

URBAN SCALING LAWS

Pierre-Alexandre Balland

Cities as complex systems



*Beneath the infrastructure
and social fabric
of every city in the world is a
set of hidden mathematical rules
common to them all.*



0:11 / 7:17



Body size and metabolic rate

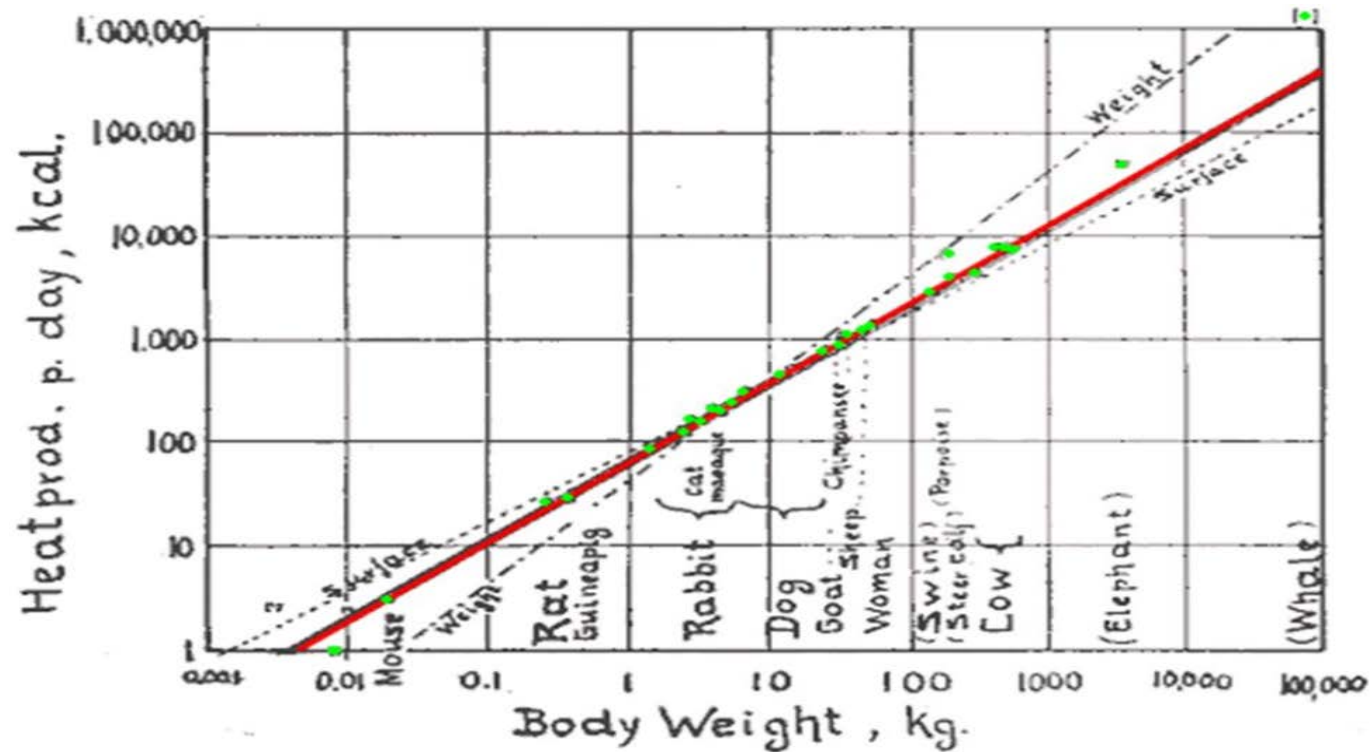


Fig. 1. Log. metabol. rate/log body weight

Kleiber's law and scaling

- Kleiber's law = an animal's metabolic rate R follows the $3/4$ power-law of its body mass M

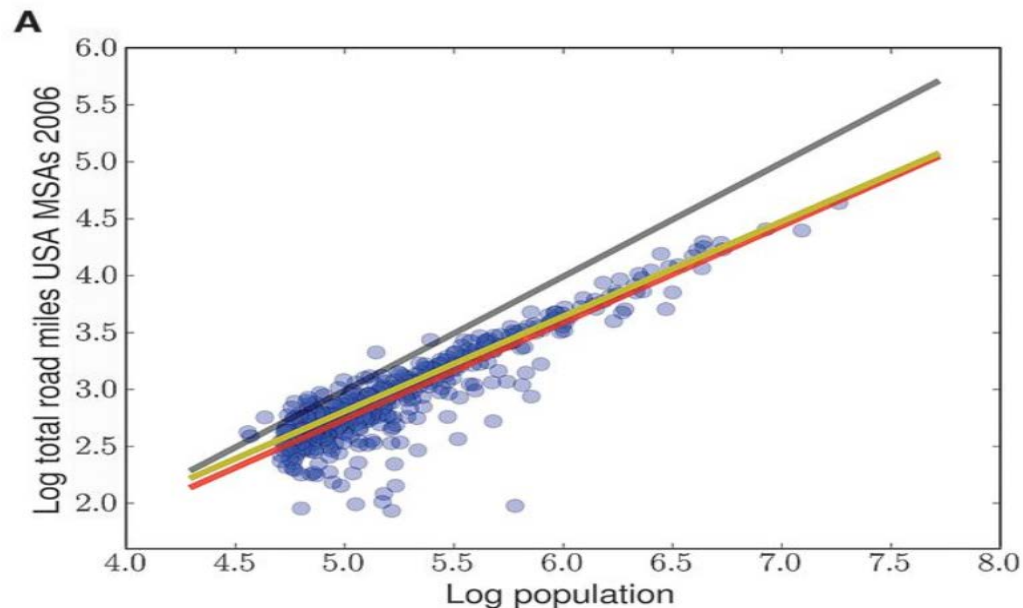
$$R \sim M^{3/4}$$

- A cat is 100 times heavier than a mouse but its metabolism is only 32 times greater
- There is in this case an economy of scale: the bigger and animal the less energy it consumes
- Metabolic rate **scales** to the $3/4$ power of the animal's mass

Scaling

Scaling defines how the properties of a (spatial) system varies with its size

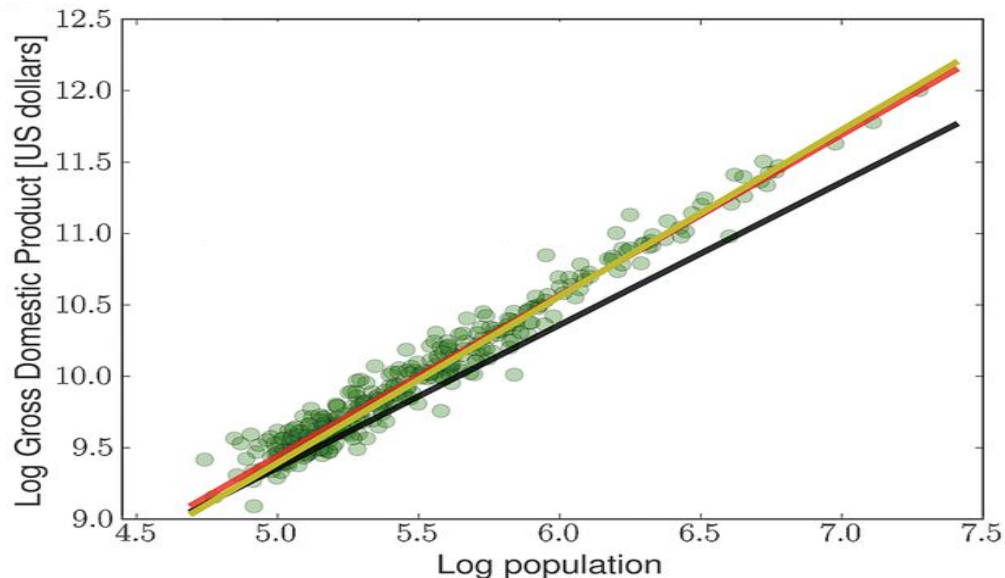
Sublinear scaling in cities (infrastructure)



Bettencourt (2013)

- Linear scaling $\beta = 1$
(black line – proportional scaling)
- Evidence of sublinear scaling with $\beta = 0.85$
(theoretical prediction: $\beta = 5/6$)
- Infrastructure (input) in a city increases at a much lower rate than its population

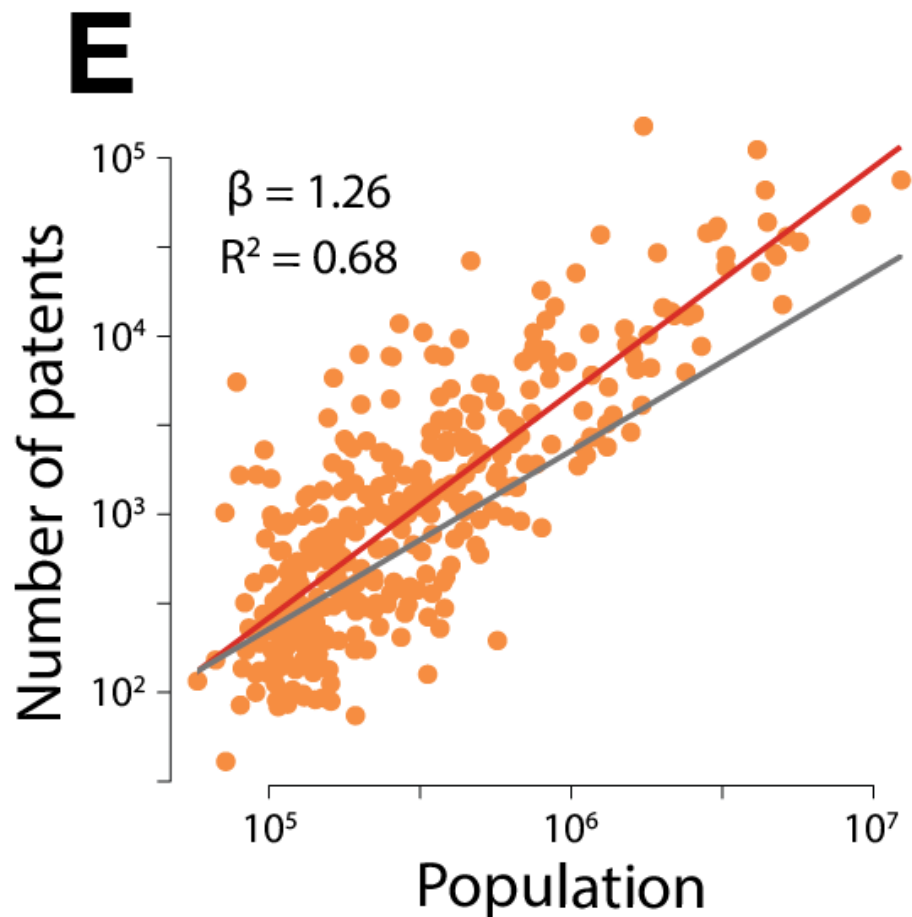
Superlinear scaling in cities (output)



Bettencourt (2013)

- Linear scaling $\beta = 1$
(black line – proportional scaling)
- Evidence of superlinear scaling with $\beta = 1.13$
(theoretical prediction: $\beta = 7/6$)
- Economic output in a city increases at a much faster rate than its population

Unequal distribution of econ. activities



Cities as complex systems

- Sublinear scaling of inputs and superlinear scaling of outputs

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- Despite their apparent complexity and diversity, a set of simple mathematical rules is common to all cities
- Superlinear scaling is unknown in the biological world
- Are cities our greatest invention?

Origins of these scaling relations

- *“A city is first and foremost a social reactor [...] it works like a star, attracting people and accelerating social interaction and social outputs in a way that is analogous to how stars compress matter and burn brighter and faster the bigger they are.” (Bettencourt – Wired magazine, 2013)*

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- Role of spatially-constrained social interactions between individuals (fuelled by density of population)
- As the number of firms or individuals increases in a city the number of potential interactions within this city **increases**
decreases

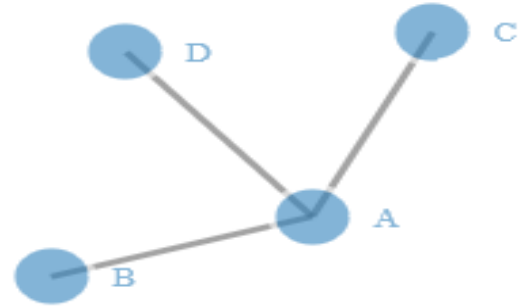
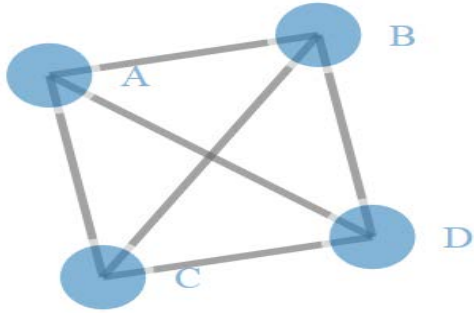
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superlinearly sublinearly

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Network density



$$\text{Network density} = \frac{\text{Actual connections}}{\text{Potential connections}}$$

$$\text{Potential connections} = \frac{n*(n-1)}{2}$$

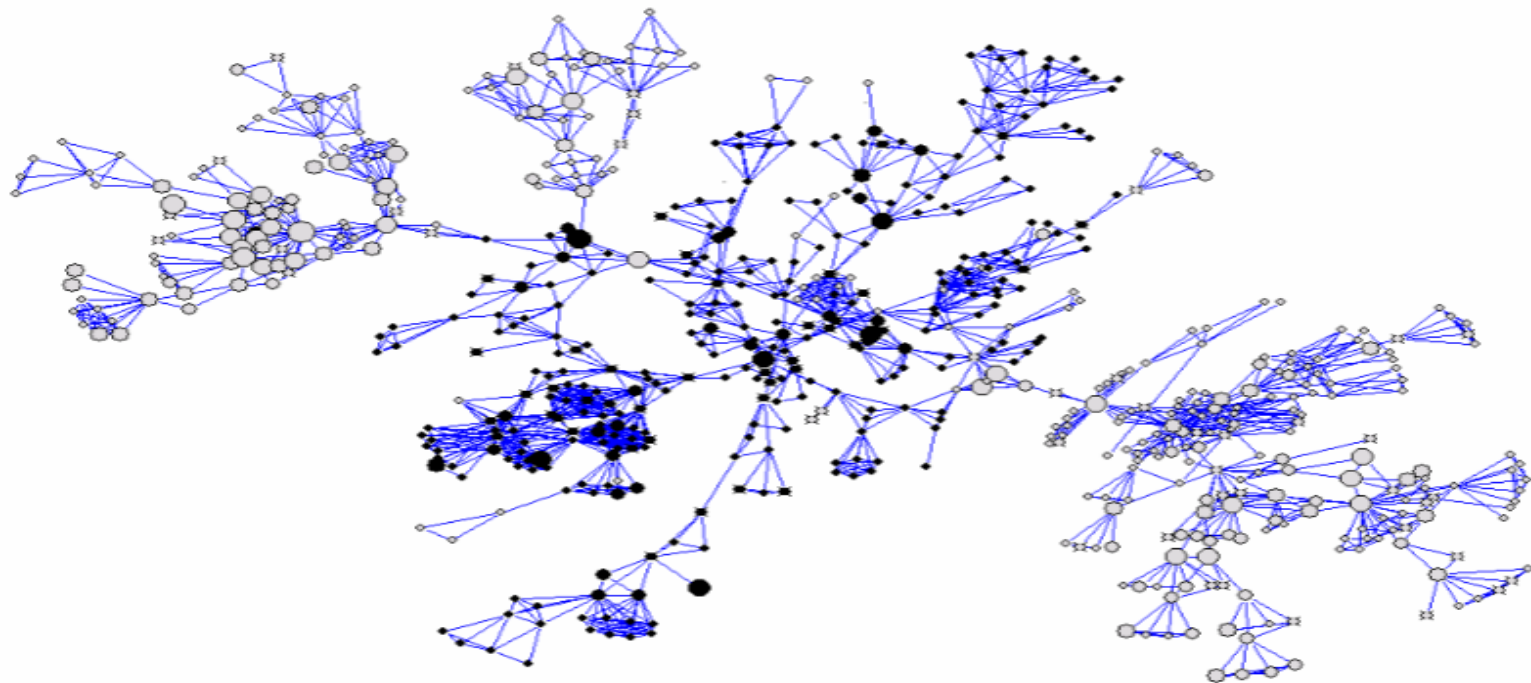
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- Role of spatially-constrained social interactions between individuals (fuelled by density of population)
- As the number of firms or individuals increases in a city the number of potential interactions within this city increases superlinearly with **$\beta = 2$** ($n \cdot n$) in the case of an undirected network

A regional system of innovation



Inter-firm network in the Silicon Valley



Fleming & Frenken 2007

Communication network

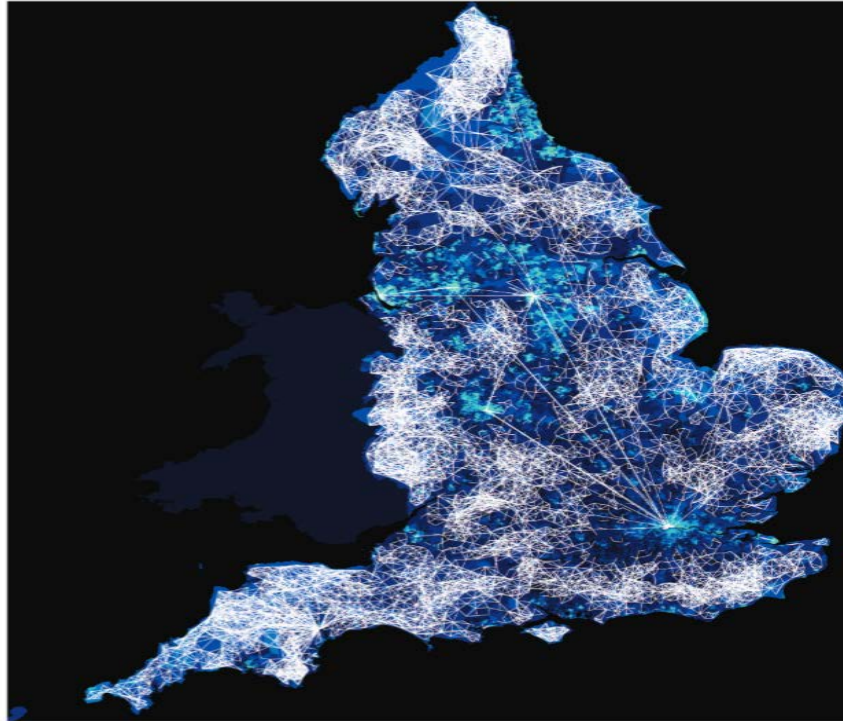
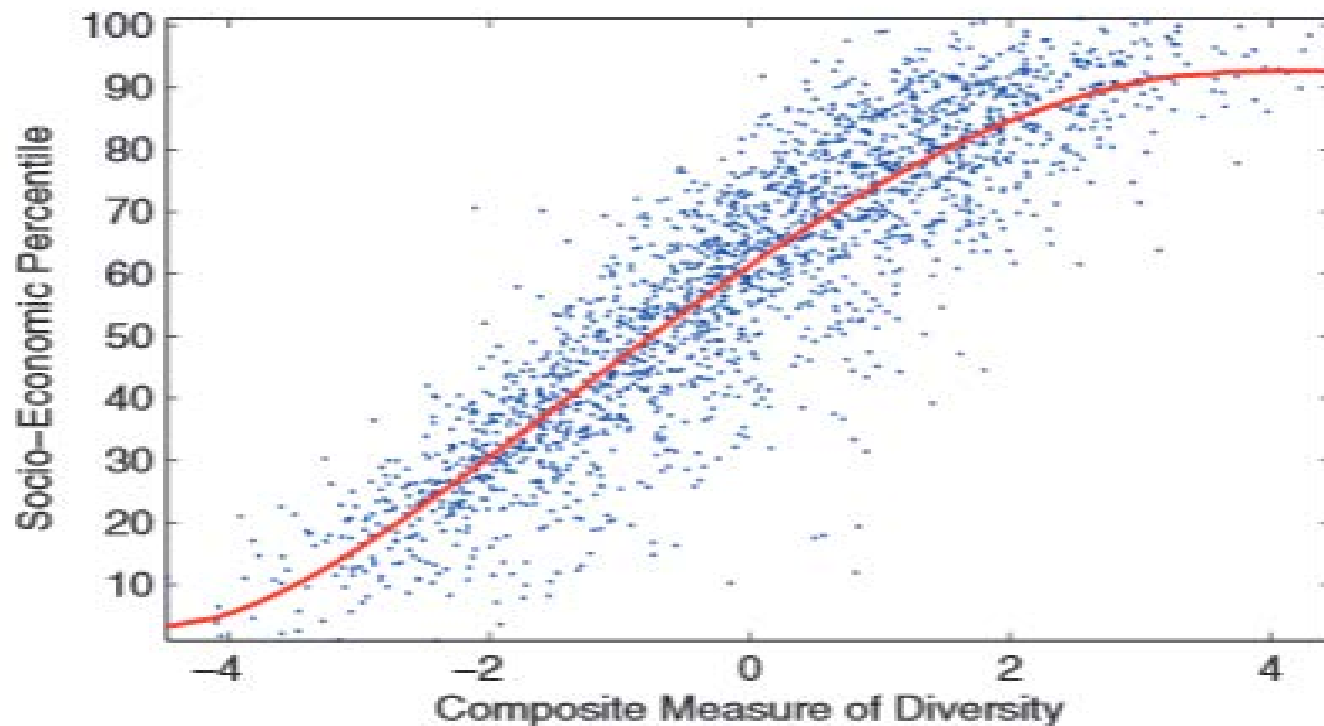


Fig. 1. An image of regional communication diversity and socioeconomic ranking for the UK. We find that communities with diverse communication patterns tend to rank higher (represented from light blue to dark blue) than the regions with more insular communication. This result implies that communication diversity is a key indicator of an economically healthy community. [(29) Crown copyright material is reproduced with the permission of the Controller of Her Majesty's Stationery Office]

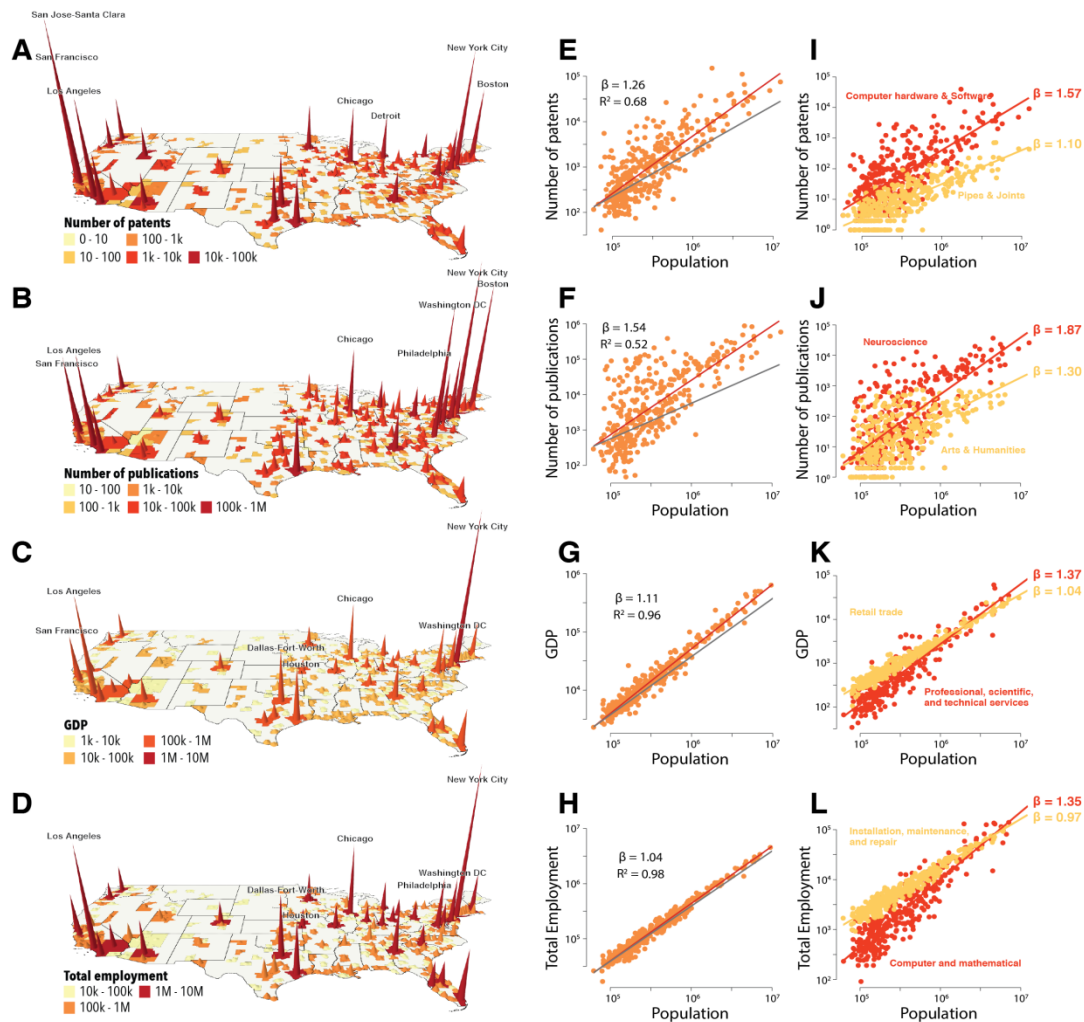
Eagle et al.,
2010

Network diversity & development




Eagle et al.,
2010

How complex economic activities scale in cities

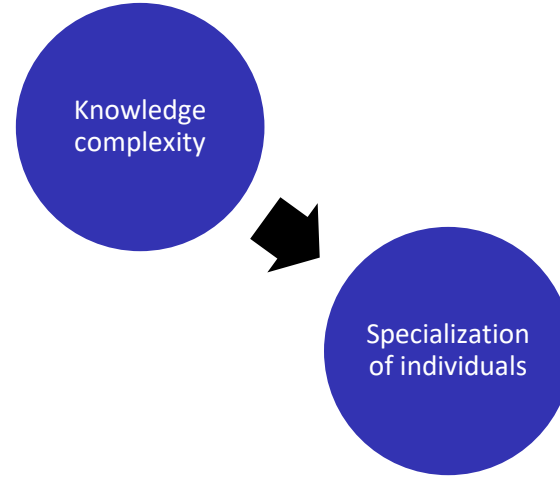


From Complexity to Spatial Inequality

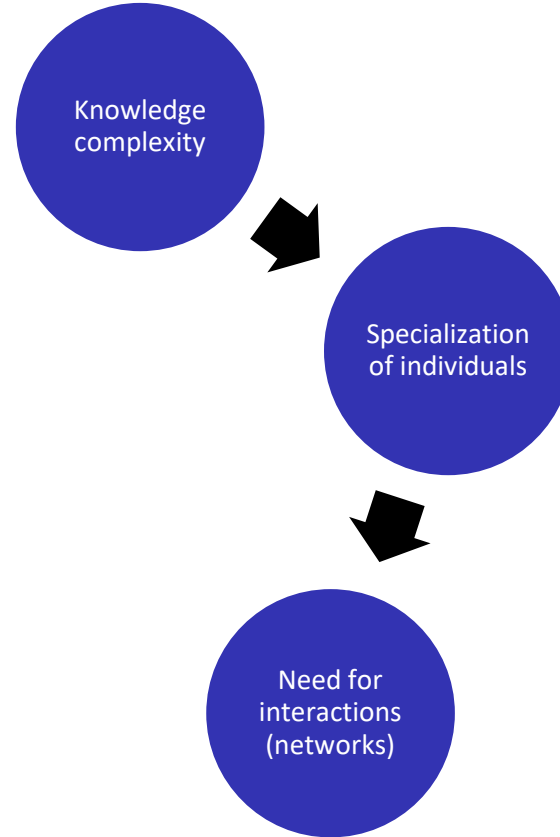


Knowledge
complexity

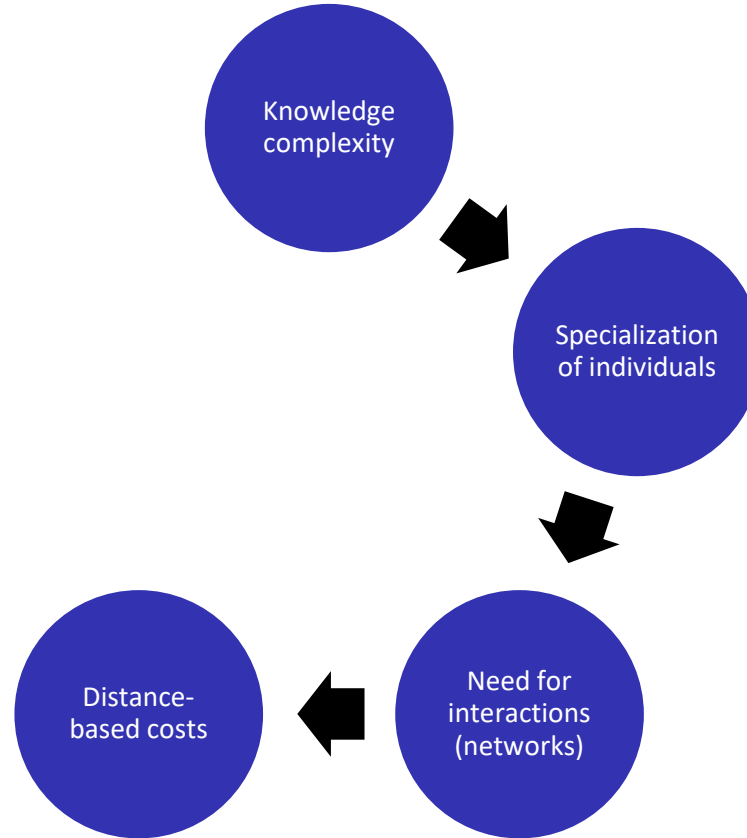
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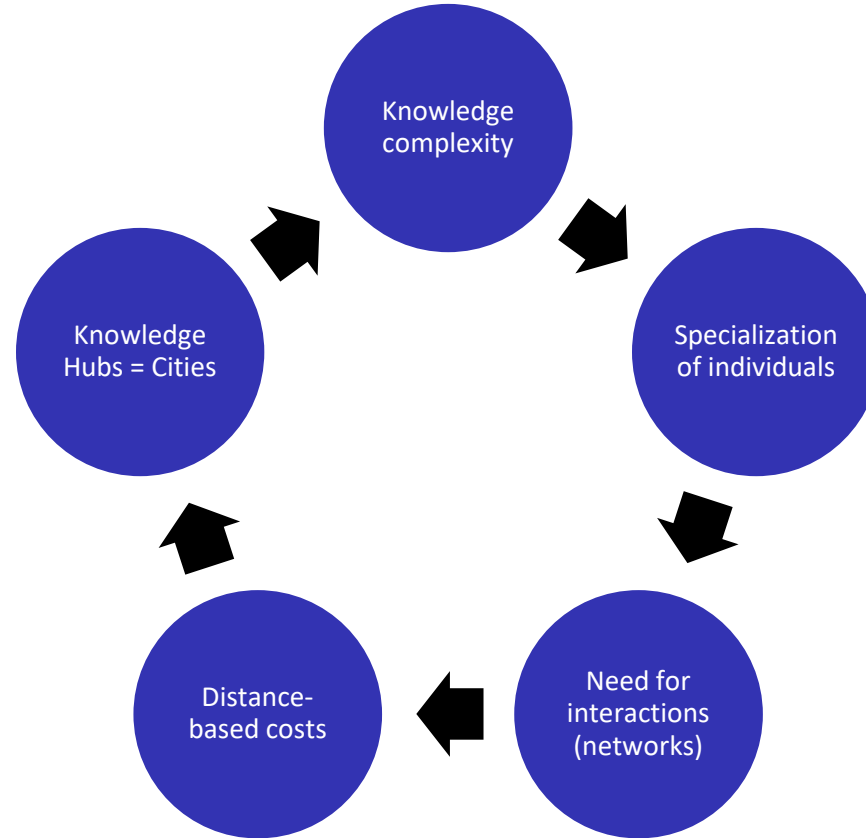
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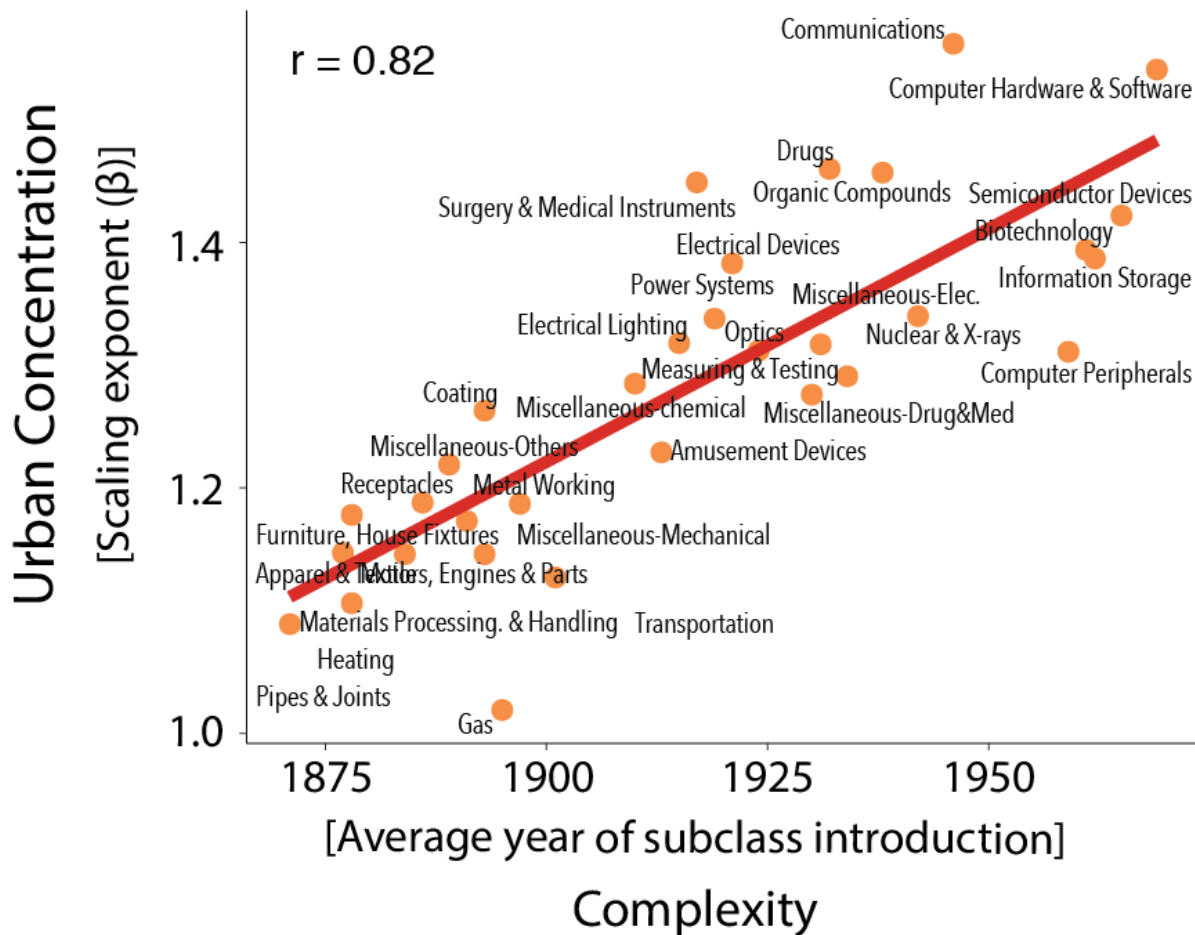


From Complexity to Spatial Inequality



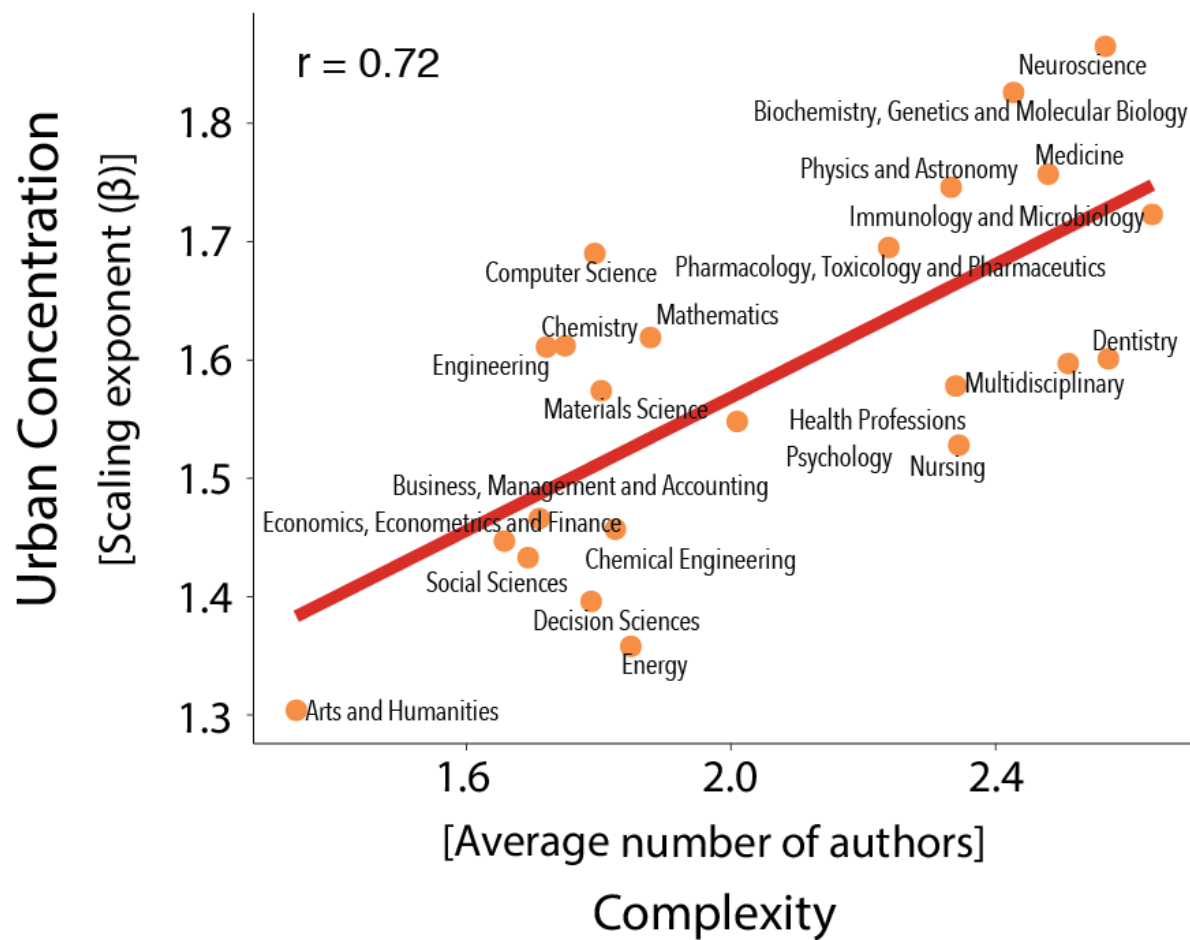
A

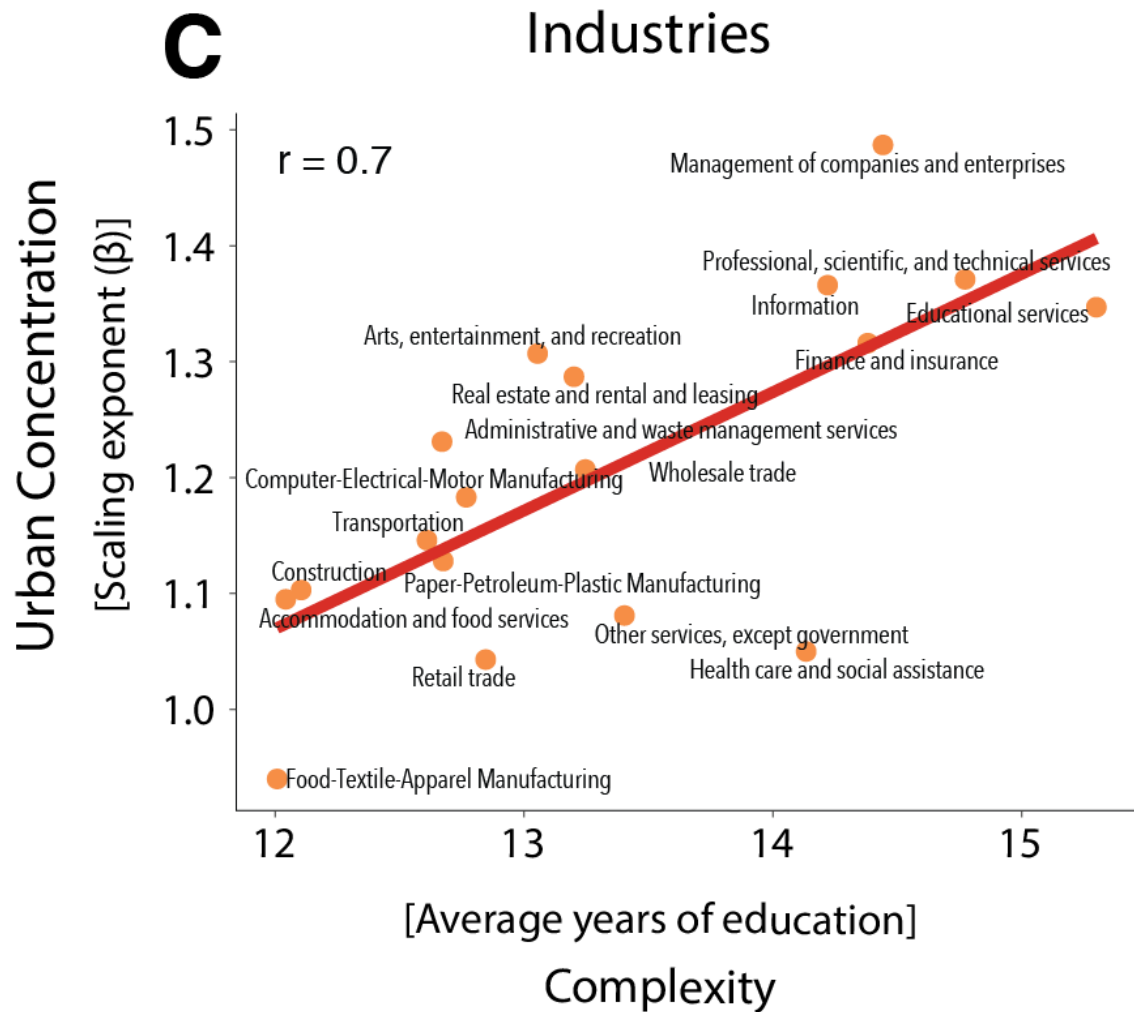
Technological Classes



B

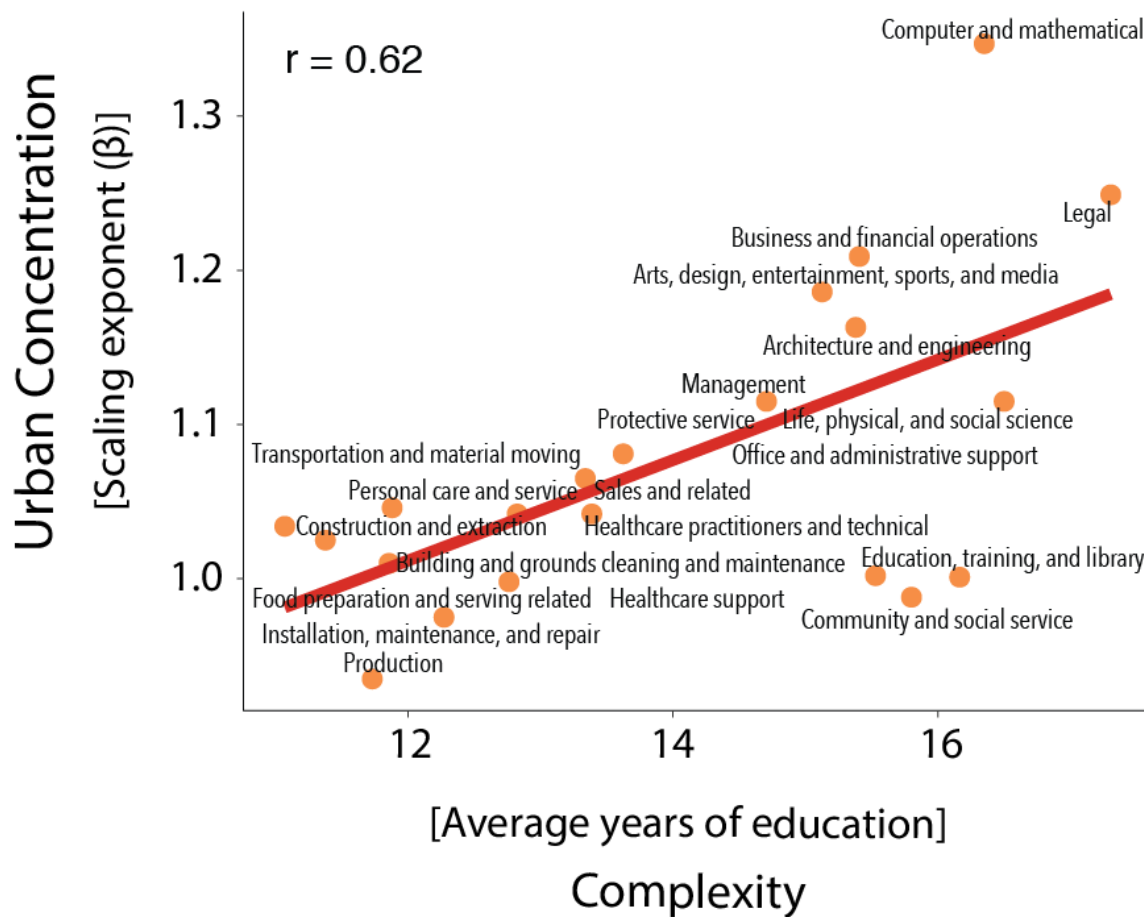
Scientific Fields





D

Occupations



The Historical Gap

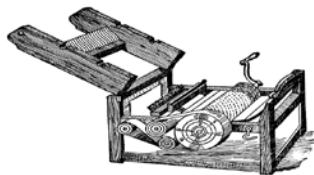
There is virtually no historical and systematic analysis on the geography of innovation and technological change prior to 1975.



The Historical Gap

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Cotton Gin



Telephone



Airplane



Biotechnology



1790

1820

1850

1880

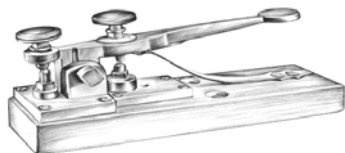
1910

1940

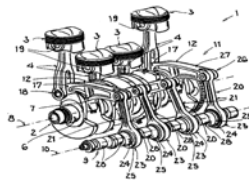
1975

2010

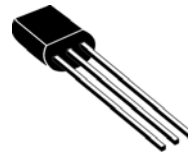
1st U.S. Patent



Telegraph



Internal combustion engine



Semiconductor



Information & Communication

Historical Patent Dataset (HistPat)

www.nature.com/articles/sdata201674 (Petrulia, Balland, Rigby; 2016)

United States Patent	[19]		[11]	4,237,224
Cohen et al.			[45]	Dec. 2, 1980

[54] **PROCESS FOR PRODUCING BIOLOGICALLY FUNCTIONAL MOLECULAR CHIMERAS**

[75] Inventors: **Stanley N. Cohen**, Portola Valley; **Herbert W. Boyer**, Mill Valley, both of Calif.

[73] Assignee: **Board of Trustees of the Leland Stanford Jr. University**, Stanford, Calif.

[21] Appl. No.: **1,021**

[22] Filed: **Jan. 4, 1979**

Related U.S. Application Data

[63] Continuation-in-part of Ser. No. 959,288, Nov. 9, 1978, which is a continuation-in-part of Ser. No. 687,430, May 17, 1976, abandoned, which is a continuation-in-part of Ser. No. 520,691, Nov. 4, 1974.

[51] **Int. Cl.³** **C12P 21/00**

[52] **U.S. Cl.** **435/68; 435/172; 435/231; 435/183; 435/317; 435/849; 435/820; 435/91; 435/207; 260/112.5 S; 260/27R; 435/212**

[58] **Field of Search** **195/1, 28 N, 28 R, 112, 195/78, 79; 435/68, 172, 231, 183**

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Primary Examiner—Alvin E. Tanenholtz
Attorney, Agent, or Firm—Bertram I. Rowland

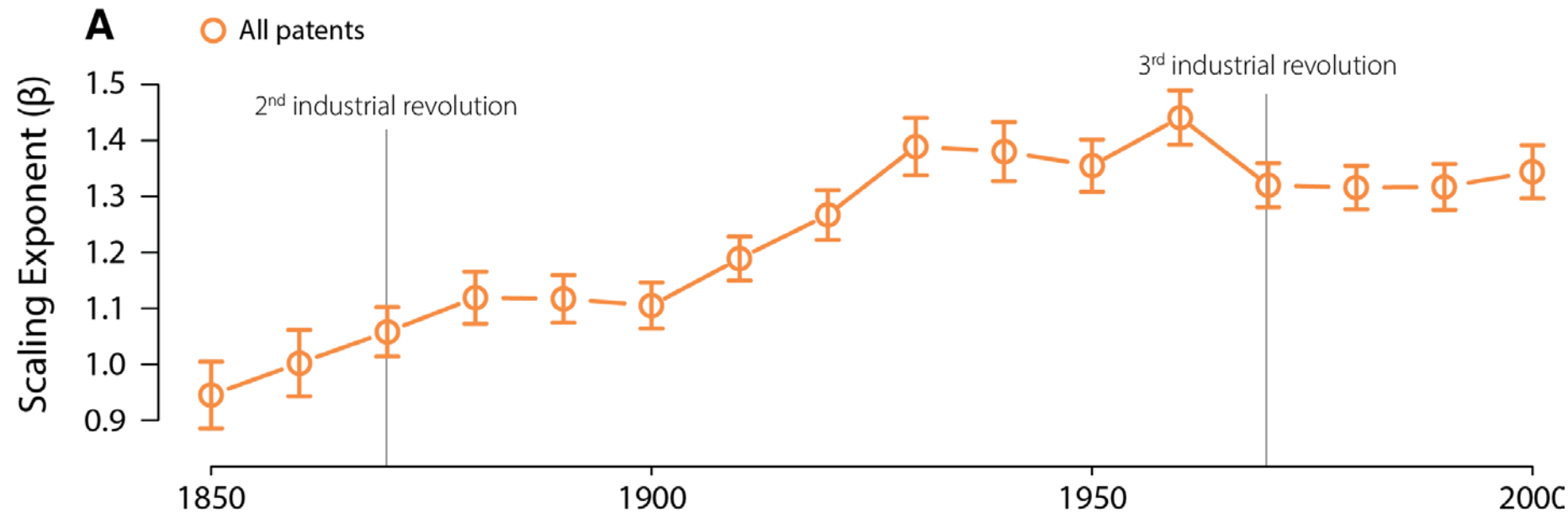
[57] ABSTRACT

Method and compositions are provided for replication and expression of exogenous genes in microorganisms. Plasmids or virus DNA are cleaved to provide linear DNA having ligatable termini to which is inserted a gene having complementary termini, to provide a biologically functional replicon with a desired phenotypic property. The replicon is inserted into a microorganism cell by transformation. Isolation of the transformants provides cells for replication and expression of the DNA molecules present in the modified plasmid. The method provides a convenient and efficient way to introduce genetic capability into microorganisms for the production of nucleic acids and proteins, such as medically or commercially useful enzymes, which may have direct usefulness, or may find expression in the production of drugs, such as hormones, antibiotics, or the like, fixation of nitrogen, fermentation, utilization of specific feedstocks, or the like.

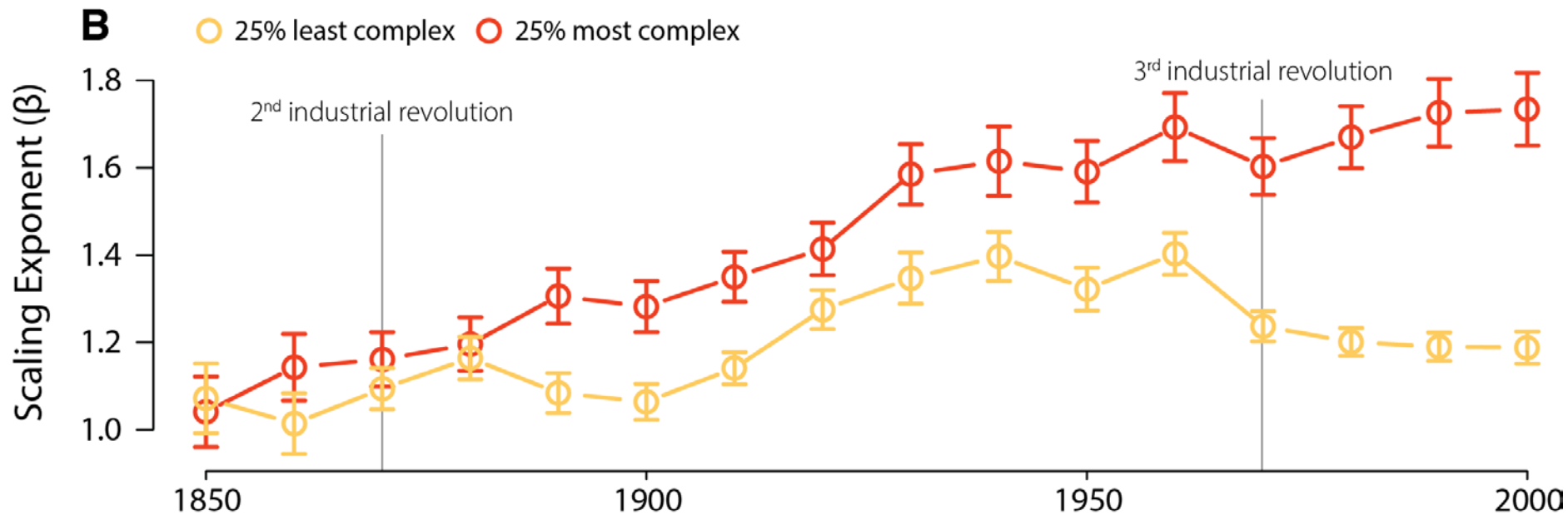
14 Claims, No Drawings

- ~ 7,000,000 US patents
- 1790 to 2016
- Geography of patents (county level – 4,000)
- And their tech classes (436 classes; 150,000 sub-classes)

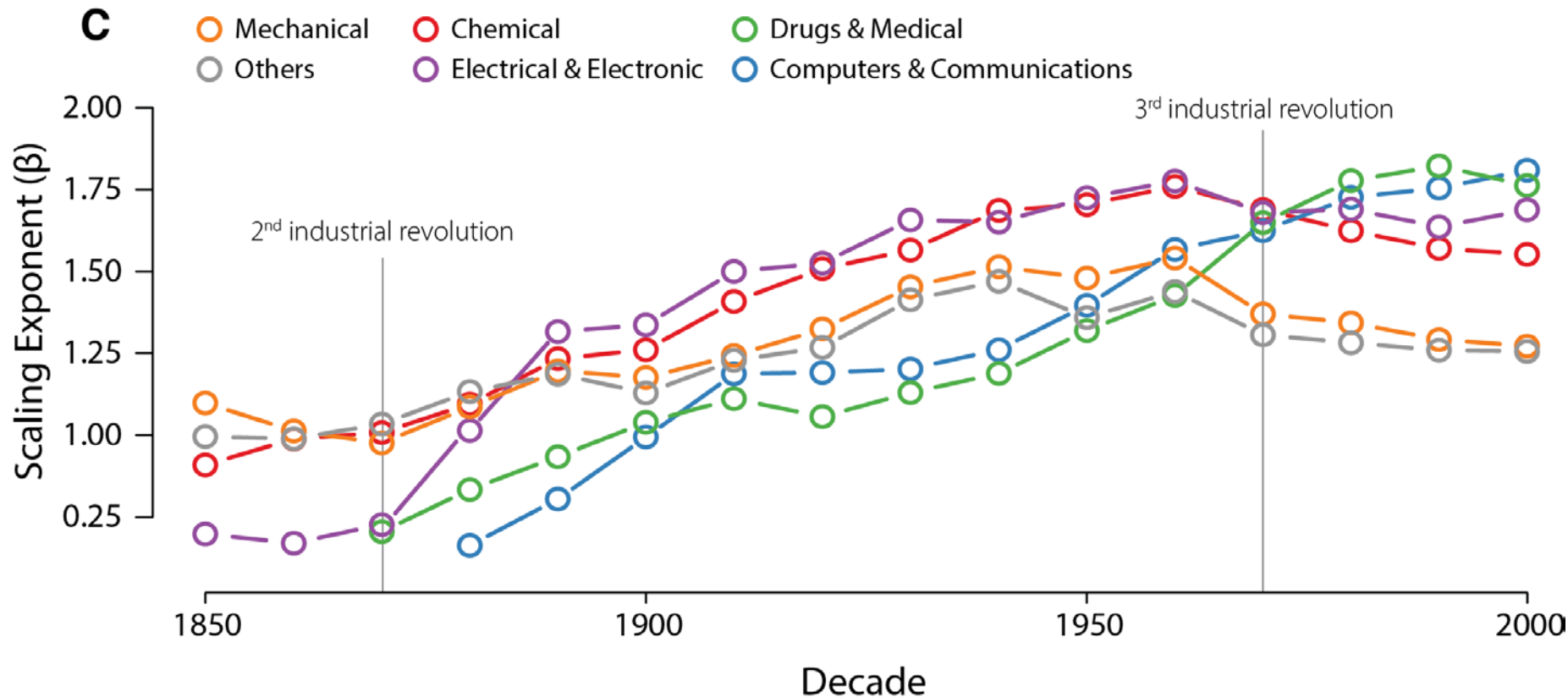
Complexity and scaling (1850-2000)



Complexity and scaling (1850-2000)



Complexity and scaling (1850-2000)



Thanks!

paballand.com

github.com/PABalland/EconGeo