Question:	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points:	10	10	10	10	10	10	10	10	10	10	10	10	120
Score:													

Answer the questions in the spaces provided after each question, in teams of two. Hand in your work to your TA or scan and submit on Canvas by 18:00 on Tuesday, 10th August.

Your names and UIDs: ______________________

1 Aim

Armageddon for the dinosaurs was most likely the result of the impact of an asteroid on Earth, 65 Ma before present [1]. To investigate this, we need to understand the "impact of such an impact." In Section 1.1, we will derive the centre of mass of an impact crater (a value we need to estimate the relationship between asteroid and crater). In Section 1.2, we will use our Moon's cratered surface as a proxy for asteroid impacts in Earth.

Key words: kinetic and potential energy, conservation of energy, centre of mass, statistical distributions, volume integration, astrophysics, geophysics.

References

[1] Walter Alvarez. *T. rex and the Crater of Doom.* Princeton University Press, 2008.

1.1 Centre of mass of a crater

We discussed in class how the kinetic energy of an incoming asteroid is transferred on impact to – mainly – potential energy to "dig" a crater, as energy is conserved:

$$m_a v_a^2/2 = m_c g h, \tag{1}$$

where for the kinetic energy on the left-hand side, we have v_a as the relative velocity between earth and asteroid of mass m_a. The potential energy on the right is the product of the mass displaced from the crater m_c, g the gravitational acceleration, and h the distance the crater dirt has to overcome to be ejected.

In principle, we would have to consider each bit of mass displaced by the crater with its location, but we are going to make two simplifying assumptions:

1. the crater is a perfect hemisphere, and
2. the material displaced by the asteroid is of constant density.

This allows us to compute the left-hand side of equation 1, *as if* all the mass m_c was at one point that is an average of the location of all points in the hemisphere. This coordinate $\mathbf{R}$ is called the *centre of mass*. If we divide the hemisphere in n small masses m_i at location r_i $(i = 1, n)$, then

$$\sum_{i=1}^{n} m_i(\mathbf{r}_i - \mathbf{R}) = 0. \tag{2}$$

In the limit that n goes to infinity, we get the integral version:

$$\iiint\limits_V \rho(\mathbf{r})(\mathbf{r} - \mathbf{R})dV = 0. \tag{3}$$

Because we assume a constant density for our crater material, the centre of mass $\mathbf{R}$ is equal to the *centroid*.

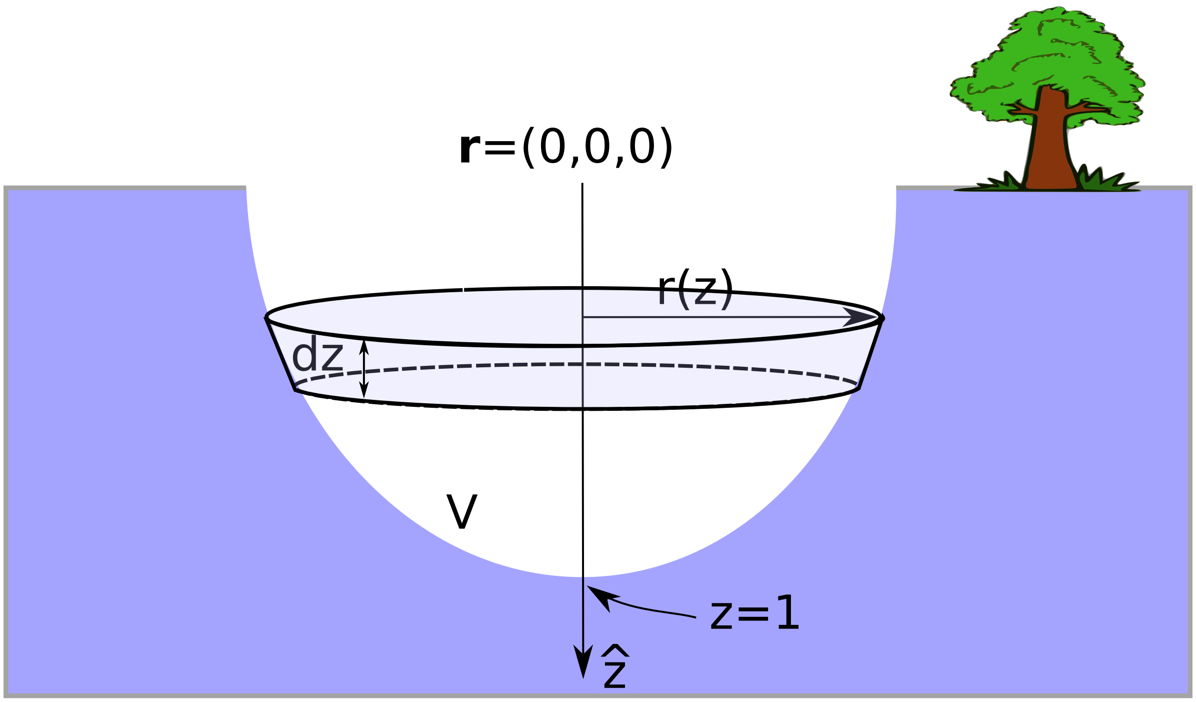

Figure 1: This hemisphere depicts an impact crater of unit radius. One way to represent the volume V is to consider discs of thickness dz and radius $r(z)$, as drawn.

1. (10 points) Show that equation 3 reduces to

$$\mathbf{R} = \frac{1}{V} \iiint_V \mathbf{r} dV. \tag{4}$$

In Figure 1, we use a cylindrical coordinate system. The vertical coordinate is z, a radial r, and an unlabeled angle around the z-axis ϕ.

2. (10 points) Explain why we can reduce the general – three-dimensional – coordinate $\mathbf{r} = z\hat{\mathbf{z}}$, in this case, so that

$$\mathbf{R} = \frac{1}{V} \iiint\limits_V z\hat{\mathbf{z}} dV. \tag{5}$$

Next, we are going to define these infinitesimal volumes and sum over all that make up the hemisphere. The elementary volume dV can be taken as a horizontal disk with thickness dz, as drawn in Figure 1.

3. (10 points) Show that $dV = \pi \left(1 - z^2\right) dz$.

4. (10 points) Integrate over V by integrating over all horizontal disks dV between $z = 0$ to $z = 1$, to determine the centre of mass of the material displaced by the crater.

5. (10 points) Draw the centre of mass **R** in Figure 1, and explain why this position makes sense to you. Comment on the horizontal and the vertical component of **R**.

1.2 Craters of the Moon

The following investigation allows you to explore the nature of the craters that can be easily seen on the surface of the Moon and to speculate what a similar bombardment of the Earth would have done to it.

6. (10 points) Select sector F4 on `https://fullmoonatlas.com/atlas/`. Using the information at the top of the page, identify the names and locations of at least two of the craters in the image. Find out their diameters and write these below.

7. (10 points) Save the moon image to file, and open it using the ImageJ software (search "ImageJ" in the start menu; see `https://imagej.nih.gov/ij/docs/index.html` for documentation and tutorials). Using the straight line tool in ImageJ, measure the diameters (in pixels) of the craters for which you already know the true diameters. What is the pixel size in km?

   Set the scale using Analyze → Set Scale, and measure/annotate other craters with the straight line tool. Each measurement can be automatically recorded in a spreadsheet by pressing "M". Save the recorded data with a ".csv" extension.

8. (10 points) Draw a histogram in Python by importing and plotting the estimated crater sizes in the csv file (We use python, because Excel is not for scientists!). See the pandas documentation for help with importing csv files, and `https://matplotlib.org/1.2.1/examples/pylab_examples/histogram_demo.html` for an example of plotting histograms. Attach your annotated map and histogram of your estimated crater sizes to this report.

9. (10 points) What do you infer from this distribution?

10. (10 points) Derive a formula, based on what you learned in the lectures, relating the diameter of the asteroid that produced the crater to the diameter of the crater. List your assumptions and **re-label the crater size axis of your original histogram with the approximate asteroid diameter.**

11. (10 points) Study sector E2 of the Moon, next. How do you account for the great differences between this sector and F4?

The gravitational acceleration of g at the surface is different on the Moon than the Earth.

12. (10 points) How would you expect a similar asteroidal bombardment to have affected the surface of the Earth? How would the presence of the Earth's atmosphere have affected the situation on Earth?