Experiment 361

September 12, 2022

Aim

The responses of resistors, capacitors and inductors to (varying) input voltages/currents can filter or amplify these input signals. Such circuits, and their digital equivalent, are commonly used in the electronics you use every day. This lab lets you investigate the theoretical properties of passive RC and LC filters, and to experimentally verify these properties.

As in all advanced lab experiments, the questions posed in this handout merely serve as a guidance to write your comprehensive lab report.

Reference

The best reference book for this lab is the stage 2 Physics 240 text (OR IS IT 340? 244?): "Linear steady state network theory" by G.E.J. Bold and J. B. Earnshaw. **should we make this available somewhere?**

For more a detailed analysis of the Bode and Nyquist plots of passive circuits try "Network analysis and synthesis" by Franklin Kuo, or any of the many other electronics texts available in the library. **Do we have something less than 50 years old for these students?**

Equipment list

- Elvis Board with power supply and USB cable
- Circuit components: two resistors $R = 10,000, 300$ and 82 Ω, a conductor $L = 10$ mH, and a capacitor $C = 22$ nF.
- Wires to connect the components on the Elvis Board.
- Digital multimeter and (access to) lab RCL meter.

Introduction

In electronics, filters are circuits which perform signal processing functions to either remove or enhance certain frequency components. Filters can be classified as either active or passive, with active filters usually containing amplifying components in the circuit and requiring an external power source. In this experiment we shall study passive filters based on combinations of resistors, inductors and capacitors.

Theory

One way to define a filter is to consider the ratio of the output to the input. This ratio is called the transfer function. In this lab, we will compare input and output voltages V, as a function of frequency f. The transfer function is

$$\mathbf{T}(f) = \frac{\mathbf{V}_{out}(f)}{\mathbf{V}_{in}(f)}. \tag{1}$$

The transfer function $\mathbf{T}$ tells us how the filter responds to a sinusoidal AC input voltage at a particular frequency f. Note that the transfer function is a vector: it has a magnitude and a phase. The **modulus**

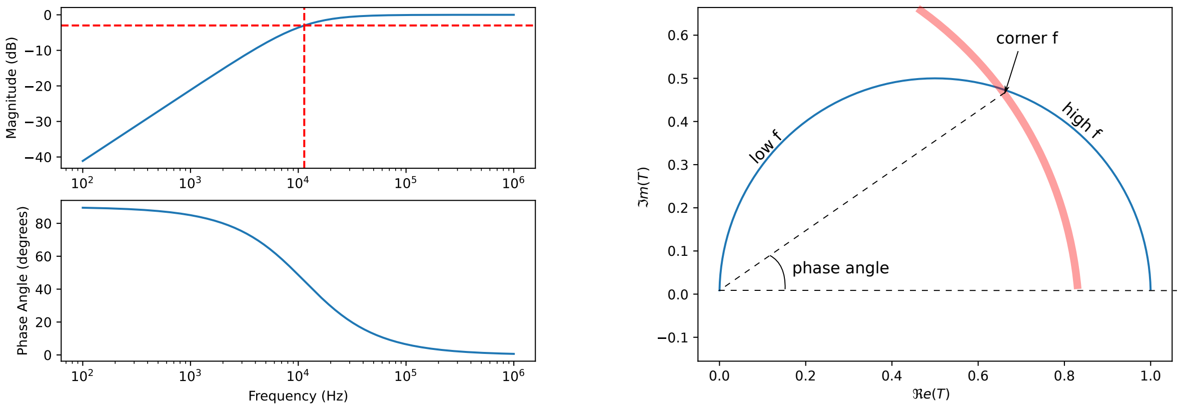

Figure 1: Bode magnitude (left, top) and phase (left, bottom) plot for a circuit/filter with a transfer function $\mathbf{T} = T\exp(i\phi)$. The magnitude plot is $T = |\mathbf{T}|$, while the phase plot is ϕ as a function of frequency f. The corner frequency is where the output magnitude of the filter changes by 3 dB. Right panel: the thin solid line in the complex plane is the Nyquist plot. Each point on this line is the real and imaginary part of $\mathbf{T} = T(\cos(\phi) + i\sin(\phi))$ at a particular frequency. The thick solid line represents the $|\mathbf{T}| = 1/\sqrt{2}$, so the intersection of the solid lines represents the corner frequency. The angle is the phase shift of the filter at this corner frequency.

of $\mathbf{T}$ tells us the voltage gain (i.e., the ratio of the amplitudes of the output and input voltages), and the **argument** tells us the phase shift introduced by the filter (i.e., the angle by which the output voltage leads the input voltage).

Bode plot

The left of Figure 1 shows the Bode plot of a filter. The bode magnitude plot displays the gain and the Bode phase plot shows the filter's phase shift in degrees between the input and output signals as a function of frequency. The magnitude plot is in decibels (dB). The transfer function can also be computed as a ratio of input and output power, which is proportional to V^2. This new transfer function is $\mathbf{T}^2$ with the unit of "bel" as the power on a $log10$ scale. The more commonly used unit is one tenth of a bel, or a decibel:

$$|\mathbf{T}|_{dB} = 10\log_{10}\left(|\mathbf{T}|^2\right) = 20\log_{10}(|\mathbf{T}|). \tag{2}$$

To characterize the filtering properties of the circuit, we define the corner or cutoff frequency as the frequency where the magntiude of the transfer function changes by $\sqrt{2}$. This frequency is annotated by the dashed line in the top left panel of Figure 1.

1. Show how the cutoff frequency is associated with a 3 dB change in gain.

Nyquist plot

Because the transfer function $\mathbf{T}$ is a complex number, an alternative way of illustrating phase and amplitude of the filter is in one panel in the complex plane. The idea is to plot the real and imaginary part of the transfer function $\mathbf{T}$ for a range of frequencies, in what is called a Nyquist plot (The right panel in Figure 1). The length of the vector $\mathbf{T}$ represents the gain of the filter at frequency f, whilst the angle of the vector with the real axis is the phase shift introduced by the filter at that same frequency f.

The Elvis II+ platform

We will build electronic circuits that act as filters to an oscillatory input voltage on a platform from National Instruments called the "Elvis Board" (see the top panel of Figure 2). The breadboard of the Elvis Board allows circuits to be tested without having to use solder to connect components. Furthermore, the Elvis

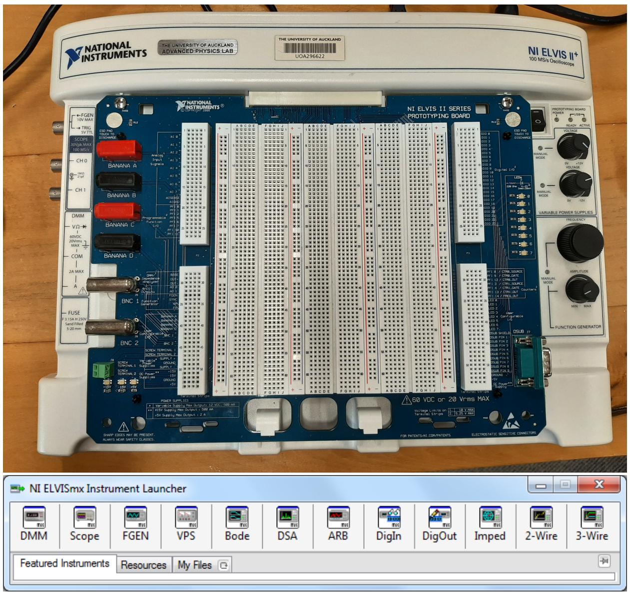

Figure 2: Top: the Elvis II+ board. Bottom: The Graphical User Interface (GUI, or "instrument launcher"), containing such useful devices as the digital multimeter (DMM), an oscilloscope (Scope), a function generator (FGEN) and a Bode analyser (Bode).

Board contains a function generator to apply an oscillating voltage to put into your circuit, and also a digital oscilloscope to interrogate the output of the signal that runs through your circuit. A USB cable connects the ELVIS II+ board to a PC with the appropriate software installed (the GUI is shown in the bottom of Figure 2), so you can control the function generator, the oscilloscope settings, and display the outputs. To access these tools inside the Elvis board, ensure that the USB plug is connected to a PC and the Elvis II+ is powered on with the switch on the side of the device. The LED indicator for the USB should display 'READY'. You can now launch the GUI called 'NI ELVISmx Instrument Launcher.'

In this experiment we will use the Elvis' built-in function generator to drive a circuit built on the breadboard. The Bode plot analyzer will be used to probe the voltage across the different components of your circuit. The Elvis board has many options to build and test a circuit. Here, we explain one way to set up your board. This requires three connections to be made between the Elvis and the circuit board under test:

2. Activate the function generator by connecting two bare wires from pin 33 (FGEN on left hand plug board of ELVIS) and pin 49 (GND on left hand plug board of ELVIS) to the input of the circuit you wish to test using alligator clips.

3. Connect a BNC cable with alligator clips between oscilloscope channel 0 (top left hand side of the Elvis) and the input of the circuit you wish to test. The black lead goes to ground, and the lead with the other colour (blue, or red) is the positive.

4. Connect a BNC cable with alligator clips between oscilloscope channel 1 (top left hand side of the Elvis) and the output of the circuit you wish to test. The black lead goes to ground, the other colour is positive.

5. Launch the Bode function analyzer program from the GUI to measure Bode amplitude and phase plots over a frequency range defined in the GUI.

6. Save your data from the graphical user interface of the ELVIS board, under the "log" button. You can then import your data into Python to analyse and plot the results. Save these plots for your report. There are many ways to import the data in python, but one of our favourites is using the pandas package. All python packages you need are installed on the lab computers, or you can use Google's COLAB to do your computing in the Cloud.

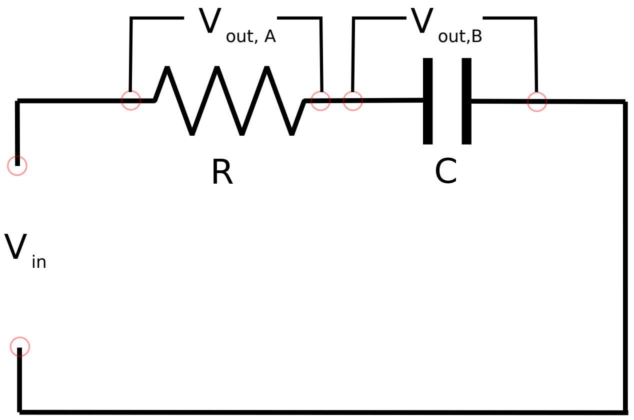

Figure 3: Circuit diagram of filters A and B. The input is V_{in}, and the outputs are $V_{out,A}$ for filter A, and $V_{out,B}$ for filter B.

Experiments

In this lab, you will explore two types of passive filters: RC and LC filters. The former contains a circuit made of resistors (R) and capacitors (C), while the latter is a circuit with inductor(s) L and capacitor(s) C.

7. Before we build any circuits and measure the transfer function, consider an alternating current. What is the impedance of R, C and L? Describe these expressions in words, as well. Explain mathematically, but also physically, the behaviour of each component in the limit of very high and very low input frequencies.

   Revisit your PHYS121 book and your course notes, if you have to.

RC Filters

The simplest of filters is one with a resistor R and a capacitor C in series (Figure 3). Such a circuit acts as a filter, because the impedance of a capacitor is frequency dependent. Let us explore both experimentally and theoretically the filtering capabilities of this simple circuits.

Filter A

Kasper, maybe switch A and B? Then you can talk about the impedance of a capacitor, decreasing with f?
Filter A is the output measured over the resistor $R = 10\ k\Omega$.

8. build the circuit of Figure 3 with a resistor $R = 10\ k\Omega$ and a capacitor $C = 22$ nF on your Elvis board.
9. Set the function generator on your Elvis board to sweep the input voltage from 100 Hz to 1MHz. **Set the number of samples high!**
10. Measure the ratio of the voltage across the resistor R over the input voltage with the Bode Analyser on your Elvis Board. What is this ratio called?
11. Save your data and plot these in python as a phase and magnitude Bode plot.
12. Explain in your report why the voltage drop across the resistor in the circuit of Figure 3 is a fraction of the input voltage:

$$\mathbf{V}_{out}(\omega) = \frac{R}{R + \frac{1}{j\omega C}}\mathbf{V}_{in}(\omega).$$

In PHYS121 you learned that the impedance of the capacitor is $Z_c = 1/(j\omega C)$, where j is the imaginary unit, and $\omega = 2\pi f$ is the angular frequency. The transfer function of Filter A is then

$$\mathbf{T}_A = \frac{\mathbf{V}_{out,A}}{\mathbf{V}_{in}} = \frac{R}{R + 1/(\mathrm{j}\omega C)} = \frac{1}{1 - \mathrm{j}\omega_0/\omega}, \tag{3}$$

where

$$\omega_0 = \frac{1}{RC}. \tag{4}$$

13. Based on what you read so far in this hand-out, show that $\omega_0 = 1/RC$ is the cutoff, or corner, angular frequency.

14. What is ω_0 for Filter A?

15. Use Python to plot a theoretical Bode amplitude plot (in dB) and Bode phase plot (in degrees) on the same figure, using a frequency range of 100Hz-1MHz.

16. On a separate figure, use Python to construct a theoretical Nyquist plot from your data.

17. Explain why the output voltage measured across the resistor is almost zero for very low frequencies.

18. What sort of output waveform ($\mathbf{V}_{out}$) would you expect to see on an oscilloscope from this filter when using a 1kHz square wave as your input waveform ($\mathbf{V}_{in}$)?

Filter A, as shown in Figure 3, is a simple high-pass phase-advance filter, commonly used for inter-stage coupling where DC isolation is required. High pass filters are characterised by their ability to allow high frequencies to pass through the filter while suppressing low frequencies.

Filter B

Filter B is the output measured over the capacitor in the same circuit ($\mathbf{V}_{out,B}$ in Figure 3).

19. Show that this output is

$$\mathbf{V}_{out,B} = \frac{1}{1 + j\omega/\omega_0}\mathbf{V}_{in}. \tag{5}$$

20. Show that the transfer function for Filter B is then

$$\mathbf{T}_B = \frac{1}{1 + j\omega/\omega_0}. \tag{6}$$

21. Make Bode plots of the theoretical and experimental transfer function $\mathbf{T}_B$.

22. What is ω_0 for Filter B?

Based on your measurements and theoretical calculations, Filter B in the circuit in Figure 3 is a simple low-pass phase-delay filter used to remove unwanted high-frequency signals.

The relationship between Filters A and B

Because the circuit used for Filters A and B is the same, the output for Filter B can also be derived from the output of Filter A, because Kirchhoff's Law says

$$\mathbf{V}_{in} = \mathbf{V}_C + \mathbf{V}_R \tag{7}$$

Therefore,

$$\mathbf{T}_B = \frac{\mathbf{V}_C}{\mathbf{V}_{in}} = \frac{\mathbf{V}_{in} - \mathbf{V}_R}{\mathbf{V}_{in}} = 1 - \frac{\mathbf{V}_R}{\mathbf{V}_{in}} = 1 - \mathbf{T}_A. \tag{8}$$

23. Plot Filters A and B together in one panel for a Bode amplitude plot, and one for a Bode phase plot.

24. As the voltage transfer function $\mathbf{T}_B$ can be derived from the voltage transfer function for Filter A, construct a Nyquist plot by by adding $-\mathbf{T}_A$ to $\mathbf{1}$ vectorially. This vector addition is equivalent to addition of complex numbers.

25. Discuss how the Bode and Nyquist plots inform us about the relations between these two filters.

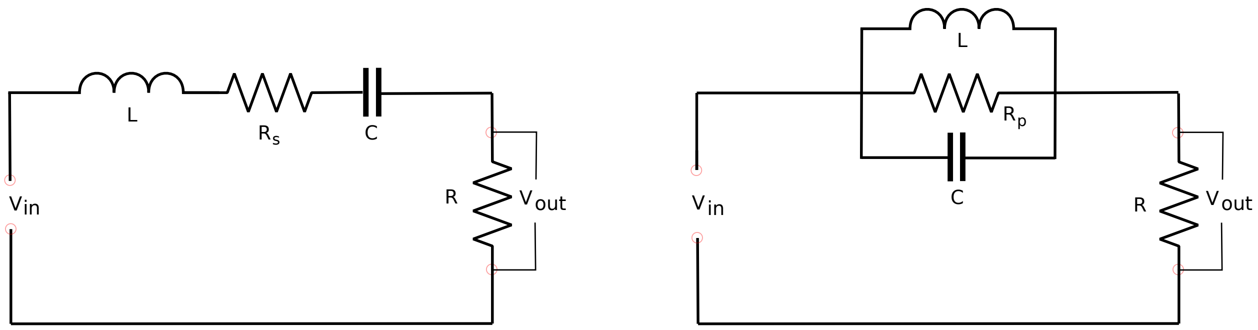

Figure 4: Circuit diagram for Filter C (left) and Filter D (right).

LC filters

We just saw that RC filters act as high- or low-pass filters. If we replace the resistor R with a inductor L, we have an LC circuit. We will see that LC filters are able to act as an electrical resonator. These resonant properties can be used to select certain frequencies out of a range of frequencies, making them useful for applications such as tuning radio transmitters and receivers, but also the scientific equipment we use in Physics.

For the LC filters, let us replace the load of $R = 10\ k\Omega$ with a smaller one: $R = 300\ \Omega$ on the Elvis Board and for your simulations.

Filter C

The left panel of Figure 4 is the circuit, where the voltage output over resistance R is Filter C. R_s represents the resistance of the inductor-capacitor series combination. There does not need to be a physical resistor there, but any inductor-capacitor in series will create a resistance of the inductor and capacitor. This resistance transfers electric energy to heat, causing electrical power loss.

26. Use the Elvis board to obtain an experimental Bode plot for Filter C.

The voltage transfer function of Filter C is

$$\mathbf{T}_C = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{R}{R + z_s}, \tag{9}$$

where the impedance of the inductor and capacitor in series is

$$z_s = Z_{R_S} + Z_C + Z_L = R_s + j\left(\omega L - \frac{1}{\omega C}\right) \tag{10}$$

From equation 9, we can see that $|\mathbf{T}_C|$ is a maximum when $|z_s|$ is a minimum.

27. Derive from equation 10 that this maximum occurs when $\omega = 1/\sqrt{LC}$. This angular frequency when the response of the filter is a maximum is called the resonant frequency $\omega_0 = 1/\sqrt{LC}$.

28. In Python create Bode magnitude plots of Filter C with the values of R, C and L of your circuit. Vary R_S until you match your observed transfer function. You may have to refine the true values of R, L and C, too!

Hopefully, your plots illustrate that the smaller the internal resistance R_s, the sharper the peak of the transfer function in the Bode amplitude plot. As we said before, R_s converts electrical energy to heat. A parameter to measure the energy lost per cycle (i.e., oscillation) is the quality factor

$$Q = \times \frac{\text{maximum energy stored}}{\text{power loss per cycle}} \tag{11}$$

A practical result of this definition is that REFERENCE WITH DERIVATION:

$$Q = \frac{f_0}{\Delta f} = \frac{\omega_0}{\Delta\omega}, \tag{12}$$

where f_0 is the resonant frequency, Δf is the resonance width or Full Width at Half Maximum (FWHM). This is the **bandwidth** over which the power is greater than half the power at the resonant frequency. Therefore, a filter with a large bandwidth has a low Q, and vice versa.

29. What is "half power" in dB?

30. Estimate the quality factor of filter Q_C from the Bode plot of your exierimental data for this filter.

We already saw that Q_C is inversely proportional to R_s. In more advanced electronics courses and books (REFERENCE!) you may derive the full relationship:

$$Q_C = \frac{\omega_0 L}{R + R_s}. \tag{13}$$

31. Even though we do not derive the equation, discuss how the resistance(s) in Filter C affect Q_C.

32. Compare Q_C with equation 13 and the information you obtained about the values of the electronic components to your estimate of Q_C from equation 12.

The concept of the quality factor is not only used in electronics, but also in laser physics to describe the energy loss in an optical cavity, in mechanical engineering, and in seismology to study the normal modes of vibration of the Earth.

33. Maybe keep this as an oral exam question: describe the filtering powers of this circuit when V_{out} is measured over the RLC combo. Could be used for Filter D too, of course.

Filter D

Filter D The circuit shown on the right in Figure 4. It shows an LC circuit, with the resistance R_p representing the resistance from the inductor and capacitor in parallel. The voltage transfer function for this circuit is given by:

$$\mathbf{T}_D = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{R}{R + z_p}, \tag{14}$$

where the impedance of the parallel components is

$$z_p \equiv \frac{1}{1/Z_{R_P} + 1/Z_C + 1/Z_L} = \frac{R_p}{1 + \mathrm{j}R_p\left(\omega C - \frac{1}{\omega L}\right)}, \tag{15}$$

where the resistance R_p is the parallel loss resistance of the inductor (the capacitor has negligible loss resistance).

34. Explain as you did in Filter C what you expect the filtering behaviour to be for Filter D.

35. Use the Elvis board to obtain a Bode plot for this filter.

36. Estimate the value of R_p, the way you did for Filter C: by fitting your data to the theoretical Bode plot.

37. Estimate the quality factor for Filter D using your measurements and equation 12

38. Maybe not surprisingly, $Q_D = \frac{R_p + R}{\omega_0 L}$. Compare this to Q_C and note the symmetry in going from a series to a parallel LC circuit. We do not derive this, but you can check if you get the same or similar value for Q_D as from your experimental Bode plot.

39. Question I do not know the answer to: Should R_p and R_s be the same?

40. Question for our colleagues: should we derive Q_c and Q_d? Does that make the lab too long or too theoretical?

S. Ruddell, S. Murdoch February 2013
S. Coen 2021
K. van Wijk, 2022