

## Section I – Value Iteration

*Note:* All solutions for this question have been generated using the spreadsheet located in **A5Q1.xlsx**

1) If we set the reward in the upper-left square to be 100, then we get the following results:

Optimal policy			Utility of starting in each square		
←	←	DONE	835.173	714.880	10.000
↑	←	←	714.880	620.808	483.756
↑	←	←	611.505	537.912	466.200

In this scenario, it is optimal for the agent to remain in the upper-left square as long as possible, while continuing to play the game as long as possible. Eventually, the discount factor will make the upper-left square give a reward approaching zero, but until that time the agent should continue to try and stay on the upper-left square.

2) If we set the reward in the upper-left square to be -3, then we see the following results:

Optimal policy			Utility of starting in each square		
→	→	DONE	2.821	7.332	10.000
→	→	↑	3.203	5.246	7.332
→	→	↑	1.913	3.407	5.039

In this scenario, it is optimal for the agent to move to the finishing square as soon as possible, because every other square on the board gives a negative reward. Further, the agent should actively avoid the upper-left square because it gives an even worse reward than the other squares with negative reward.

3) If we set the reward in the upper-left square to be 0, then we see the following results:

Optimal policy			Utility of starting in each square		
→	→	DONE	6.240	7.341	10.000
↑	↑	↑	4.367	5.339	7.341
↑	↑	↑	2.706	3.543	5.060

In this scenario, it is still optimal for the agent to move to the finishing square as soon as possible, because many of the other squares on the board give a negative reward. However, it is permissible to go through the upper-left square on the way to the final square because it gives a reward of zero, which is still better than a negative reward.

4) If we set the reward in the upper-left square to be 3, then we see the following results:

Optimal policy		
←	←	DONE
↑	←	←
↑	←	←

Utility of starting in each square		
24.193	19.429	10.000
19.429	15.704	11.947
15.335	12.421	9.745

In this scenario, it is again optimal for the agent to remain in the upper-left square as long as possible. However, the discount factor of 0.9 means that the game does not have to continue for very long before the upper-left square has a very small reward. At this point, the agent should try to end the game immediately by moving to the upper-right square.

## Section II – Games

We begin with the following normal form game:

	N	C	J
N	73, 25	57, 42	66, 32
C	80, 26	35, 12	32, 54
J	28, 27	63, 31	54, 29

**Step #1:** We notice that strategy N for the second agent is dominated, and this agent would have no reason to employ this strategy. Therefore, the strategy can be eliminated.

	C	J
N	57, 42	66, 32
C	35, 12	32, 54
J	63, 31	54, 29

**Step #2:** At this point, we see that strategy C for the first agent is dominated and can be eliminated.

	C	J
N	57, 42	66, 32
J	63, 31	54, 29

**Step #3:** Strategy J for the second agent is also dominated and subsequently eliminated.

	C
N	57, 42
J	63, 31

**Step #4:** Finally, strategy N for the first agent is dominated, and is thus eliminated.

	C
J	63, 31

After eliminating all dominated strategies, we are left with only strategy J for the first agent and strategy C for the second agent. This is a Nash equilibrium because given the other player's strategy, neither player would change their own. If the first agent uses strategy J, then the second agent cannot choose a

strategy better than C (31 is larger than 27 and 29). Similarly, if the second agent chooses strategy C, then the first agent cannot do any better than using strategy J (63 is greater than 35 and 57).

### Section III – Cross Validation

In leave-one-out cross validation, a learner is given a training data set that contains all but one point from the entire data set. The remaining point is used as a test point for the learner after it has finished processing the training data. Supposing that the data set contains 100 positive and 100 negative examples, then there are two possible cases:

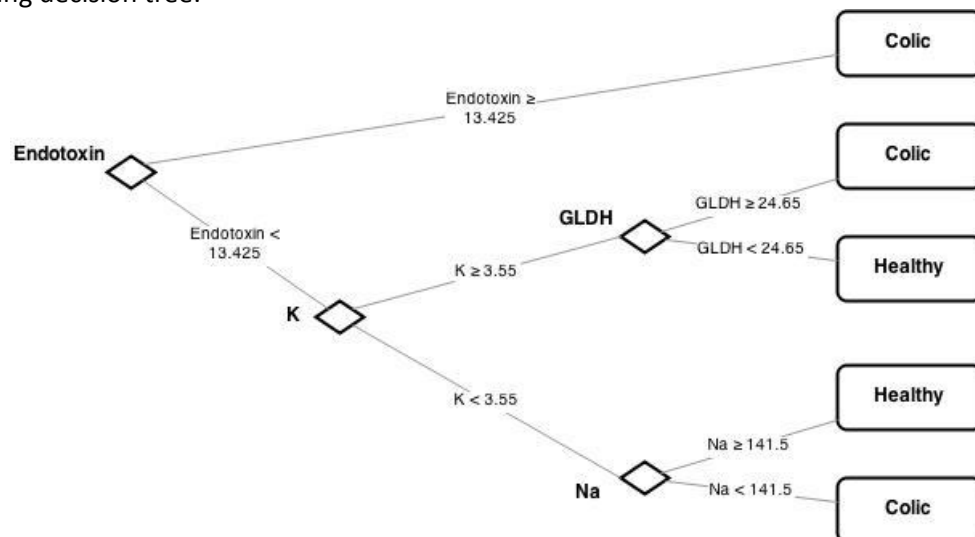
- i. The test data contains a single positive example, leaving the training data with 99 positive examples and 100 negative examples. In this case, the majority classifier will predict that a data point is negative, as this is the majority label of the training set.
- ii. The test data contains a single negative example, leaving the training data with 99 negative examples and 100 positive examples. In this case, the majority classifier will predict that a data point is positive, as this is the majority label of the training set.

In either case, the majority classifier will predict the label of the test data point incorrectly. Removing one of the points from the training data to use for testing immediately forces the majority classifier to return the opposite label of the point left out for testing. For this reason, the majority classifier will never predict the correct label, resulting in a score of 0% instead of the expected 50%.

### Section IV – Equine Colic Diagnosis

All of the solutions for this section are found by using the code in *Learner.py*, along with the *equine* and *test/train* flags. The output of the algorithm is found in **Appendix A5Q4**.

- 1) After running on the training data, the decision tree learning algorithm finishes with the following decision tree:



Output of the tree construction portion of the algorithm is found in **Appendix A5Q4**

- 2) The decision tree classifies 100% of the training instances correctly. Results of the classification of the training data is as follows:

Correct Labels	Total Training Points	Accuracy
132	132	100.00%

- 3) The decision tree also classifies 100% of the test instances correctly. Results of the classification of the test data is found below:

Correct Labels	Total Test Points	Accuracy
13	13	100.00%

- 4) The information gain metric was used in determining which attribute to split the data on at each stage of learning the decision tree. This attribute was chosen by finding the attribute and node test threshold that provided the largest information gain at each step. By using the attribute and threshold that provide the highest information gain, we reduce the amount of entropy in the data set by the largest amount possible at each stage of the tree construction.

## Section V – Math Student Performance

All of the solutions for this section are found by using the code in *Learner.py*, along with the *math* and *test/train* flags. The output of the algorithm is found in **Appendix A5Q5**.

- 1) The output of the decision tree construction for Math Student Performance is found in **Appendix A5Q5**
- 2) The decision tree classifies 100% of the training instances correctly. Results of the classification of the training data is as follows:

Correct Labels	Total Training Points	Accuracy
249	249	100.00%

- 3) However, the decision tree only classifies 56.9% (83/146) of the test instances correctly. Results of the classification of the test data is found below:

Correct Labels	Total Test Points	Accuracy
83	146	56.9%

- 4) There is a large discrepancy between the accuracy of the learner on the training data and the accuracy on the test data. As expected, the learner is 100% accurate on the training data used to construct the decision tree, but it is only 56.9% accurate on the test data. This difference is most likely explained by overfitting, meaning that the decision tree is too tightly coupled to the training data and therefore likely to be inaccurate when predicting the label of other unseen data.

As an experiment, the size of the training data was reduced by 50% to a total of 126 data points. Surprisingly, the accuracy of the decision tree built from this training data was about 65% on the same test data. This behavior could be evidence of overfitting on the original training data; as

the size of the training data is reduced, the decision tree becomes less tightly coupled to the training set and is more likely to predict the label of an unseen data point correctly. With less training data, the decision tree is less likely to split on thresholds made by outliers of particular attributes and is therefore more likely to classify new data based on more general patterns in the training data.

Additionally, this decision tree learner is permitted to reuse the same attributes for splitting at different points in the tree. This means that the learner will keep splitting until it finds data sets with no entropy, and therefore fit the training data perfectly. It is possible that restricting the learner by not allowing it to reuse attribute would prevent it from fitting the decision tree too closely to the training data.

## Appendix A5Q4

Phase #1: Building decision tree for Equine Colic classification

Selecting best attribute:

Attribute: Endotoxin

Threshold: 13.425

Information Gain: 0.6174924611847552

Selecting best attribute:

Attribute: K

Threshold: 3.55

Information Gain: 0.5088358471784984

Selecting best attribute:

Attribute: Na

Threshold: 141.5

Information Gain: 0.4394969869215134

Phase #2: Classification of Equine Colic disease  
on training data

Correct labels: 132

Total data points: 132

Accuracy: 100.0%

Selecting best attribute:

Attribute: GLDH

Threshold: 24.65

Information Gain: 0.15649106290570153

Decision Tree:

Colic

4, 13.425

Colic

8, 24.65

Healthy

0, 3.55

Healthy

1, 141.5

Colic

Phase #2: Classification of Equine Colic disease  
on test data

Correct labels: 13

Total data points: 13

Accuracy: 100.0%

## Appendix A5Q5

*Note:* The output of the actual decision tree has been excluded here due to its size.

Phase #1: Building decision tree for Math Student Performance  
classification

Selecting best attribute:

Attribute: Previous failures

Threshold: 0.5

Information Gain: 0.05213871314315577

Selecting best attribute:

Attribute: Days of class missed

Threshold: 21.5

Information Gain: 0.027582979546835862

Selecting best attribute:

Attribute: Travel time

Threshold: 3.5

Information Gain: 0.021773290406922996

Selecting best attribute:

Attribute: Frequency of seeing friends

Threshold: 2.5

Information Gain: 0.018252760451768935

Selecting best attribute:

Attribute: Health status

Threshold: 1.5

Information Gain: 0.0923778015608967

Selecting best attribute:

Attribute: Days of class missed

Threshold: 5.0

Information Gain: 0.06699672745561203

Selecting best attribute:

Attribute: Home

Threshold: 0.5

Information Gain: 0.10002566891096987

Selecting best attribute:

Attribute: Quality of family relationships

Threshold: 4.5

Information Gain: 0.1277217544981898

Selecting best attribute:

Attribute: Health status

Threshold: 2.5

Information Gain: 0.1271921109730324

Selecting best attribute:

Attribute: Father's education

Threshold: 2.5

Information Gain: 0.1270528389126364

Selecting best attribute:

Attribute: Free time after school

Threshold: 4.5

Information Gain: 0.16715678748009255

Selecting best attribute:

Attribute: Extra-curricular activities

Threshold: 0.5

Information Gain: 0.2516291673878229

Selecting best attribute:

Attribute: Gender of caretaker

Threshold: 0.5

Information Gain: 0.23645279766002802

Selecting best attribute:

Attribute: Free time after school

Threshold: 2.5

Information Gain: 0.34758988139079705

Selecting best attribute:

Attribute: Sex

Threshold: 0.5

Information Gain: 0.4199730940219749

Selecting best attribute:

Attribute: School Attended

Threshold: 0.5

Information Gain: 0.9182958340544896

Selecting best attribute:

Attribute: Age

Threshold: 17.5

Information Gain: 0.4067153767696875

Selecting best attribute:

Attribute: Father's education

Threshold: 1.5

Information Gain: 0.45810589515712374

Selecting best attribute:

Attribute: Home

Threshold: 0.5

Information Gain: 0.9182958340544896

Selecting best attribute:

Attribute: Father's education

Threshold: 1.5

Information Gain: 0.0456445428229465

Selecting best attribute:

Attribute: Free time after school

Threshold: 2.5

Information Gain: 0.15007189740414284

Selecting best attribute:

Attribute: Health status

Threshold: 4.5

Information Gain: 0.4591479170272448

Selecting best attribute:

Attribute: Age

Threshold: 16.5

Information Gain: 0.9182958340544896

Selecting best attribute:

Attribute: Quality of family relationships

Threshold: 3.5

Information Gain: 0.2045218714794187

Selecting best attribute:

Attribute: Sex

Threshold: 0.5

Information Gain: 0.9182958340544896

Selecting best attribute:

Attribute: Days of class missed

Threshold: 5.5

Information Gain: 0.22837518378373278

Selecting best attribute:

Attribute: School Attended

Threshold: 0.5

Information Gain: 1.0

Selecting best attribute:

Attribute: Weekend alcohol consumption

Threshold: 2.5

Information Gain: 0.03490346627367713

Selecting best attribute:

Attribute: Age

Threshold: 17.5

Information Gain: 0.12346046720702553

Selecting best attribute:

Attribute: Sex

Threshold: 0.5

Information Gain: 0.1435890623688481

Selecting best attribute:

Attribute: Days of class missed

Threshold: 1.0

Information Gain: 0.11157078888372529

Selecting best attribute:

Attribute: Age

Threshold: 16.5

Information Gain: 0.46956521111470706

Selecting best attribute:  
Attribute: Gender of caretaker  
Threshold: 0.5  
Information Gain: 0.7219280948873623

Selecting best attribute:  
Attribute: Health status  
Threshold: 3.5  
Information Gain: 0.19087450462110933

Selecting best attribute:  
Attribute: Mother's education  
Threshold: 3.5  
Information Gain: 0.2516291673878229

Selecting best attribute:  
Attribute: Father's education  
Threshold: 3.5  
Information Gain: 0.4591479170272448

Selecting best attribute:  
Attribute: Family Size  
Threshold: 0.5  
Information Gain: 0.8112781244591328

Selecting best attribute:  
Attribute: Romantic relationship  
Threshold: 0.5  
Information Gain: 0.11629598989239365

Selecting best attribute:  
Attribute: Family educational support  
Threshold: 0.5  
Information Gain: 0.6500224216483541

Selecting best attribute:  
Attribute: Family educational support  
Threshold: 0.5  
Information Gain: 0.22478750958935978

Selecting best attribute:  
Attribute: School Attended  
Threshold: 0.5  
Information Gain: 0.19811742113040354

Selecting best attribute:  
Attribute: Age  
Threshold: 19.0  
Information Gain: 0.31668908831502096

Selecting best attribute:  
Attribute: Weekday alcohol consumption  
Threshold: 1.5  
Information Gain: 0.7219280948873623

Selecting best attribute:  
Attribute: Home  
Threshold: 0.5  
Information Gain: 0.1011360148320043

Selecting best attribute:  
Attribute: Mother's education  
Threshold: 3.5  
Information Gain: 0.43359411726054453

Selecting best attribute:

Attribute: Father's education  
Threshold: 3.0  
Information Gain: 0.9182958340544896

Selecting best attribute:  
Attribute: Health status  
Threshold: 1.5  
Information Gain: 0.10879408669903734

Selecting best attribute:  
Attribute: Mother's education  
Threshold: 3.5  
Information Gain: 0.08930861846377391

Selecting best attribute:  
Attribute: Family Size  
Threshold: 0.5  
Information Gain: 0.182308984608897

Selecting best attribute:  
Attribute: Gender of caretaker  
Threshold: 0.5  
Information Gain: 0.24545967395890878

Selecting best attribute:  
Attribute: Mother's education  
Threshold: 1.5  
Information Gain: 0.19350684337293445

Selecting best attribute:  
Attribute: Age  
Threshold: 16.5  
Information Gain: 0.31976006206417584

Selecting best attribute:  
Attribute: Mother's education  
Threshold: 2.5  
Information Gain: 1.0

Selecting best attribute:  
Attribute: Family Size  
Threshold: 0.5  
Information Gain: 0.22600024438491662

Selecting best attribute:  
Attribute: Weekend alcohol consumption  
Threshold: 3.5  
Information Gain: 0.2780719051126377

Selecting best attribute:  
Attribute: Study time  
Threshold: 1.5  
Information Gain: 0.7219280948873623

Selecting best attribute:  
Attribute: Extra educational support  
Threshold: 0.5  
Information Gain: 0.7219280948873623

Selecting best attribute:  
Attribute: Primary caretaker  
Threshold: 0.5  
Information Gain: 0.13536509514407458

Selecting best attribute:  
Attribute: Weekend alcohol consumption

Threshold: 2.5  
Information Gain: 0.4089866249299394

Selecting best attribute:  
Attribute: Study time  
Threshold: 1.5  
Information Gain: 0.5032583347756457

Selecting best attribute:  
Attribute: Family educational support  
Threshold: 0.5  
Information Gain: 0.6500224216483541

Selecting best attribute:  
Attribute: Health status  
Threshold: 4.5  
Information Gain: 0.13104163868572294

Selecting best attribute:  
Attribute: Previous failures  
Threshold: 1.5

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Phase #2: Classification of Math Student Performance  
On training data

Correct labels: 249  
Total data points: 249  
Accuracy: 100.0%

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Information Gain: 0.19482707682211142

Selecting best attribute:  
Attribute: Romantic relationship  
Threshold: 0.5  
Information Gain: 0.24902249956730627

Selecting best attribute:  
Attribute: Sex  
Threshold: 0.5  
Information Gain: 0.7219280948873623

Selecting best attribute:  
Attribute: Quality of family relationships  
Threshold: 3.5  
Information Gain: 0.3219280948873623

Selecting best attribute:  
Attribute: Father's education  
Threshold: 1.5  
Information Gain: 1.0

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Phase #2: Classification of Math Student Performance  
On test data

Correct labels: 83  
Total data points: 146  
Accuracy: 56.849315068493155%

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