Experiment 361

August 3, 2022

Aim

The responses of resistors, capacitors and inductors to (varying) input voltages/currents can filter or amplify these input signals. Such circuits are commonly used in the electronics you use every day. This lab lets you investigate the theoretical properties of passive RC and LC filters, and to experimentally verify these properties.

As in all advanced lab experiments, the questions posed in this handout merely serve as a guidance to write your comprehensive lab report.

Reference

The best reference book for this lab is the stage 2 Physics 240 text (OR IS IT 340? 244?): "Linear steady state network theory" by G.E.J. Bold and J. B. Earnshaw. **should we make this available somewhere?**

For more a detailed analysis of the Bode and Nyquist plots of passive circuits try "Network analysis and synthesis" by Franklin Kuo, or any of the many other electronics texts available in the library. **Do we have something less than 50 years old for these students?**

Introduction

In electronics, filters are circuits which perform signal processing functions to either remove or enhance certain frequency components. Filters can be classified as either active or passive, with active filters usually containing amplifying components in the circuit and requiring an external power source. In this experiment we shall study passive filters based on combinations of resistors, inductors and capacitors.

Theory

One way to define a certain type of filter is to consider the ratio of the output to the input voltage. This ratio is called the transfer function

$$\mathbf{T}(f) = \frac{\mathbf{V}_{out}(f)}{\mathbf{V}_{in}(f)}. \tag{1}$$

$\mathbf{T}$ tells us how the filter responds to a sinusoidal AC input voltage at a particular frequency f. Note that the transfer function is a vector: it has a magnitude and a phase. The **modulus** of $\mathbf{T}$ tells us the voltage gain (i.e., the ratio of the amplitudes of the output and input voltages), and the **argument** tells us the phase shift (i.e., the angle by which the output voltage leads the input voltage).

Bode plot

A Bode plot, shown in Figure 1, contains both a plot of the circuit's gain-vs-frequency (the Bode magnitude plot), as well as the phase-vs-frequency (the Bode phase plot) in degrees, between the input and output signals. The magnitude plot is in decibels (dB). The transfer function can also be computed as a ratio of input and output power, proportional to V^2. This new transfer function is $\mathbf{T}^2$ with the unit of "bel" as the power on a $log10$ scale. The more commonly used unit is the tenth of a bel, or decibel:

$$|\mathbf{T}|_{dB} = 10\log_{10}\left(|\mathbf{T}|^2\right) = 20\log_{10}|\mathbf{T}|. \tag{2}$$

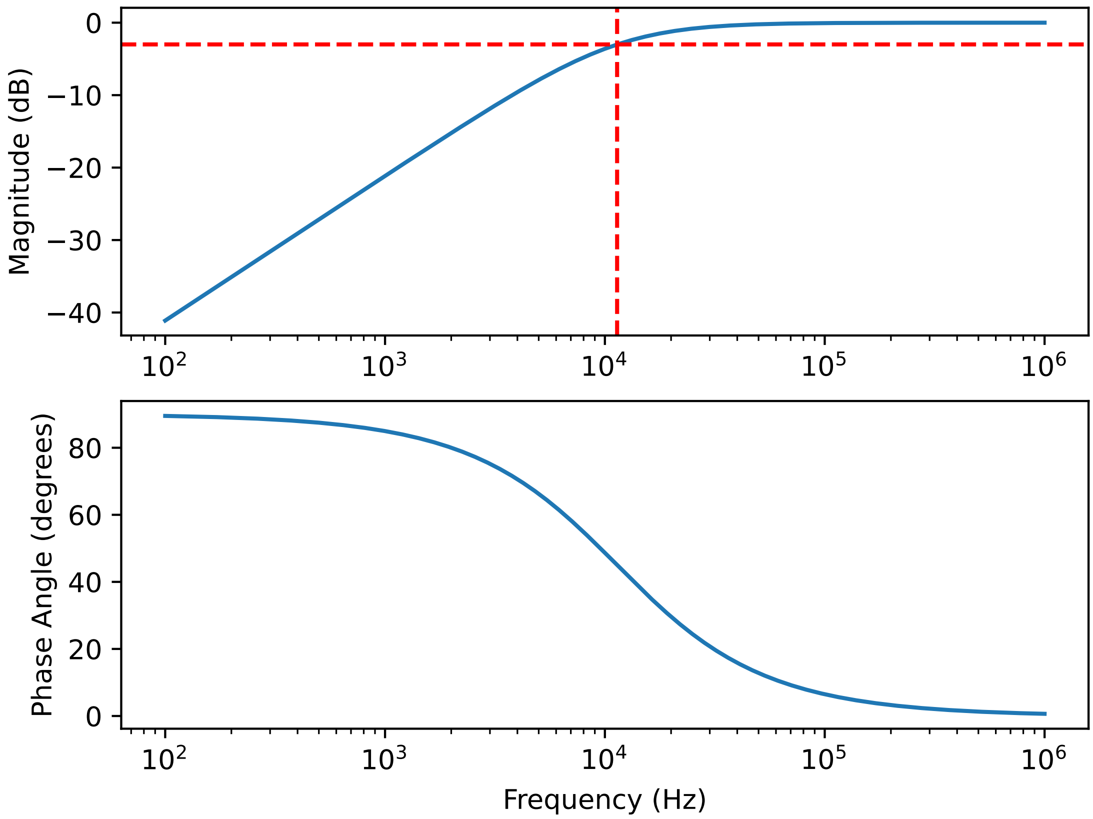

Figure 1: Bode magnitude (top) and phase (bottom) plot for a circuit/filter with a transfer function $\mathbf{T} = T \exp(i\phi)$. The magnitude plot is $T = \text{abs}(\mathbf{T})$, while the phase plot is ϕ as a function of frequency f. The corner frequency is where the output magnitude of the filter changes by 3 dB.

To characterize the filtering properties of the circuit, we define the corner or cutoff frequency as the frequency where the magntiude of the transfer function changes by $\sqrt{2}$. This frequency is annotated by the dashed line in Figure 1.

1. Show how the cutoff frequency corresponds to a change of power by a factor of 2, which equals 3 dB.

Nyquist plot

Because the transfer function $\mathbf{T}$ is a complex number, an alternative way of illustrating phase and amplitude of the filter is in one plot in the complex plane. The idea is to plot the real and imaginary part of the transfer function $\mathbf{T}$ for a range of frequencies, in what is called a Nyquist plot (Figure 2). The length of the vector $\mathbf{T}$ represents the gain of the filter at frequency f, whilst the angle of the vector with the real axis is the phase shift introduced by the filter at that same frequency f.

The Elvis II+ platform

We will build circuits that act as a filter to an oscillatory input signal on a platform from National Instruments called the “Elvis Board” (see the top panel of Figure 3). The breadboard of the Elvis Board allows circuits to be tested without having to use solder to connect components. Furthermore, the Elvis Board contains a function generator to apply an oscillating voltage to put into your circuit, and also a digital oscilloscope to interrogate the output of the signal that runs through your circuit. A USB cable connects the ELVIS II+

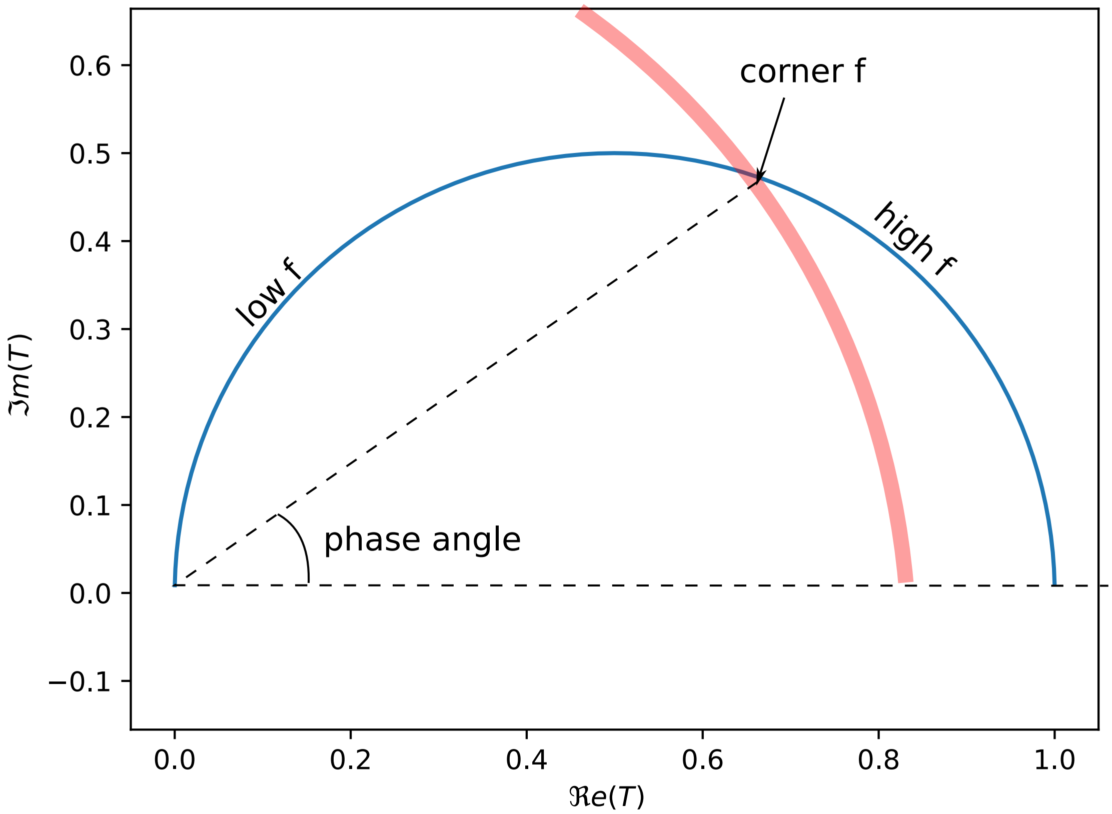

Figure 2: The thin solid line in the complex plane is the Nyquist plot. Each point on this line is the real and imaginary part of $\mathbf{T} = T(\cos(\phi) + i\sin(\phi))$ at a particular frequency. The thick solid line represents the $\mathrm{abs}(\mathbf{T}) = 1/\sqrt{2}$, so the intersection of the solid lines represents the corner frequency. The angle is the phase shift of the filter at this corner frequency.

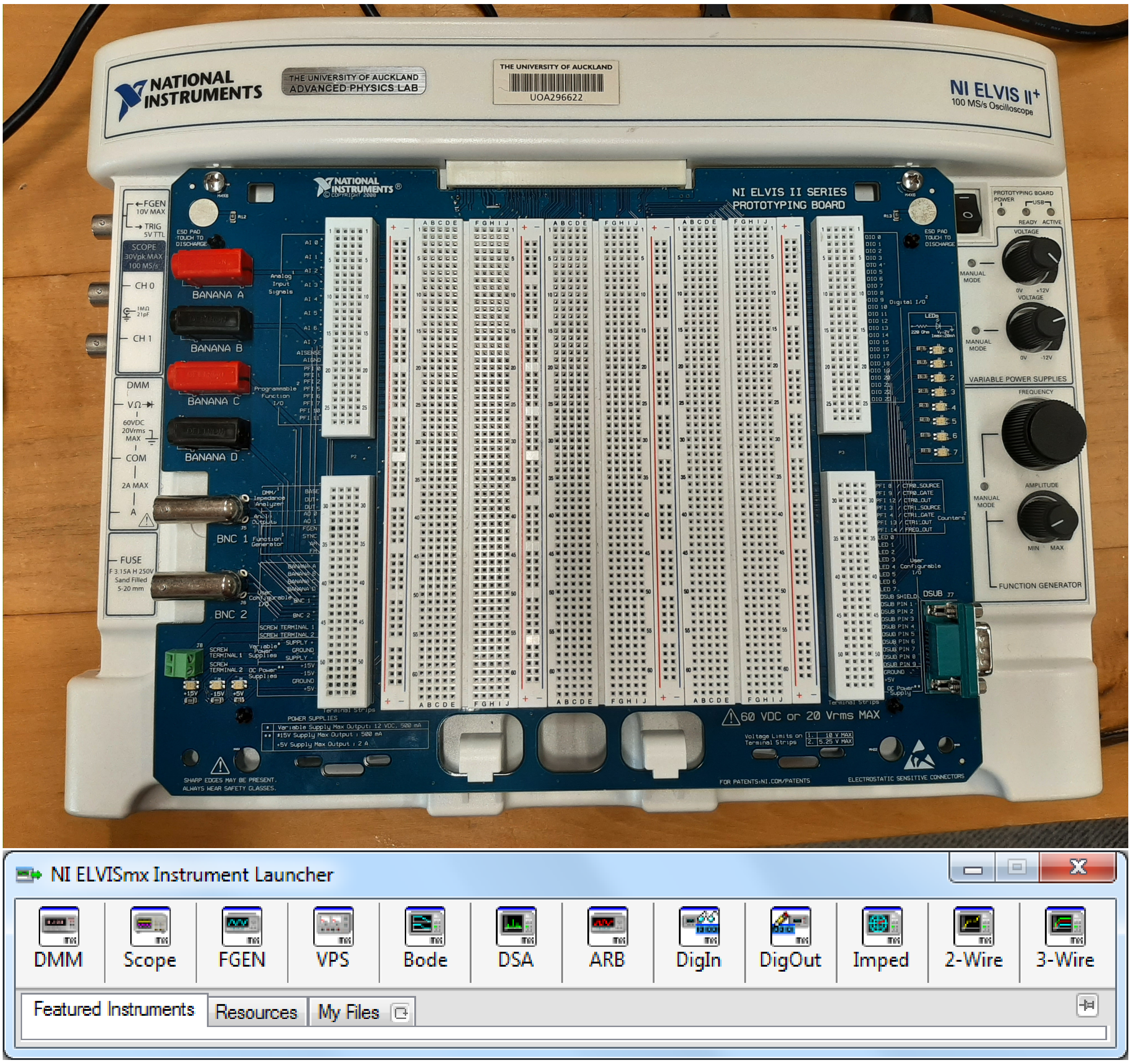

Figure 3: Top: photograph of the Elvis II+ board (top). Bottom: The graphical user interface or "instrument launcher," containing such useful devices as the digital multimeter (DMM), an oscilloscope (Scope), a function generator (FGEN) and a Bode analyser (Bode).

board to a PC with the appropriate software installed (the GUI is shown in the bottom of Figure 3), so you can control the function generator, the oscilloscope settings, and display the outputs. To access these tools inside the Elvis board, ensure that the USB plug is connected to a PC and the Elvis II+ is powered on with the switch on the side of the device. The LED indicator for the USB should display 'READY'. You can now launch the GUI called 'NI ELVISmx Instrument Launcher'

In this experiment we will use the Elvis' built-in function generator to drive a circuit built on the breadboard. The Bode plot analyzer will be used to probe the voltage across the different components of your circuit. The Elvis board has many options to build and test a circuit. Here, we explain one way to set up your board. This requires three connections to be made between the Elvis and the circuit board under test:

- Activate the function generator by connecting two bare wires from pin 33 (FGEN on left hand plug board of ELVIS) and pin 49 (GND on left hand plug board of ELVIS) to the input of the circuit you wish to test using alligator clips.
- Connect a BNC cable with alligator clips between oscilloscope channel 0 (top left hand side of the Elvis) and the input of the circuit you wish to test. The black lead goes to ground, and the lead with the other colour (blue, or red) is the positive.
- Connect a BNC cable with alligator clips between oscilloscope channel 1 (top left hand side of the Elvis) and the output of the circuit you wish to test. The black lead goes to ground, the other colour is positive.
- Launch the Bode function analyzer program from the GUI to measure Bode amplitude and phase plots over a frequency range defined in the GUI.

Note: Save your data from the graphical user interface of the ELVIS board, under the "log" button. You can then import your data into Python to analyse and plot the results. Save these plots for your report. There are many ways to import the data in python, but one of our favourites is using the pandas package. All python packages you need are installed on the lab computers, or you can use Google's COLAB to do your computing in the Cloud.

Experiments

In this lab, you will explore two types of passive filters: RC and LC filters. The former contains a circuit made of resistors (R) and capacitors (C), while the latter is a circuit with inductor(s) L and capacitor(s) C.

RC Filters

The simplest of filters is one with a resistor R and a capacitor C in series (Figure 4). Such a circuit acts as a filter, because the impedance of a capacitor is frequency dependent. Let us explore both experimentally and theoretically the filtering capabilities of this simple circuits.

- build the circuit of Figure 4 with a resistor $R = 10\ k\Omega$ and a capacitor $C = 22$ nF on your Elvis board.
- Set the function generator on your Elvis board to sweep the input voltage from 100 Hz to 1MHz.
- Measure the ratio of the voltage across the resistor R over the input voltage with the Bode Analyser on your Elvis Board. What is this ratio called?
- Save your data and plot these in python as a phase and magnitude Bode plot.

In PHYS121 you learned that the impedance of the capacitor can be written as $Z_c = 1/(j\omega C)$, where j is the imaginary unit, and $\omega = 2\pi f$ is the angular frequency.

1. Explain in your report why the voltage drop across the resistor in the circuit of Figure 4 is a fraction of the input voltage:

$$\mathbf{V}_{out}(\omega) = \frac{R}{R + \frac{1}{j\omega C}} \mathbf{V}_{in}(\omega).$$

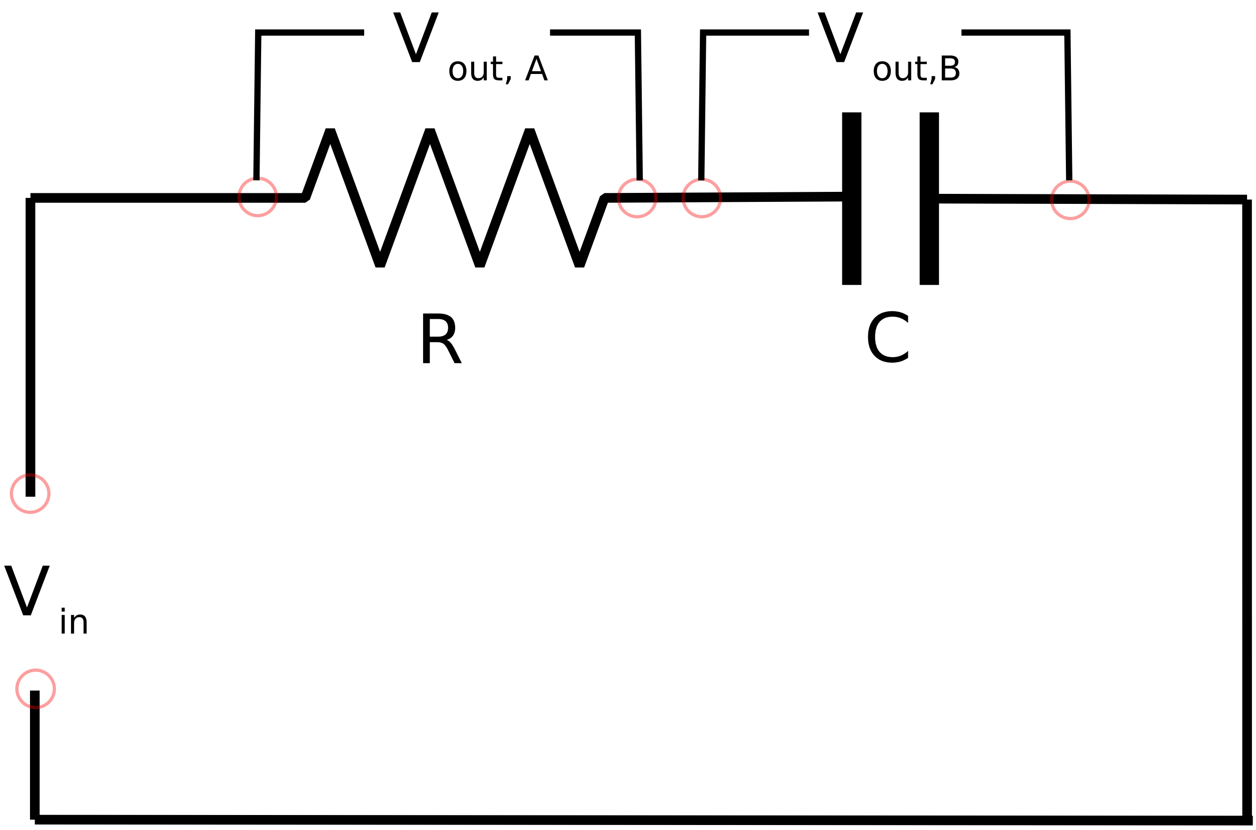

Figure 4: Circuit diagram of filters A and B. The input is V_{in}, and the outputs are $V_{out,A}$ for filter A, and $V_{out,B}$ for filter B.

The transfer function of Filter A is then

$$\mathbf{T}_A = \frac{\mathbf{V}_{out,A}}{\mathbf{V}_{in}} = \frac{R}{R + 1/(\mathrm{j}\omega C)} = \frac{1}{1 - \mathrm{j}\omega_0/\omega}, \tag{3}$$

where

$$\omega_0 = \frac{1}{RC}. \tag{4}$$

1. Based on what you read so far in this hand-out, show that $\omega_0 = 1/RC$ is the cutoff, or corner, angular frequency.
2. What is ω_0 for Filter A?
3. Use Python to plot a theoretical Bode amplitude plot (in dB) and Bode phase plot (in degrees) on the same figure, using a frequency range of 100Hz-1MHz.
4. On a separate figure, use Python to construct a theoretical Nyquist plot from your data.
5. Explain why the output voltage measured across the resistor is almost zero for very low frequencies.
6. What sort of output waveform ($\mathbf{V}_{out}$) would you expect to see on an oscilloscope from this filter when using a 1kHz square wave as your input waveform ($\mathbf{V}_{in}$)?

Filter A, as shown in Figure 4, is a simple high-pass phase-advance filter, commonly used for inter-stage coupling where DC isolation is required. High pass filters are characterised by their ability to allow high frequencies to pass through the filter while suppressing low frequencies.

We can also consider the voltage measured over the capacitor in the same circuit as the output ($\mathbf{V}_{out,B}$ in Figure 4). This output is

$$\mathbf{V}_{out,B}(\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}\mathbf{V}_{in}(\omega) = \frac{1}{1 + RCj\omega}\mathbf{V}_{in}(\omega) = \frac{1}{1 + j\omega/\omega_0}\mathbf{V}_{in}(\omega) \tag{5}$$

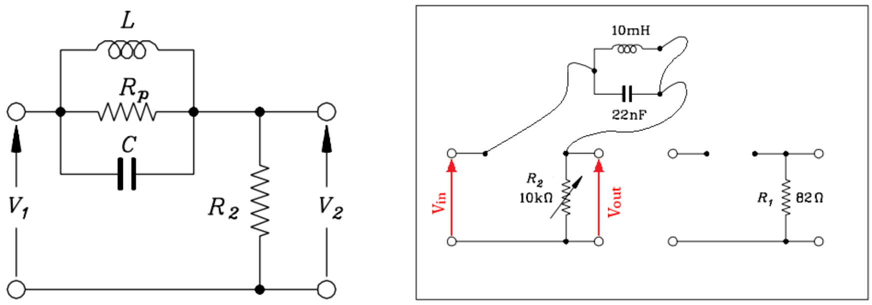

Figure 5: Circuit diagram for filter C (left) and experimental setup (right).

and the transfer function for Filter B is then

$$\mathbf{T}_B = \frac{\mathbf{V}_{out,B}}{\mathbf{V}_{in}} = \frac{1}{1 + \mathrm{j}\omega/\omega_0}. \tag{6}$$

1. Make Bode plots of the theoretical and experimental transfer function $\mathbf{T}_B$.

Because the circuit used for Filters A and B is the same, the output for Filter B can also be derived from the output of Filter A:

$$\mathbf{V}_{in}(f) = \mathbf{V}_C(f) + \mathbf{V}_R(f) \tag{7}$$

Therefore,

$$\mathbf{T}_B(f) = \frac{\mathbf{V}_C(f)}{\mathbf{V}_{in}(f)} = \frac{\mathbf{V}_{in}(f) - \mathbf{V}_R(f)}{\mathbf{V}_{in}(f)} = 1 - \frac{\mathbf{V}_R(f)}{\mathbf{V}_{in}(f)} = 1 - \mathbf{T}_A(f). \tag{8}$$

1. Construct Bode amplitude and phase plots with Filters A and B in each panel.

2. As the voltage transfer function $\mathbf{T}_B$ can be derived from the voltage transfer function for Filter A, construct a Nyquist plot by by adding $-\mathbf{T}_A$ to $\mathbf{1}$ vectorially. This vector addition is equivalent to addition of complex numbers.

3. Discuss how the Bode and Nyquist plots inform us about the relations between these two filters.

Based on your measurements and theoretical calculations, Filter B in the circuit in Figure 4 is a simple low-pass phase-delay filter used to remove unwanted high-frequency signals.

1 LC filters

LC filters have different characteristics to those of RC filters, due to the inclusion of an inductor L in the circuit. These filters are able to act as an electrical resonator, storing energy oscillating at the circuit's resonant frequency. These resonant properties can be used to select certain frequencies out of a range of frequencies, making them useful for applications such as tuning radio transmitters and receivers.

HERE? Theoretical LC filters are unachievable in the real world, as the capacitor and inductor combination will always possess some resistance, and the output signal will be damped even at resonance.

HERE? The **Q factor** (or quality factor) is a measure of the "sharpness" of the resonance. Conversely, it determines how damped an oscillator is. The definition of Q is the full width at half of the maximum value. See experiment 263 and its handout for more details. The Q of an LC filter is not infinite due to the internal resistance of the inductor and capacitor combination. EQUATION HERE. A circuit having a low Q factor (Q$<1/2$) is said to be overdamped, and will not oscillate. A circuit with a high Q factor (Q$>1/2$) is said to be underdamped and will oscillate with a decay of the amplitude of the signal. When Q$=1/2$ the circuit is said to be critically damped.

1.1 Filter C

The circuit shown in Fig. 5 shows a real LC circuit, with the resistance R_p representing the resistance from the inductor and capacitor in parallel. The voltage transfer function for this circuit is given by:

$$\mathbf{T}_C = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{R_2}{R_2 + z_p}, \tag{9}$$

where the impedance of the parallel components is

$$z_p \equiv \frac{1}{1/Z_R + 1/Z_C + 1/Z_L} = \frac{R_p}{1 + \mathrm{j}R_p\left(\omega C - \frac{1}{\omega L}\right)}. \tag{10}$$

It is clear that $|\mathbf{T}_C|$ is a minimum when $|z_p|$ is a maximum. This will occur at the resonant frequency, $\omega_0 = 1/\sqrt{LC}$.

We can define Q_p as the Q factor of the parallel inductor-capacitor combination, which gives a measure of how damped the system is. Q_p is given by

$$Q_p = \frac{R_p}{\omega_0 L} = \omega_0 R_p C. \tag{11}$$

The Q factor of filter C (different from Q_p) can then be defined as

$$Q_C = Q_p\sqrt{1 - 2|\mathbf{T}_{Cmin}|^2}, \tag{12}$$

where $|\mathbf{T}_{Cmin}|$ is the minimum value of $|\mathbf{T}_C|$.

(6) The resistance R_p of filter C (see Fig. 5) is the parallel loss resistance of the inductor (the capacitor has negligible loss resistance). Set R_2 to have a value of 300Ω and use the Elvis to obtain a Bode plot for the filter.

(7) Determine the frequencies f_{min} and f_{max} corresponding to the greatest positive and negative phase shifts between the input and output signals.

(8) Use this plot to find the value of R_p, and hence determine Q_p. Find the value of Q_C, and interpret it in relation to the damping of the circuit.

(9) Using your knowledge of the resistances in this circuit, obtain a theoretical Bode plot for this filter with Python. Confirm that it agrees with what you have found experimentally.

(10) Create a Nyquist plot containing both theoretical and experimental data, and see whether the two agree.

1.2 Filter D

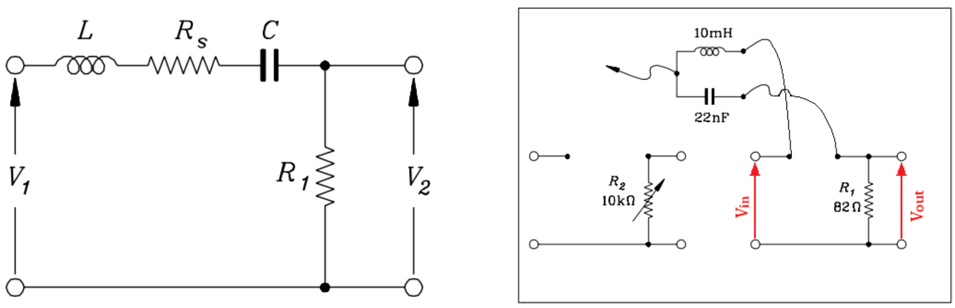

Figure 6: Circuit diagram for filter D (left) and experimental setup (right).

In the circuit of Filter D (see Fig. 6), R_s represents the power loss resistance of the inductor-capacitor series combination. The voltage transfer function of Filter D is given by:

$$\mathbf{T}_D = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{R_1}{R_1 + z_s}, \tag{13}$$

where

$$z_s = R_s\left[1 + \mathrm{j}\left(\frac{\omega L}{R_s} - \frac{1}{\omega R_s C}\right)\right]. \tag{14}$$

From equation 13, we can see that $\mathbf{T}_D$ is a maximum when z_s is a minimum, and equation 14 shows us that this occurs at the resonant frequency, when $\omega = \omega_0 = 1/\sqrt{LC}$. If we define Q_s to be the quality factor of the series inductor-capacitor combination, then by definition

$$Q_s = \frac{\omega_0 L}{R_s} = \frac{1}{\omega_0 R_s C}. \tag{15}$$

The Q factor of filter D is given by:

$$Q_D = \frac{\omega_0 L}{R_1 + R_s} = \frac{1}{\omega_0 (R_1 + R_s) C} = Q_s \frac{R_s}{R_1} |\mathbf{T}_{Dmax}|. \tag{16}$$

(11) Use the Elvis to obtain an experimental Bode plot for the circuit shown in Fig. 6.

(12) Find R_s and hence determine the value of Q_s. Find the value of Q_D and interpret this in relation to how damped the circuit is.

(13) Use Python to construct a theoretical Bode plot and a Nyquist plot for this circuit, using your experimental values for the circuit components. Check whether these agree with the experimental plots.

S. Ruddell, S. Murdoch February 2013
S. Coen 2021
K. van Wijk, 2022