

Due Date: March 5, 2015 at 11:59pm.

Submission Instructions: All assignments are to be submitted through LEARN, in the Dropbox labelled *Assignment 4 Submissions* in the Assignment 4 folder. Late assignments will be accepted up until March 7th at 11:59pm.. Please read the course policy on assignments submitted after the official due date. *No assignments will be accepted, for any reason, after 11:59pm on March 7th.*

Lead TA: Nam Pham (npham@uwaterloo.ca). Office hours are Fridays from 9:00am-10:30am in DC2302F.

Announcements: The following exercises are to be done individually.

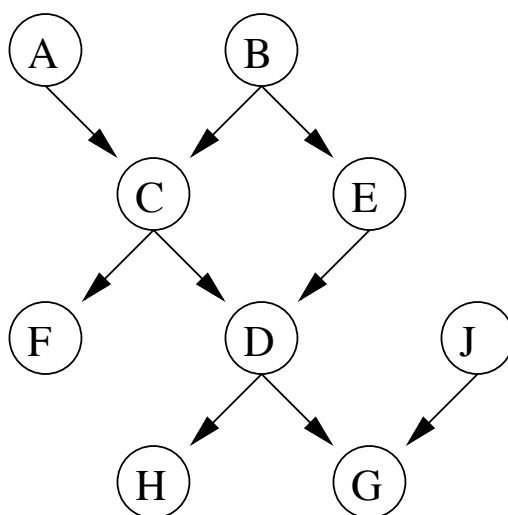
1. Warmup Exercises (20 points)

1. (5 points) A town has one hundred taxis, eighty-five green taxis owned by the Green Cab company and 15 blue taxis owned by the Blue Cab Company. On February 27th, John Doe was struck by a speeding taxi, and the only witness testified that the taxi was blue rather than green. John Doe sued the Blue Cab Company. The judge instructed the jury and the lawyers at the start of the case that the reliability of a witness must be assumed to be 80% in a case of this sort. The judge also reminded the jury that in order for the Blue Cab Company to be held liable there must be the “preponderance of the evidence” on the side of the plaintiff (John Doe). This means that the probability he was actually hit by a Blue taxi must be at least 50%.

The lawyer for John Doe argued that the Blue Cab Company should pay, since the witness’s statement gives a probability of 80% that Doe was struck by a blue taxi. The lawyer for the Blue Cab Company argued as follows. If the witness was shown all the taxis in town, he would have incorrectly identified 20% of the 85 green taxis (i.e. seventeen of them) as blue, and correctly identified 80% of the 15 blue taxis (i.e. twelve of them) as blue. Thus, of the 29 identifications of a taxi as blue, only twelve would be correct and seventeen would be incorrect. Thus, the preponderance of evidence is in favor of the defendant – most likely Mr. Doe was hit by a green taxi.

Formulate the second lawyer’s argument rigorously in terms of Bayes’ Rule. Which argument do you think is correct? If neither is correct, what is a good argument for this case?

2. (15 points) Consider the following Bayes net (we leave out the Conditional Probability Tables).



For each of the questions below, state whether the independence relation holds. Give a justification for your answer using the d-separation criterion: if d-separation holds, state why each *undirected* path is blocked; if it does not hold, describe one undirected path which is unblocked. (Note: there are no more than two undirected paths between any pair of nodes).

- (a) A and B are independent
- (b) A and B are independent given D
- (c) A and E are independent
- (d) A and E are independent given C
- (e) A and E are independent given B
- (f) A and E are independent given B and C
- (g) A and E are independent given D
- (h) A and E are independent given F
- (i) F and H are independent
- (j) J and E are independent
- (k) J and E are independent given G
- (l) J and E are independent given A
- (m) G and A are independent
- (n) G and A are independent given C
- (o) G and A are independent given C and D

2. Variable Elimination (0 points)

This part of the assignment is worth zero points. However, your implementation will be used in the rest of the assignment. *If you do the implementation then upload a copy of it with your assignment.*

Implement the variable elimination algorithm by coding the following four functions in the programming language of your choice.

- a. **restrictedFactor=restrict(factor, variable, value)**: a function that restricts a variable to some value in a given factor
- b. **productFactor = multiply(factor1, factor2)**: a function that multiplies two factors
- c. **resultFactor = sumout(factor, variable)**: a function that sums out a variable given a factor
- d. **resultFactor = inference(factorList, queryVariables, orderedListOfHiddenVariables, evidenceList)**: a function that computes $Pr(queryVariable \mid evidenceList)$ by variable elimination. This function should restrict the factors in *factorList* according to the evidence in *evidenceList*. Next, it should sum out the hidden variables from the product of the factors in *factorList*. The variables should be summed out in the order given in *orderedListOfHiddenVariables*. Finally, the answer should be normalized. Note that you might want to implement an additional function **normalize(factor)** to help you do this.

Here are some useful tips for this part of the assignment.

Tip Factors are essentially multi-dimensional arrays. Therefore, you may want to use this data structure. However, you are free to use any data structure that you want.

Tip Test each function individually using the simple examples we covered in class. Debugging the entire variable elimination algorithm at once can be tricky.

Hand in a printout of your code. While there are no marks for the code, in Question 3 part marks will be given for using your variable elimination implementation to do the calculations.

3. Bayes Nets (80 points)

Your dog Fido has been howling for the last three hours and you want to decide whether or not to take him to the vet, or just put in earplugs and go back to sleep. You have the following information available to you:

- You know that Fido often howls if he is genuinely sick. However, he is a healthy dog and is only sick 5% of the time. If Fido is really sick he probably has not eaten much of his dinner and has food left in his bowl. In the past you have observed that when Fido is sick then 60% of the time he does not eat. However, about 10% of the time, when Fido is healthy he still does not eat his dinner.

- You also know that Fido often howls if there is a full moon or the neighbour's dog howls. The neighbour's dog sometimes howls at the full moon and sometimes howls when your neighbour is away. However, the neighbour's dog is never affected by Fido's howls. You know that (since you live on Earth) there is a full moon once out of every twenty-eight days. You have observed that your neighbour travels three days out of every ten. You also know that if there is no full moon and the neighbour is at home then the dog never howls, but if there is no full moon and the neighbour is away then the dog howls with probability 0.5. If there is a full moon then the dog is more likely to howl; if the neighbour is at home then it howls with probability 0.4, but if the neighbour is also away then the probability of howling increases to 0.8.
- Finally, you know that if all the triggers are there (i.e. full moon, howling neighbour's dog, Fido sick) then Fido will howl with probability 0.99, but if none of the triggers are there, then Fido will not howl. If Fido is sick then he is likely to howl. In particular, if he is sick (but there are no other triggers) then Fido will howl with probability 0.5. If Fido is sick and there is another trigger then he is even more likely to howl – if the neighbour's dog is also howling then Fido will howl $\frac{3}{4}$'s of the time, while if there is a full moon then Fido howls 90% of the time. If Fido is *not* sick then he is less likely to howl. The full moon and the neighbour's dog will only cause him to howl with probability 0.65, while if there is only a full moon then he will howl 40% of the time, and if there is no full moon, but the neighbour's dog is making noise, then he howl's with probability 0.2.

1. (32 points) Given the information about Fido, construct a Bayes Network. Show the graph and the conditional probability tables. The network should encode all the information stated above. It should contain six nodes corresponding to the following binary random variables:

- *FH* – Fido howls
- *FS* - Fido is sick
- *FB* - there is food left in Fido's food bowl
- *FM* - there is a full moon
- *NA* - the neighbour is away
- *NDG* - the neighbour's dog howls

The edges in your Bayes Network should accurately capture the probabilistic dependencies between these variables.

For the next set of questions indicate what queries (i.e. $Pr(\text{vars} \mid \text{evidence})$) you used to compute the probability. Whether you answer the queries by hand or by using your code, provide a printout of the factors computed at each step of variable elimination. If you think that some of the probabilities will not change, then you do not have to redo the calculations,

but you do need to provide an explanation. Note that a maximum of two thirds of the marks are earned if you answer correctly the question by doing the computations by hand instead of using your program.

2. (12 points) What is the prior probability that Fido will howl (i.e. $\Pr(FH)$)?
3. (12 points) You can hear Fido howling, and you are a little concerned that he is sick. You look out the window and see that the moon is full. What is probability that Fido is sick?
4. (12 points) You next walk to the kitchen to see if there is any food left in Fido's bowl. You note that the bowl is full – that is Fido has not eaten. How does your belief change about Fido being sick now that you know there is a full moon and Fido has not eaten?
5. (12 points) Finally, you decide to call your neighbour to see if they are home or not. The phone rings and rings so you conclude that your neighbour is away. Now, given this information, how does your belief about Fido being sick change?