CS 486 – Assignment #4
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Section I: Warmup Exercises

1) The lawyer from the Blue Cab Company must show that the preponderance of the evidence is in favor of the defendant, that is, the probability that the plaintiff was actually hit by a Blue taxi is less than 50%. Therefore, he must show that the probability of the plaintiff being struck by a Blue car given that the witness claims it was Blue is less than 50%, or:

$$P(Car = Blue \mid Witness = Blue) < 0.5$$

From the presented information, we gather the following probabilities:

$$P(Car = Blue) = 0.15$$

$$P(Car = Green) = 0.85$$

$$P(Witness = Blue \mid Car = Blue) = P(Witness = Green \mid Car = Green) = 0.8$$

$$P(Witness = Blue \mid Car = Green) = P(Witness = Green \mid Car = Blue) = 0.2$$

With this information, we can construct the following joint probability table using chain rule:

	Witness = Blue	**Witness = Green**	
Car = Blue	0.12	0.03	0.15
Car = Green	0.17	0.68	0.85
	0.29	0.71	1.00

Finally, we can now use this probabilities to find the preponderance of the evidence on the side of the plaintiff using Bayes' Rule:

$$P(Car = Blue \mid Witness = Blue) = \frac{P(Witness = Blue \mid Car = Blue) \times P(Car = Blue)}{P(Witness = Blue)}$$

$$P(Car = Blue \mid Witness = Blue) = \frac{0.80 \times 0.15}{0.29}$$

$$P(Car = Blue \mid Witness = Blue) = \frac{0.12}{0.29} = 0.41 < 0.5$$

Bayes' Rule shows that the preponderance of the evidence on the side of the plaintiff is less than 50% and is therefore in favor of the defendant. The argument made by the lawyer for the Blue Cab Company is correct and it is more likely that Mr. Doe was struck by a Green taxi.

2) The following table shows the information needed to determine if each independence relation holds. In cases where the relation does hold, each path for the relation is shown and the

reasoning for the blocking on that path is given, based on the d-separation criteria. In cases where the relation does not hold, then only one unblocked path is shown.

	Blocked?	Path	Reasoning	Result
A	True	A -> C -> B	Node C – Case #3	Relation holds
	True	A -> C -> D -> E -> B	Node D – Case #3	
B	False	A -> C -> D -> E -> B		Relation does not hold
C	True	A -> C -> B -> E	Node C – Case #3	Relation holds
	True	A -> C -> D -> E	Node D – Case #3	
D	False	A -> C -> B -> E		Relation does not hold
E	True	A -> C -> B -> E	Node B – Case #2	Relation holds
	True	A -> C -> D -> E	Node D – Case #3	
F	True	A -> C -> B -> E	Node B – Case #2	Relation holds
	True	A -> C -> D -> E	Node C – Case #1	
G	False	A -> C -> D - >E		Relation does not hold
H	False	A -> C -> B -> E	Notice that descendant of C, F is in the evidence	Relation does not hold
I	False	F -> C -> D -> H		Relation does not hold
J	True	J -> G -> D -> E	Node G – Case #3	Relations holds
	True	J -> G -> D -> C -> B -> E	Node G – Case #3	
K	False	J -> G -> D -> E		Relation does not hold
L	True	J -> G -> D -> E	Node G – Case #3	Relation does not hold
	True	J -> G -> D -> C -> B -> E	Node G – Case #3	
M	False	G -> D -> C -> A		Relation does not hold
N	False	G -> D -> E -> B -> C -> A		Relation does not hold
O	True	G -> D -> C -> A	Node C/D – Case #1	Relation holds
	True	G -> D -> E -> B -> C -> A	Node D – Case #1	

Section II: Variable Elimination

The code for variable elimination has been included in the file *Factors.py*. A factor has been implemented as a dictionary that maps a list of label objects to a probability value. A brief overview of each of the five main functions in given below:

a. **Restrict (factor, variable, value):** Iterate through all entries in the factor; if an entry contains the variable with given value, remove the variable and add the result to a new factor.
b. **Multiply (factor1, factor2):** Find the variables that the two factors have in common. Iterate through pairs created by taking one entry from each factor. If there is a pair of entries that contain the common variables, multiply the values at those entries and add the result to a new factor.
c. **Sumout (factor, variable):** Iterate through each entry in the factor; if any entry contains either the variable or its negation, remove that variable from the entry. Take the remaining variables and insert in a new factor, taking the probability from the original entry.
d. **Normalize (factor):** Add together the probabilities from every entry in the factor, then divide all entries by the resulting sum.
e. **Inference (factorlist, query, hidden, evidence):** computes the probability of the query, given the evidence. This is done in a few steps:
    i. *Restriction:* Restrict every factor in the factorlist to each of the evidence variables.

ii. *Multiplication and Sum Out:* For every hidden variable, find the factors that contain the variable and multiply them together. Take the product and sum out the hidden variable to obtain a new factor.
iii. *Normalization:* The result of the first two steps is a list of factors that only contain information about the query variable. Take the product of these remaining factors and normalize to produce a single final factor.
iv. *Query:* Return the value for the query variable found in the final factor.

Section III – Bayes Nets

For the questions asking for particular probabilities, the output of the code at each stage of variable elimination has been included as an appendix.

1) Using the provided information, we construct the following probability tables:

FS	0.50
~FS	0.95

FM	0.036
~FM	0.964

NA	0.3
~NA	0.7

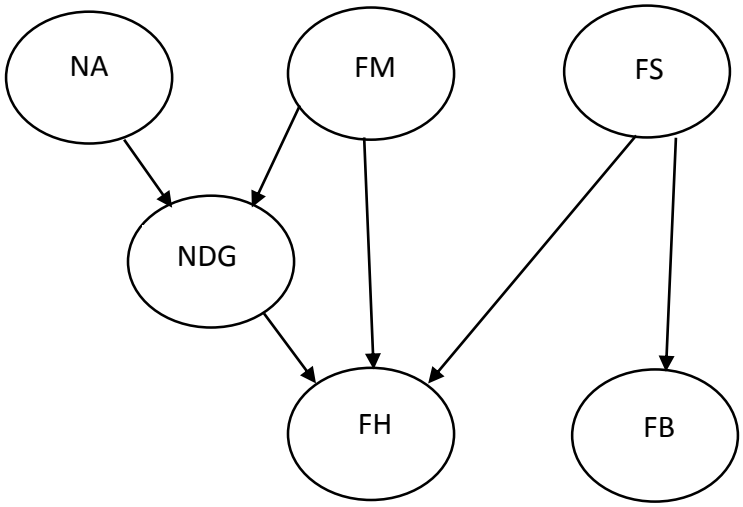
	NA, FM	**~NA, FM**	**NA, ~FM**	**~NA, ~FM**
NDG	0.009	0.010	0.145	0.000
~NDG	0.002	0.015	0.145	0.675

	FS	**~FS**
FB	0.03	0.095
~FB	0.02	0.855

	NDG, FS	**NDG, ~FS**	**~NDG, FS**	**~NDG, ~FS**
FH, FM	0.0010	0.0117	0.0008	0.0065
FH, ~FM	0.0055	0.0276	0.0205	0.0000
~FH, FM	0.0000	0.0063	0.0008	0.0010
~FH, ~FM	0.0018	0.1102	0.0205	0.7790

Note: The FM variable should be moved into the columns with NDG and FS, instead of in the rows with FH. The table is presented this way for readability.

Further, we can draw the resulting Bayes' Net:



2) We want to find the prior probability that Fido is howling, that is, the probability that he is howling without any observations. We make a query with the following parameters:
   i. *Query:* FH
   ii. *Hidden:* (NDG, NA, FB, FS, FM)
   iii. *Evidence:* None

   The query returns with the result:

$$P(FH) = 0.003027425077$$

   The probability that Fido is howling is about 0.303%. The output from the inference code is included in **Appendix Q3P2**.

3) We want to find the probability that Fido is sick, given that we hear him howling and we observe a full moon. We make a query with the following parameters:
   i. *Query:* FS
   ii. *Hidden:* (NDG, NA, FB)
   iii. *Evidence:* (FH, FM)

   The query returns with the result:

$$P(FS \mid FH, FM) = 0.0002773987764$$

   The probability that Fido is sick given there is a full moon and he is howling is about 0.028%. This is reasonable because the full moon not only causes Fido to howl sometimes, but it also causes the neighbor's dog to howl sometimes, further increasing the likelihood that Fido is howling. Fido does howl when he is sick, but with the observed evidence, it seems much more likely that he is howling for other reasons. The output from the inference code is included in **Appendix Q3P3**.

4) In addition to seeing the full moon and hearing Fido howl, we also observer that there is still food in his bowl. Now, what is the probability that he is sick? We make a query with the following parameters:
   i. *Query:* FS
   ii. *Hidden:* (NDG, NA)
   iii. *Evidence:* (FH, FM, FB)

   The query returns with the result:

$$P(FS \mid FH, FM, FB) = 0.001662087353$$

   The probability Fido is sick given our first observation and the fact that there is food in the bowl is about 0.166%. The increase in the likelihood that he is sick is explained by the fact that Fido often does not eat if he is sick. However, given all three of these observations, the likelihood of Fido howling is still not very high because he the full moon causes the neighbor's dog to howl sometimes, which in turn causes Fido to howl; the food in his bowl could just be a coincidence. The output from the inference code is included in **Appendix Q3P4**.

5) In addition to seeing the food in his bowl and our original observations, we now discover that the neighbor is away. Now, what is the probability that he is sick? We make a query with the following parameters:
   i. *Query:* FS
   ii. *Hidden:* (NDG)
   iii. *Evidence:* (FH, FM, FB, NA)

   The query returns with the result:

$$P(FS \mid FH, FM, FB, NA) = 0.001484646167$$

   The probability Fido is sick given all of these observations is about 0.149%. The discovery that the neighbor is away decreases the likelihood that Fido is sick because the neighbor's dog is more likely to howl when the owner is away; this decreases the likelihood that Fido is howling because he is sick. The full moon and the neighbor being away both cause Fido to howl for a reason other than being sick, either directly or indirectly. As we see many alternative reasons for Fido howling, we can conclude that he is not howling because he is sick and that the food left in the bowl is only a coincedence. The output from the inference code is included in **Appendix Q3P5**.

Section III – Appendices

All code output is presented in columns to conserve space. Each appendix contains the output at each of the variable elimination steps for the solutions presented in Section II. For all of the following appendices, we assume that we start with the following factors:

```
[NDG, ~FM, NA] -> 0.145
[~NDG, ~FM, NA] -> 0.145
[~NDG, FM, NA] -> 0.002
[NDG, ~FM, ~NA] -> 0
[~NDG, FM, ~NA] -> 0.015
[NDG, FM, ~NA] -> 0.01
[~NDG, ~FM, ~NA] -> 0.675
[NDG, FM, NA] -> 0.009

[FM] -> 0.0357
[~FM] -> 0.964

[FH, ~FM, ~NDG, FS] -> 0.0205
[FH, FM, ~NDG, FS] -> 0.0008
[~FH, FM, NDG, ~FS] -> 0.00632
[~FH, ~FM, ~NDG, ~FS] -> 0.7790
[~FH, ~FM, ~NDG, FS] -> 0.0205
[~FH, FM, ~NDG, FS] -> 0.00077
[~FH, ~FM, NDG, FS] -> 0.0018
[~FH, ~FM, NDG, ~FS] -> 0.1102
```

```
[FH, ~FM, NDG, FS] -> 0.00554
[~FH, FM, NDG, FS] -> 0
[FH, FM, NDG, ~FS] -> 0.01173
[~FH, FM, ~NDG, ~FS] -> 0.000969
[FH, FM, ~NDG, ~FS] -> 0.00646
[FH, FM, NDG, FS] -> 0.001
[FH, ~FM, NDG, ~FS] -> 0.02755
[FH, ~FM, ~NDG, ~FS] -> 0

[~FS] -> 0.95
[FS] -> 0.05

[FS, FB] -> 0.03
[FS, ~FB] -> 0.02
[~FS, ~FB] -> 0.855
[~FS, FB] -> 0.095

[~NA] -> 0.7
[NA] -> 0.3
```

Appendix Q3P2 – Code Output

```
Q2
Query: [FH]
Hidden: [NDG, NA, FB, FS, FM]
Evidence: []
-------------------------------------
1: Restrict Factors based on Evidence []
Factors after restrictions:
[NDG, ~FM, NA] -> 0.145
```

```
[~NDG, ~FM, NA] -> 0.145
[~NDG, FM, NA] -> 0.002
[NDG, ~FM, ~NA] -> 0
[~NDG, FM, ~NA] -> 0.015
[NDG, FM, ~NA] -> 0.01
[~NDG, ~FM, ~NA] -> 0.675
[NDG, FM, NA] -> 0.009
```

[FM] -> 0.0357
[~FM] -> 0.964

[FH, ~FM, ~NDG, FS] -> 0.0205
[FH, FM, ~NDG, FS] -> 0.0008
[~FH, FM, NDG, ~FS] -> 0.00632
[~FH, ~FM, ~NDG, ~FS] -> 0.7790
[~FH, ~FM, ~NDG, FS] -> 0.0205
[~FH, FM, ~NDG, FS] -> 0.00077
[~FH, ~FM, NDG, FS] -> 0.0018
[~FH, ~FM, NDG, ~FS] -> 0.1102
[FH, ~FM, NDG, FS] -> 0.00554
[~FH, FM, NDG, FS] -> 0
[FH, FM, NDG, ~FS] -> 0.01173
[~FH, FM, ~NDG, ~FS] -> 0.000969
[FH, FM, ~NDG, ~FS] -> 0.00646
[FH, FM, NDG, FS] -> 0.001
[FH, ~FM, NDG, ~FS] -> 0.02755
[FH, ~FM, ~NDG, ~FS] -> 0

[~FS] -> 0.95
[FS] -> 0.05

[FS, FB] -> 0.03
[FS, ~FB] -> 0.02
[~FS, ~FB] -> 0.855
[~FS, FB] -> 0.095

[~NA] -> 0.7
[NA] -> 0.3

2: Multiply Factors with hidden variables and sum out from the product
Hidden Variable: NDG
Multiplying:
[NDG, ~FM, NA] -> 0.145
[~NDG, ~FM, ~NA] -> 0.675
[~NDG, FM, NA] -> 0.002
[~NDG, FM, ~NA] -> 0.015
[~NDG, ~FM, NA] -> 0.145
[NDG, FM, NA] -> 0.009
[NDG, ~FM, ~NA] -> 0
[NDG, FM, ~NA] -> 0.01

[~FH, ~FM, ~NDG, FS] -> 0.0205
[~FH, ~FM, ~NDG, ~FS] -> 0.7790
[~FH, FM, ~NDG, ~FS] -> 0.000969
[~FH, FM, ~NDG, FS] -> 0.00077
[FH, ~FM, NDG, FS] -> 0.00554
[FH, FM, ~NDG, FS] -> 0.0008
[FH, FM, NDG, FS] -> 0.001
[FH, ~FM, ~NDG, FS] -> 0.0205
[FH, ~FM, NDG, ~FS] -> 0.02755
[FH, FM, ~NDG, ~FS] -> 0.00646
[FH, FM, NDG, ~FS] -> 0.01173
[~FH, FM, NDG, FS] -> 0
[FH, ~FM, ~NDG, ~FS] -> 0
[~FH, ~FM, NDG, ~FS] -> 0.1102
[~FH, FM, NDG, ~FS] -> 0.00632
[~FH, ~FM, NDG, FS] -> 0.0018

Product:
[FH, FM, NA, ~NDG, ~FS] -> 0.00001292
[~FH, ~FM, ~NA, ~NDG, FS] -> 0.0138375
[FH, ~FM, ~NA, NDG, ~FS] -> 0.00000
[FH, ~FM, ~NA, ~NDG, FS] -> 0.0138375
[~FH, FM, NA, ~NDG, FS] -> 0.00000154
[~FH, ~FM, ~NA, NDG, FS] -> 0.0000
[FH, FM, ~NA, NDG, FS] -> 0.00001
[~FH, FM, ~NA, NDG, FS] -> 0.00
[FH, FM, NA, NDG, FS] -> 0.000009
[FH, FM, ~NA, ~NDG, FS] -> 0.0000120
[~FH, FM, NA, NDG, ~FS] -> 0.00005688
[~FH, FM, ~NA, ~NDG, ~FS] -> 0.000014535
[~FH, ~FM, ~NA, NDG, ~FS] -> 0.0000
[~FH, FM, ~NA, NDG, ~FS] -> 0.0000632
[FH, FM, ~NA, NDG, ~FS] -> 0.0001173
[~FH, FM, NA, NDG, FS] -> 0.000
[FH, ~FM, ~NA, ~NDG, ~FS] -> 0.000
[FH, ~FM, NA, ~NDG, ~FS] -> 0.000
[~FH, FM, NA, ~NDG, ~FS] -> 0.000001938
[~FH, ~FM, NA, ~NDG, FS] -> 0.0029725
[FH, ~FM, NA, NDG, ~FS] -> 0.00399475
[FH, ~FM, NA, ~NDG, FS] -> 0.0029725
[FH, FM, ~NA, ~NDG, ~FS] -> 0.00009690
[~FH, FM, ~NA, ~NDG, FS] -> 0.00001155
[~FH, ~FM, NA, NDG, FS] -> 0.0002610
[FH, ~FM, ~NA, NDG, FS] -> 0.00000
[FH, ~FM, NA, NDG, FS] -> 0.00080330
[FH, FM, NA, NDG, ~FS] -> 0.00010557
[~FH, ~FM, ~NA, ~NDG, ~FS] -> 0.5258250
[FH, FM, NA, ~NDG, FS] -> 0.0000016
[~FH, ~FM, NA, NDG, ~FS] -> 0.0159790
[~FH, ~FM, NA, ~NDG, ~FS] -> 0.1129550

Sumout: NDG
Result:
[FH, FM, NA, FS] -> 0.0000106
[~FH, FM, NA, ~FS] -> 0.000058818
[~FH, ~FM, ~NA, ~FS] -> 0.5258250
[FH, ~FM, ~NA, FS] -> 0.0138375
[~FH, FM, NA, FS] -> 0.00000154
[~FH, ~FM, ~NA, FS] -> 0.0138375
[FH, FM, ~NA, FS] -> 0.0000220
[FH, FM, NA, ~FS] -> 0.00011849
[FH, ~FM, NA, ~FS] -> 0.00399475
[~FH, FM, ~NA, FS] -> 0.00001155
[~FH, ~FM, NA, ~FS] -> 0.1289340
[~FH, ~FM, NA, FS] -> 0.0032335
[FH, ~FM, NA, FS] -> 0.00377580
[FH, ~FM, ~NA, ~FS] -> 0.00000
[~FH, FM, ~NA, ~FS] -> 0.000077735
[FH, FM, ~NA, ~FS] -> 0.00021420

Hidden Variable: NA
Multiplying:
[~NA] -> 0.7
[NA] -> 0.3

[FH, FM, ~NA, FS] -> 0.0000220
[FH, ~FM, ~NA, FS] -> 0.0138375
[~FH, ~FM, ~NA, ~FS] -> 0.5258250
[~FH, FM, NA, FS] -> 0.00000154
[~FH, FM, ~NA, FS] -> 0.00001155
[~FH, FM, NA, ~FS] -> 0.000058818
[FH, FM, ~NA, ~FS] -> 0.00021420
[FH, FM, NA, ~FS] -> 0.00011849
[~FH, ~FM, NA, ~FS] -> 0.1289340
[FH, FM, NA, FS] -> 0.0000106
[FH, ~FM, NA, ~FS] -> 0.00399475
[~FH, FM, ~NA, ~FS] -> 0.000077735
[FH, ~FM, ~NA, ~FS] -> 0.00000
[~FH, ~FM, ~NA, FS] -> 0.0138375
[~FH, ~FM, NA, FS] -> 0.0032335
[FH, ~FM, NA, FS] -> 0.00377580

Product:
[~NA, FH, FS, ~FM] -> 0.00968625
[NA, FH, ~FS, FM] -> 0.000035547
[~NA, ~FH, FS, ~FM] -> 0.00968625
[NA, FH, FS, ~FM] -> 0.001132740
[NA, FH, ~FS, ~FM] -> 0.001198425

```
[~NA, FH, ~FS, FM] -> 0.000149940
[NA, ~FH, ~FS, FM] -> 0.0000176454
[~NA, FH, FS, FM] -> 0.00001540
[~NA, FH, ~FS, ~FM] -> 0.000000
[~NA, ~FH, FS, FM] -> 0.000008085
[NA, ~FH, FS, FM] -> 4.62E-7
[NA, ~FH, ~FS, ~FM] -> 0.03868020
[~NA, ~FH, ~FS, ~FM] -> 0.36807750
[NA, ~FH, FS, ~FM] -> 0.00097005
[~NA, ~FH, ~FS, FM] -> 0.0000544145
[NA, FH, FS, FM] -> 0.00000318

Sumout: NA
Result:
[FH, ~FS, FM] -> 0.000185487
[FH, ~FS, ~FM] -> 0.001198425
[~FH, ~FS, ~FM] -> 0.40675770
[FH, FS, FM] -> 0.00001858
[~FH, FS, FM] -> 0.000008547
[~FH, FS, ~FM] -> 0.01065630
[~FH, ~FS, FM] -> 0.0000720599
[FH, FS, ~FM] -> 0.010818990

Hidden Variable: FB
Multiplying:
[FS, FB] -> 0.03
[FS, ~FB] -> 0.02
[~FS, FB] -> 0.095
[~FS, ~FB] -> 0.855

Product:
[FS, FB] -> 0.03
[FS, ~FB] -> 0.02
[~FS, FB] -> 0.095
[~FS, ~FB] -> 0.855

Sumout: FB
Result:
[~FS] -> 0.950
[FS] -> 0.05

Hidden Variable: FS
Multiplying:
[~FS] -> 0.95
[FS] -> 0.05

[~FH, FS, ~FM] -> 0.01065630
[FH, FS, FM] -> 0.00001858
[~FH, ~FS, ~FM] -> 0.40675770
[FH, FS, ~FM] -> 0.010818990
[FH, ~FS, FM] -> 0.000185487
[~FH, FS, FM] -> 0.000008547
[~FH, ~FS, FM] -> 0.0000720599
[FH, ~FS, ~FM] -> 0.001198425
[FS] -> 0.05
[~FS] -> 0.950

Product:
[FH, ~FS, ~FM] -> 0.001081578562
[FH, FS, FM] -> 4.6450E-8
[~FH, ~FS, ~FM] -> 0.3670988242
[~FH, ~FS, FM] -> 0.00006503405975
[~FH, FS, ~FM] -> 0.000026640750
[~FH, FS, FM] -> 2.13675E-8
[FH, FS, ~FM] -> 0.0000270474750
[FH, ~FS, FM] -> 0.0001674020175

Sumout: FS
Result:
[~FH, ~FM] -> 0.3671254650
[FH, ~FM] -> 0.001108626037
[~FH, FM] -> 0.00006505542725
[FH, FM] -> 0.0001674484675

Hidden Variable: FM
Multiplying:
[FM] -> 0.0357
[~FM] -> 0.964

[~FH, FM] -> 0.00006505542725
[~FH, ~FM] -> 0.3671254650
[FH, FM] -> 0.0001674484675
[FH, ~FM] -> 0.001108626037

Product:
[~FH, ~FM] -> 0.3539089483
[~FH, FM] -> 0.000002322478753
[FH, ~FM] -> 0.001068715500
[FH, FM] -> 0.000005977910290

Sumout: FM
Result:
[~FH] -> 0.3539112708
[FH] -> 0.001074693410
--------------------------------------
3: Multiply remaining factors:
[~FH] -> 0.3539112708
[FH] -> 0.001074693410

Result:
[~FH] -> 0.3539112708
[FH] -> 0.001074693410
--------------------------------------
4: Normalizing result:
[~FH] -> 0.9969725750
[FH] -> 0.003027425077
--------------------------------------
P ([FH] | []) = 0.003027425077 or
0.3027425077%
```

Appendix Q3P3 – Code Output

```
Q3
Query: [FS]
Hidden: [NDG, NA, FB]
Evidence: [FH, FM]
--------------------------------------
1: Restrict Factors based on Evidence [FH,
FM]
Factors after restrictions:
[NDG, NA] -> 0.009
[~NDG, ~NA] -> 0.015
[~NDG, NA] -> 0.002
[NDG, ~NA] -> 0.01

[~NDG, ~FS] -> 0.00646
[NDG, FS] -> 0.001
[~NDG, FS] -> 0.0008
[NDG, ~FS] -> 0.01173

[FS] -> 0.05
[~FS] -> 0.95

[~FS, ~FB] -> 0.855
[~FS, FB] -> 0.095
[FS, FB] -> 0.03
[FS, ~FB] -> 0.02

[~NA] -> 0.7
[NA] -> 0.3
```

```
--------------------------------------
2: Multiply Factors with hidden variables
and sum out from the product
Hidden Variable: NDG
Multiplying:
[NDG, NA] -> 0.009
[NDG, ~NA] -> 0.01
[~NDG, ~NA] -> 0.015
[~NDG, NA] -> 0.002

[NDG, ~FS] -> 0.01173
[NDG, FS] -> 0.001
[~NDG, ~FS] -> 0.00646
[~NDG, FS] -> 0.0008

Product:
[NA, ~NDG, FS] -> 0.0000016
[~NA, ~NDG, ~FS] -> 0.00009690
[~NA, NDG, FS] -> 0.00001
[NA, ~NDG, ~FS] -> 0.00001292
[NA, NDG, FS] -> 0.000009
[~NA, ~NDG, FS] -> 0.0000120
[~NA, NDG, ~FS] -> 0.0001173
[NA, NDG, ~FS] -> 0.00010557

Sumout: NDG
Result:
[NA, FS] -> 0.0000106
[NA, ~FS] -> 0.00011849
[~NA, ~FS] -> 0.00021420
[~NA, FS] -> 0.0000220

Hidden Variable: NA
Multiplying:
[~NA] -> 0.7
[NA] -> 0.3

[NA, FS] -> 0.0000106
[~NA, FS] -> 0.0000220
[NA, ~FS] -> 0.00011849
[~NA, ~FS] -> 0.00021420

Product:
[~NA, FS] -> 0.00001540
[NA, ~FS] -> 0.000035547
[~NA, ~FS] -> 0.000149940
[NA, FS] -> 0.00000318

Sumout: NA
Result:
[FS] -> 0.00001858
[~FS] -> 0.000185487

Hidden Variable: FB
Multiplying:
[FS, FB] -> 0.03
[FS, ~FB] -> 0.02
[~FS, FB] -> 0.095
[~FS, ~FB] -> 0.855

Product:
[FS, FB] -> 0.03
[FS, ~FB] -> 0.02
[~FS, FB] -> 0.095
[~FS, ~FB] -> 0.855

Sumout: FB
Result:
[FS] -> 0.05
[~FS] -> 0.950
--------------------------------------
3: Multiply remaining factors:
[FS] -> 0.05
[~FS] -> 0.95

[FS] -> 0.00001858
[~FS] -> 0.000185487

[FS] -> 0.05
[~FS] -> 0.950

Result:
[FS] -> 1.6582650E-9
[~FS] -> 0.000005976252025
--------------------------------------
4: Normalizing result:
[FS] -> 0.0002773987764
[~FS] -> 0.9997226011
--------------------------------------
P ([FS] | [FH, FM]) = 0.0002773987764 or
0.02773987764%
```

Appendix Q3P4 – Code Output

```
Q4
Query: [FS]
Hidden: [NDG, NA]
Evidence: [FH, FM, FB]
--------------------------------------
1: Restrict Factors based on Evidence [FH,
FM, FB]
Factors after restrictions:
[NDG, NA] -> 0.009
[~NDG, NA] -> 0.002
[~NDG, ~NA] -> 0.015
[NDG, ~NA] -> 0.01

[~NDG, ~FS] -> 0.00646
[NDG, ~FS] -> 0.01173
[NDG, FS] -> 0.001
[~NDG, FS] -> 0.0008

[~FS] -> 0.95
[FS] -> 0.05

[FS] -> 0.03
[~FS] -> 0.095

[NA] -> 0.3
[~NA] -> 0.7

--------------------------------------
2: Multiply Factors with hidden variables
and sum out from the product
Hidden Variable: NDG
Multiplying:
[NDG, NA] -> 0.009
[~NDG, ~NA] -> 0.015
[NDG, ~NA] -> 0.01
[~NDG, NA] -> 0.002

[~NDG, FS] -> 0.0008
[~NDG, ~FS] -> 0.00646
[NDG, ~FS] -> 0.01173
[NDG, FS] -> 0.001
```

```
Product:
[FS, NDG, ~NA] -> 0.00001
[FS, ~NDG, ~NA] -> 0.0000120
[~FS, NDG, ~NA] -> 0.0001173
[~FS, NDG, NA] -> 0.00010557
[FS, NDG, NA] -> 0.000009
[~FS, ~NDG, ~NA] -> 0.00009690
[~FS, ~NDG, NA] -> 0.00001292
[FS, ~NDG, NA] -> 0.0000016

Sumout: NDG
Result:
[FS, NA] -> 0.0000106
[~FS, ~NA] -> 0.00021420
[~FS, NA] -> 0.00011849
[FS, ~NA] -> 0.0000220

Hidden Variable: NA
Multiplying:
[NA] -> 0.3
[~NA] -> 0.7

[FS, ~NA] -> 0.0000220
[~FS, ~NA] -> 0.00021420
[~FS, NA] -> 0.00011849
[FS, NA] -> 0.0000106

Product:
[~FS, ~NA] -> 0.000149940
[FS, NA] -> 0.00000318
[FS, ~NA] -> 0.00001540
[~FS, NA] -> 0.000035547

Sumout: NA
Result:
[FS] -> 0.00001858
[~FS] -> 0.000185487

--------------------------------------
3: Multiply remaining factors:
[~FS] -> 0.95
[FS] -> 0.05

[FS] -> 0.03
[~FS] -> 0.095

[FS] -> 0.00001858
[~FS] -> 0.000185487

Result:
[FS] -> 9.949590E-10
[~FS] -> 5.976252025E-7

--------------------------------------
4: Normalizing result:
[~FS] -> 0.9983379125
[FS] -> 0.001662087353

--------------------------------------
P ([FS] | [FH, FM, FB]) = 0.001662087353 or
0.1662087353%
```

Appendix Q3P5 – Code Output

```
Q5
Query: [FS]
Hidden: [NDG]
Evidence: [FH, FM, FB, NA]
--------------------------------------
1: Restrict Factors based on Evidence [FH,
FM, FB, NA]
Factors after restrictions:
[NDG] -> 0.009
[~NDG] -> 0.002

[~NDG, ~FS] -> 0.00646
[NDG, ~FS] -> 0.01173
[NDG, FS] -> 0.001
[~NDG, FS] -> 0.0008

[~FS] -> 0.95
[FS] -> 0.05

[FS] -> 0.03
[~FS] -> 0.095

--------------------------------------
2: Multiply Factors with hidden variables
and sum out from the product
Hidden Variable: NDG
Multiplying:
[NDG] -> 0.009
[~NDG] -> 0.002

[~NDG, FS] -> 0.0008
[~NDG, ~FS] -> 0.00646
[NDG, FS] -> 0.001
[NDG, ~FS] -> 0.01173

Product:
[~FS, ~NDG] -> 0.00001292
[~FS, NDG] -> 0.00010557
[FS, NDG] -> 0.000009
[FS, ~NDG] -> 0.0000016

Sumout: NDG
Result:
[FS] -> 0.0000106
[~FS] -> 0.00011849

--------------------------------------
3: Multiply remaining factors:
[FS] -> 0.05
[~FS] -> 0.95

[~FS] -> 0.095
[FS] -> 0.03

[FS] -> 0.0000106
[~FS] -> 0.00011849

Result:
[FS] -> 1.702890E-10
[~FS] -> 1.145297680E-7

--------------------------------------
4: Normalizing result:
[~FS] -> 0.9985153539
[FS] -> 0.001484646167

--------------------------------------
P ([FS] | [FH, FM, FB, NA]) = 0.001484646167
or 0.1484646167%
```