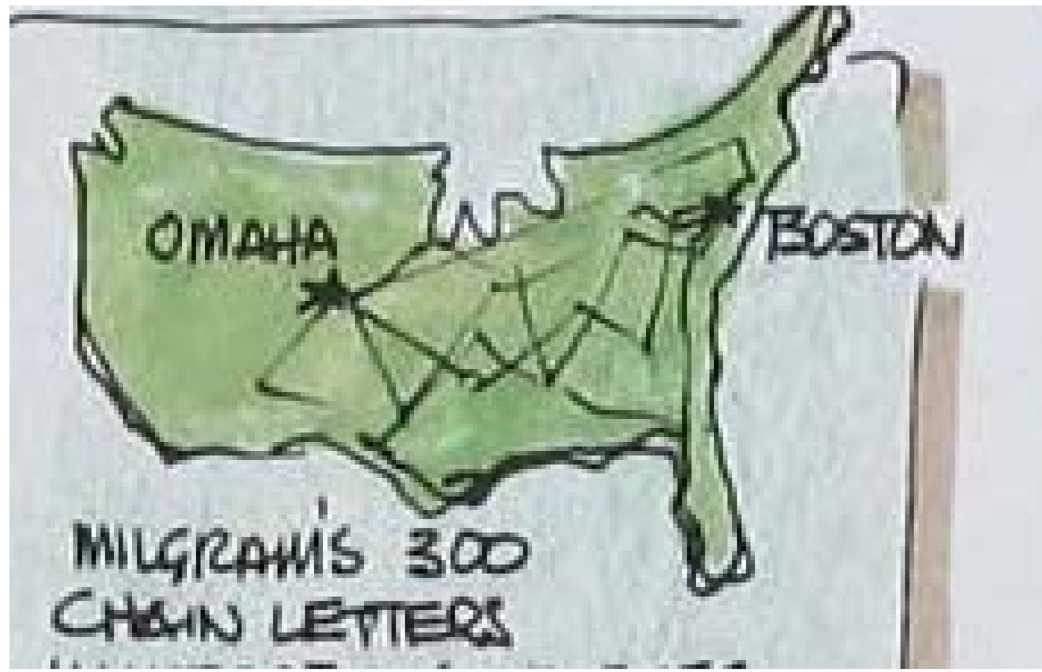
Multiplex networks

- Emma (1) helps Mason (2)
- Emma (1) is friends with Mason (2)
- Emma (1) works with Mason (2)
- ...

The small world experiment of Milgram







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- Roughly a third of the letters eventually arrived at the target
- In a median of six steps (6 degrees of separation)

Small world Milgram – 2 lessons

- Short paths are ubiquitous in human networks (it is a small world after all)
- Individuals are good at finding these short paths (search process: what if no name? no occupation, etc...)

The Bacon number

https://oracleofbacon.org/

Structural features of networks

- Real-world networks are characterized by:

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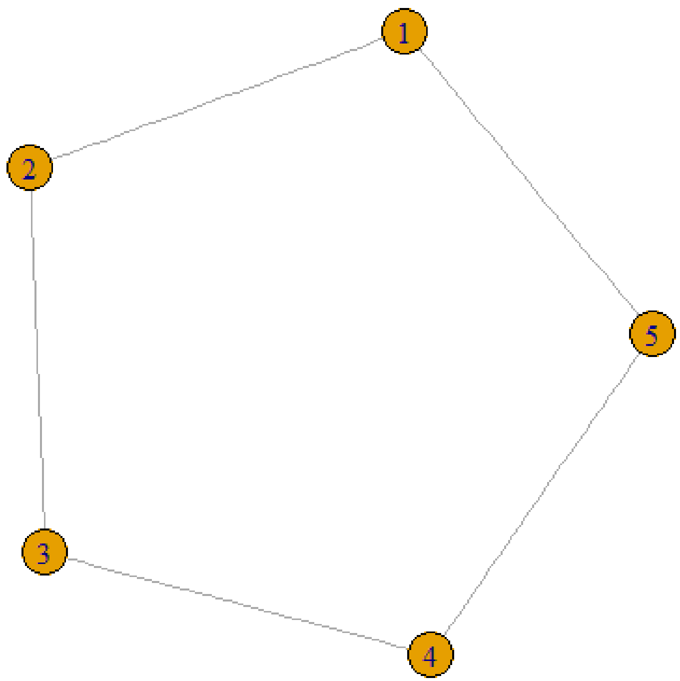
Compute average path length



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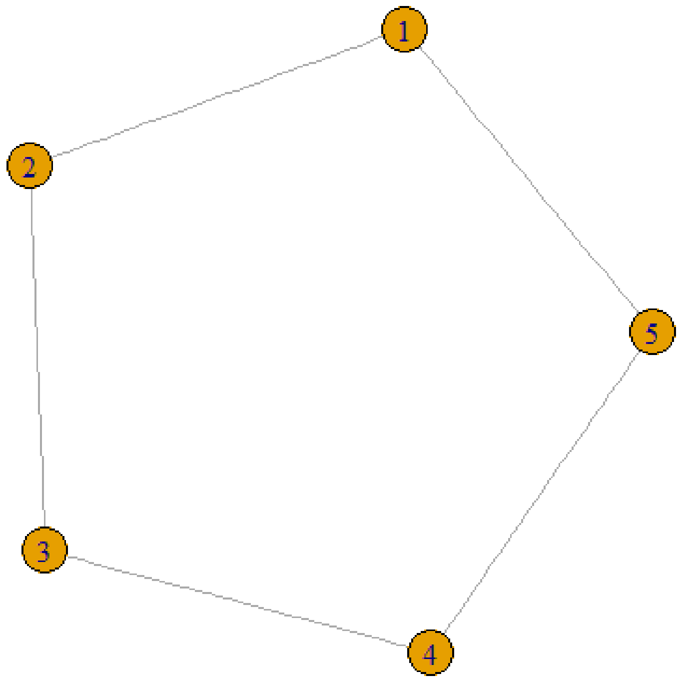
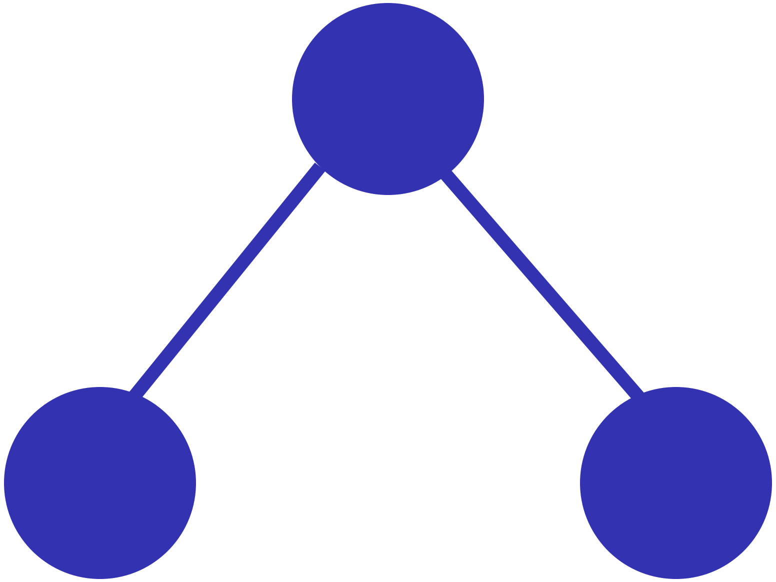


Average path length = (1+2+2+1)/4 = 1.5

Erdős–Rényi Random Graph model

- Early attempt to generate random graphs
- A random network consists of N nodes where each node pair is connected with the same probability p
- Path length in random graphs is short (similar to path length in real-world networks)
- Often used as a benchmark for network metrics
- As p increases the network tends to form a giant component (percolation)

Clustering in networks

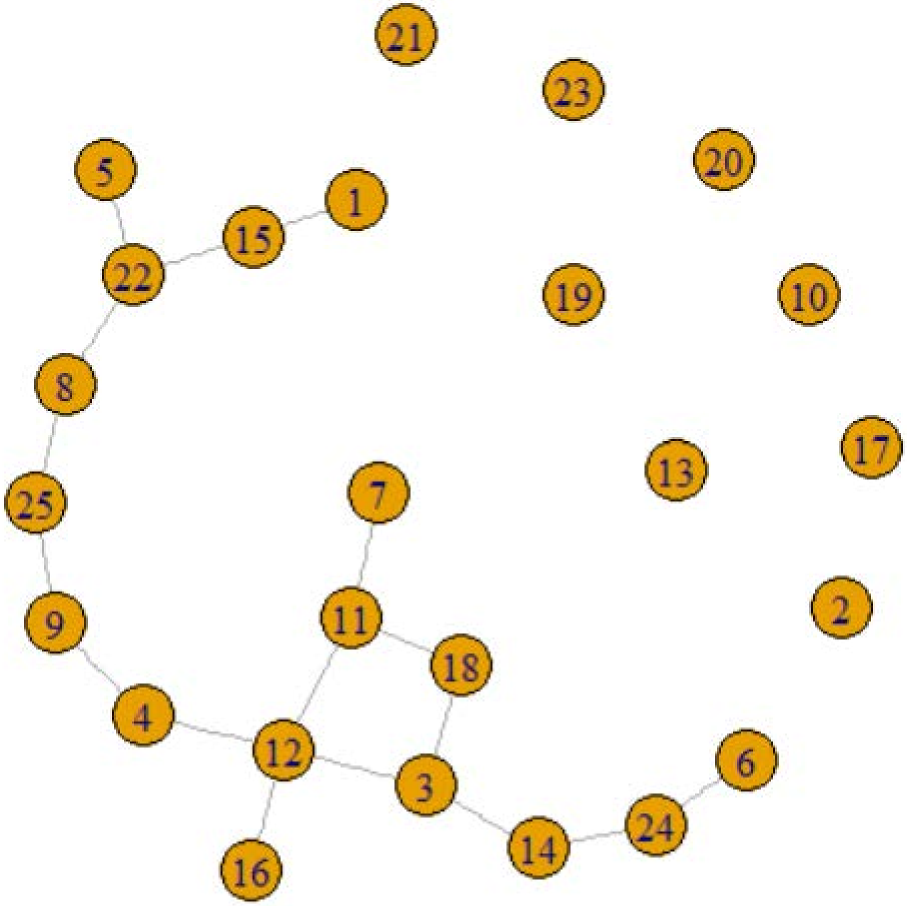


Structural features of networks

- Real-world networks are characterized by:

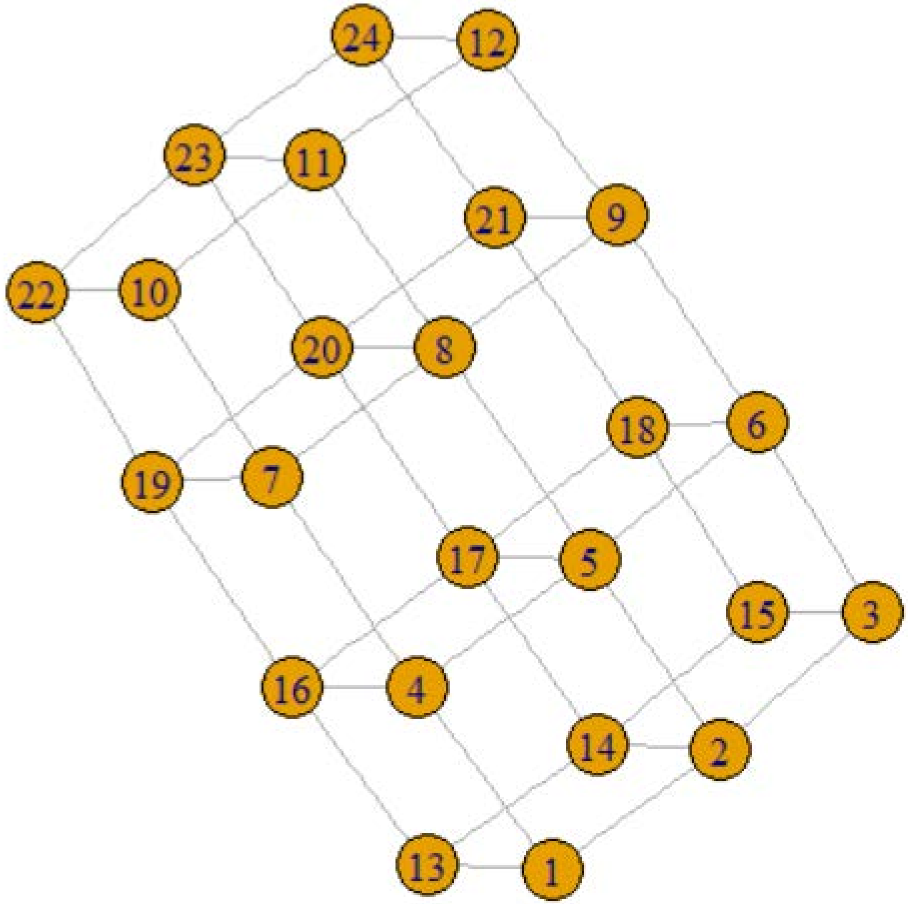
  (1) <u>Small</u> average path length
  (2) <u>High</u> clustering coefficient [equation]

Random network



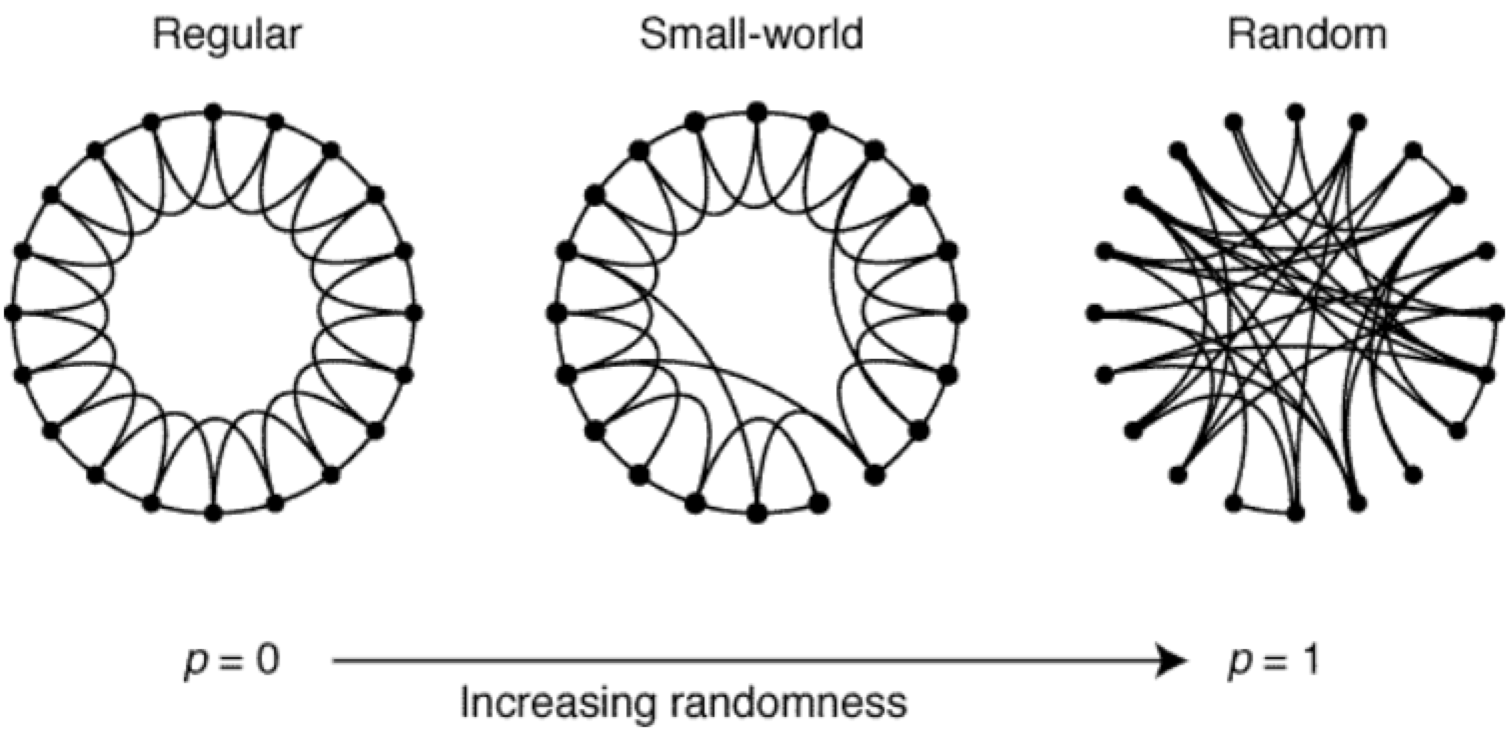
(1) Small average path length

Lattice



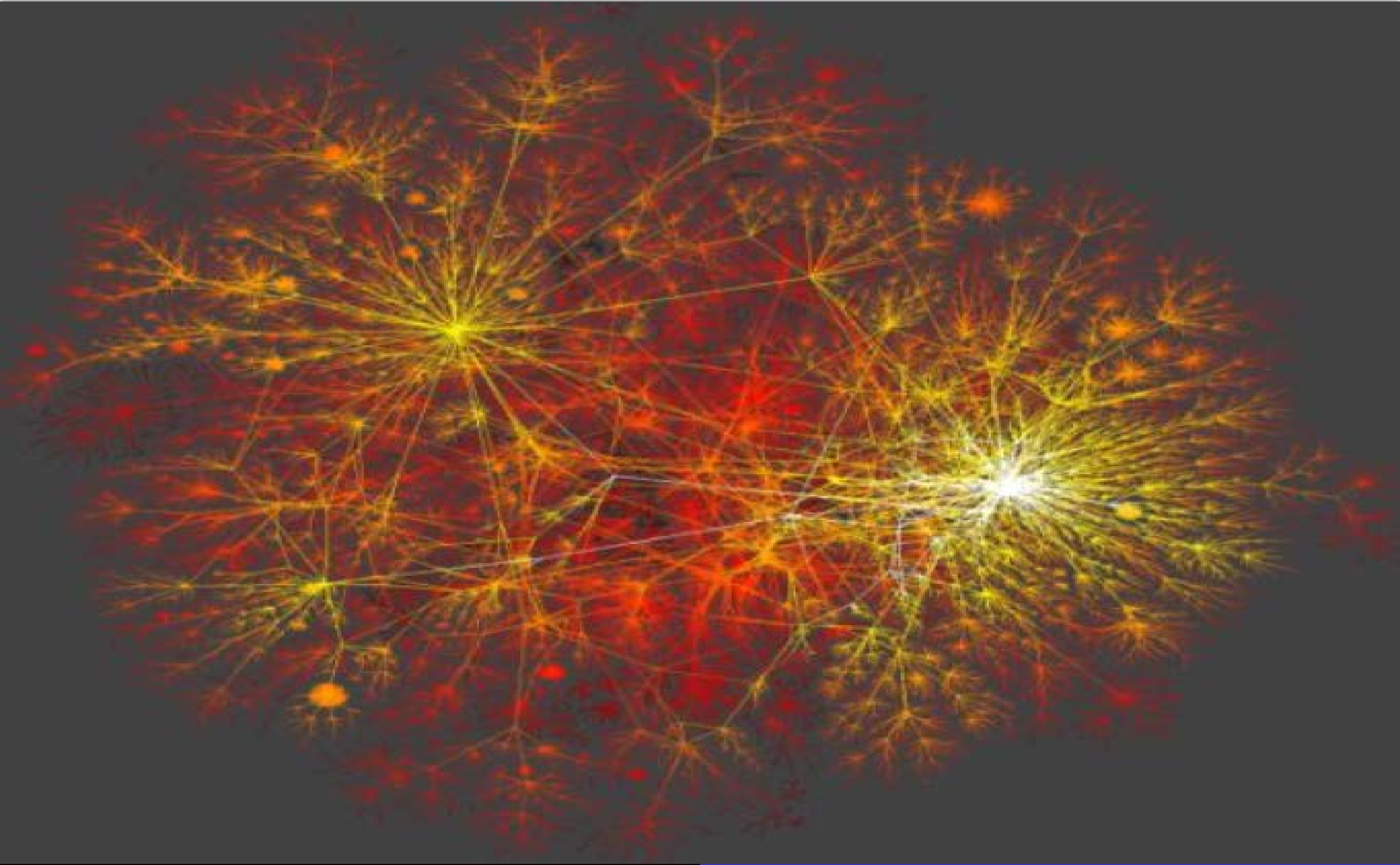
(2) <u>High</u> clustering coefficient

Small world model



Watts, D. J., & Strogatz, S. H. (1998). Collective dynamics of ‘small-world’ networks. Nature, 393(6684), 440-442.

Structural unequality



Structural features of networks

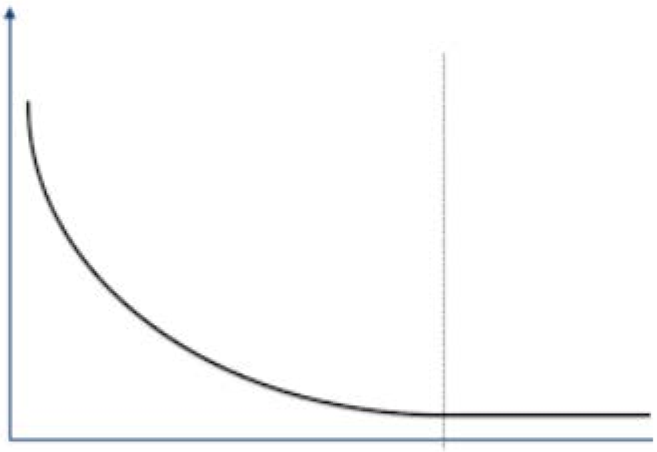
- Real-world networks are characterized by:

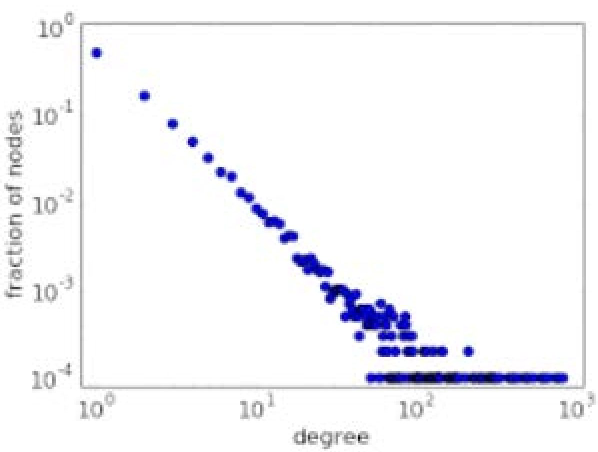
  (1) Small average path length

  (2) High clustering coefficient

  (3) Unequal degree distribution

Long tails and power law distribution





Barabási-Albert model

- A few hubs have a lot of connections
- Most of the nodes have very few
- Barabási-Albert model generates these type of networks by "preferential attachment“
- New network members prefer to make a connection to the more popular existing members

Implementation of the BA model

- The model starts with two nodes connected by an edge
- At each step, a new node is added
- A new node picks an existing node to connect to randomly, but with some bias
- A node's chance of being selected is proportional to the number of connections it already has (degree)

Complex networks

- Non-trivial topology
- Inherently dynamical properties – emergent behavior
- High structural heterogeneity