Centrality and Power

Pierre-Alexandre Balland

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Network design and aesthetics

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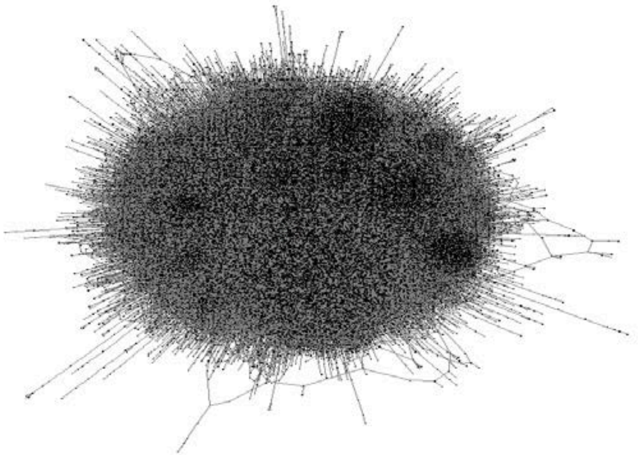
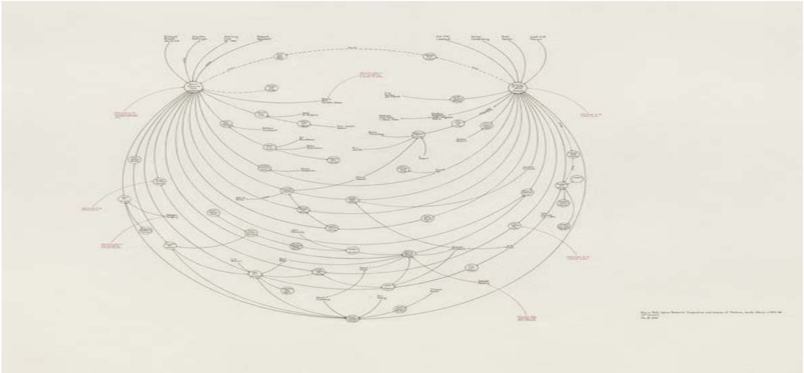
- You don’t (necessarily) need a computer for that: start with pen and paper
- When working with a computer (R), don’t let the computer decide for you
- Maintain a humanist approach of data: do you understand your story?
- Learn to throw things out (as in writing, and life)

Network design and aesthetics

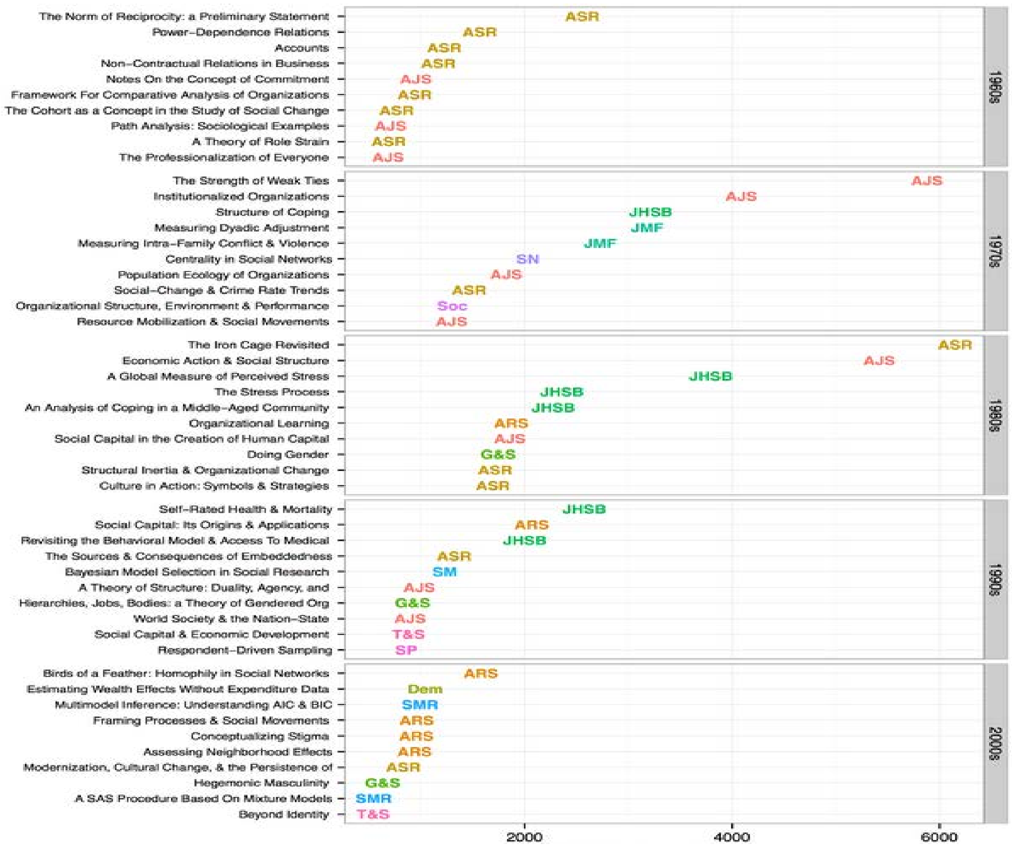




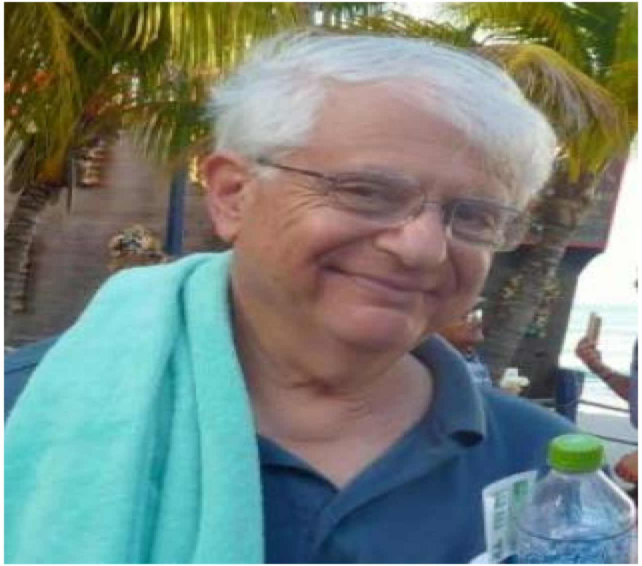
Centrality in networks

- Basic (key) network statistics
- Micro-property (actor level)
- Indicates the relative positions of actors/nodes in a network
- Describe the way an actor is embedded in a relational context (constraints? opportunities?)
- In sociology power (centrality) is a relational concept

Sociology’s most cited articles



The Strength of Weak Ties



Mark Granovetter

- Granovetter surveyed around 300 professional, technical, and managerial workers in total
- Analyzed the type of ties between the job seeker and the contact person who provided the necessary information
- Tie strength was measured in terms of how often they saw the contact person during the period of the job transition, using the following assignment
- often = at least once a week
- occasionally = more than once a year but less than twice a week
- rarely = once a year or less
- “not a friend, an acquaintance.”

Weak ties and embeddedness

- Strong ties are not bridges
- Through transitivity, cluster of actors form and tend to exhibit high level of homophily
- Weak ties can connect these different groups of actors
- Weak ties are important channels of information and new ideas
- Weak ties reduce average path length in a network
- 1985 paper on embeddedness launched new economic sociology
- Economic relations between individuals or firms are **embedded** in **social networks** and do not exist in an abstract idealized market – these embeddings provides economic opportunities and constraints

Structural holes and good ideas

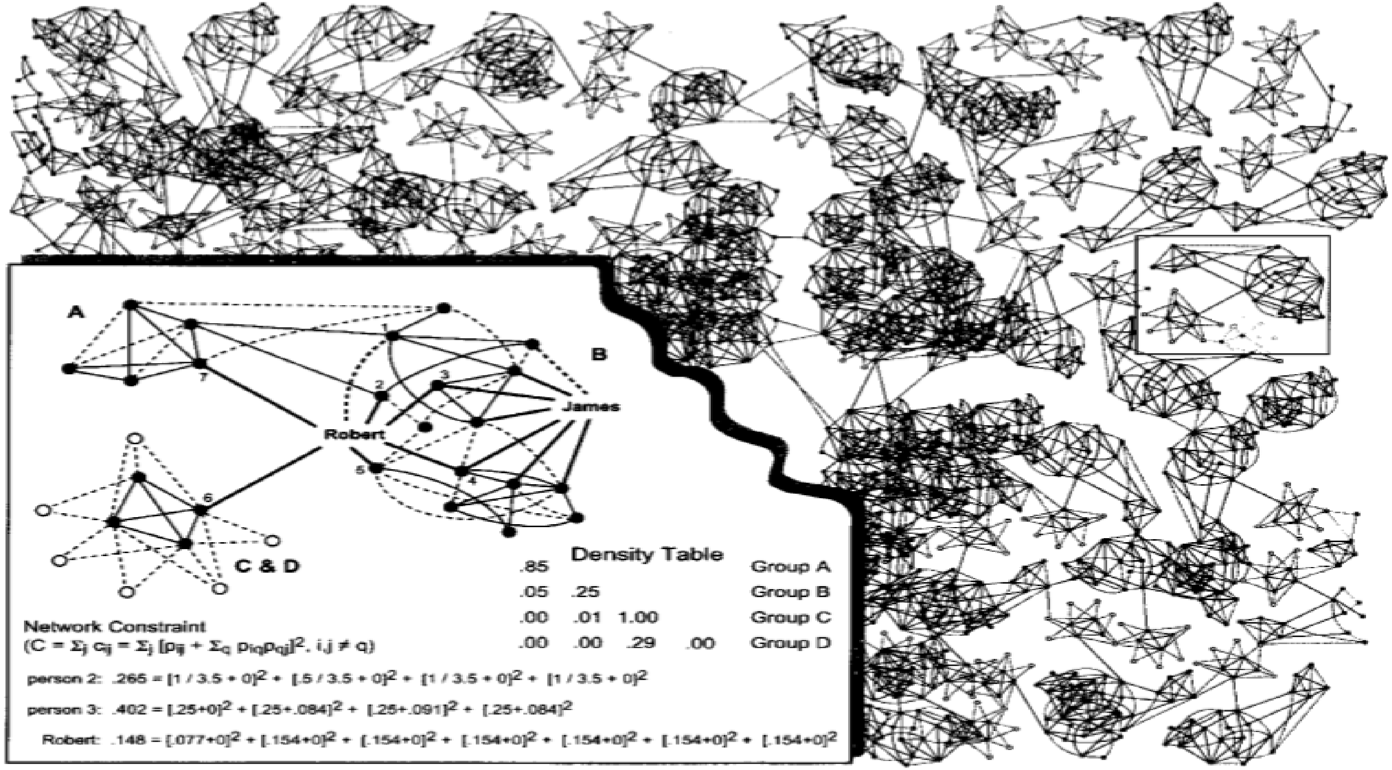
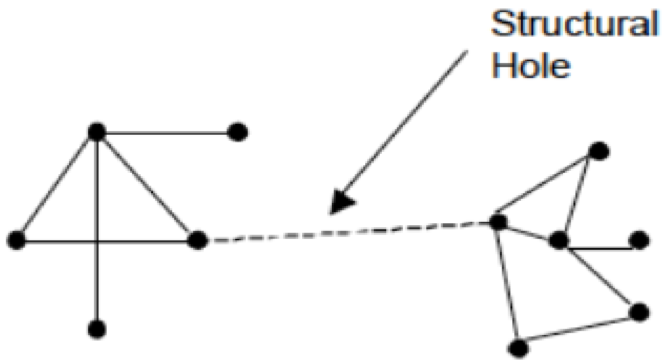


FIG. 1.—The small world of markets and organizations

Burt and Coleman



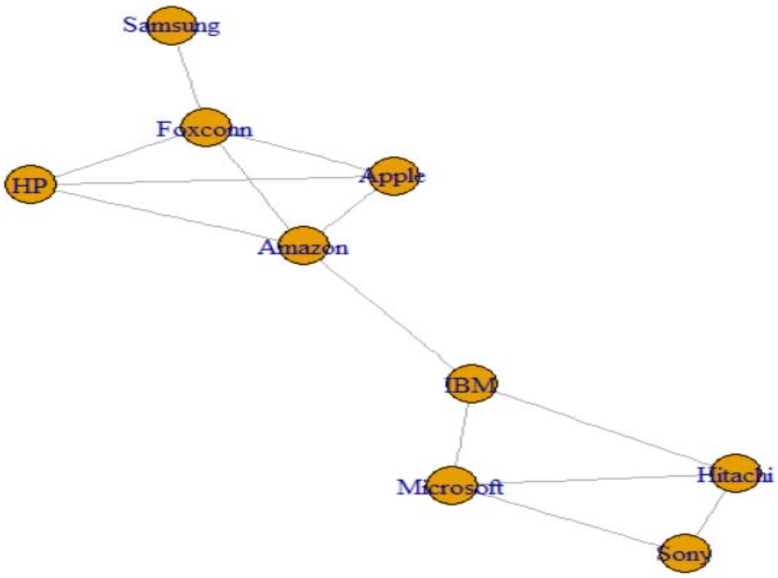
- Structural holes are a better predictor of performance than network closure (information is less redundant, and can be controled)

- Coleman considers that closure is more important, as closure allows information to flow:

  - 1. More quickly

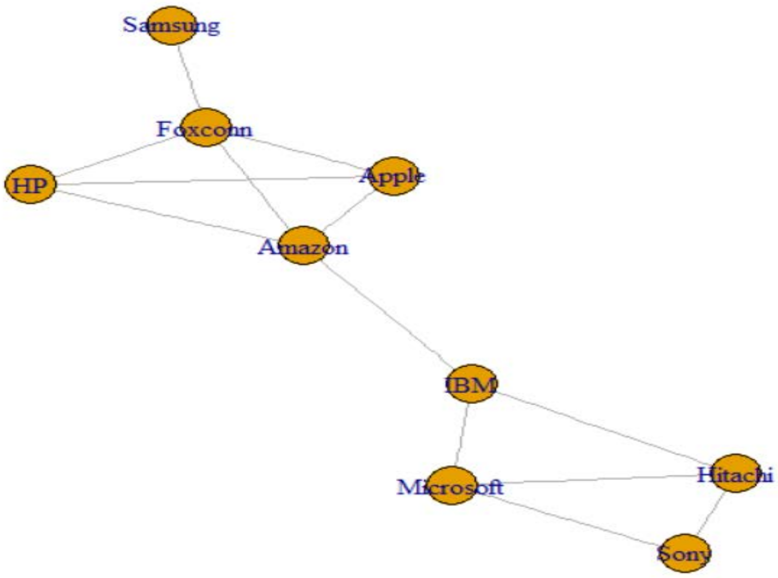
  - 2. Allows to build trust (important for strategic information)

Degree centrality



- Number of direct ties of a given node
- Normalized degree centrality = centrality / (n-1)
- n = number of nodes
- Degree centrality of Amazon?

Degree centrality

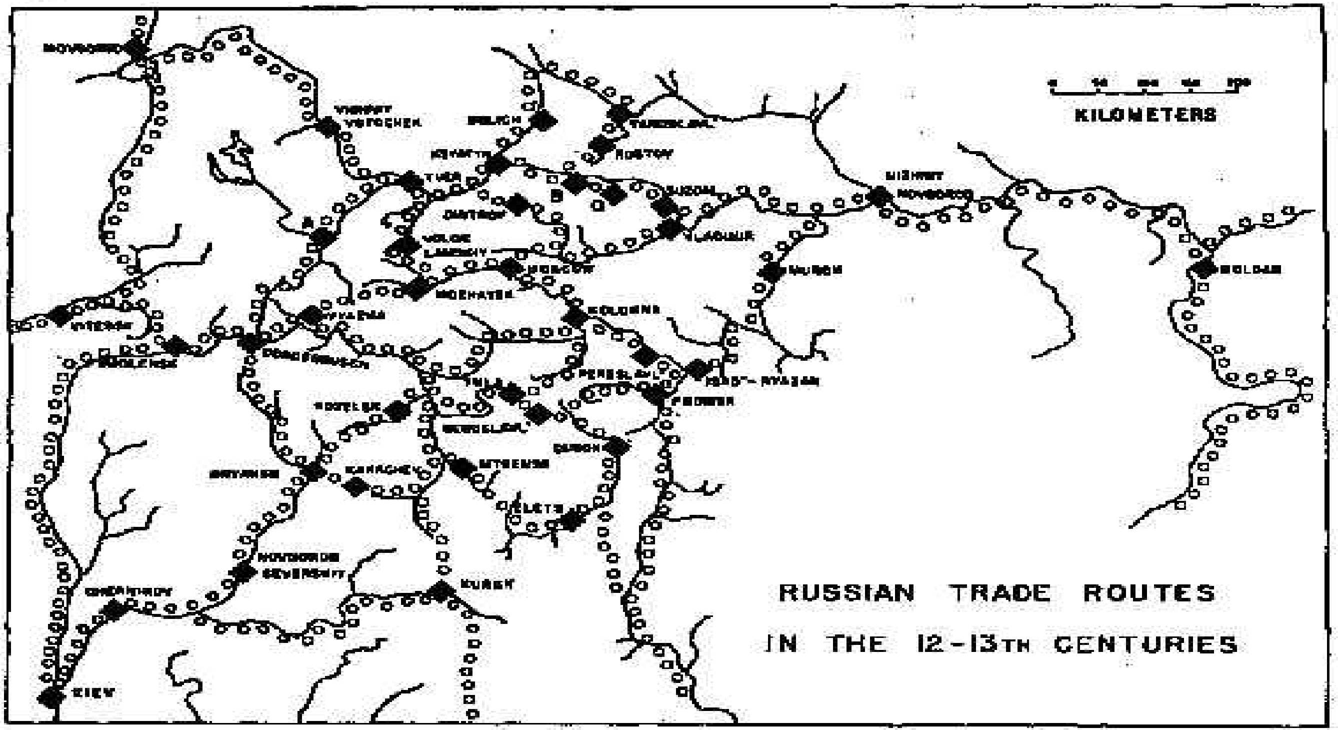


- Number of direct ties of a given node
- Normalized degree centrality = centrality / (n-1)
- n = number of nodes
- Degree centrality of Amazon = 4 (or 4/8 = 0.5)

actor	degree centrality	normalized dc
Amazon	4	0.500
Apple	3	0.375
Foxconn	4	0.500
Hitachi	3	0.375
HP	3	0.375
IBM	3	0.375
Microsoft	3	0.375
Samsung	1	0.125
Sony	2	0.250

Medieval River Trade Network of Russia

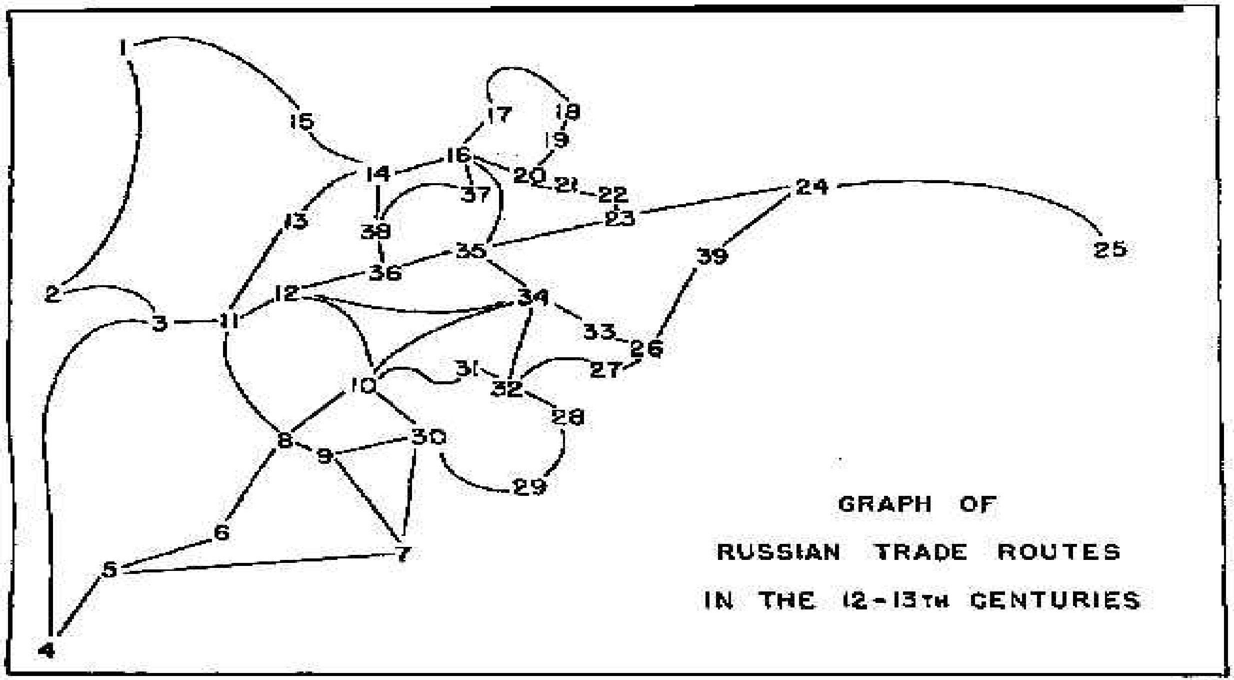
Figure 1. ***Russian trade routes in the 12th - 13th centuries.***



(Pitts 1965)

Graph of Russian trade routes

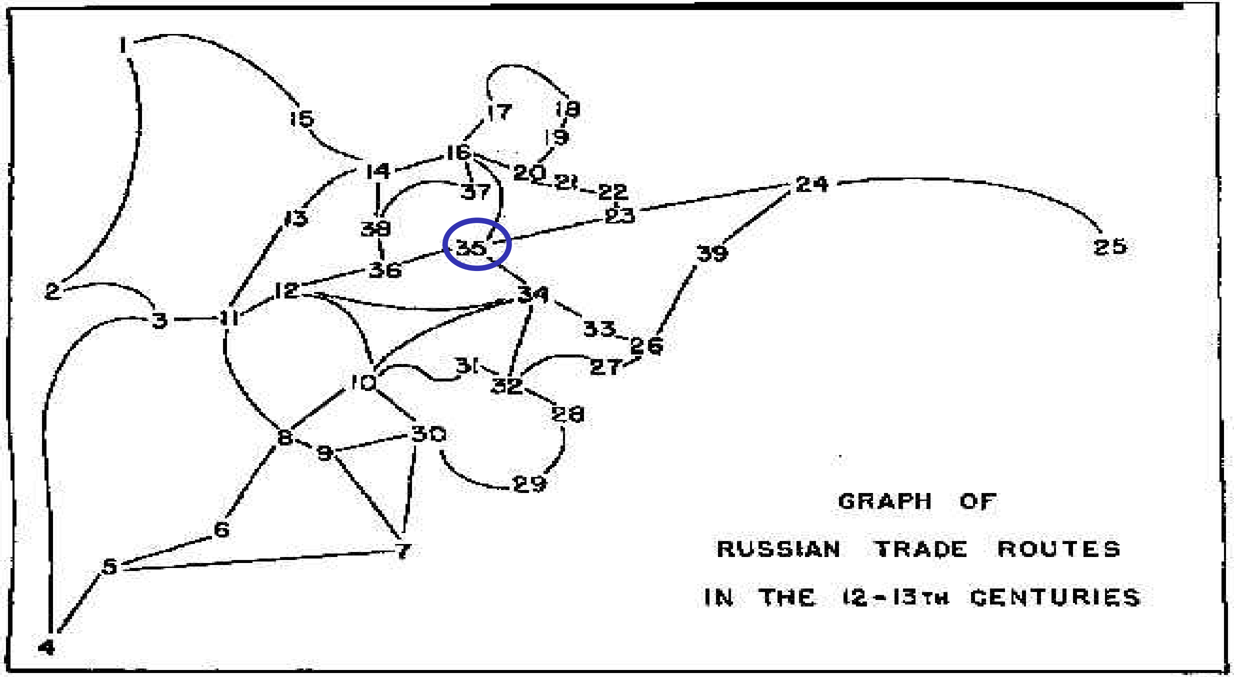
Figure 2. *Graph of Russian trade routes in the 12th - 13th centuries.*



(Pitts 1965)

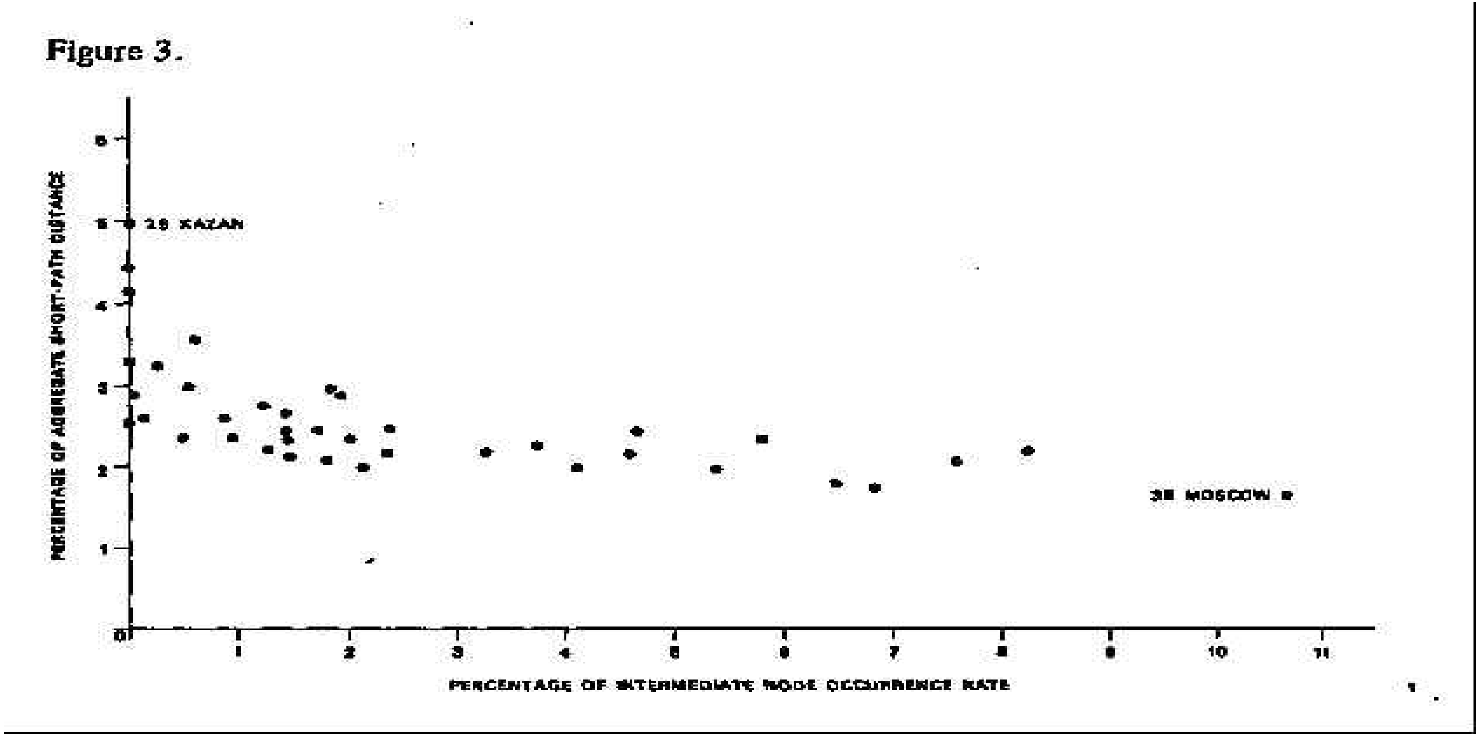
And centrality of Moscow

Figure 2. *Graph of Russian trade routes in the 12th - 13th centuries.*



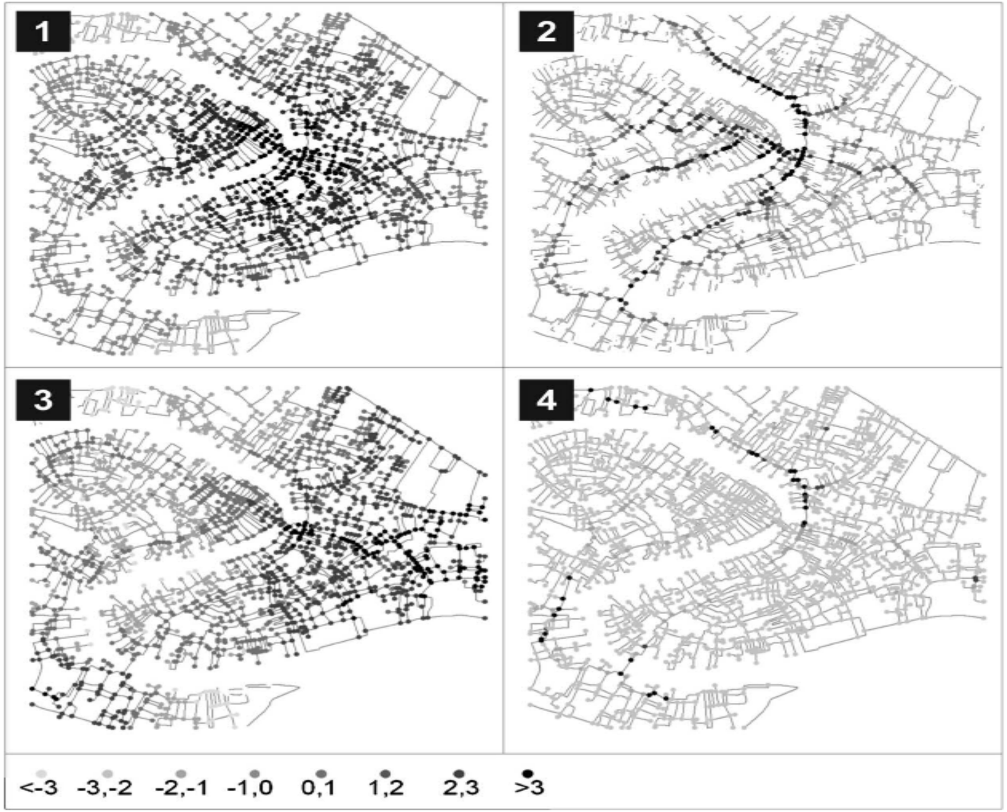
(Pitts 1965)

Centrality of urban places



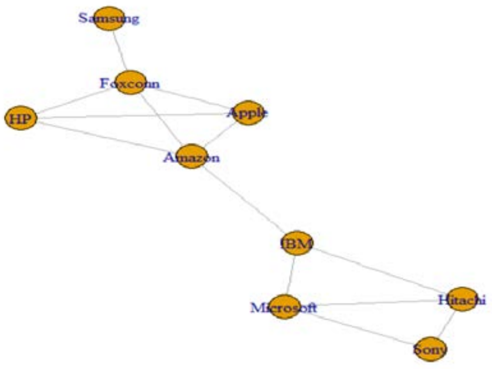
(Pitts 1965)

Spatial dist. of node centrality in Venice



- Study of urban street patterns
- Edges = streets
- Node = connection between 2 streets
- Vertices are formed when edges cross
- Planar network # from a relational network

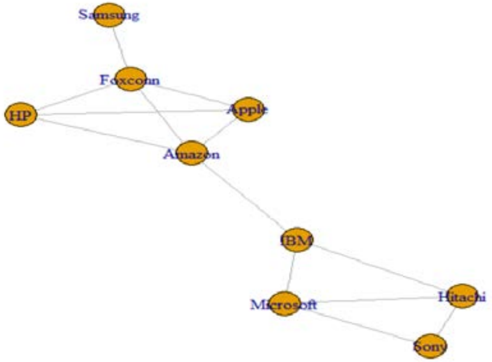
Closeness centrality



$$C_C(n_i) = \frac{1}{\sum_j g_{ij}}$$

- Inverse of the sum of geodesic distances to all other nodes
- Geodesic distance = shortest path between two nodes (total number of steps)
- Sum of geodesic distances = sum of the shortest path to all nodes
- Closeness cent * (n-1) to normalize
- Closeness centrality of Amazon?

Closeness centrality



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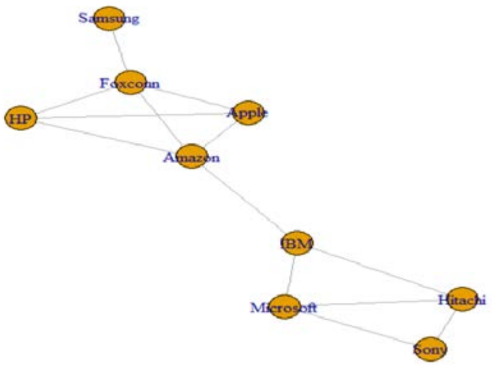
row.names	Amazon	Apple	Foxconn	Hitachi	HP	IBM	Microsoft	Samsung	Sony
Amazon	0	1	1	2	1	1	2	2	3
Apple	1	0	1	3	1	2	3	2	4
Foxconn	1	1	0	3	1	2	3	1	4
Hitachi	2	3	3	0	3	1	1	4	1
HP	1	1	1	3	0	2	3	2	4
IBM	1	2	2	1	2	0	1	3	2
Microsoft	2	3	3	1	3	1	0	4	1
Samsung	2	2	1	4	2	3	4	0	5
Sony	3	4	4	1	4	2	1	5	0

Closeness centrality



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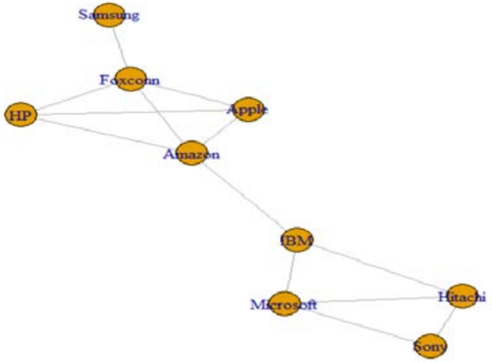
Apple	1	0	1	3	1	2	3	2	4
Foxconn	1	1	0	3	1	2	3	1	4
Hitachi	2	3	3	0	3	1	1	4	1
HP	1	1	1	3	0	2	3	2	4
IBM	1	2	2	1	2	0	1	3	2
Microsoft	2	3	3	1	3	1	0	4	1
Samsung	2	2	1	4	2	3	4	0	5
Sony	3	4	4	1	4	2	1	5	0

Closeness centrality



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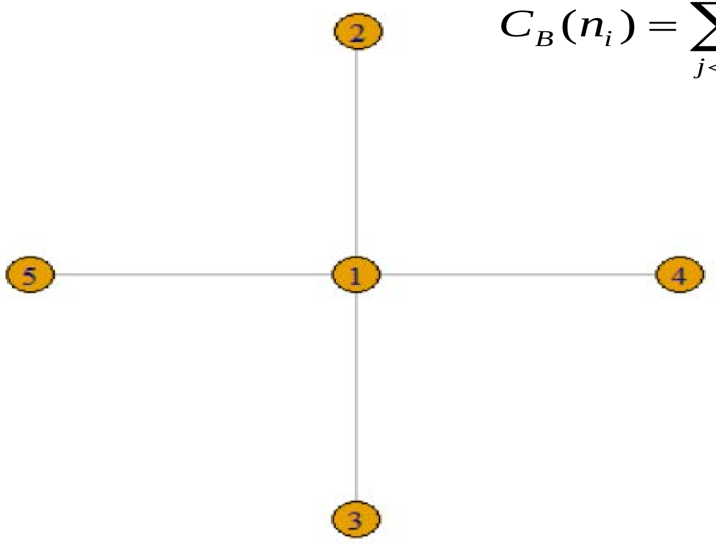
n =

actor	closeness centrality	normalized cc
Amazon	0.07692308	0.6153846
Apple	0.05882353	0.4705882
Foxconn	0.06250000	0.5000000
Hitachi	0.05555556	0.4444444
HP	0.05882353	0.4705882
IBM	0.07142857	0.5714286
Microsoft	0.05555556	0.4444444
Samsung	0.04347826	0.3478261
Sony	0.04166667	0.3333333

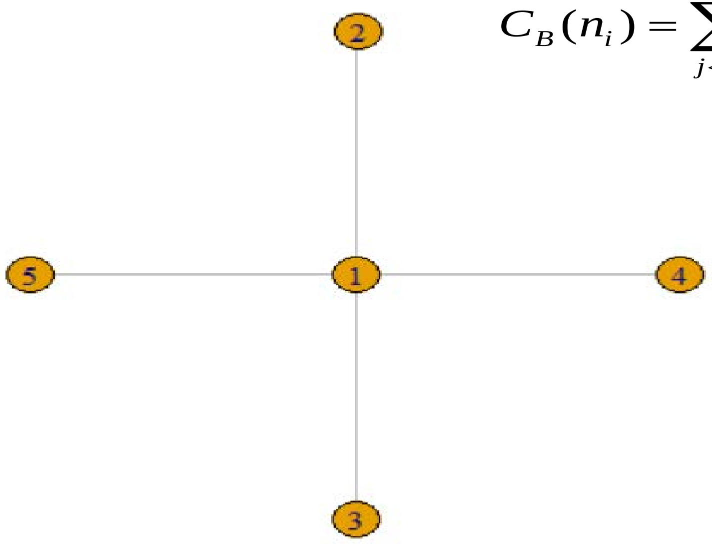
Betweenness centrality



$$C_B(n_i) = \sum_{j<k} \frac{g_{jk}(n_i)}{g_{jk}}$$

- Number of times that a node lies along the shortest path between two others
- Normalized betweenness centrality = bet. cent. / ((n-1)*(n-2)/2)
- n = number of nodes
- Betweenness centrality of actor 1?

Betweenness centrality

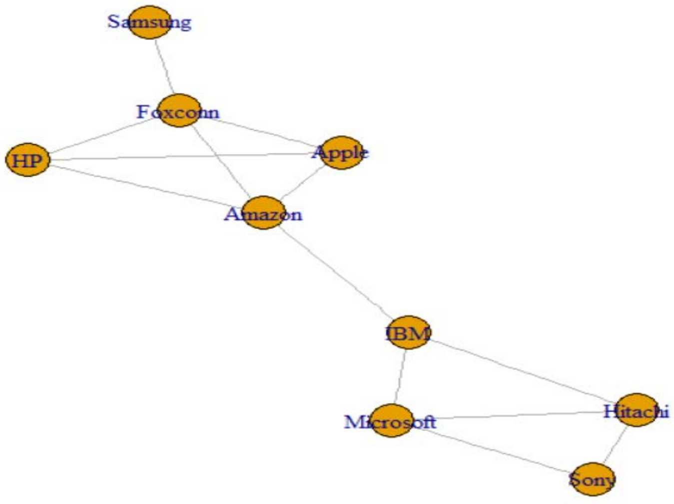


$$C_B(n_i) = \sum_{j<k} \frac{g_{jk}(n_i)}{g_{jk}}$$

- Number of times that a node lies along the shortest path between two others
- Normalized betweenness centrality = bet. cent. / ((n-1)*(n-2)/2)
- n = number of nodes
- Betweenness centrality of actor 1 = 6

	betweenness	bc normalized
1	6	1
2	0	0
3	0	0
4	0	0
5	0	0

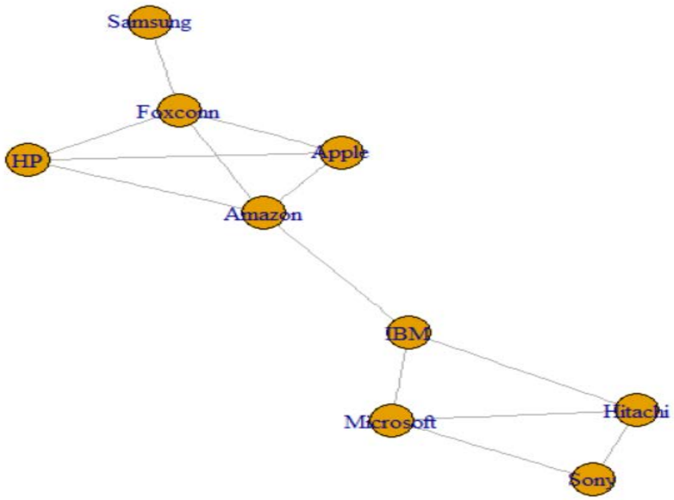
Betweenness centrality



- Number of times that a node lies along the shortest path between two others
- Normalized betweenness centrality = bet. cent. / ((n-1)*(n-2)/2)
- n = number of nodes
- Betweenness centrality of Amazon

	row.names	betweenness	bet normalized
1	Amazon	16	0.5714286
2	Apple	0	0.0000000
3	Foxconn	7	0.2500000
4	Hitachi	3	0.1071429
5	HP	0	0.0000000
6	IBM	15	0.5357143
7	Microsoft	3	0.1071429
8	Samsung	0	0.0000000
9	Sony	0	0.0000000

Eigenvector centrality



- A node is central if it is linked to other central nodes
- Eigenvector centrality of a node is computed as a function of the centralities of its neighbors
- To find eigenvector centrality, we multiply our n*n adjacency matrix by our n*1 vector of degree centrality
- The result is another n*1 vector (v1)
- We multiply our matrix by v1...
- Until we reach an equilibrium

$$A\mathbf{x} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix}$$

Eigenvector centrality

$$\begin{matrix} 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{matrix} \times \begin{matrix} 4 \\ 3 \\ 4 \\ 3 \\ 3 \\ 3 \\ 3 \\ 1 \\ 2 \end{matrix} =$$

Eigenvector centrality

$$\begin{matrix} 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{matrix} \times \begin{matrix} 4 \\ 3 \\ 4 \\ 3 \\ 3 \\ 3 \\ 3 \\ 1 \\ 2 \end{matrix} = \begin{matrix} 13 \\ 11 \\ 11 \\ 8 \\ 11 \\ 10 \\ 8 \\ 4 \\ 6 \end{matrix}$$

Eigenvector centrality

$$
\begin{matrix}
0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0
\end{matrix}
\times
\begin{matrix} 4 \\ 3 \\ 4 \\ 3 \\ 3 \\ 3 \\ 3 \\ 1 \\ 2 \end{matrix}
=
\begin{matrix} 13 \\ 11 \\ 11 \\ 8 \\ 11 \\ 10 \\ 8 \\ 4 \\ 6 \end{matrix}
$$

- Then we iterate again until we reach an equilibrium
- This equilibrium can also be found by computing the eigenvectors of our n*n adjacency matrix
- Eigenvector centrality is the eigenvector associated with the largest eigenvalue